Thermodynamics of Hot and Dense Deconfined Matter in HTL Resummed Perturbation Theory

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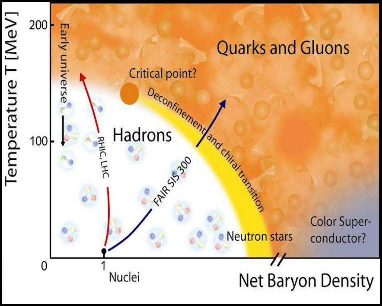
Content:

- Introduction
- QCD Perturbation Theory
  - BPT, DRQCD, HTLpt
- Pressure of Hot and Dense matter (QGP)
  - PQCD/HTL Resummed PT Vs. LQCD
- Other Relevant Thermodynamic Quantities of QGP
  - HTL Resummed PT Vs. LQCD
- Conclusion

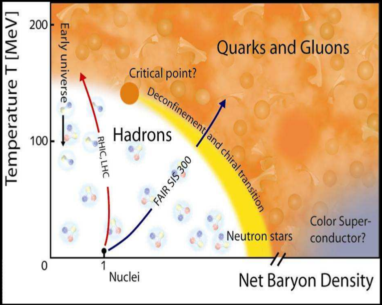
Present-day understanding of the QCD phase diagram



- Theory (QCD and models)
- Numerical Experiments (LQCD)
- Mini Bang (Laboratory Expt.)

☛ Complex: Difficult to span the whole PD by a single effort !

Present-day understanding of the QCD phase diagram



- Theory (QCD and models)
- Numerical Experiments (LQCD)
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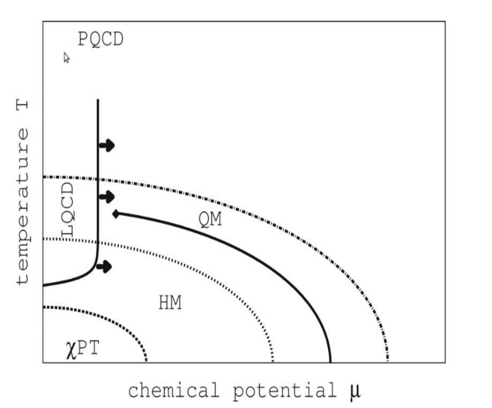
☛ Complex: Difficult to span the whole PD by a single effort !

- Higher the collision energy the closer is the created system located near temperature axis
- A crossover at RHIC and LHC; No true phase transition; LQCD also exhibited.
- Preliminary to attribute a definite position for critical point in phase diagram (BES-I at RHIC and LQCD)
- Data appear to demand an explanation beyond a purely hadronic scenario ⇒ non-hadronic source (partonic degrees)
- Collective behaviours ⇒ Circumstantial evidences

Existing Theoretical Approaches



- All regions of the PD by the first principle QCD calculations !
- Not yet!
- Interface of Nuclear Physics and Particle Physics

Interface of particle physics & high-energy nuclear physics

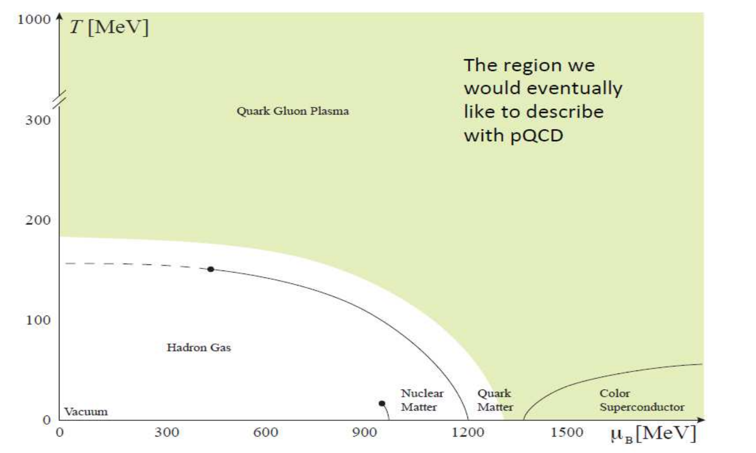
- Draws heavily from QCD: perterbative and non-perterbative
- Overlaps with:
  - Thermal Field Theory
  - Relativistic Fluid Dynamics
  - Kinetic or Transport Theory
  - Quantum Collision Theory
  - String Theory
  - Statistical Mechanics & Thermodynamics

Present Talk

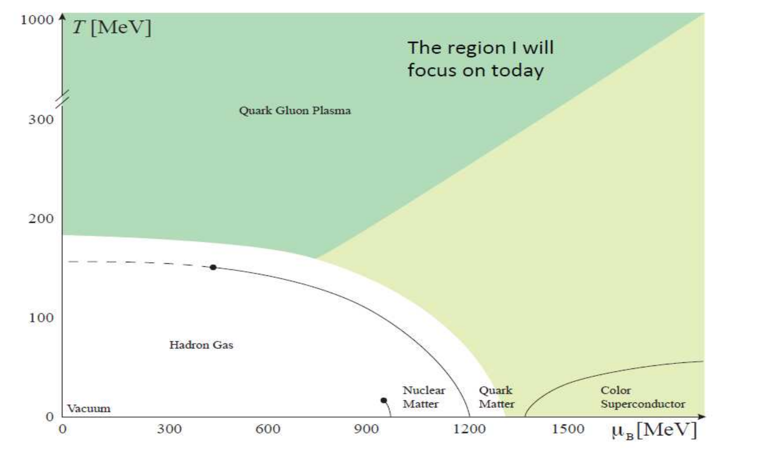
- Peturbative Thermal QCD to study EOS and Thermodynamics of Hot and Dense Matter

- Precise knowledge of equation of state (pressure) of QCD matter at high density and temperature has important significance for the analysis of HIC experiments.

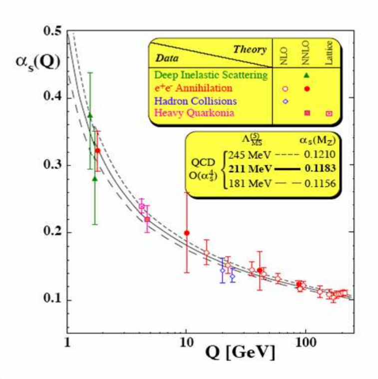
Perturbative QCD (weak coupling expansion) Domain:



Perturbative QCD: Domain of Present Talk



Perturbative QCD (weak coupling expansion):



- At high temp. and/or high density matter is simple !!
- QCD interactions weaken at high energy
- Simplicity to emerge in extreme (asymptotic) situations

- Any quantity ➥ By expanding in α_s around free theory
- ☛ Both Static + Dynamic quantities

Scale separation at high temperature $T \gg$ any intrinsic mass scale and and $g < 1$

- **Hard Scale:**
  - Thermal fluctuations: Momenta $\sim T$; Length $\sim 1/T$;
  - Purely perturbative contribution to QCD thermodynamics (g^{2n})
- **Soft (Electric Scale):**
  - Static chromoelectric fluctuations: Momenta $\sim gT$; Length $\sim 1/gT$;
  - Debye screening mass of A_0
  - Resummation of an infinite subset of diagrams
  - Odd powers of g and $\log$ creep in (viz., $g,\ g^2,\ g^3, g^4 \log, \cdots$)
- **Ultra-soft (Magnetic Scale):**
  - Static chromomagnetic fluctuations: Momenta $\sim g^2T$; Length $\sim 1/g^2T$;
  - Magnetic mass
  - Generates non-perturbative contribution to pressure starting at 4-loop order (Linde Problem)

