The 2nd Conference on Heavy Ion Collisions
in the LHC Era and Beyond

E-by-E description of jet energy loss with **MUSIC**, **MARTINI** and **UrQMD**

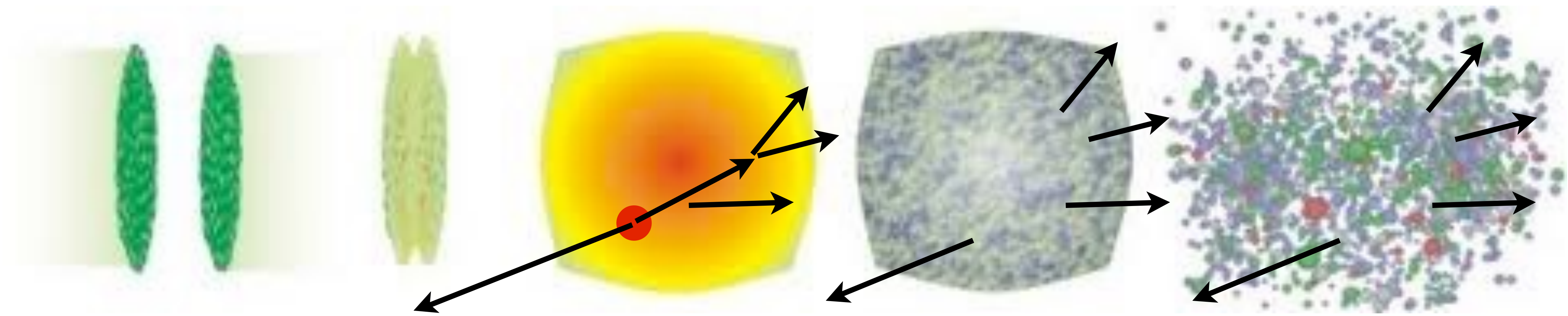
Sangwook Ryu

In collaboration with
Jean-Francois Paquet, Gabriel Denicol, Bjoern Schenke, Chun Shen, Sangyong Jeon and Charles Gale

McGill

Introduction & Motivation

Heavy Ion Collision at a Glance



Why heavy ion collision?

Quark gluon plasma can form in heavy ion collisions

Good place to study
the nature of QCD matter with high temperature

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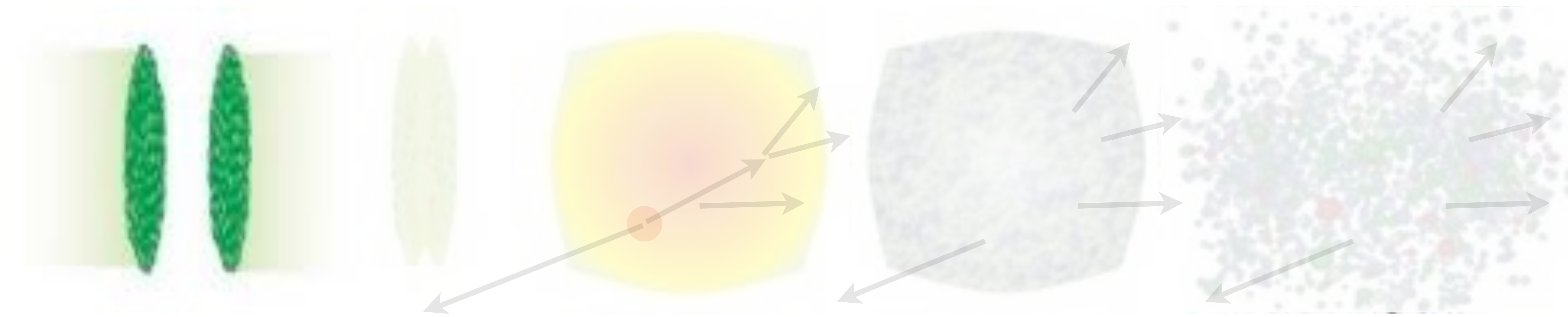


