Hot QCD and QCD-like theories on the lattice

Marco Panero

University of Turin and INFN, Turin

"The Second Conference on Heavy Ion Collisions in the LHC Era and Beyond"
Quy Nhơn, Việt Nam, 26-31 July 2015

Outline

Outline

Why do we need to study QCD and QCD-like theories on the lattice?

- We need *ab initio* theoretical predictions for QCD under the conditions probed in heavy-ion collisions
- Weak-coupling expansions in thermal QCD become accurate only at high temperatures, . . .
- . . . cannot capture the physics of long-wavelength modes **[A. Linde, 1980]** . . .
- . . . nor the dynamics in the hadronic phase
- The lattice regularization **[K. G. Wilson, 1974]** provides *the* only known mathematically well-defined, non-perturbative formulation of QCD
- Thanks to steady theoretical, algorithmic and computer-power progress, lattice QCD computations are now producing accurate predictions
- Furthermore, the lattice investigation of QCD-like theories (possibly combined with different theoretical approaches: weak- or strong-coupling calculations, effective theories, phenomenological models, . . .) can provide analytical understanding of the physics, too

Why do we need to study QCD and QCD-like theories on the lattice?

- We need *ab initio* theoretical predictions for QCD under the conditions probed in heavy-ion collisions
- Weak-coupling expansions in thermal QCD become accurate only at high temperatures, . . .
- . . . cannot capture the physics of long-wavelength modes **[A. Linde, 1980]** . . .
- . . . nor the dynamics in the hadronic phase
- The lattice regularization **[K. G. Wilson, 1974]** provides *the* only known mathematically well-defined, non-perturbative formulation of QCD
- Thanks to steady theoretical, algorithmic and computer-power progress, lattice QCD computations are now producing accurate predictions
- Furthermore, the lattice investigation of QCD-like theories (possibly combined with different theoretical approaches: weak- or strong-coupling calculations, effective theories, phenomenological models, . . .) can provide analytical understanding of the physics, too

Why do we need to study QCD and QCD-like theories on the lattice?

- We need *ab initio* theoretical predictions for QCD under the conditions probed in heavy-ion collisions
- Weak-coupling expansions in thermal QCD become accurate only at high temperatures, . . .
- . . . cannot capture the physics of long-wavelength modes **[A. Linde, 1980]** . . .
- . . . nor the dynamics in the hadronic phase
- The lattice regularization **[K. G. Wilson, 1974]** provides *the* only known mathematically well-defined, non-perturbative formulation of QCD
- Thanks to steady theoretical, algorithmic and computer-power progress, lattice QCD computations are now producing accurate predictions
- Furthermore, the lattice investigation of QCD-like theories (possibly combined with different theoretical approaches: weak- or strong-coupling calculations, effective theories, phenomenological models, . . .) can provide analytical understanding of the physics, too

Why do we need to study QCD and QCD-like theories on the lattice?

- We need *ab initio* theoretical predictions for QCD under the conditions probed in heavy-ion collisions
- Weak-coupling expansions in thermal QCD become accurate only at high temperatures, . . .
- . . . cannot capture the physics of long-wavelength modes **[A. Linde, 1980]** . . .
- . . . nor the dynamics in the hadronic phase
- The lattice regularization **[K. G. Wilson, 1974]** provides *the* only known mathematically well-defined, non-perturbative formulation of QCD
- Thanks to steady theoretical, algorithmic and computer-power progress, lattice QCD computations are now producing accurate predictions
- Furthermore, the lattice investigation of QCD-like theories (possibly combined with different theoretical approaches: weak- or strong-coupling calculations, effective theories, phenomenological models, . . .) can provide analytical understanding of the physics, too

Why do we need to study QCD and QCD-like theories on the lattice?

- We need *ab initio* theoretical predictions for QCD under the conditions probed in heavy-ion collisions
- Weak-coupling expansions in thermal QCD become accurate only at high temperatures, . . .
- . . . cannot capture the physics of long-wavelength modes **[A. Linde, 1980]** . . .
- . . . nor the dynamics in the hadronic phase
- The lattice regularization **[K. G. Wilson, 1974]** provides *the* only known mathematically well-defined, non-perturbative formulation of QCD
- Thanks to steady theoretical, algorithmic and computer-power progress, lattice QCD computations are now producing accurate predictions
- Furthermore, the lattice investigation of QCD-like theories (possibly combined with different theoretical approaches: weak- or strong-coupling calculations, effective theories, phenomenological models, . . .) can provide analytical understanding of the physics, too

Why do we need to study QCD and QCD-like theories on the lattice?

- We need *ab initio* theoretical predictions for QCD under the conditions probed in heavy-ion collisions
- Weak-coupling expansions in thermal QCD become accurate only at high temperatures, ...
- ...cannot capture the physics of long-wavelength modes **[A. Linde, 1980]** ...
- ...nor the dynamics in the hadronic phase
- The lattice regularization **[K. G. Wilson, 1974]** provides *the* only known mathematically well-defined, non-perturbative formulation of QCD
- Thanks to steady theoretical, algorithmic and computer-power progress, lattice QCD computations are now producing accurate predictions
- Furthermore, the lattice investigation of QCD-like theories (possibly combined with different theoretical approaches: weak- or strong-coupling calculations, effective theories, phenomenological models, ...) can provide analytical understanding of the physics, too

Why do we need to study QCD and QCD-like theories on the lattice?

- We need *ab initio* theoretical predictions for QCD under the conditions probed in heavy-ion collisions
- Weak-coupling expansions in thermal QCD become accurate only at high temperatures, ...
- ...cannot capture the physics of long-wavelength modes **[A. Linde, 1980]** ...
- ...nor the dynamics in the hadronic phase
- The lattice regularization **[K. G. Wilson, 1974]** provides *the* only known mathematically well-defined, non-perturbative formulation of QCD
- Thanks to steady theoretical, algorithmic and computer-power progress, lattice QCD computations are now producing accurate numerical predictions
- Furthermore, the lattice investigation of QCD-like theories (possibly combined with different theoretical approaches: weak- or strong-coupling calculations, effective theories, phenomenological models, ...) can provide analytical understanding of the physics, too

Why do we need to study QCD and QCD-like theories on the lattice?

- We need *ab initio* theoretical predictions for QCD under the conditions probed in heavy-ion collisions
- Weak-coupling expansions in thermal QCD become accurate only at high temperatures, . . .
- . . . cannot capture the physics of long-wavelength modes **[A. Linde, 1980]** . . .
- . . . nor the dynamics in the hadronic phase
- The lattice regularization **[K. G. Wilson, 1974]** provides *the* only known mathematically well-defined, non-perturbative formulation of QCD
- Thanks to steady theoretical, algorithmic and computer-power progress, lattice QCD computations are now producing accurate numerical predictions
- Furthermore, the lattice investigation of QCD-like theories (possibly combined with different theoretical approaches: weak- or strong-coupling calculations, effective theories, phenomenological models, . . .) can provide analytical understanding of the physics, too

Why do we need to study QCD and QCD-like theories on the lattice?