Exsisting Approach at high T & μ

- Bare Perturbation Theory (BPT)
  - Hard Scale; contribution (g^{2n})
  - BPT breaks down due to Infrared divergence !
  - Requires separation of scales
- Possible Works Around:
  - Dimensional Reduction (DR) : An effective theory
  - Resummation : An effective theory

Dimensional Reduction: An Effective Theory

Dimensional Reduction:

- Separation of scales: T (Hard), gT (Elec. Screen.), g^2T (Mag. Screen.)
- Except zero bosonic mode ($n = 0$) all other d.o.fs get large effective mass $[\omega_n^b = 2n\pi T;\ \omega_n^f = (2n+1)\pi T]$
- Integrate out non-static massive modes ($n \neq 0$) and zero static mode ($n = 0$) remains intact
- A 3-dim effective theory of static electric modes
- P_{QCD} = Hard+Soft+Ultra-soft $= P_E + P_M + P_G$

Requires Matching:

- P_E and its coeffs: involves scale T and obtained in BPT thru 1PI diagrams $\sim g^2(2\pi T)$
- P_M and its coeffs: involves scale gT and obtained as $(gT)^3$ and higher order
- P_G: involves scale g^2T obtained by fitting LQCD data ($\sim g^6$)

Exsisting PQCD/DR Results at high T & μ

$P/P_{SB} = 1$	Stefan-Boltzmann ideal gas
$+\ g^2$	1-loop (Shuryak 78, Chin 78)
$+\ g^3$	2-loop (Kapusta 79)
$+\ g^4 \ln(1/g)$	2-loop (Toimela 83)
$+\ g^4$	3-loop (Arnold, Zhai 94)
$+\ g^5$	3-loop (Zhai, Kastening 95)
$+\ g^6 \ln(1/g)$	3-loop (Kajantie et al. 03; Vourinen 03)
$+\ g^6$	*not perturbatively computable* (Linde 80)
$+\ g^7$	
$+\ \cdots$	

☛ Efforts took 25 years (1978-2003)

Exsisting PQCD/DR Results at high T & μ

P/P_S

Exsisting PQCD Results at T & $\mu \neq 0$ (Vourinen, PRD68, 2003)

$$\mathcal{F} = -\frac{d_A \pi^2}{45} T^4 \left[\mathcal{F}_0 + \mathcal{F}_2 \frac{\alpha_s}{\pi} + \mathcal{F}_3 \left(\frac{\alpha_s}{\pi}\right)^{3/2} + \mathcal{F}_4 \left(\frac{\alpha_s}{\pi}\right)^2 + \mathcal{F}_5 \left(\frac{\alpha_s}{\pi}\right)^{5/2} + \mathcal{F}_6 \left(\left(\frac{\alpha_s}{\pi}\right)^3 \log\left(\frac{\alpha_s}{\pi}\right) \right) + \cdots \right],$$

$$\mathcal{F}_0 = 1 + \frac{21}{32} N_f \left(1 + \frac{120}{7}\hat{\mu}^2 + \frac{240}{7}\hat{\mu}^4 \right),$$

$$\mathcal{F}_2 = -\frac{15}{4} \left[1 + \frac{5N_f}{12} \left(1 + \frac{72}{5}\hat{\mu}^2 + \frac{144}{5}\hat{\mu}^4 \right) \right],$$

$$\mathcal{F}_3 = 30 \left[1 + \tfrac{1}{6} \left(1 + 12\hat{\mu}^2 \right) N_f \right]^{3/2},$$

en 03)

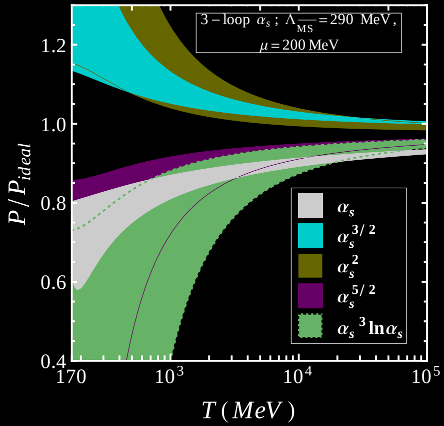
nde 80)

$+ \ \cdots$

- Efforts took 25 years (1978-2003)

Exsisting PQCD/DR Results at high T & μ

$$P/P_{SB} = \left[1 + g^2 + g^3 + g^4\ln(1/g) + g^4 + g^5 + g^6\ln(1/g) + g^6 + \cdots\right]$$



- IR Divergence at $\sim g^3$: Bare PT breaks down
  - At T & $\mu > 0$; $\int d^4K \to T\sum_{k_0}\int d^3k$; Matsubara Mode: $\omega_n^f = (2n+1)\pi T - i\mu$; $\omega_n^b = 2n\pi T$
  - Quark's are harmless: lowest Matsubara mode $\omega_n^f = \pi T$; $n_F(k) \to \frac{1}{2}$ as $k \to 0$
  - Gluons are IR sensitive: lowest Matsubara mode: $\omega_n^b = 0$; $n_B(k) \sim T/k \to \infty$ as $k \to 0$
  - $\omega_n^b = 2\pi nT$; $n = 0$, $\omega_n = 0$; zero bosonic mode can propagate over distance $\gg 1/T$

Message from weak coupling expansion PQCD/DR

- Severe convergence problem spoils pert. exp. due to infrared problem [not specific to QCD; exists in QED and scalar theories]
- Observable sensitive to infrared problem at $T \,\&\, \mu \neq 0$ in PQCD
- g^6-coefficient is tuned to fit LQCD data ➥ pressure at all T (DR)
- Band for a given α_s order is very wide for the scale (πT to $4\pi T$).

Aim:

- A more convergent gauge-invariant scheme for $T > 2T_c$
- A framework that should describe dynamical properties of the QGP
- Improvement ➥ HTL resummation

HTL perturbation Theory (HTLpt): [Andersen Braaten and Strickland, 99 → 02]

- Assumption: $T \gg$ any intrinsic mass scale of the theory and $g < 1$

☛ Typical momenta of a particle in a heat bath $\sim T$ (hard scale)
☛ Due to interaction massless particles acquire mass $\sim gT$ (soft scale)
☛ Scales are well separated in weak coupling $(T \gg gT)$

- Observation: There are thermal corrections from all orders of PT;

  Thermal Corr.= $\frac{g^2T^2}{P^2}$ $\times$ Tree Level

- Lesson: Corrections to be taken into account if a physical quantity is sensitive to the soft scale $(\sim gT)$

- Resum: HTL N-point fns. in geom. series; satisfy Ward identity; replace those in bare-PT ➥ reorganisation of BPT

Hard Thermal Loop Action [Andersen Braaten and Strickland, 99 → 02]

