Hot QCD and QCD-like theories on the lattice

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Outline

1. Motivation
2. Generalities about lattice gauge theory
3. Results from lattice QCD at finite temperature
4. Results from QCD-like theories at finite temperature
Motivation

Generalities about lattice gauge theory

Results from lattice QCD at finite temperature

Results from QCD-like theories at finite temperature
Why do we need to study QCD and QCD-like theories on the lattice?

- We need *ab initio* theoretical predictions for QCD under the conditions probed in heavy-ion collisions
- Weak-coupling expansions in thermal QCD become accurate only at high temperatures, ...
- ... cannot capture the physics of long-wavelength modes [A. Linde, 1980] ...
- ... nor the dynamics in the hadronic phase
- The lattice regularization [K. G. Wilson, 1974] provides the only known mathematically well-defined, non-perturbative formulation of QCD
- Thanks to steady theoretical, algorithmic and computer-power progress, lattice QCD computations are now producing accurate predictions
- Furthermore, the lattice investigation of QCD-like theories (possibly combined with different theoretical approaches: weak- or strong-coupling calculations, effective theories, phenomenological models, ...) can provide analytical understanding of the physics, too
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$$U_\mu(x) = \exp[i g a A_\mu(x)]$$

GAUGE FIELDS ON LINKS

$\psi(x)$

MATTER FIELDS ON SITES
Basic ideas

Regularize the path integrals by discretizing the theory on a Euclidean lattice of spacing $a$ [K. G. Wilson, 1974]

Define gauge and matter fields on lattice elements, build a gauge-invariant lattice action and observables

$$S = -\frac{1}{g^2} \sum_\Box \text{Tr}(U_\Box + U_\Box^\dagger) + \sum_{x,y,f} a^4 \bar{\psi}_f(x) M^f_{x,y} \psi_f(y)$$

$$M^f_{x,y} = m \delta_{x,y} - \frac{1}{2a} \sum_{\mu} \left[ (r - \gamma_\mu) U_\mu(x) \delta_{x+a_\mu,y} + (r + \gamma_\mu) U^\dagger_\mu(y) \delta_{x-a_\mu,y} \right]$$
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Continuum action recovered for $a \rightarrow 0$
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Suitable for numerical simulation: Sample configuration space according to a statistical weight proportional to $\exp(-S)$, compute expectation values

$$
\langle \mathcal{O} \rangle = \frac{\int \prod d\psi(x) d\bar{\psi}(x) \prod dU_\mu(x) \mathcal{O} \exp(-S)}{\int \prod d\psi(x) d\bar{\psi}(x) \prod dU_\mu(x) \exp(-S)}
$$
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1. Importance sampling made possible by real positive statistical weight
2. Natural interpretation as a thermal QFT at equilibrium
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Debunking some common misconceptions

- “Lattice QCD is only an approximation of QCD”
- “The results depend on the details of your discretization”
- “You can never recover the correct rotational and translational symmetries of the original continuum theory”
- “You always have undesired additional quark species (doublers)”
- “It only works / it is only defined at strong coupling”
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- “Lattice QCD is only an approximation of QCD”— False
  - In the physical, large-volume and continuum limits, it is the mathematically rigorous non-perturbative definition of QCD

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- “The results depend on the *details* of your discretization”
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- “Lattice QCD is only an approximation of QCD”— False
- “The results depend on the details of your discretization”— False
  - The intermediate results do depend on the discretization details, those extrapolated to the continuum limit do not
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- “The results depend on the details of your discretization”— False
- “You can never recover the correct rotational and translational symmetries of the original continuum theory”— False
  - When the lattice spacing $a \to 0$ the continuum theory, with its full symmetries, emerges as a good low-energy effective description of the lattice model
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  - It is defined at any value of the coupling; the continuum limit $a \to 0$ is taken at weak coupling
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  - Moore’s law and algorithmic progress came to the rescue: for standard lattice QCD computations, quenched calculations are now obsolete
1 Motivation

2 Generalities about lattice gauge theory

3 Results from lattice QCD at finite temperature

4 Results from QCD-like theories at finite temperature
Equilibrium properties

- Chiral symmetry restoration and deconfinement
- Equation of state in QCD (see also talk by C. Ratti on Friday at 11:00)
Chiral symmetry restoration and deconfinement

1. Chiral symmetry restoration (at $\mu = 0$) is a crossover taking place at $T_c \simeq 155$ MeV [S. Borsányi et al., 2010] [A. Bazavov et al., 2011]