- We need *ab initio* theoretical predictions for QCD under the conditions probed in heavy-ion collisions
- Weak-coupling expansions in thermal QCD become accurate only at high temperatures, ...
- ...cannot capture the physics of long-wavelength modes **[A. Linde, 1980]** ...
- ...nor the dynamics in the hadronic phase
- The lattice regularization **[K. G. Wilson, 1974]** provides *the* only known mathematically well-defined, non-perturbative formulation of QCD
- Thanks to steady theoretical, algorithmic and computer-power progress, lattice QCD computations are now producing accurate numerical predictions
- Furthermore, the lattice investigation of QCD-like theories (possibly combined with different theoretical approaches: weak- or strong-coupling calculations, effective theories, phenomenological models, ...) can provide analytical understanding of the physics, too

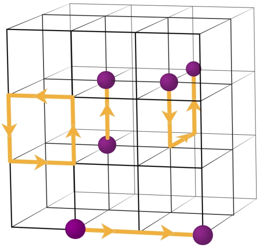
Outline

Basic ideas

Regularize the path integrals by discretizing the theory on a Euclidean lattice of spacing a **[K. G. Wilson, 1974]**

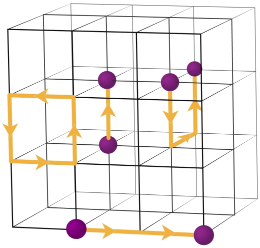
Basic ideas

Regularize the path integrals by discretizing the theory on a Euclidean lattice of spacing a **[K. G. Wilson, 1974]**



Basic ideas

Regularize the path integrals by discretizing the theory on a Euclidean lattice of spacing *a* **[K. G. Wilson, 1974]**

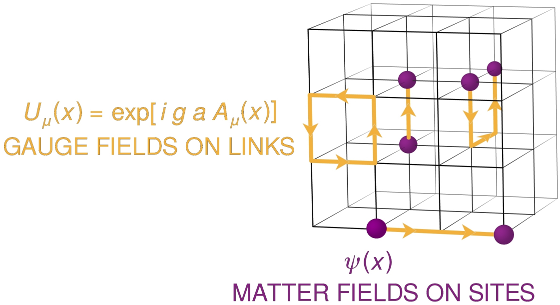


$\psi(x)$

MATTER FIELDS ON SITES

Basic ideas

Regularize the path integrals by discretizing the theory on a Euclidean lattice of spacing a **[K. G. Wilson, 1974]**



Basic ideas

Regularize the path integrals by discretizing the theory on a Euclidean lattice of spacing a **[K. G. Wilson, 1974]**

Define gauge and matter fields on lattice elements, build a gauge-invariant lattice action and observables

$$S = -\frac{1}{g^2} \sum_{\Box} \mathrm{Tr}(U_{\Box} + U_{\Box}^{\dagger}) + \sum_{x,y,f} a^4 \bar{\psi}_f(x) M^f_{x,y} \psi_f(y)$$

$$M^f_{x,y} = m\delta_{x,y} - \frac{1}{2a} \sum_{\mu} \left[(r - \gamma_\mu) U_\mu(x) \delta_{x+a\hat{\mu},y} + (r + \gamma_\mu) U^{\dagger}_{\mu}(y) \delta_{x-a\hat{\mu},y} \right]$$

Basic ideas

Regularize the path integrals by discretizing the theory on a Euclidean lattice of spacing a **[K. G. Wilson, 1974]**

Define gauge and matter fields on lattice elements, build a gauge-invariant lattice action and observables

Continuum action recovered for $a \to 0$

Basic ideas

Regularize the path integrals by discretizing the theory on a Euclidean lattice of spacing a **[K. G. Wilson, 1974]**

Define gauge and matter fields on lattice elements, build a gauge-invariant lattice action and observables

Continuum action recovered for $a \to 0$

A gauge-invariant, non-perturbative regularization

Basic ideas

Regularize the path integrals by discretizing the theory on a Euclidean lattice of spacing a **[K. G. Wilson, 1974]**

Define gauge and matter fields on lattice elements, build a gauge-invariant lattice action and observables

Continuum action recovered for $a \to 0$

A gauge-invariant, non-perturbative regularization

Suitable for numerical simulation: Sample configuration space according to a *statistical weight* proportional to $\exp(-S)$, compute expectation values

$$\langle \mathcal{O} \rangle = \frac{\int \prod \mathrm{d}\psi(x)\mathrm{d}\bar{\psi}(x) \prod \mathrm{d}U_\mu(x) \mathcal{O} \exp(-S)}{\int \prod \mathrm{d}\psi(x)\mathrm{d}\bar{\psi}(x) \prod \mathrm{d}U_\mu(x) \exp(-S)}$$

Basic ideas

Regularize the path integrals by discretizing the theory on a Euclidean lattice of spacing a **[K. G. Wilson, 1974]**

Define gauge and matter fields on lattice elements, build a gauge-invariant lattice action and observables

Continuum action recovered for $a \to 0$

A gauge-invariant, non-perturbative regularization

Suitable for numerical simulation: Sample configuration space according to a *statistical weight* proportional to $\exp(-S)$, compute expectation values

Observations:

1. Importance sampling made possible by *real positive* statistical weight
2. Natural interpretation as a *thermal* QFT at equilibrium

Basic ideas

Regularize the path integrals by discretizing the theory on a Euclidean lattice of spacing a **[K. G. Wilson, 1974]**

Define gauge and matter fields on lattice elements, build a gauge-invariant lattice action and observables

Continuum action recovered for $a \to 0$

A gauge-invariant, non-perturbative regularization

Suitable for numerical simulation: Sample configuration space according to a *statistical weight* proportional to $\exp(-S)$, compute expectation values

Observations:

1. Importance sampling made possible by *real positive* statistical weight
2. Natural interpretation as a *thermal* QFT at equilibrium

Basic ideas

Regularize the path integrals by discretizing the theory on a Euclidean lattice of spacing a **[K. G. Wilson, 1974]**

Define gauge and matter fields on lattice elements, build a gauge-invariant lattice action and observables

Continuum action recovered for $a \to 0$

A gauge-invariant, non-perturbative regularization

Suitable for numerical simulation: Sample configuration space according to a *statistical weight* proportional to $\exp(-S)$, compute expectation values

Observations:

1. Importance sampling made possible by *real positive* statistical weight
2. Natural interpretation as a *thermal* QFT at equilibrium

Basic ideas

Regularize the path integrals by discretizing the theory on a Euclidean lattice of spacing a **[K. G. Wilson, 1974]**

Define gauge and matter fields on lattice elements, build a gauge-invariant lattice action and observables

Continuum action recovered for $a \to 0$

A gauge-invariant, non-perturbative regularization

Suitable for numerical simulation: Sample configuration space according to a *statistical weight* proportional to $\exp(-S)$, compute expectation values

Observations:

1. Importance sampling made possible by *real positive* statistical weight
2. Natural interpretation as a *thermal* QFT at equilibrium

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"
- "The results depend on the *details* of your discretization"
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"
- "You always have undesired additional quark species (*doublers*)"
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"
- "The results depend on the *details* of your discretization"
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"
- "You always have undesired additional quark species (*doublers*)"
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
  - In the physical, large-volume and continuum limits, it is *the* mathematically rigorous non-perturbative definition of QCD
- "The results depend on the *details* of your discretization"
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"
- "You always have undesired additional quark species (*doublers*)"
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"
- "You always have undesired additional quark species (*doublers*)"
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"— *False*
  - The *intermediate* results do depend on the discretization details, those extrapolated to the *continuum limit* do not
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"
- "You always have undesired additional quark species (*doublers*)"
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"— *False*
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"
- "You always have undesired additional quark species (*doublers*)"
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"— *False*
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"— *False*
  - When the lattice spacing $a \to 0$ the continuum theory, with its full symmetries, emerges as *a good low-energy effective description* of the lattice model
- "You always have undesired additional quark species (*doublers*)"
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"— *False*
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"— *False*
- "You always have undesired additional quark species (*doublers*)"
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"— *False*
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"— *False*
- "You always have undesired additional quark species (*doublers*)"— *False*
  - They are easily removed e.g. by adding a Wilson term (or in more sophisticated ways)
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"— *False*
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"— *False*
- "You always have undesired additional quark species (*doublers*)"— *False*
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"— *False*
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"— *False*
- "You always have undesired additional quark species (*doublers*)"— *False*
- "It only works / it is only defined at *strong* coupling"— *False*
  - It is defined at any value of the coupling; the continuum limit $a \to 0$ is taken at *weak* coupling
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"— *False*
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"— *False*
- "You always have undesired additional quark species (*doublers*)"— *False*
- "It only works / it is only defined at *strong* coupling"— *False*
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"— *False*
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"— *False*
- "You always have undesired additional quark species (*doublers*)"— *False*
- "It only works / it is only defined at *strong* coupling"— *False*
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"— *False*
  - Moore's law and algorithmic progress came to the rescue: for standard lattice QCD computations, quenched calculations are now *obsolete*

Outline

Equilibrium properties

- Chiral symmetry restoration and deconfinement
- Equation of state in QCD (see also talk by C. Ratti on Friday at 11:00)

Equilibrium properties

- Chiral symmetry restoration and deconfinement
  1. Chiral symmetry restoration (at $\mu = 0$) is a *crossover* taking place at $T_c \simeq 155$ MeV **[S. Borsányi et al., 2010] [A. Bazavov et al., 2011]**
  2. Disconnected chiral susceptibilities of the form

$$\chi_{\mathrm{m}} = \int \mathrm{d}^4x \langle \mathrm{m}(x)\mathrm{m}(0)\rangle \qquad \text{with } \mathrm{m} \in \{i\bar{\psi}_f\tau^a\gamma_5\psi_f,\ \bar{\psi}_f\tau^a\psi_f,\ i\bar{\psi}_f\gamma_5\psi_f,\ \bar{\psi}_f\psi_f\}$$

     show that $\mathrm{SU_A}(n_f)$ is restored, $\mathrm{U_A}(1)$ remains broken **[M. I. Buchoff et al., 2013] [T. Bhattacharya et al., 2014]**
  3. Deconfinement occurs in the same temperature range
- Equation of state in QCD (see also talk by C. Ratti on Friday at 11:00)

Equilibrium properties

- Chiral symmetry restoration and deconfinement
  1. Chiral symmetry restoration (at $\mu = 0$) is a *crossover* taking place at $T_c \simeq 155$ MeV **[S. Borsányi et al., 2010] [A. Bazavov et al., 2011]**
  2. Disconnected chiral susceptibilities of the form

$$\chi_{\mathrm{m}} = \int \mathrm{d}^4 x \langle \mathrm{m}(x)\mathrm{m}(0)\rangle \qquad \text{with } \mathrm{m} \in \{ i\bar{\psi}_f \tau^a \gamma_5 \psi_f,\ \bar{\psi}_f \tau^a \psi_f,\ i\bar{\psi}_f \gamma_5 \psi_f,\ \bar{\psi}_f \psi_f \}$$

     show that $\mathrm{SU_A}(n_f)$ is restored, $\mathrm{U_A}(1)$ remains broken **[M. I. Buchoff et al., 2013] [T. Bhattacharya et al., 2014]**
  3. Deconfinement occurs in the same temperature range
- Equation of state in QCD (see also talk by C. Ratti on Friday at 11:00)

Equilibrium properties

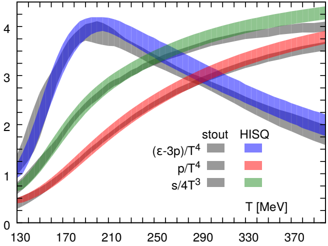
- Chiral symmetry restoration and deconfinement
  1. Chiral symmetry restoration (at $\mu = 0$) is a *crossover* taking place at $T_c \simeq 155$ MeV **[S. Borsányi et al., 2010] [A. Bazavov et al., 2011]**
  2. Disconnected chiral susceptibilities of the form

$$\chi_{\mathrm{m}} = \int \mathrm{d}^4 x \langle \mathrm{m}(x)\mathrm{m}(0)\rangle \qquad \text{with } \mathrm{m} \in \left\{ i\bar{\psi}_f \tau^a \gamma_5 \psi_f,\ \bar{\psi}_f \tau^a \psi_f,\ i\bar{\psi}_f \gamma_5 \psi_f,\ \bar{\psi}_f \psi_f \right\}$$

     show that $\mathrm{SU}_{\mathrm{A}}(n_f)$ is restored, $\mathrm{U}_{\mathrm{A}}(1)$ remains broken **[M. I. Buchoff et al., 2013] [T. Bhattacharya et al., 2014]**
  3. Deconfinement occurs in the same temperature range
- Equation of state in QCD (see also talk by C. Ratti on Friday at 11:00)