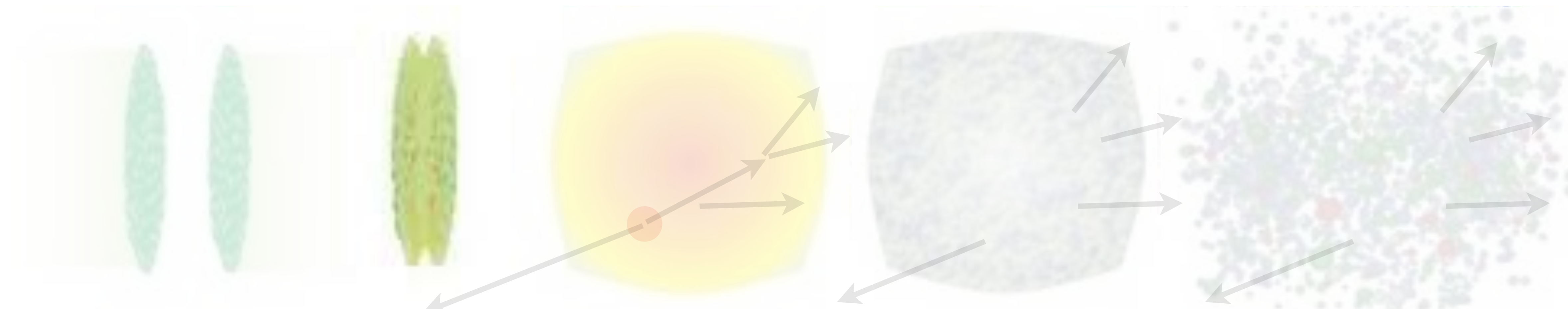
Each stage is governed by different physics

* sub-nucleonic fluctuations
* viscous hydrodynamics
* jet-medium interactions
* hadronic re-scattering in the late stage

We attempt for the first time to develop an event generator with all those features

Introduction & Motivation

Heavy Ion Collision at a Glance



Relevant initial partonic processes

Soft processes

Small-x partons and low momentum transfer

These thermalize into sQGP

Hard processes

Large-x partons and high momentum transfer

These produce high-p_T jets

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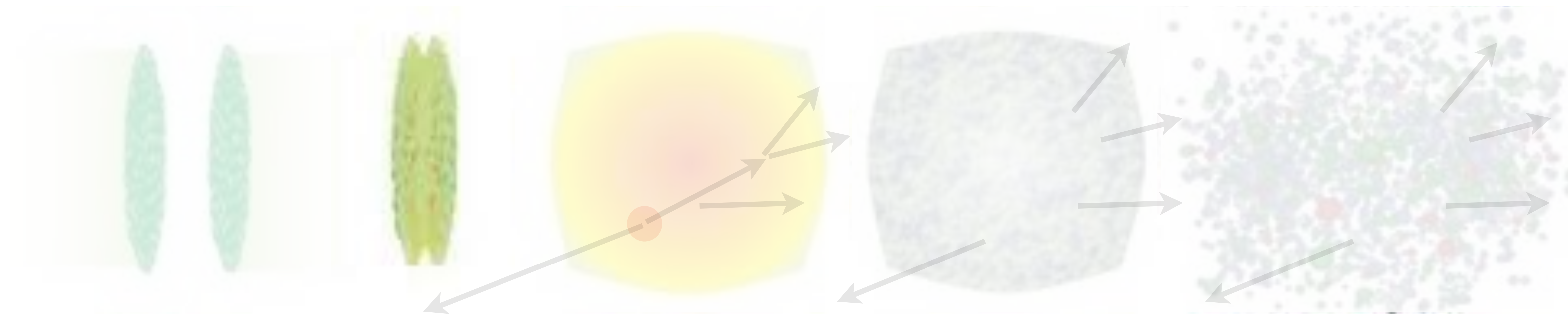
Heavy Ion Collision at a Glance