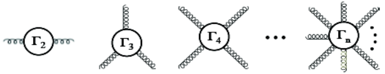
- Can express an infinite number of HTL-dressed n-point functions concisely in terms of an HTL effective action, $\mathcal{L}_{\rm HTL}$
- Expanding $\mathcal{L}_{\rm HTL}$ to quadratic order in A gives dressed propagator (2-point function)
- Expanding to cubic order in A gives the dressed gluon three-vertex
- Expanding to quartic order in A gives dressed gluon four-vertex
- And so on . . . contains an infinite number of higher order vertices which all exactly satisfy the appropriate Slavnov-Taylor identities

$$\mathcal{L} = \left(\mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm HTL}\right)\Big|_{g_s \to \sqrt{\delta} g_s} + \Delta\mathcal{L}_{\rm HTL}$$

[Andersen, Braaten, and MS, 99 → 02]

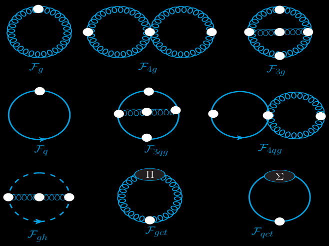
$$\mathcal{L}_{\rm QCD} = -\frac{1}{2}{\rm Tr}\left[G_{\mu\nu}G^{\mu\nu}\right] + i\bar{\psi}\gamma^\mu D_\mu \psi + \mathcal{L}_{\rm gf} + \mathcal{L}_{\rm gh} + \Delta\mathcal{L}_{\rm QCD}$$

$$\mathcal{L}_{\rm HTL} = -\frac{1}{2}(1-\delta) m_D^2 {\rm Tr}\left(G_{\mu\alpha}\left\langle \frac{y^\alpha y^\beta}{(y\cdot D)^2}\right\rangle_y G^\mu{}_\beta\right) + (1-\delta)\, i m_q^2 \bar{\psi}\gamma^\mu \left\langle \frac{y_\mu}{y\cdot D}\right\rangle_y \psi\,,$$



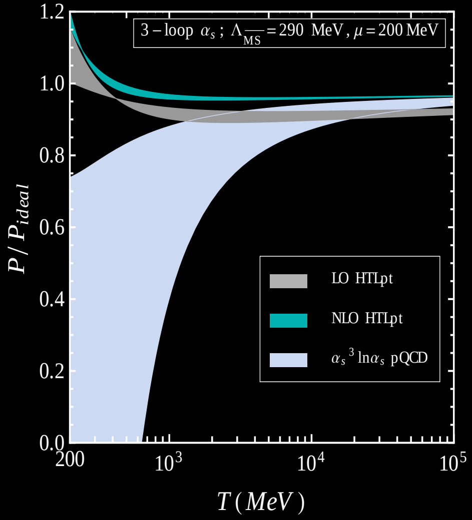
- $n^{\rm th}$ order loop expansion in HTLpt $= \delta^{n-1}$ expansion in the partition function; then $\delta \to 1$

N. Haque and MGM, arXiv:1007.2076[hep-ph]; N. Haque, MGM, M. Strickland, PRD87, 105007 (2013); JHEP 130, 184 (2013)



- Leading Order (LO) ➤ (One Loop)
- Next-To-Leading Order (NLO) ➤ (Two Loop)

N. Haque and MGM, arXiv:1007.2076[hep-ph]; N. Haque, MGM, M. Strickland, PRD87, 105007 (2013); JHEP 130, 184 (2013)

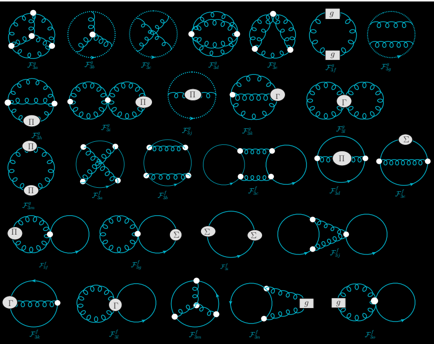


Message from NLO HTLpt (2-loop)

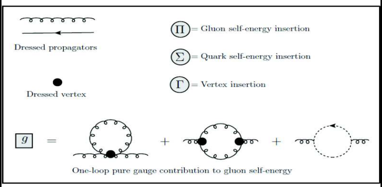
- Resummation causes overcounting: unlike pQCD, loop and coupling expansion in HTLpt are not symmetrical ➤ higher loops contribute to the lower loop order
- ☛ NLO (2-loop) calculation corrects the overcounting in LO (1-loop)
- ☛ NLO (2-loop) pressure obtained here is nominally accurate in g^5 at low T and no $g^6 \ln g$ in comparision to PQCD
- A NNLO (3-loop) calculation in HTLpt is essential to cure overcounting and convergence problems in NLO

Three Loop HTLpt: NNLO calculation of $\mathcal{P}(T,\mu)$

N. Haque, J. Andersen, MGM, M. Strickland, N. Su, PRD90 (2014); JHEP 1502 (2014) 011



- Total 49 diagrams to compute in 3-loop
- Various Insertions:



- One-Loop running $\alpha_s(1.5\text{GeV}) = 0.326$ [Bazavov et al]

- Mass Prescription(Braaten-Nieto):

$$\hat{m}_D^2 = \frac{\alpha_s}{3\pi}\Bigg\{c_A + \frac{c_A^2\alpha_s}{12\pi}\left(5+22\gamma_E+22\ln\frac{\hat{\Lambda}_g}{2}\right) + s_F\left(1+12\hat{\mu}^2\right) + \frac{c_A s_F\alpha_s}{12\pi}\Big(\left(9+132\hat{\mu}^2\right)+22\left(1+12\hat{\mu}^2\right)\gamma_E + 2\left(7+132\hat{\mu}^2\right)\ln\frac{\hat{\Lambda}_q}{2}+4\aleph(z)\Big) + \frac{s_F^2\alpha_s}{3\pi}\left(1+12\hat{\mu}^2\right)\left(1-2\ln\frac{\hat{\Lambda}_q}{2}+\aleph(z)\right) - \frac{3}{2}\frac{s_{2F}\alpha_s}{\pi}\left(1+12\hat{\mu}^2\right)\Bigg\}$$