Equilibrium properties

- Chiral symmetry restoration and deconfinement
- Equation of state in QCD (see also talk by C. Ratti on Friday at 11:00)



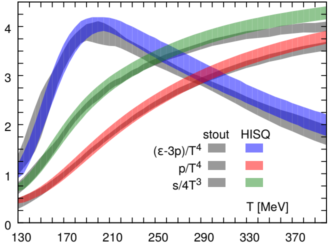
[R. A. Soltz et al., 2015]

See also **[A. Bazavov et al., 2012] [S. Borsányi et al., 2013]**

1. Confined phase: Consistency with hadron-resonance-gas model
2. Abrupt increase in thermodynamic potentials signals the "liberation" of colored degrees of freedom around $T \simeq 160$ MeV
3. Just above deconfinement: Significant deviations from conformality, interactions not quite "small"
4. Deconfined phase: Slow approach to hard-thermal-loop resummed perturbation theory predictions

Equilibrium properties

- Chiral symmetry restoration and deconfinement
- Equation of state in QCD (see also talk by C. Ratti on Friday at 11:00)



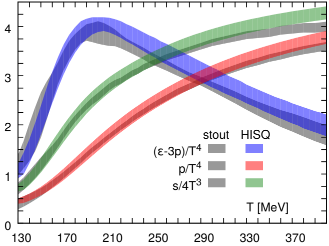
[R. A. Soltz et al., 2015]

See also **[A. Bazavov et al., 2012] [S. Borsányi et al., 2013]**

1. Confined phase: Consistency with hadron-resonance-gas model
2. Abrupt increase in thermodynamic potentials signals the "liberation" of colored degrees of freedom around $T \simeq 160$ MeV
3. Just above deconfinement: Significant deviations from conformality, interactions not quite "small"
4. Deconfined phase: Slow approach to hard-thermal-loop resummed perturbation theory predictions

Equilibrium properties

- Chiral symmetry restoration and deconfinement
- Equation of state in QCD (see also talk by C. Ratti on Friday at 11:00)



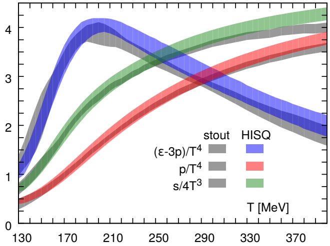
[R. A. Soltz et al., 2015]

See also **[A. Bazavov et al., 2012] [S. Borsányi et al., 2013]**

1. Confined phase: Consistency with hadron-resonance-gas model
2. Abrupt increase in thermodynamic potentials signals the "liberation" of colored degrees of freedom around $T \simeq 160$ MeV
3. Just above deconfinement: Significant deviations from conformality, interactions not quite "small"
4. Deconfined phase: Slow approach to hard-thermal-loop resummed perturbation theory predictions

Equilibrium properties

- Chiral symmetry restoration and deconfinement
- Equation of state in QCD (see also talk by C. Ratti on Friday at 11:00)



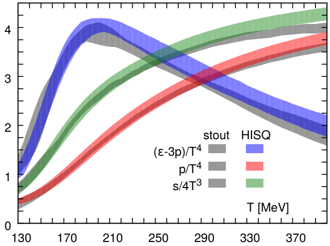
[R. A. Soltz et al., 2015]

See also **[A. Bazavov et al., 2012] [S. Borsányi et al., 2013]**

1. Confined phase: Consistency with hadron-resonance-gas model
2. Abrupt increase in thermodynamic potentials signals the "liberation" of colored degrees of freedom around $T \simeq 160$ MeV
3. Just above deconfinement: Significant deviations from conformality, interactions not quite "small"
4. Deconfined phase: Slow approach to hard-thermal-loop resummed perturbation theory predictions

Equilibrium properties

- Chiral symmetry restoration and deconfinement
- Equation of state in QCD (see also talk by C. Ratti on Friday at 11:00)



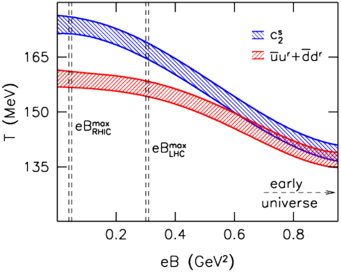
[R. A. Soltz et al., 2015]

See also **[A. Bazavov et al., 2012] [S. Borsányi et al., 2013]**

1. Confined phase: Consistency with hadron-resonance-gas model
2. Abrupt increase in thermodynamic potentials signals the "liberation" of colored degrees of freedom around $T \simeq 160$ MeV
3. Just above deconfinement: Significant deviations from conformality, interactions not quite "small"
4. Deconfined phase: Slow approach to hard-thermal-loop resummed perturbation theory predictions

Equilibrium properties

- Chiral symmetry restoration and deconfinement
- Equation of state in QCD (see also talk by C. Ratti on Friday at 11:00)
- Strong (electro)magnetic fields induce a slight decrease in T_c



[G. S. Bali et al., 2011]

Equilibrium properties

- Chiral symmetry restoration and deconfinement
- Equation of state in QCD (see also talk by C. Ratti on Friday at 11:00)
- Strong (electro)magnetic fields induce a slight decrease in T_c
- Freeze-out conditions from fluctuations of conserved charges (baryon number B, electric charge Q, strangeness S) **[F. Karsch, 2012]**

$$T_{\mathrm{fr}} = 144(10)\ \mathrm{MeV},\ \mu_{\mathrm{fr}}^{B} = 102(6)\ \mathrm{MeV}\ \text{at RHIC (STAR, } \sqrt{s} = 39\ \mathrm{GeV)}$$

[S. Borsányi et al., 2011] [A. Bazavov et al., 2012] [S. Borsányi et al., 2014]

Quarkonium melting

The sequential disappearance of more and more strongly bound quarkonium states was proposed long ago as a possible QGP "thermometer" **[T. Matsui and H. Satz, 1986]**

Quarkonium melting

The sequential disappearance of more and more strongly bound quarkonium states was proposed long ago as a possible QGP "thermometer" **[T. Matsui and H. Satz, 1986]**

General strategy of the lattice computation:

1. Heavy quarks can be treated non-relativistically (NRQCD)
2. Compute correlation functions of sources with desired quantum numbers

$$G_E(\tau) \simeq \int_{-2M}^{\infty} \frac{d\omega}{2\pi} \exp(-\omega\tau)\rho(\omega)$$

3. Invert to extract spectral function $\rho(\omega)$

Quarkonium melting

The sequential disappearance of more and more strongly bound quarkonium states was proposed long ago as a possible QGP "thermometer" **[T. Matsui and H. Satz, 1986]**

General strategy of the lattice computation:

1. Heavy quarks can be treated non-relativistically (NRQCD)
2. Compute correlation functions of sources with desired quantum numbers

$$G_E(\tau) \simeq \int_{-2M}^{\infty} \frac{d\omega}{2\pi} \exp(-\omega\tau)\rho(\omega)$$

3. Invert to extract spectral function $\rho(\omega)$

Quarkonium melting

The sequential disappearance of more and more strongly bound quarkonium states was proposed long ago as a possible QGP "thermometer" **[T. Matsui and H. Satz, 1986]**

General strategy of the lattice computation:

1. Heavy quarks can be treated non-relativistically (NRQCD)
2. Compute correlation functions of sources with desired quantum numbers

$$G_{\mathrm{E}}(\tau) \simeq \int_{-2M}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \exp(-\omega\tau)\rho(\omega)$$

3. Invert to extract spectral function $\rho(\omega)$

Quarkonium melting

The sequential disappearance of more and more strongly bound quarkonium states was proposed long ago as a possible QGP "thermometer" **[T. Matsui and H. Satz, 1986]**

General strategy of the lattice computation:

1. Heavy quarks can be treated non-relativistically (NRQCD)
2. Compute correlation functions of sources with desired quantum numbers

$$G_E(\tau) \simeq \int_{-2M}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \exp(-\omega\tau)\rho(\omega)$$

3. Invert to extract spectral function $\rho(\omega)$

Quarkonium melting

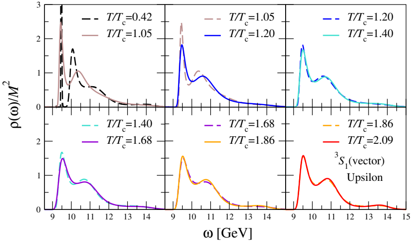
The sequential disappearance of more and more strongly bound quarkonium states was proposed long ago as a possible QGP "thermometer" **[T. Matsui and H. Satz, 1986]**

General strategy of the lattice computation:

1. Heavy quarks can be treated non-relativistically (NRQCD)
2. Compute correlation functions of sources with desired quantum numbers

$$G_{\mathrm{E}}(\tau) \simeq \int_{-2M}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \exp(-\omega\tau)\rho(\omega)$$

3. Invert to extract spectral function $\rho(\omega)$



Bottomonium excitation melting **[G. Aarts et al., 2011]**

Transport coefficients

Describe QGP response to long-wavelength / low-frequency perturbations in energy and momentum density and other conserved charges **[H. B. Meyer, 2011]**

Transport coefficients

Describe QGP response to long-wavelength / low-frequency perturbations in energy and momentum density and other conserved charges **[H. B. Meyer, 2011]**

Example: Shear (η) and bulk (ζ) viscosities

$$T^{\mu\nu} = (\epsilon + p)u^{\mu}u^{\nu} + pg^{\mu\nu} - \mathbb{P}^{\mu i}\mathbb{P}^{\nu j}\left[\eta\left(\partial_i u_j + \partial_j u_i - \frac{2}{3}g_{ij}\partial_k u^k\right) + \zeta g_{ij}\partial_k u^k\right]$$

Transport coefficients

Describe QGP response to long-wavelength / low-frequency perturbations in energy and momentum density and other conserved charges **[H. B. Meyer, 2011]**

Difficult to access on a Euclidean lattice ⇒ Indirectly reconstructed from Kubo formulæ

Transport coefficients

Describe QGP response to long-wavelength / low-frequency perturbations in energy and momentum density and other conserved charges **[H. B. Meyer, 2011]**

Difficult to access on a Euclidean lattice ⇒ Indirectly reconstructed from Kubo formulæ

Example: Shear viscosity

$$\eta = \pi \lim_{\omega \to 0} \lim_{\mathbf{k} \to 0} \frac{\rho(\omega, \mathbf{k})}{\omega}$$

with ρ the spectral function, related to a suitable (e.g. $T^{\mu\nu}$) Euclidean correlator via

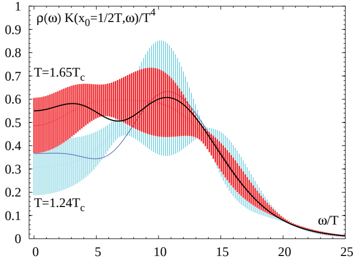
$$G_{\mathrm{E}}(t, \mathbf{k}) = \int_0^\infty \mathrm{d}\omega \, \rho(\omega, \mathbf{k}) \, \frac{\cosh\left[\omega\left(t - \frac{1}{2T}\right)\right]}{\sinh\left(\frac{\omega}{2T}\right)}$$

Transport coefficients

Describe QGP response to long-wavelength / low-frequency perturbations in energy and momentum density and other conserved charges **[H. B. Meyer, 2011]**

Difficult to access on a Euclidean lattice ⇒ Indirectly reconstructed from Kubo formulæ

Typically, numerically *very* challenging



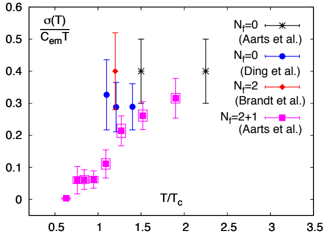
[H. B. Meyer, 2007]

Transport coefficients

Describe QGP response to long-wavelength / low-frequency perturbations in energy and momentum density and other conserved charges **[H. B. Meyer, 2011]**

Difficult to access on a Euclidean lattice ⇒ Indirectly reconstructed from Kubo formulæ

Another example: Electric conductivity



[H.-T. Ding, F. Karsch and S. Mukherjee, 2015], with results from **[G. Aarts et al., 2007] [H.-T. Ding et al., 2010] [B. B. Brandt et al., 2012] [G. Aarts et al., 2014]**

Outline

Equation of state for different gauge groups

Insight from the comparison of different $\mathrm{SU}(N)$ gauge groups

Equation of state for different gauge groups

Insight from the comparison of different $\mathrm{SU}(N)$ gauge groups—particularly in the large-N limit

Equation of state for different gauge groups

Insight from the comparison of different $\mathrm{SU}(N)$ gauge groups—particularly in the large-N limit

Why QCD at large N?

- The large-N limit of QCD (at fixed $\lambda = g^2 N$) has interesting phenomenological implications **[G. 't Hooft, 1974]**
- It plays a crucial rôle in the holographic gauge/string duality **[J. Maldacena, 1998]**
- Important applications at finite temperature **[J. Casalderrey-Solana et al., 2014]** (see also talk by W. van der Schee on Thursday at 11:30)
- What can the lattice say? **[M. P., 2012]** **[B. Lucini and M. P., 2012]** **[B. Lucini and M. P., 2013]**

Equation of state for different gauge groups

Insight from the comparison of different $\mathrm{SU}(N)$ gauge groups—particularly in the large-N limit

Why QCD at large N?