Relevant initial partonic processes

Soft processes

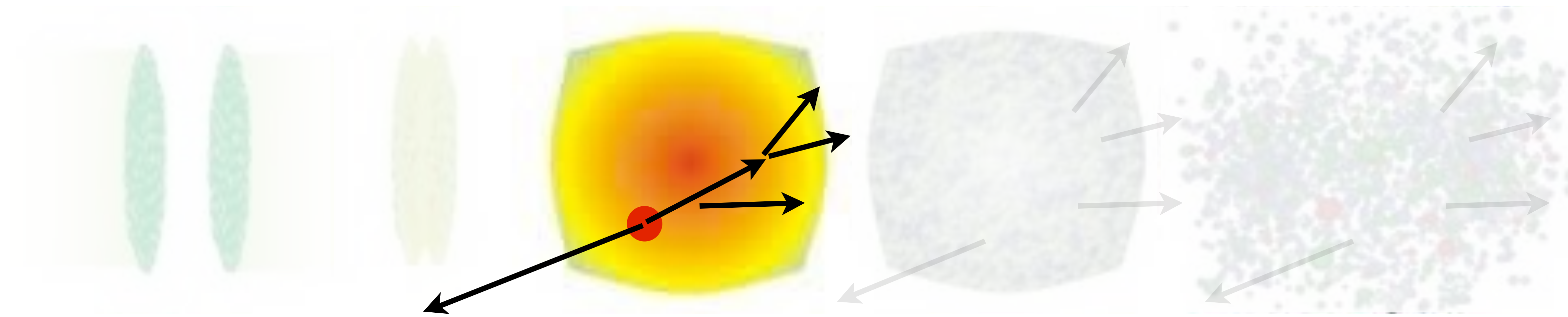
Provide initial condition for the sQGP evolution

Hard processes

Provide additional probe (jet-medium interactions)

Introduction & Motivation

Heavy Ion Collision at a Glance



Evolution of QGP

Hydrodynamic expansion of the sQGP matter

Viscosity is important

Energy loss of the high-p_T jet partons

Energy loss through elastic and inelastic processes

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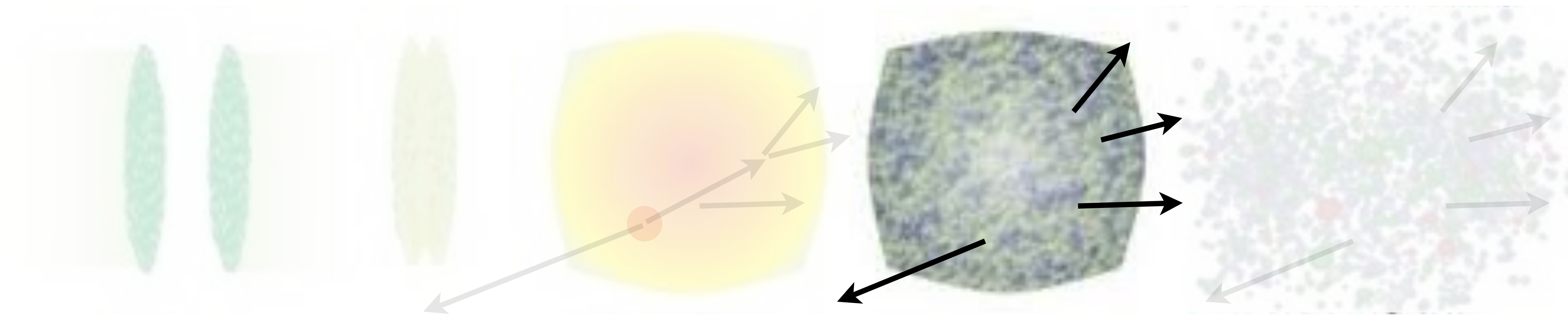