2. Disconnected chiral susceptibilities of the form

$$\chi_m = \int d^4x \langle m(x)m(0) \rangle$$

with $m \in \{i\bar{\psi}_f\tau^a\gamma_5\psi_f, \bar{\psi}_f\tau^a\psi_f, i\bar{\psi}_f\gamma_5\psi_f, \bar{\psi}_f\psi_f\}$

show that $SU_A(n_f)$ is restored, $U_A(1)$ remains broken [M. I. Buchoff et al., 2013] [T. Bhattacharya et al., 2014]

3. Deconfinement occurs in the same temperature range

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![Graph showing the equation of state in QCD](image)

[R. A. Soltz et al., 2015]

See also [A. Bazavov et al., 2012] [S. Borsányi et al., 2013]

1. Confined phase: Consistency with hadron-resonance-gas model
2. Abrupt increase in thermodynamic potentials signals the “liberation” of colored degrees of freedom around $T \approx 160$ MeV
3. Just above deconfinement: Significant deviations from conformality, interactions not quite “small”
4. Deconfined phase: Slow approach to hard-thermal-loop resummed perturbation theory predictions
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![Graph showing thermodynamic potentials vs. temperature]

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\[
\frac{(\varepsilon - 3p)}{T^4} = \frac{p}{T^4} = \frac{s}{4T^3}
\]

\[\begin{array}{c}
130 & 170 & 210 & 250 & 290 & 330 & 370 \\
T\ [\text{MeV}] & & & & & & \\
\end{array}\]

Stout HISQ

\[\text{[R. A. Soltz et al., 2015]}\]

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- Strong (electro)magnetic fields induce a slight decrease in $T_c$

\[ T_c \]

[G. S. Bali et al., 2011]
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- Chiral symmetry restoration and deconfinement
- Equation of state in QCD (see also talk by C. Ratti on Friday at 11:00)
- Strong (electro)magnetic fields induce a slight decrease in $T_c$
- Freeze-out conditions from fluctuations of conserved charges (baryon number $B$, electric charge $Q$, strangeness $S$) [F. Karsch, 2012]

$$T_{fr} = 144(10) \text{ MeV}, \quad \mu_{fr}^B = 102(6) \text{ MeV} \text{ at RHIC (STAR, } \sqrt{s} = 39 \text{ GeV})$$

[S. Borsányi et al., 2011] [A. Bazavov et al., 2012] [S. Borsányi et al., 2014]
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Quarkonium melting

The sequential disappearance of more and more strongly bound quarkonium states was proposed long ago as a possible QGP “thermometer” [T. Matsui and H. Satz, 1986]
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General strategy of the lattice computation:

1. Heavy quarks can be treated non-relativistically (NRQCD)
2. Compute correlation functions of sources with desired quantum numbers
   \[ G_E(\tau) \sim \int_{-2M}^{\infty} \frac{d\omega}{2\pi} \exp(-\omega \tau) \rho(\omega) \]
3. Invert to extract spectral function \( \rho(\omega) \)
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Bottomonium excitation melting [G. Aarts et al., 2011]
Transport coefficients

Describe QGP response to long-wavelength / low-frequency perturbations in energy and momentum density and other conserved charges [H. B. Meyer, 2011]
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Example: Shear ($\eta$) and bulk ($\zeta$) viscosities

\[ T^{\mu \nu} = (\epsilon + p) u^\mu u^\nu + p g^{\mu \nu} - P^\mu_i P^\nu_j \left[ \eta \left( \partial_i u_j + \partial_j u_i - \frac{2}{3} g_{ij} \partial_k u^k \right) + \zeta g_{ij} \partial_k u^k \right] \]
Describe QGP response to long-wavelength / low-frequency perturbations in energy and momentum density and other conserved charges \([\text{H. B. Meyer, 2011}]\)

Difficult to access on a Euclidean lattice ⇒ Indirectly reconstructed from Kubo formulæ
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\[
\eta = \pi \lim_{\omega \to 0} \lim_{k \to 0} \frac{\rho(\omega, k)}{\omega}
\]

with \( \rho \) the spectral function, related to a suitable (e.g. \( T^{\mu \nu} \)) Euclidean correlator via

\[
G_E(t, k) = \int_0^\infty d\omega \rho(\omega, k) \frac{\cosh \left( \omega \left( t - \frac{1}{2T} \right) \right)}{\sinh \left( \frac{\omega}{2T} \right)}
\]
Transport coefficients

Describe QGP response to long-wavelength / low-frequency perturbations in energy and momentum density and other conserved charges \cite{Meyer2011}.

Difficult to access on a Euclidean lattice $\Rightarrow$ Indirectly reconstructed from Kubo formulæ.

Typically, numerically very challenging.