- The large-N limit of QCD (at fixed $\lambda = g^2 N$) has interesting phenomenological implications **[G. 't Hooft, 1974]**
- It plays a crucial rôle in the holographic gauge/string duality **[J. Maldacena, 1998]**
- Important applications at finite temperature **[J. Casalderrey-Solana et al., 2014]** (see also talk by W. van der Schee on Thursday at 11:30)
- What can the lattice say? **[M. P., 2012] [B. Lucini and M. P., 2012] [B. Lucini and M. P., 2013]**

Equation of state for different gauge groups

Insight from the comparison of different $\mathrm{SU}(N)$ gauge groups—particularly in the large-N limit

Why QCD at large N?

The large-N limit of QCD (at fixed $\lambda = g^2 N$) has interesting phenomenological implications **[G. 't Hooft, 1974]**

It plays a crucial rôle in the holographic gauge/string duality **[J. Maldacena, 1998]**

$$\lambda = \frac{R^4}{l_s^4} \qquad \frac{\lambda}{N} = 4\pi g_s$$

Important applications at finite temperature **[J. Casalderrey-Solana et al., 2014]** (see also talk by W. van der Schee on Thursday at 11:30)

What can the lattice say? **[M. P., 2012] [B. Lucini and M. P., 2012] [B. Lucini and M. P., 2013]**

Equation of state for different gauge groups

Insight from the comparison of different $\mathrm{SU}(N)$ gauge groups—particularly in the large-N limit

Why QCD at large N?

- The large-N limit of QCD (at fixed $\lambda = g^2 N$) has interesting phenomenological implications **[G. 't Hooft, 1974]**
- It plays a crucial rôle in the holographic gauge/string duality **[J. Maldacena, 1998]**: *classical* string limit
- Important applications at finite temperature **[J. Casalderrey-Solana et al., 2014]** (see also talk by W. van der Schee on Thursday at 11:30)
- What can the lattice say? **[M. P., 2012] [B. Lucini and M. P., 2012] [B. Lucini and M. P., 2013]**

Equation of state for different gauge groups

Insight from the comparison of different $\mathrm{SU}(N)$ gauge groups—particularly in the large-N limit

Why QCD at large N?

The large-N limit of QCD (at fixed $\lambda = g^2 N$) has interesting phenomenological implications **[G. 't Hooft, 1974]**

It plays a crucial rôle in the holographic gauge/string duality **[J. Maldacena, 1998]**: *classical* string limit

Important applications at finite temperature **[J. Casalderrey-Solana et al., 2014]** (see also talk by W. van der Schee on Thursday at 11:30)

What can the lattice say? **[M. P., 2012] [B. Lucini and M. P., 2012] [B. Lucini and M. P., 2013]**

Equation of state for different gauge groups

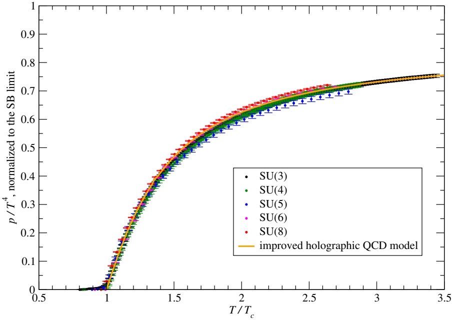
Insight from the comparison of different $\mathrm{SU}(N)$ gauge groups—particularly in the large-N limit

Why QCD at large N?

- The large-N limit of QCD (at fixed $\lambda = g^2 N$) has interesting phenomenological implications **[G. 't Hooft, 1974]**
- It plays a crucial rôle in the holographic gauge/string duality **[J. Maldacena, 1998]**: *classical* string limit
- Important applications at finite temperature **[J. Casalderrey-Solana et al., 2014]** (see also talk by W. van der Schee on Thursday at 11:30)
- What can the lattice say? **[M. P., 2012] [B. Lucini and M. P., 2012] [B. Lucini and M. P., 2013]**

Equation of state for different gauge groups

Insight from the comparison of different $\mathrm{SU}(N)$ gauge groups—particularly in the large-N limit

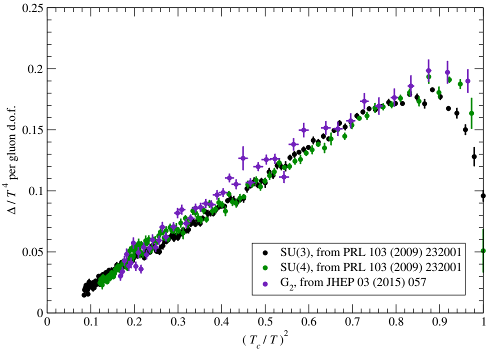


[M. P., 2009], with a comparison with the holographic model from **[U. Gürsoy et al., 2008]**

See also **[B. Lucini, M. Teper and U. Wenger, 2003]** **[B. Bringoltz and M. Teper, 2005]** **[S. Datta and S. Gupta, 2010]** **[A. Mykkänen, K. Rummukainen and M. P., 2012]**

Equation of state for different gauge groups

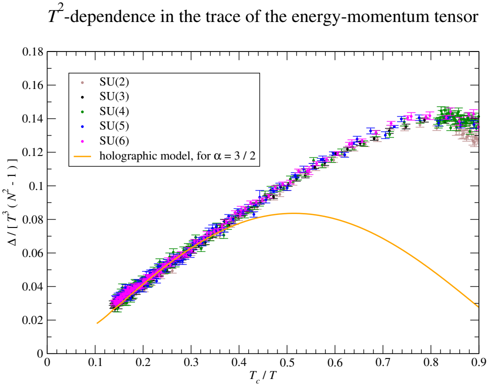
Comparison of different gauge groups: $\mathrm{SU}(N)$ *versus* G_2 **[M. Bruno et al., 2014]**



See also **[M. Pepe and U.-J. Wiese, 2006] [G. Cossu et al., 2007] [C. Bonati, 2015]**

Equation of state for different gauge groups

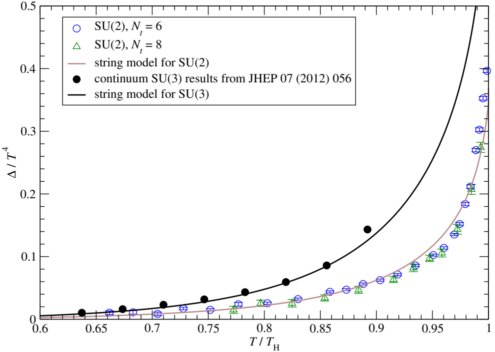
Insight from the comparison of different $\mathrm{SU}(N)$ gauge groups—particularly in the large-N limit