$$
\begin{aligned}
\mathcal{P}_{\rm NNLO} = \frac{d_A\pi^2T^4}{45}\Big[\!\!\Big[& 1+\frac{7}{4}\frac{d_F}{d_A}\left(1+\frac{120}{7}\hat\mu^2+\frac{240}{7}\hat\mu^4\right)-\frac{15}{4}\hat m_D^3-\frac{s_F\alpha_s}{\pi}\Big[\frac{5}{8}\left(5+72\hat\mu^2+144\hat\mu^4\right) \\
& +90\hat m_q^2\hat m_D-\frac{15}{2}\left(1+12\hat\mu^2\right)\hat m_D-\frac{15}{2}\Big(2\ln\frac{\hat\Lambda}{2}-1-\aleph(z)\Big)\hat m_D^3\Big]+s_{2F}\left(\frac{\alpha_s}{\pi}\right)^2\Big[-\frac{45}{2}\hat m_D\left(1+12\hat\mu^2\right) \\
& +\frac{15}{64}\Big\{35-32\left(1-12\hat\mu^2\right)\frac{\zeta'(-1)}{\zeta(-1)}+472\hat\mu^2+1328\hat\mu^4+64\Big(6(1+8\hat\mu^2)\aleph(1,z)+3i\hat\mu(1+4\hat\mu^2)\aleph(0,z) \\
& -36i\hat\mu\aleph(2,z)\Big)\Big\}\Big]+\left(\frac{s_F\alpha_s}{\pi}\right)^2\Big[\frac{5}{4\hat m_D}\left(1+12\hat\mu^2\right)^2+30\left(1+12\hat\mu^2\right)\frac{\hat m_q^2}{\hat m_D}+\frac{25}{12}\Big\{\frac{1}{20}\left(1+168\hat\mu^2+2064\hat\mu^4\right) \\
& +\Big(1+\frac{72}{5}\hat\mu^2+\frac{144}{5}\hat\mu^4\Big)\ln\frac{\hat\Lambda}{2}+\frac{3\gamma_E}{5}\left(1+12\hat\mu^2\right)^2-\frac{8}{5}(1+12\hat\mu^2)\frac{\zeta'(-1)}{\zeta(-1)}-\frac{34}{25}\frac{\zeta'(-3)}{\zeta(-3)}-\frac{72}{5}\Big[3\aleph(3,2z) \\
& +8\aleph(3,z)-12\hat\mu^2\aleph(1,2z)-2(1+8\hat\mu^2)\aleph(1,z)+12i\hat\mu\left(\aleph(2,z)+\aleph(2,2z)\right)-i\hat\mu(1+12\hat\mu^2)\aleph(0,z)\Big]\Big\} \\
& -\frac{15}{2}\left(1+12\hat\mu^2\right)\left(2\ln\frac{\hat\Lambda}{2}-1-\aleph(z)\right)\hat m_D\Big]+\frac{c_A\alpha_s}{3\pi}\frac{s_F\alpha_s}{\pi}\Big[\frac{15}{2\hat m_D}\left(1+12\hat\mu^2\right)+90\frac{\hat m_q^2}{\hat m_D} \\
& -\frac{235}{16}\Big\{\Big(1+\frac{792}{47}\hat\mu^2+\frac{1584}{47}\hat\mu^4\Big)\ln\frac{\hat\Lambda}{2}-\frac{24\gamma_E}{47}\left(1+12\mu^2\right)+\frac{319}{940}\Big(1+\frac{2040}{319}\hat\mu^2+\frac{38640}{319}\hat\mu^4\Big)-\frac{268}{235}\frac{\zeta'(-3)}{\zeta(-3)} \\
& -\frac{144}{47}\left(1+12\hat\mu^2\right)\ln\hat m_D-\frac{44}{47}\Big(1+\frac{156}{11}\hat\mu^2\Big)\frac{\zeta'(-1)}{\zeta(-1)}-\frac{72}{47}\Big[4i\hat\mu\aleph(0,z)+\Big(5-92\hat\mu^2\Big)\aleph(1,z)+144i\hat\mu\aleph(2,z) \\
& +52\aleph(3,z)\Big]\Big\}+\frac{315}{4}\Big\{\Big(1+\frac{132}{7}\hat\mu^2\Big)\ln\frac{\hat\Lambda}{2}+\frac{11}{7}\left(1+12\hat\mu^2\right)\gamma_E+\frac{9}{14}\Big(1+\frac{132}{9}\hat\mu^2\Big)+\frac{2}{7}\aleph(z)\Big\}\hat m_D\Big] \\
& +\frac{c_A\alpha_s}{3\pi}\Big[-\frac{15}{4}+\frac{45}{2}\hat m_D-\frac{135}{2}\hat m_D^2-\frac{495}{4}\Big(\ln\frac{\hat\Lambda_g}{2}+\frac{5}{22}+\gamma_E\Big)\Big]+\left(\frac{c_A\alpha_s}{3\pi}\right)^2\Big[\frac{45}{4\hat m_D}-\frac{165}{8}\Big(\ln\frac{\hat\Lambda_g}{2} \\
& -\frac{72}{11}\ln\hat m_D-\frac{84}{55}-\frac{6}{11}\gamma_E-\frac{74}{11}\frac{\zeta'(-1)}{\zeta(-1)}+\frac{19}{11}\frac{\zeta'(-3)}{\zeta(-3)}\Big)+\frac{1485}{4}\Big(\ln\frac{\hat\Lambda_g}{2}-\frac{79}{44}+\gamma_E-\ln 2-\frac{\pi^2}{11}\Big)\hat m_D\Big]\Big]\!\!\Big]
\end{aligned}
$$

$$\mathcal{P}_{\rm NNLO} = \frac{d_A \pi^2 T^4}{45}\left[\!\!\left[1 + \frac{7}{4}\frac{d_F}{d_A}\left(1 + \frac{120}{7}\hat\mu^2 + \frac{240}{7}\hat\mu^4\right) - \frac{15}{4}\hat m_D^3 - \frac{s_F\alpha_s}{\pi}\left[\frac{5}{8}\left(5 + 72\hat\mu^2 + 144\hat\mu^4\right)\right.\right.\right.$$

$$+90\hat m_q^2 \hat m_D - \frac{15}{2}\left(1+12\hat\mu^2\right)\hat m_D - \frac{15}{2}\left(2\ln\frac{\hat\Lambda}{2} - 1 - \aleph(z)\right)\hat m_D^3\Big] + s_{2F}\left(\frac{\alpha_s}{\pi}\right)^2\Big[-\frac{45}{2}\hat m_D\left(1+12\hat\mu^2\right)$$

$$+\frac{15}{64}\Big\{35 - 32\left(1-12\hat\mu^2\right)\frac{\zeta'(-1)}{\zeta(-1)} + 472\hat\mu^2 + 1328\hat\mu^4 + 64\Big(6(1+8\hat\mu^2)\aleph(1,z) + 3i\hat\mu(1+4\hat\mu^2)\aleph(0,z)$$

$$-36i\hat\mu\aleph(2,z)\Big)\Big\}\Big] + \left(\frac{s_F \alpha \cdots}{\pi}\right. \cdots \Big\{\frac{1}{20}\left(1+168\hat\mu^2+2064\hat\mu^4\right)$$

$$+\left(1+\frac{72}{5}\hat\mu^2+\frac{144}{5}\hat\mu^4\right)\ln \cdots \frac{\zeta'(-3)}{\zeta(-3)} - \frac{72}{5}\Big[3\aleph(3,2z)$$

$$+8\aleph(3,z) - 12\hat\mu^2\aleph(1,2z) \cdots + 12\hat\mu^2)\,\aleph(0,z)\Big]\Big\}$$

$$-\frac{15}{2}\left(1+12\hat\mu^2\right)\left(2\ln\frac{\hat\Lambda}{2} \cdots 90\frac{\hat m_q^2}{\hat m_D}\right.$$

$$-\frac{235}{16}\Big\{\Big(1+\frac{792}{47}\hat\mu^2 + \frac{158\cdots}{4\cdots} \cdots + \frac{38640}{319}\hat\mu^4\Big) - \frac{268}{235}\frac{\zeta'(-3)}{\zeta(-3)}$$

$$-\frac{144}{47}\left(1+12\hat\mu^2\right)\ln\hat m_D \cdots 2\hat\mu^2\Big)\,\aleph(1,z) + 144i\hat\mu\aleph(2,z)$$

$$+52\aleph(3,z)\Big]\Big\} + \frac{315}{4}\Big\{\Big(1 \cdots \hat\mu^2\Big) + \frac{2}{7}\aleph(z)\Big\}\hat m_D\Big]$$

$$+\frac{c_A\alpha_s}{3\pi}\left[-\frac{15}{4} + \frac{45}{2}\hat m_D - \frac{135}{2}\hat m_D^2 - \frac{495}{4}\left(\ln\frac{\hat\Lambda_g}{2} + \frac{5}{22} + \gamma_E\right)\right] + \left(\frac{c_A\alpha_s}{3\pi}\right)^2\Big[\frac{45}{4\hat m_D} - \frac{165}{8}\Big(\ln\frac{\hat\Lambda_g}{2}$$

$$-\frac{72}{11}\ln\hat m_D - \frac{84}{55} - \frac{6}{11}\gamma_E - \frac{74}{11}\frac{\zeta'(-1)}{\zeta(-1)} + \frac{19}{11}\frac{\zeta'(-3)}{\zeta(-3)}\Big) + \frac{1485}{4}\left(\ln\frac{\hat\Lambda_g}{2} - \frac{79}{44} + \gamma_E - \ln 2 - \frac{\pi^2}{11}\right)\hat m_D\Big]\!\!\Big]$$