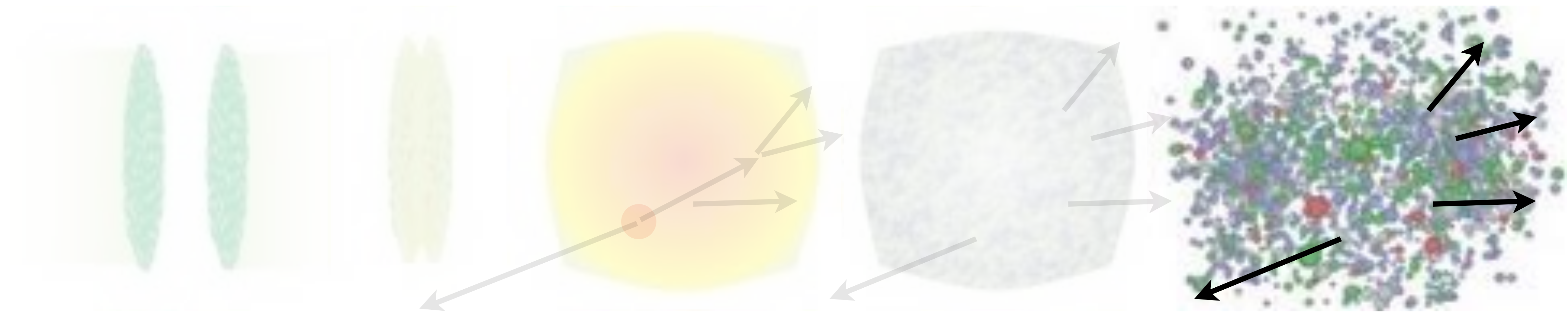
Switching to hadronic degrees of freedom

Phase transition to hadronic gas

- As the system expands, energy density drops
- Quarks and gluons combine to form hadrons
- Equation of state has the information on the transition