Analogous results in $D = 2 + 1$ spacetime dimensions **[M. Caselle et al., 2011]**

Equation of state for different gauge groups

Insight from the comparison of different $\mathrm{SU}(N)$ gauge groups—particularly in the large-N limit



Hadron resonance gas and a string model for glueballs **[M. Caselle et al., 2015]**

See also **[H. B. Meyer, 2009] [M. Caselle et al., 2011] [S. Borsányi et al., 2012]** and talk by H. Stöcker on Monday at 11:00

Transport and real-time phenomena from effective field theories

The practical (or fundamental) challenges in the direct lattice QCD study of certain transport coefficients can be bypassed combining lattice simulations with effective field theories

Transport and real-time phenomena from effective field theories

The practical (or fundamental) challenges in the direct lattice QCD study of certain transport coefficients can be bypassed combining lattice simulations with effective field theories

Some examples:

1.
2. The momentum broadening of a hard quark moving through the QGP is described by the jet quenching parameter

Transport and real-time phenomena from effective field theories

The practical (or fundamental) challenges in the direct lattice QCD study of certain transport coefficients can be bypassed combining lattice simulations with effective field theories

Some examples:

1. Using heavy-quark effective theory, the heavy quark momentum diffusion coefficient κ can be extracted from the spectral function $\rho_{\mathrm{E}}(\omega)$ associated with the correlator of "chromoelectric field insertions" onto a Polyakov loop **[S. Caron-Huot, M. Laine and G. D. Moore, 2009]**

$$\kappa = 2T \lim_{\omega \to 0} \frac{\rho_{\mathrm{E}}(\omega)}{\omega}$$

2. The momentum broadening of a hard quark moving through the QGP is described by the jet quenching parameter

Transport and real-time phenomena from effective field theories

The practical (or fundamental) challenges in the direct lattice QCD study of certain transport coefficients can be bypassed combining lattice simulations with effective field theories

Some examples:

1. Using heavy-quark effective theory, the heavy quark momentum diffusion coefficient κ can be extracted from the spectral function $\rho_{\mathrm{E}}(\omega)$ associated with the correlator of "chromoelectric field insertions" onto a Polyakov loop **[S. Caron-Huot, M. Laine and G. D. Moore, 2009]**

$$\kappa = 2T \lim_{\omega \to 0} \frac{\rho_{\mathrm{E}}(\omega)}{\omega}$$

   Results indicate $\kappa/T^3 \simeq 2.5$ for $T_c \lesssim T \lesssim 2T_c$ **[D. Banerjee et al., 2011]**

2. The momentum broadening of a hard quark moving through the QGP is described by the jet quenching parameter

Transport and real-time phenomena from effective field theories

The practical (or fundamental) challenges in the direct lattice QCD study of certain transport coefficients can be bypassed combining lattice simulations with effective field theories

Some examples:

1. Heavy quark momentum diffusion coefficient κ from heavy-quark effective field theory
2. The momentum broadening of a hard quark moving through the QGP is described by the jet quenching parameter

$$\hat{q} = \frac{\langle p_\perp^2 \rangle}{L} = \int \frac{\mathrm{d}^2 p_\perp}{(2\pi)^2} p_\perp^2 C(p_\perp)$$

Transport and real-time phenomena from effective field theories

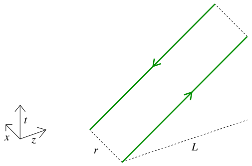
The practical (or fundamental) challenges in the direct lattice QCD study of certain transport coefficients can be bypassed combining lattice simulations with effective field theories

Some examples:

1. Heavy quark momentum diffusion coefficient κ from heavy-quark effective field theory
2. The momentum broadening of a hard quark moving through the QGP is described by the jet quenching parameter

$$\hat{q} = \frac{\langle p_\perp^2 \rangle}{L} = \int \frac{\mathrm{d}^2 p_\perp}{(2\pi)^2} p_\perp^2 C(p_\perp)$$

$C(p_\perp)$, the differential parton-plasma constituents collision rate, is related to two-point correlator of *light-cone Wilson lines*

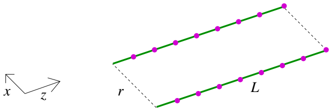


Transport and real-time phenomena from effective field theories

The practical (or fundamental) challenges in the direct lattice QCD study of certain transport coefficients can be bypassed combining lattice simulations with effective field theories

Some examples:

1. Heavy quark momentum diffusion coefficient κ from heavy-quark effective field theory
2. The momentum broadening of a hard quark moving through the QGP is described by the jet quenching parameter. Non-perturbative *soft* contributions to $\hat{q}$ can be extracted from lattice simulations of a dimensionally reduced effective theory **[S. Caron-Huot, 2009] [M. Laine, 2012] [J. Ghiglieri et al., 2013]** (see also **[B. Brandt et al., 2014]**)



Transport and real-time phenomena from effective field theories

The practical (or fundamental) challenges in the direct lattice QCD study of certain transport coefficients can be bypassed combining lattice simulations with effective field theories

Some examples:

1. Heavy quark momentum diffusion coefficient κ from heavy-quark effective field theory
2. The momentum broadening of a hard quark moving through the QGP is described by the jet quenching parameter. Non-perturbative *soft* contributions to $\hat{q}$ can be extracted from lattice simulations of a dimensionally reduced effective theory **[S. Caron-Huot, 2009] [M. Laine, 2012] [J. Ghiglieri et al., 2013]** (see also **[B. Brandt et al., 2014]**). Evidence for rather large non-perturbative effects: $\hat{q} \simeq 6$ GeV^2/fm at RHIC **[M. P., K. Rummukainen and A. Schäfer, 2014]**

The numerical sign problem in QCD at large density

At finite quark chemical potential μ, the determinant of the Dirac operator in lattice QCD is generally *complex*

The numerical sign problem in QCD at large density

At finite quark chemical potential μ, the determinant of the Dirac operator in lattice QCD is generally *complex*: importance sampling at large densities fails

The numerical sign problem in QCD at large density

At finite quark chemical potential μ, the determinant of the Dirac operator in lattice QCD is generally *complex*: importance sampling at large densities fails

Traditional workarounds include Taylor expansions around $\mu = 0$ (see talk by C. Schmidt today at 11:30), analytical continuations form imaginary μ, . . .