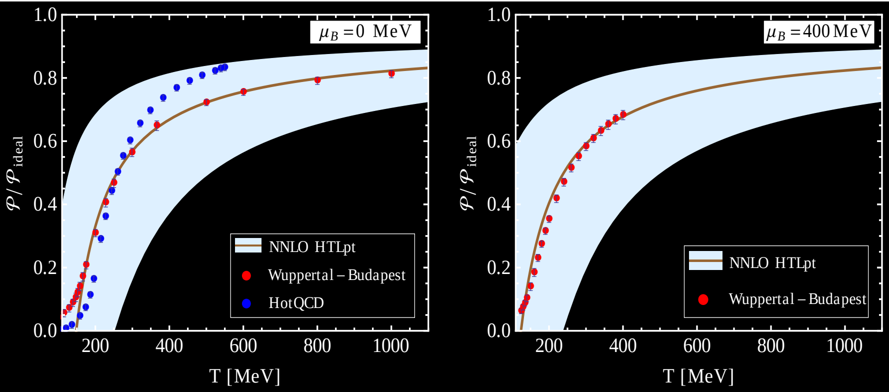
COMPLETELY ANALYTIC

AND

GAUGE INDEPENDENT EXPRESSION

NNLO HTL Pressure $\mathcal{P}(T,\mu)$:

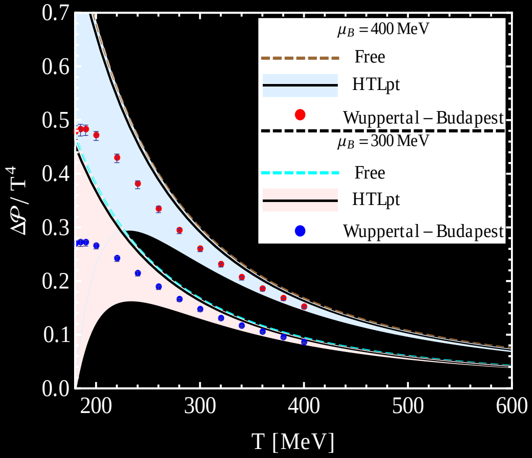
N. Haque, J. Andersen, MGM, M. Strickland, N. Su, PRD90(2014)



- $\mathcal{P}(T,\mu=0) \rightarrow$ Andersen et al JHEP 8(2011)053
- LQCD data: Brosásnyi et al, JHEP 11 (2010)077; JHEP08 (2012) 053

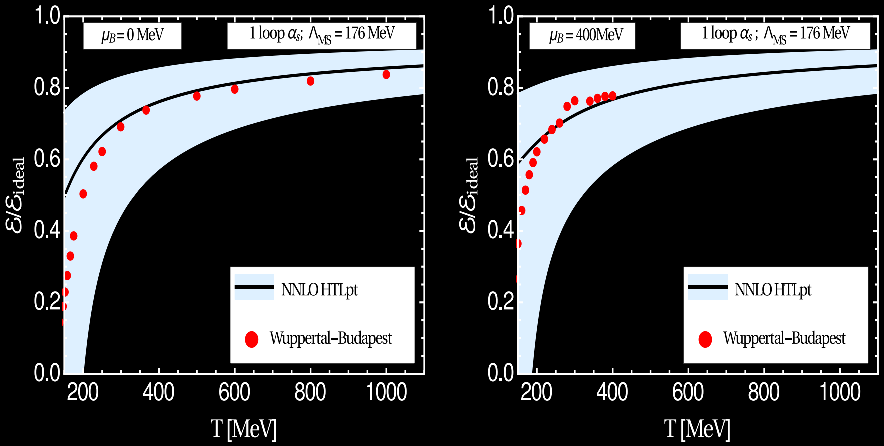
NNLO $\Delta\mathcal{P}(T,\mu) = \mathcal{P}(T,\mu) - \mathcal{P}(T,0)$:

N. Haque, J. Andersen, MGM, M. Strickland, N. Su, PRD90(2014)



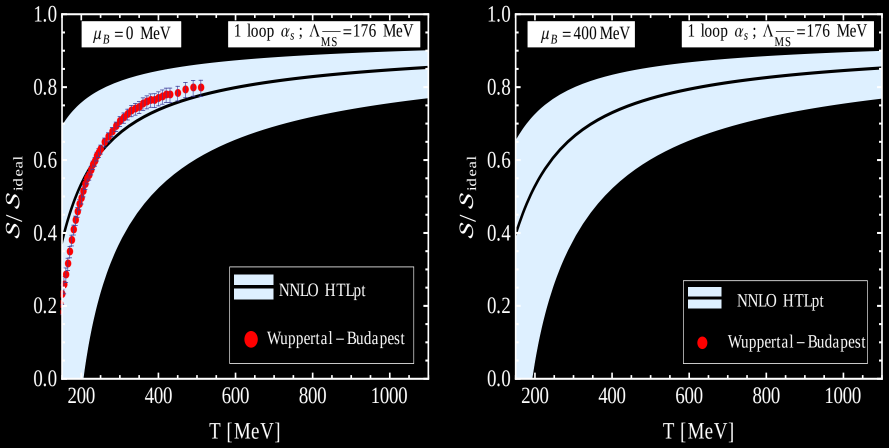
NNLO Energy Density:

N. Haque, A. Bandyopadhyay, J. Andersen, MGM, M. Strickland, N. Su, JHEP (2014)



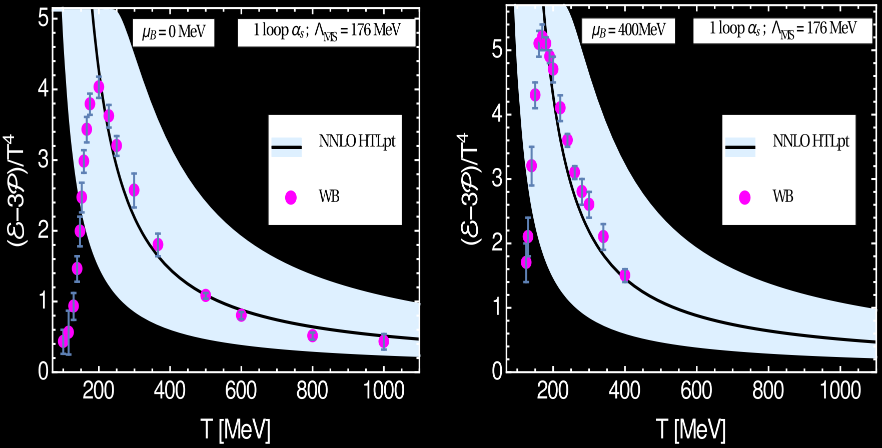
NNLO Entropy Density:

N. Haque, A. Bandyopadhyay, J. Andersen, MGM, M. Strickland, N. Su, JHEP (2014)



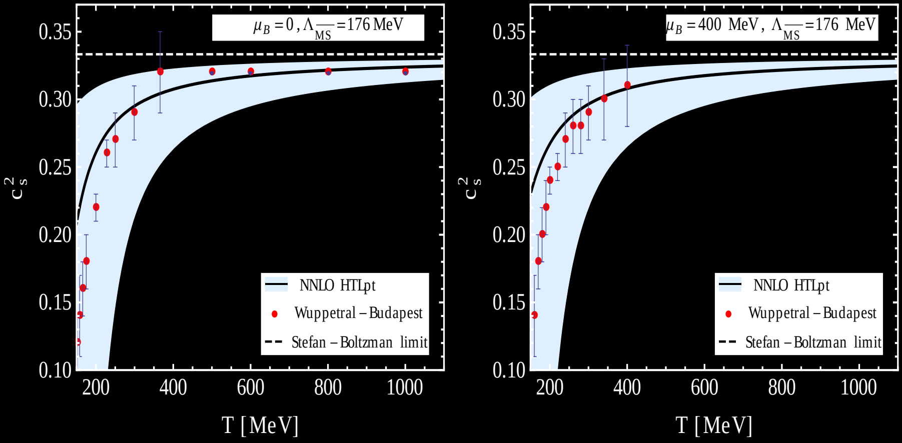
NNLO Trace Anomaly: $(\mathcal{E} - 3\mathcal{P})/T^4$:

N. Haque, A. Bandyopadhyay, J. Andersen, MGM, M. Strickland, N. Su, JHEP (2014)



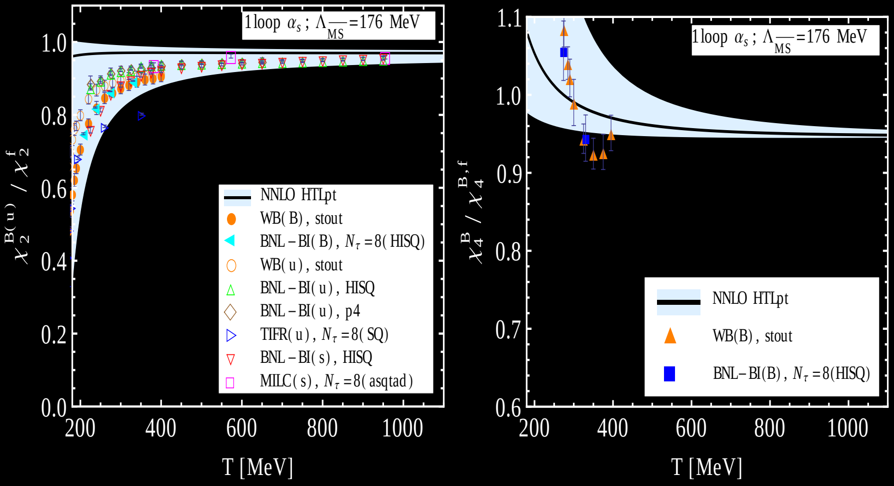
NNLO speed of sound $c_s^2 = dP/d\epsilon$:

N. Haque, A. Bandyopadhyay, J. Andersen, MGM, M. Strickland, N. Su, JHEP (2014)



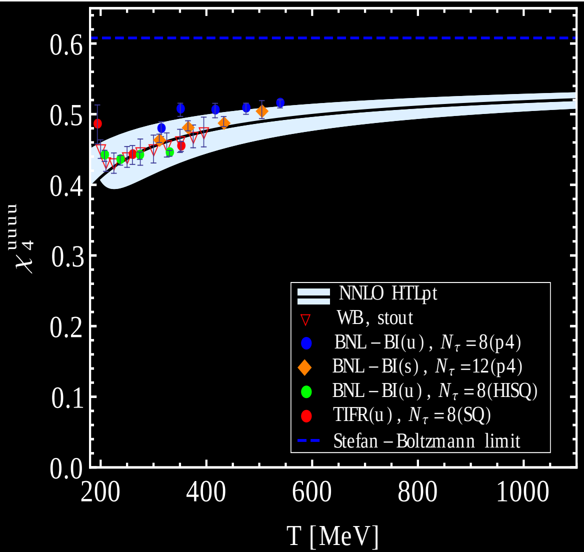
NNLO Baryon No. Susceptibilities: χ_2^B and χ_4^B

N. Haque, A. Bandyopadhyay, J. Andersen, MGM, M. Strickland, N. Su, JHEP (2014)



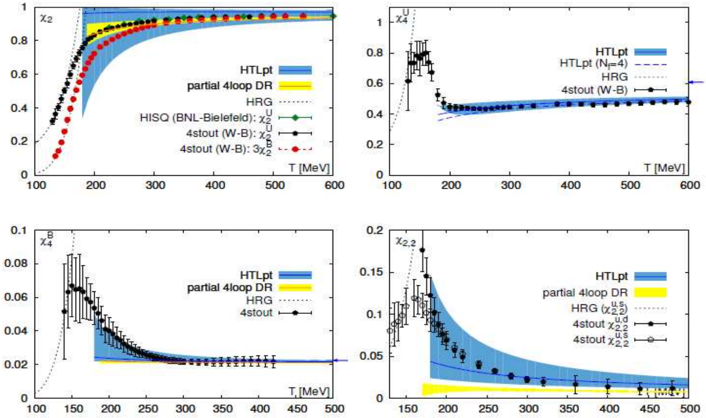
Fourth order diagonal quark number fluctuations

N. Haque, A. Bandyopadhyay, J. Andersen, MGM, M. Strickland, N. Su, JHEP (2014)



From S. Borsani Lattice2015 talk: arXiv:1507:046271; WB Collaboration

At high temperature: lattice vs Hard Thermal Loops



HTL results: [Haque et al 1309.3968,1402.6907] Dimensional reduction: [1307.8098]

Lattice results: [WB: 1507.04627]

Comparison with new LQCD Data [Ding et al: arXiv:1507:06637]

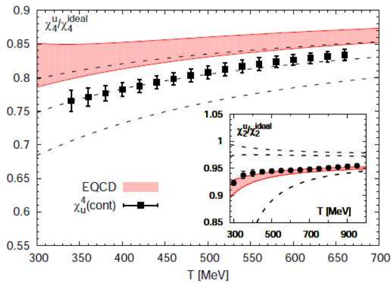


FIG. 4. The continuum extrapolated result for χ_4^u compared to perturbative EQCD calculations shown as the shaded band. The width of the band corresponds to the variation of the renormalization scale from πT to $4\pi T$. The dashed lines correspond to the 3-loop HTL calculations evaluated for the renormalization scale $\Lambda = 4\pi T$, $2\pi T$ and πT (from top to bot-

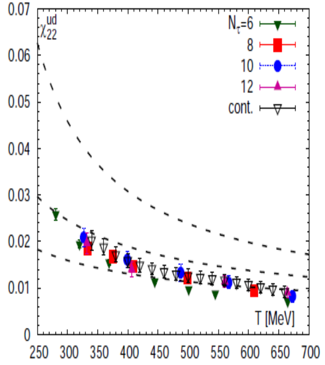


FIG. 5. The fourth order off-diagonal susceptibility χ_{22}^{ud} (left)