Introduction & Motivation

Heavy Ion Collision at a Glance



Hadronic rescattering

Switching to transport theory

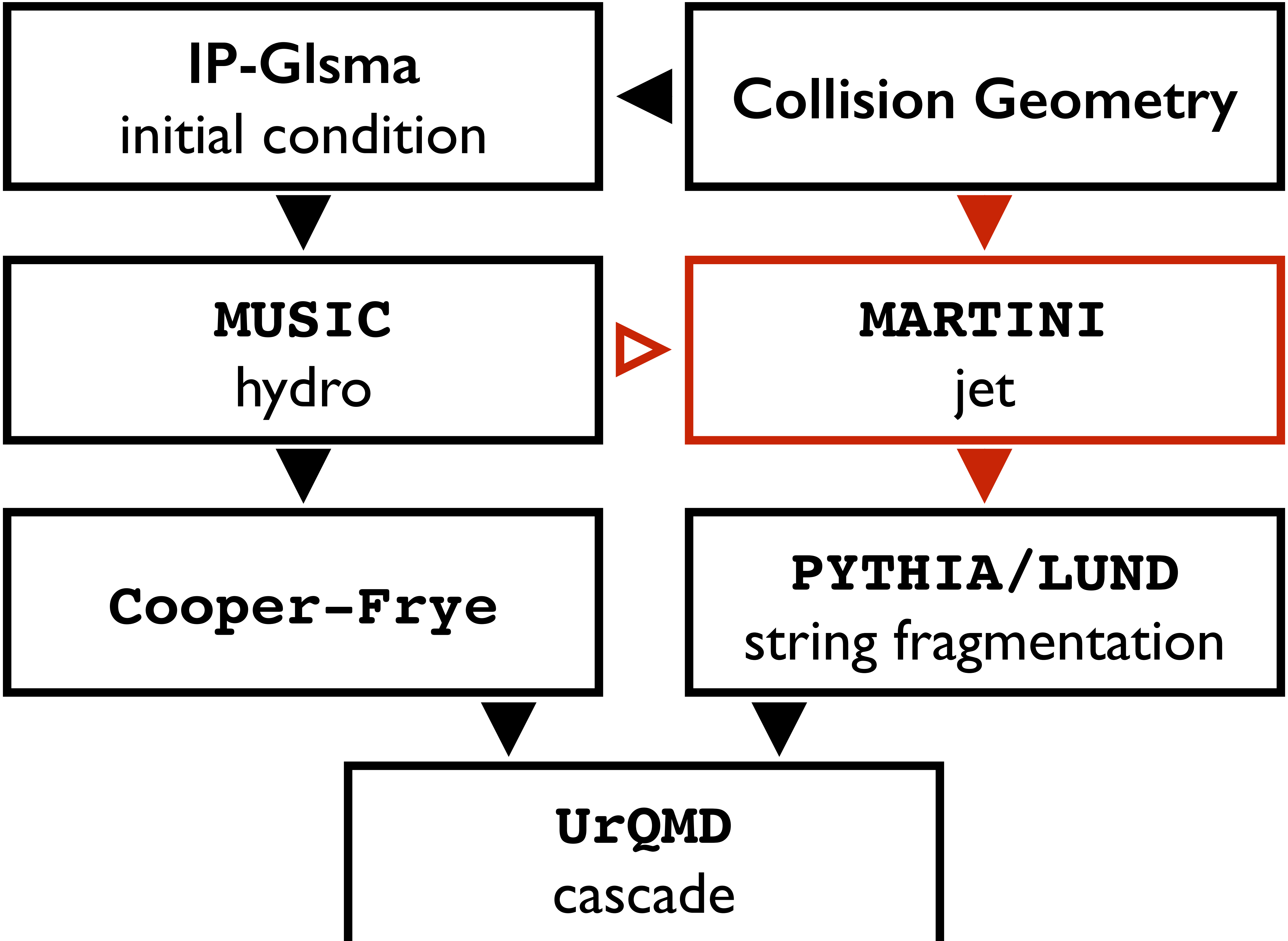
The mean free path increases

$$\lambda_{\mathrm{mfp}} \sim L_{\mathrm{macro}} \sim L_{\mathrm{size}}$$

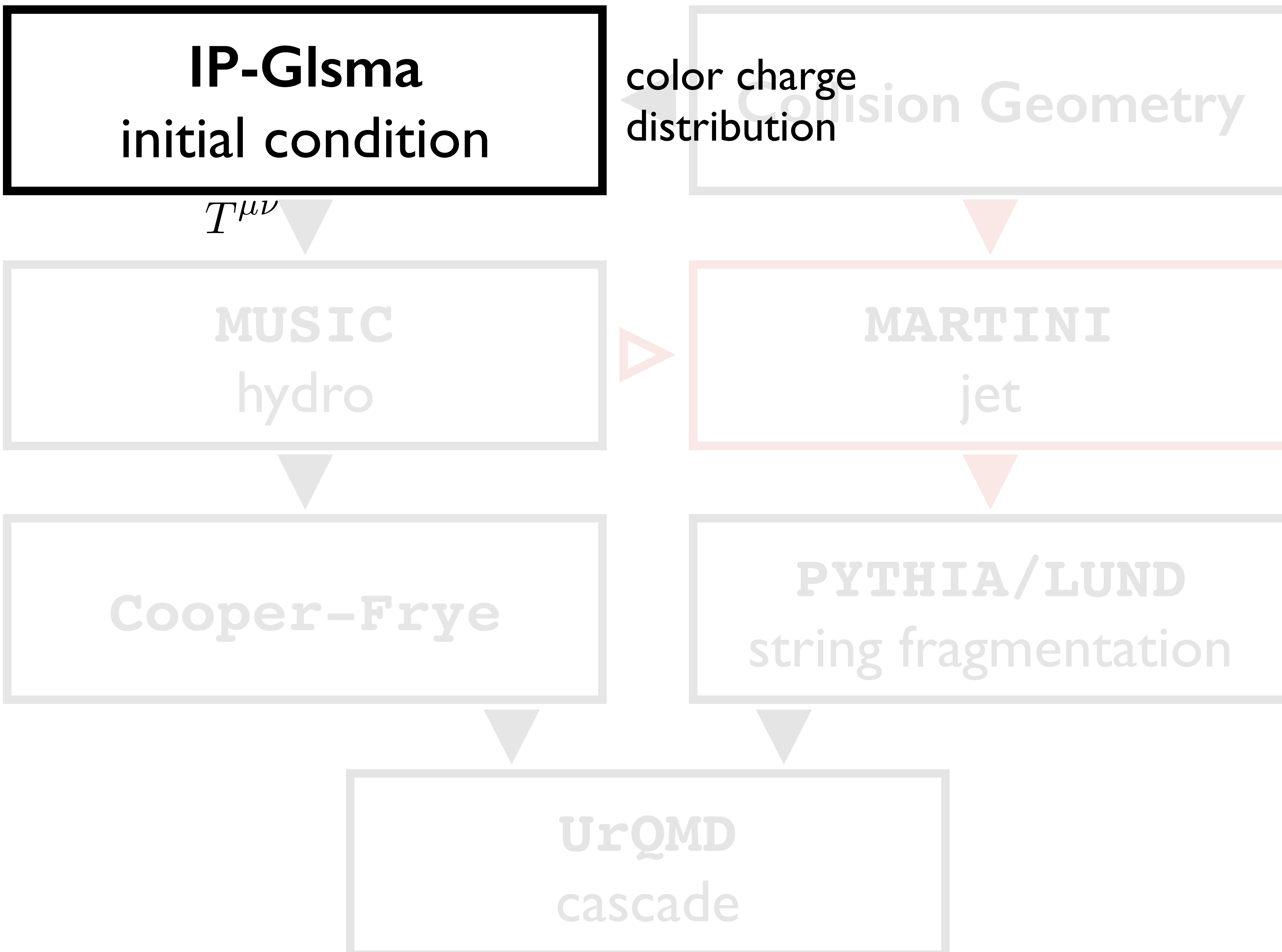
Transport approach is necessary

Important in describing jet energy loss in hadronic phase

Model Structure



Model Structure



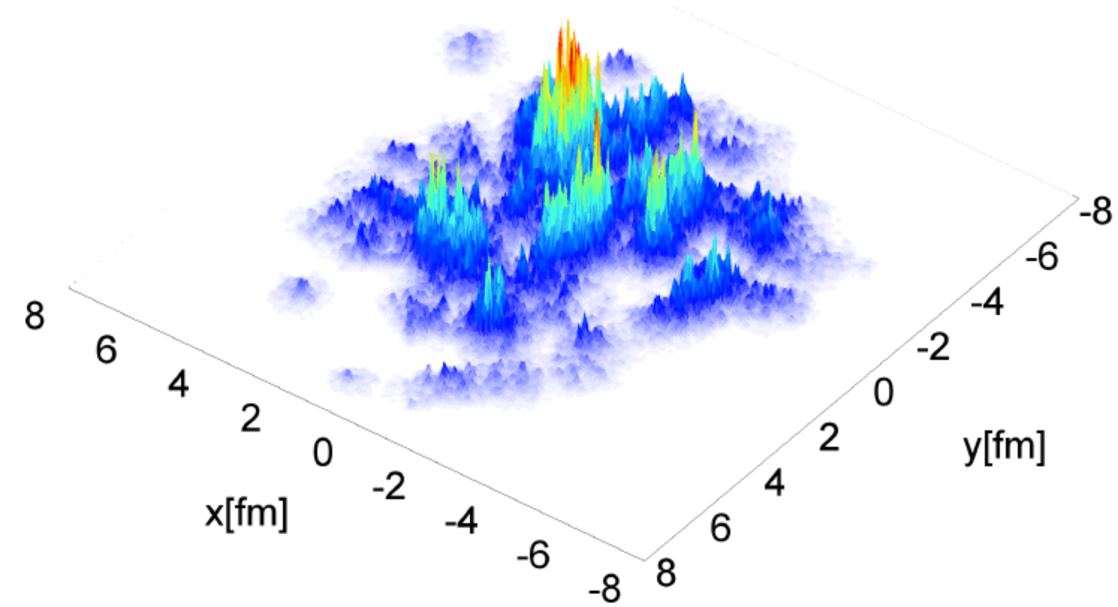
Model : IP-Glasma I.C.

B. Schenke, P. Tribedy and R. Venugopalan (2012)

Classical YM dynamics with color sources in nuclei

color charge distribution

$$\langle \rho^a(\mathbf{x}'_T)\, \rho^a(\mathbf{x}''_T) \rangle$$
$$= g^2 \mu_A^2 \delta^{ab} \delta^2(\mathbf{x}'_T - \mathbf{x}''_T)$$



▼ gluon field from each nucleus

$$A^i_{(1,2)}(\mathbf{x}_T)$$
$$= \frac{i}{g} U_{(1,2)}(\mathbf{x}_T)\, \partial_i U^\dagger_{(1,2)}(\mathbf{x}_T)$$

$$U_{(1,2)}(\mathbf{x}_T) = \mathcal{P} \exp\left[-ig \int dx^{\pm} \frac{\rho_{(1,2)}(\mathbf{x}_T, x^{\pm})}{\nabla_T^2 - m^2} \right]$$