The numerical sign problem in QCD at large density

At finite quark chemical potential μ, the determinant of the Dirac operator in lattice QCD is generally *complex*: importance sampling at large densities fails

Traditional workarounds include Taylor expansions around $\mu = 0$ (see talk by C. Schmidt today at 11:30), analytical continuations form imaginary μ, . . . but, typically, are reliable only for $\mu \lesssim T$

The numerical sign problem in QCD at large density

At finite quark chemical potential μ, the determinant of the Dirac operator in lattice QCD is generally *complex*: importance sampling at large densities fails

Traditional workarounds include Taylor expansions around $\mu = 0$ (see talk by C. Schmidt today at 11:30), analytical continuations form imaginary μ, ... but, typically, are reliable only for $\mu \lesssim T$

Additional qualitative insight can be obtained from QCD-like theories which are free from the sign problem:

1. $\mathrm{SU}(2)$-QCD **[S. Hands et al., 1999]**
2. G_2-QCD **[A. Maas et al., 2012]**
3. Abelian theories admitting an exact *dual* formulation **[Y. Delgado Mercado, C. Gattringer and A. Schmidt, 2013]** (see also **[M. P., 2005]**)
4. $\mathrm{SO}(N)$-QCD: orbifold mapping to ordinary QCD at large N **[P. Kovtun, M. Ünsal and L. G. Yaffe, 2004] [A. Cherman, M. Hanada and D. Robles-Llana, 2010]**

The numerical sign problem in QCD at large density

At finite quark chemical potential μ, the determinant of the Dirac operator in lattice QCD is generally *complex*: importance sampling at large densities fails

Traditional workarounds include Taylor expansions around $\mu = 0$ (see talk by C. Schmidt today at 11:30), analytical continuations form imaginary μ, ...but, typically, are reliable only for $\mu \lesssim T$

Additional qualitative insight can be obtained from QCD-like theories which are free from the sign problem:

1. $\mathrm{SU}(2)$-QCD **[S. Hands et al., 1999]**
2. G_2-QCD **[A. Maas et al., 2012]**
3. Abelian theories admitting an exact *dual* formulation **[Y. Delgado Mercado, C. Gattringer and A. Schmidt, 2013]** (see also **[M. P., 2005]**)
4. $\mathrm{SO}(N)$-QCD: orbifold mapping to ordinary QCD at large N **[P. Kovtun, M. Ünsal and L. G. Yaffe, 2004] [A. Cherman, M. Hanada and D. Robles-Llana, 2010]**

The numerical sign problem in QCD at large density

At finite quark chemical potential μ, the determinant of the Dirac operator in lattice QCD is generally *complex*: importance sampling at large densities fails

Traditional workarounds include Taylor expansions around $\mu = 0$ (see talk by C. Schmidt today at 11:30), analytical continuations form imaginary μ, . . . but, typically, are reliable only for $\mu \lesssim T$

Additional qualitative insight can be obtained from QCD-like theories which are free from the sign problem:

1. $\mathrm{SU}(2)$-QCD **[S. Hands et al., 1999]**
2. G_2-QCD **[A. Maas et al., 2012]**
3. Abelian theories admitting an exact *dual* formulation **[Y. Delgado Mercado, C. Gattringer and A. Schmidt, 2013]** (see also **[M. P., 2005]**)
4. $\mathrm{SO}(N)$-QCD: orbifold mapping to ordinary QCD at large N **[P. Kovtun, M. Ünsal and L. G. Yaffe, 2004] [A. Cherman, M. Hanada and D. Robles-Llana, 2010]**

The numerical sign problem in QCD at large density

At finite quark chemical potential μ, the determinant of the Dirac operator in lattice QCD is generally *complex*: importance sampling at large densities fails

Traditional workarounds include Taylor expansions around $\mu = 0$ (see talk by C. Schmidt today at 11:30), analytical continuations form imaginary μ, ... but, typically, are reliable only for $\mu \lesssim T$

Additional qualitative insight can be obtained from QCD-like theories which are free from the sign problem:

1. $\mathrm{SU}(2)$-QCD **[S. Hands et al., 1999]**
2. G_2-QCD **[A. Maas et al., 2012]**
3. Abelian theories admitting an exact *dual* formulation **[Y. Delgado Mercado, C. Gattringer and A. Schmidt, 2013]** (see also **[M. P., 2005]**)
4. $\mathrm{SO}(N)$-QCD: orbifold mapping to ordinary QCD at large N **[P. Kovtun, M. Ünsal and L. G. Yaffe, 2004] [A. Cherman, M. Hanada and D. Robles-Llana, 2010]**

The numerical sign problem in QCD at large density

At finite quark chemical potential μ, the determinant of the Dirac operator in lattice QCD is generally *complex*: importance sampling at large densities fails

Traditional workarounds include Taylor expansions around $\mu = 0$ (see talk by C. Schmidt today at 11:30), analytical continuations form imaginary μ, ... but, typically, are reliable only for $\mu \lesssim T$

Additional qualitative insight can be obtained from QCD-like theories which are free from the sign problem:

1. $\mathrm{SU}(2)$-QCD **[S. Hands et al., 1999]**
2. G_2-QCD **[A. Maas et al., 2012]**
3. Abelian theories admitting an exact *dual* formulation **[Y. Delgado Mercado, C. Gattringer and A. Schmidt, 2013]** (see also **[M. P., 2005]**)
4. $\mathrm{SO}(N)$-QCD: orbifold mapping to ordinary QCD at large N **[P. Kovtun, M. Ünsal and L. G. Yaffe, 2004] [A. Cherman, M. Hanada and D. Robles-Llana, 2010]**

Concluding remarks

- Lattice calculations are providing increasingly accurate first-principle predictions for many physical quantities relevant for QCD under extreme conditions
- The lattice determination of equilibrium properties is by now settled, and recent works are addressing quantities related to fluctuations, transport, real-time dynamics, et c.
- Lattice simulations of QCD-like theories can shed light onto a number of open theoretical problems

Concluding remarks

- Lattice calculations are providing increasingly accurate first-principle predictions for many physical quantities relevant for QCD under extreme conditions
- The lattice determination of equilibrium properties is by now settled, and recent works are addressing quantities related to fluctuations, transport, real-time dynamics, et c.
- Lattice simulations of QCD-like theories can shed light onto a number of open theoretical problems

Concluding remarks

- Lattice calculations are providing increasingly accurate first-principle predictions for many physical quantities relevant for QCD under extreme conditions
- The lattice determination of equilibrium properties is by now settled, and recent works are addressing quantities related to fluctuations, transport, real-time dynamics, et c.
- Lattice simulations of QCD-like theories can shed light onto a number of open theoretical problems

Concluding remarks

- Lattice calculations are providing increasingly accurate first-principle predictions for many physical quantities relevant for QCD under extreme conditions
- The lattice determination of equilibrium properties is by now settled, and recent works are addressing quantities related to fluctuations, transport, real-time dynamics, et c.
- Lattice simulations of QCD-like theories can shed light onto a number of open theoretical problems

Thanks for your attention!