- N. Haque, A. Bandyopadhyay, J. Andersen, MGM, M. Strickland, N. Su, JHEP (2014)

Message from NNLO (3-loop) HTLpt

- HTLpt is a state-of-the-art calculation for thermodynamic (and also for dynamic) quantities for deconfined hot and dense matter
- NNLO (3-loop) HTLpt improves convergence & overcounting problems in NLO (2-loop) HTLpt
- NNLO $\mathcal{P}(T,\mu)$, $\mathcal{E}$, $\Delta = (\mathcal{E} - P)/T^4$, c_s^2 and QNSs $(\chi_2,\ \chi_4)$ in HTLpt agree with LQCD $T \geq 200$ MeV
- All these quantities at $T \leq 200$ deviate from LQCD because of T^2 (non-ideal), which is non-perturbative in nature
- Very recent LQCD data on QNS are rejoice for NNLO HTLpt.
  - Work is in progress for the full $\mu - T$ plane (requires ring summation at low T and high μ)
  - Needs $\log$ resummation to reduce further the renor. scale dependent band!

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(Saha Inst., India)

Najmul Haque
(Kent State Univ. USA)

Michael Strickland
(Kent State Univ. USA)

Nan Su
(Frankfurt U., Germany)

THANK YOU

Application of HTLpt: an improved perturbation theory

- This state-of-the-art machinary can be extended for $T = 0$ but any μ, appropriate for FAIR perspective. Work is in progress for both NLO & NNLO!
  - No LQCD data at $\mu \neq 0$ and $T = 0$ (Difficult task) !

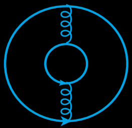
- HTLpt is important
  - **for various particle productions (l^+l^-, γ, $\cdots$) in QGP**
  - **energy-loss/gain for high energetic particles in QGP**
  - **one and two body potential in QGP**
  - **mesonic correlation function for binary states in QGP**
- Difficult to trade around phase transition line; No chiral symmetry broken/restoration and confinement/deconfinement information!

- Beyond scope to discuss all

Fluctuations of conserved charges

N. Haque, A. Bandyopadhyay, J. Andersen, MGM, M. Strickland, N. Su, JHEP (2015)

In three loop HTLpt case, we have a diagram:



The flavor of two fermionic loop are not same always.

$\Rightarrow$ Off-diagonal susceptibility is non-zero.

$\Rightarrow$ Quark number flutuations and baryon number fluctuations are not proportional to each other.

Fluctuations of conserved charges

- Fluctuations and correlations of conserved charges are sensitive probes of deconfinement
- Quark Number fluctuation for three flavor system is defined as

$$\chi^{uds}_{ijk}(T) = \frac{\partial^{i+j+k}\mathcal{P}}{\partial\mu_u^i\partial\mu_d^j\partial\mu_s^k}$$

- We can also define Baryon Number fluctuations as

$$\chi^{B}_{n}(T) = \frac{\partial^{n}\mathcal{P}}{\partial\mu_B^n}$$

with $\mu_B = \mu_u + \mu_d + \mu_s$ for three-flavor system.

Fluctuations of conserved charges

7.1 Baryon number susceptibilities

We begin by considering the baryon number susceptibilities. The $n^{\rm th}$-order baryon number susceptibility is defined as

$$\chi_B^n(T) \equiv \left.\frac{\partial^n \mathcal{P}}{\partial \mu_B^n}\right|_{\mu_B=0} . \tag{7.2}$$

For a three flavor system consisting of (u, d, s), the baryon number susceptibilities can be related to the quark number susceptibilities [15]

$$\chi_2^B = \frac{1}{9}\left[\chi_2^{uu} + \chi_2^{dd} + \chi_2^{ss} + 2\chi_2^{ud} + 2\chi_2^{ds} + 2\chi_2^{us}\right] , \tag{7.3}$$

and

$$\chi_4^B = \frac{1}{81}\Big[\chi_4^{uuuu} + \chi_4^{dddd} + \chi_4^{ssss} + 4\chi_4^{uuud} + 4\chi_4^{uuus} + 4\chi_4^{dddu} + 4\chi_4^{ddds} + 4\chi_4^{sssu} + 4\chi_4^{sssd} + 6\chi_4^{uudd} + 6\chi_4^{ddss} + 6\chi_4^{uuss} + 12\chi_4^{uuds} + 12\chi_4^{ddus} + 12\chi_4^{ssud}\Big] . \tag{7.4}$$

If we treat all quarks as having the same chemical potential $\mu_u = \mu_d = \mu_s = \mu = \frac{1}{3}\mu_B$, eqs. (7.3) and (7.4) reduce to $\chi_2^B = \chi_2^{uu}$ and $\chi_4^B = \chi_4^{uuuu}$. This allows us to straightforwardly compute the baryon number susceptibility by computing derivatives of (4.5) with respect to μ.

$$\chi_2^{ud} = \chi_2^{ds} = \chi_2^{su} = 0 , \tag{7.5}$$

and, as a result, the single quark second order susceptibility is proportional to the baryon number susceptibility

$$\chi_2^{uu} = \frac{1}{3}\chi_2^B . \tag{7.6}$$

For the fourth order susceptibility, there is only one non-zero off-diagonal susceptibility, namely $\chi_4^{uudd} = \chi_4^{uuss} = \chi_4^{ddss}$, which is related to the diagonal susceptibility, e.g. $\chi_4^{uuuu} = \chi_4^{dddd} = \chi_4^{ssss}$, as

$$\chi_4^{uuuu} = 27\chi_4^B - 6\chi_4^{uudd} . \tag{7.7}$$

IR problem in pQCD

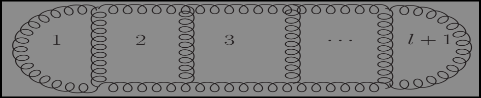
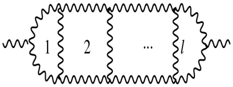


Figure : Divergent $(l+1)$-order loop diagrams

$$g^{2l}\left(T\int d^3k\right)^{l+1}k^{2l}(k^2+m^2)^{-3l}$$

- For $l=3$: $g^6T^4\ln\frac{T}{m}$
- For $l>3$: $g^6T^4\left(\frac{g^2T}{m}\right)^{l-3}$
- For $l\geq 3$: one needs to calculate infinite number of diagrams for g^6

IR problem in pQCD



Divergent l-loop contribution to the 2-point function

$$\underbrace{g^{2l}}_{vertex}\underbrace{\left(\int d^3p\right)^l}_{loop\ integral}\underbrace{p^{2l}}_{vertex}\underbrace{\left(p^2+m^2\right)^{-3l+1}}_{propagators}.$$

- For $l=2:\ g^4T^2\ln\frac{T}{m}$

☛ magnetic mass: $m=g^2T$

- For $l\geq 3:\ g^6T^2\ln\frac{T}{m}$; $\qquad g^4T^2\left(\frac{g^2T}{m}\right)^{l-2}$
- For ≥ 3; one needs to calculate infinite number of diagrams for g^6

Dimensional Reduction

Electrostatic QCD (EQCD)

- Result: 3-dimensional effective theory over distances $\gtrsim 1/gT$: [Braaten,Nieto]

$$\mathcal{L}_{EQCD} = \frac{1}{4} F_{ij}^a F_{ij}^a + \mathrm{Tr}[D_i, A_0]^2 + m_E^2 \mathrm{Tr}[A_0^2] + \lambda_E^{(1)} (\mathrm{Tr}[A_0^2])^2 + \lambda_E^{(2)} \mathrm{Tr}[A_0^4] + \cdots$$

where $F_{ij}^a = \partial_i A_j^a - \partial_j A_i^a + g_E f^{abc} A_i^b A_j^c$ and $D_i = \partial_i - i g_E A_i$.