\cite{Meyer2007}
Transport coefficients

Describe QGP response to long-wavelength / low-frequency perturbations in energy and momentum density and other conserved charges [H. B. Meyer, 2011]

Difficult to access on a Euclidean lattice ⇒ Indirectly reconstructed from Kubo formulæ

Another example: Electric conductivity

\[ \frac{\sigma(T)}{\epsilon_{\text{em}}T} \]

\[ (\text{Aarts et al.)} \quad (\text{Ding et al.)} \quad (\text{Brandt et al.)} \quad (\text{Aarts et al.)} \]

\[ N_f=0 \quad N_f=0 \quad N_f=2 \quad N_f=2+1 \]

[H.-T. Ding, F. Karsch and S. Mukherjee, 2015], with results from

[G. Aarts et al., 2007] [H.-T. Ding et al., 2010] [B. B. Brandt et al., 2012] [G. Aarts et al., 2014]
Motivation

Generalities about lattice gauge theory

Results from lattice QCD at finite temperature

Results from QCD-like theories at finite temperature
Equation of state for different gauge groups

Insight from the comparison of different $SU(N)$ gauge groups
Equation of state for different gauge groups

Insight from the comparison of different $SU(N)$ gauge groups—particularly in the large-$N$ limit
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**Why QCD at large $N$?**

- The large-$N$ limit of QCD (at fixed $\lambda = g^2 N$) has interesting phenomenological implications [G. 't Hooft, 1974]
- It plays a crucial rôle in the holographic gauge/string duality [J. Maldacena, 1998]
- Important applications at finite temperature [J. Casalderrey-Solana et al., 2014] (see also talk by W. van der Schee on Thursday at 11:30)
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$$\lambda = \frac{R^4}{l_s^4} \quad \frac{\lambda}{N} = 4\pi g_s$$

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[M. P., 2009], with a comparison with the holographic model from [U. Gürsoy et al., 2008]

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Comparison of different gauge groups: \( SU(N) \) versus \( G_2 \) [M. Bruno et al., 2014]

See also [M. Pepe and U.-J. Wiese, 2006] [G. Cossu et al., 2007] [C. Bonati, 2015]
Equation of state for different gauge groups

Insight from the comparison of different $SU(N)$ gauge groups—particularly in the large-$N$ limit

$T^2$-dependence in the trace of the energy-momentum tensor

Analogous results in $D = 2 + 1$ spacetime dimensions [M. Caselle et al., 2011]
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Hadron resonance gas and a string model for glueballs [M. Caselle et al., 2015]

See also [H. B. Meyer, 2009] [M. Caselle et al., 2011] [S. Borsányi et al., 2012] and talk by H. Stöcker on Monday at 11:00
Transport and real-time phenomena from effective field theories

The practical (or fundamental) challenges in the direct lattice QCD study of certain transport coefficients can be bypassed combining lattice simulations with effective field theories.
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Results indicate $\kappa / T^3 \approx 2.5$ for $T_c \lessapprox T \lessapprox 2T_c$ [D. Banerjee et al., 2011]

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$$\hat{q} = \frac{\langle p_\perp^2 \rangle}{L} = \int \frac{d^2 p_\perp}{(2\pi)^2} p_\perp^2 C(p_\perp)$$
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$C(p_{\perp})$, the differential parton-plasma constituents collision rate, is related to two-point correlator of light-cone Wilson lines.
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1. Heavy quark momentum diffusion coefficient $\kappa$ from heavy-quark effective field theory.

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2. The momentum broadening of a hard quark moving through the QGP is described by the jet quenching parameter. Non-perturbative soft contributions to $\hat{q}$ can be extracted from lattice simulations of a dimensionally reduced effective theory [S. Caron-Huot, 2009] [M. Laine, 2012] [J. Ghiglieri et al., 2013] (see also [B. Brandt et al., 2014]). Evidence for rather large non-perturbative effects: $\hat{q} \simeq 6 \text{ GeV}^2/\text{fm}$ at RHIC [M. P., K. Rummukainen and A. Schäfer, 2014].
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Additional qualitative insight can be obtained from QCD-like theories which are free from the sign problem:

1. SU(2)-QCD [S. Hands et al., 1999]
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Concluding remarks

- Lattice calculations are providing increasingly accurate first-principle predictions for many physical quantities relevant for QCD under extreme conditions.
- The lattice determination of equilibrium properties is by now settled, and recent works are addressing quantities related to fluctuations, transport, real-time dynamics, et c.
- Lattice simulations of QCD-like theories can shed light onto a number of open theoretical problems.
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Thanks for your attention!