- Higher order operators do not (yet) contribute:

$$\frac{\delta p_{\mathrm{QCD}}(T)}{T} \sim g^2 \frac{D_k D_l}{(2\pi T)^2} \mathcal{L}_{EQCD} \sim g^2 \frac{(gT)^2}{(2\pi T)^2} (gT)^3 \sim g^7 T^3 .$$

Dimensional Reduction

Eff. gauge coupling g_E^2 und mass m_E^2

- Four matching coefficients have to be determined:

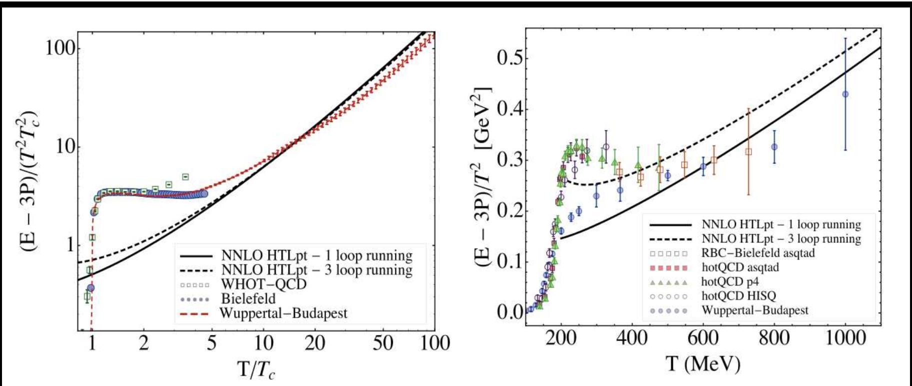
$$m_E^2 = T^2 \left[\#g^2 + \#g^4 + \#g^6 + \ldots\right],$$
$$g_E^2 = T\left[\#g^2 + \#g^4 + \#g^6 + \#g^8 + \ldots\right],$$
$$\lambda_E^{(1/2)} = T\left[\#g^4 + \ldots\right].$$

- 2-loop correction [Laine,Schröder]‘05
- Coefficients can be determined by matching: require the same result in QCD and EQCD.
- Many possibilities, Here: Computation of self-energies $\Pi_{\mu\nu}$ on both sides.

Symmetries in QCD

Symmetry	Vacuum	High T	Low T, high μ	Order parameter	Consequences
(Local) color $SU(3)$	Unbroken	Unbroken	Broken	Diquark condensate	Color superconductivity
$Z(3)$ center symmetry	Unbroken	Broken	Broken	Polyakov loop	Confinement/ deconfinement
Scale invariance	Anomaly			Gluon condensate	Scale (Λ_{QCD}), running coupling
Chiral symmetry $U_L(N_f) \times U_R(N_f) = U_V(1) \times SU_V(N_f) \times SU_A(N_f) \times U_A(1)$					
$U_V(1)$	Unbroken	Unbroken	Unbroken	—	Baryon number conservation
Flavor $SU_V(N_f)$	Unbroken	Unbroken	Unbroken	—	Multiplets
Chiral $SU_A(N_f)$	Broken	Unbroken	Broken	Quark condensate	Goldstone bosons, no degenerate states with opposite parity
$U_A(1)$	Anomaly			Topological susceptibility	Violation of intrinsic parity

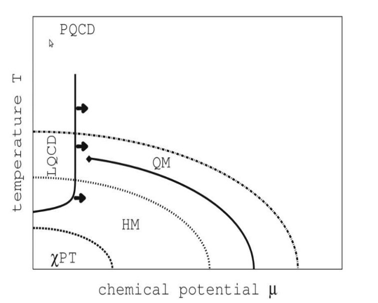
Trace Anomaly



- pQCD only scale is T; ϵ, $P \sim T^4$
- Non-ideal behaviour $\sim T^2$ in addition to T^4 (ideal behaviour)

☛ Effective model should incorporate this feature and QCD symmetries

Chiral Perturbation Theory



- χ-PT: a systematic approach to describe strongly interacting system (lightest hadrons) at low T and μ
- It accounts smallness up/down quark mass and broken chiral symmetry
- It does not work when hadron resonances start influencing the properties of strongly interacting system

Thermodynamic Pressure of massless QCD at T $\& \; \mu \neq 0$

- Thermodynamic observables via partition function in path integral representation and Eucledian space-time:

$$\mathcal{Z}(T,\mu) \equiv \mathrm{Tr}(e^{-\beta H}) \to \int \mathcal{D}A^a \, \mathcal{D}c \, \mathcal{D}\bar{c} \, \mathcal{D}\psi \, \mathcal{D}\bar{\psi} \, \exp\left[-\mathcal{S}_E\right]$$

- $\mathcal{S}_E = \int_0^\beta d\tau \, \int d^{d-1}\mathbf{x} \, (\mathcal{L}_E - \mu\mathcal{N})$ and $\beta \equiv 1/T;\; T =$ Temperature

- Pressure: $P = \lim_{V\to\infty} \frac{T}{V} \ln \mathcal{Z}$

- Other Quantities using Thermodynamic Relations

Why BAND ?

Running coupling constant corresponding to one loop beta function

$$\alpha_s(\Lambda) = \frac{12\pi}{11C_A - 2N_f} \frac{1}{\ln\left(\frac{\Lambda^2}{\Lambda^2_{\overline{MS}}}\right)}$$

- $C_A =$ Color factor associated with gluon emmision from a gluon. For $SU(N_C)$ gauge theory, $C_A = N_c$.
- $N_f =$ Number of flavor,
- $\Lambda_{\overline{MS}} =$ QCD scale. For one loop beta function with $N_f = 3,\ \Lambda_{\overline{MS}} = 176$ MeV(from Lattice).
- $\Lambda =$ Renormalization scale which is $\sim 2\pi T$ at finite temperature. We choose here the center value as $2\pi\sqrt{T^2 + \mu^2/\pi^2}$ and we varied the center value by a factor of 2.

Other Thermodynamic quantities

$$\begin{aligned}
\text{Entropy density} \qquad \mathcal{S}(T,\mu) &= \frac{\partial \mathcal{P}}{\partial T}\,, \\
\text{Number density} \qquad n_i(T,\mu_i) &= \frac{\partial \mathcal{P}}{\partial \mu_i}\,, \\
\text{Energy density} \qquad \mathcal{E}(T,\mu) &= T\frac{\partial \mathcal{P}}{\partial T} + \mu\frac{\partial \mathcal{P}}{\partial \mu} - \mathcal{P} \\
\text{Speed of sound} \qquad c_s^2(T,\mu) &= \frac{\partial \mathcal{P}}{\partial \mathcal{E}} \\
\text{Trace anomaly} \qquad I(T,\mu) &= \mathcal{E} - 3\mathcal{P}
\end{aligned}$$