▼ initial gluon field after collision

$$A^i(\tau = +0) = A^i_{(1)} + A^i_{(2)}$$
$$A^\eta(\tau = +0) = \frac{ig}{2} [A^i_{(1)}, A^i_{(2)}]$$

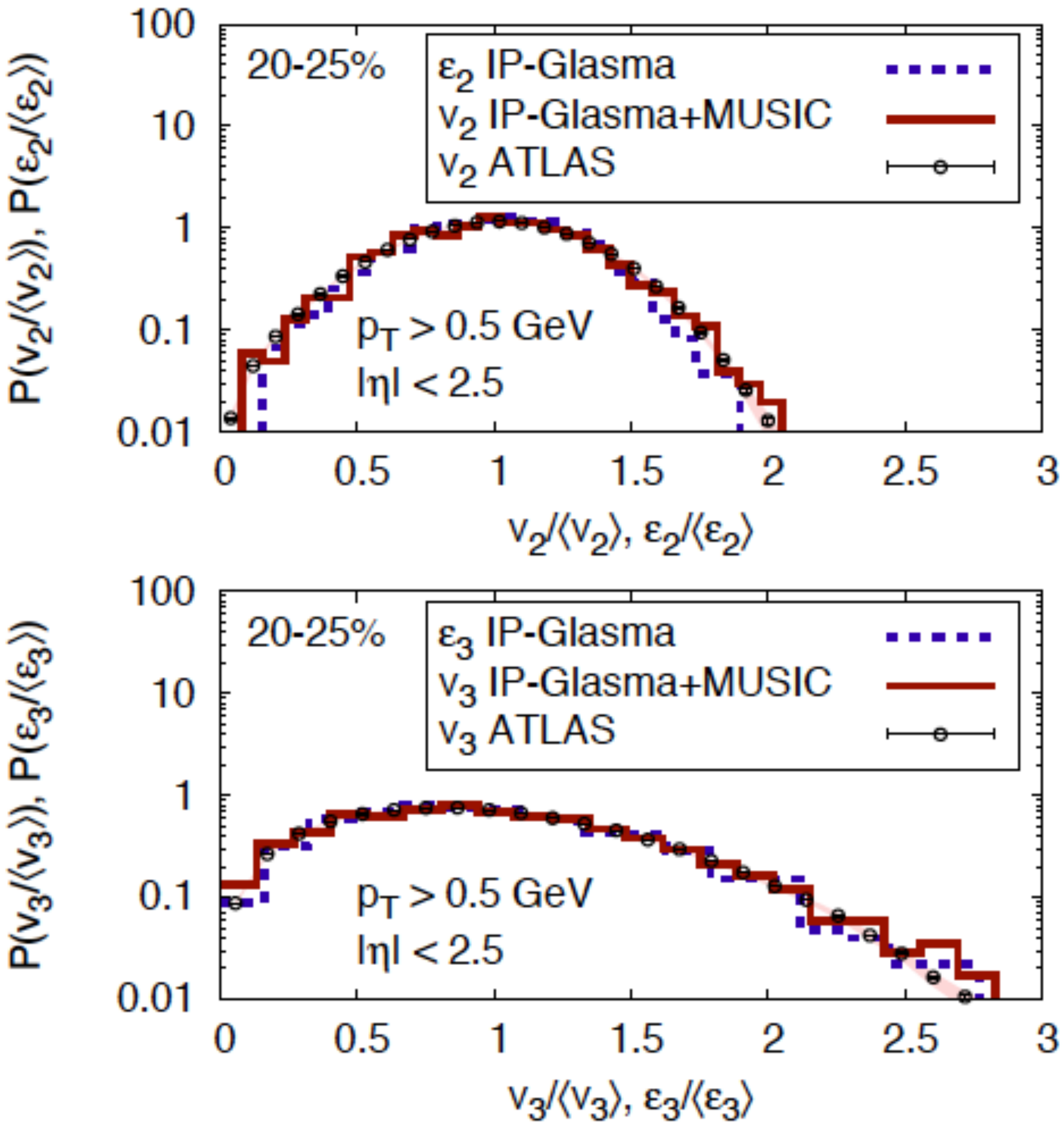
▶ energy density profile at $\tau = \tau_0$

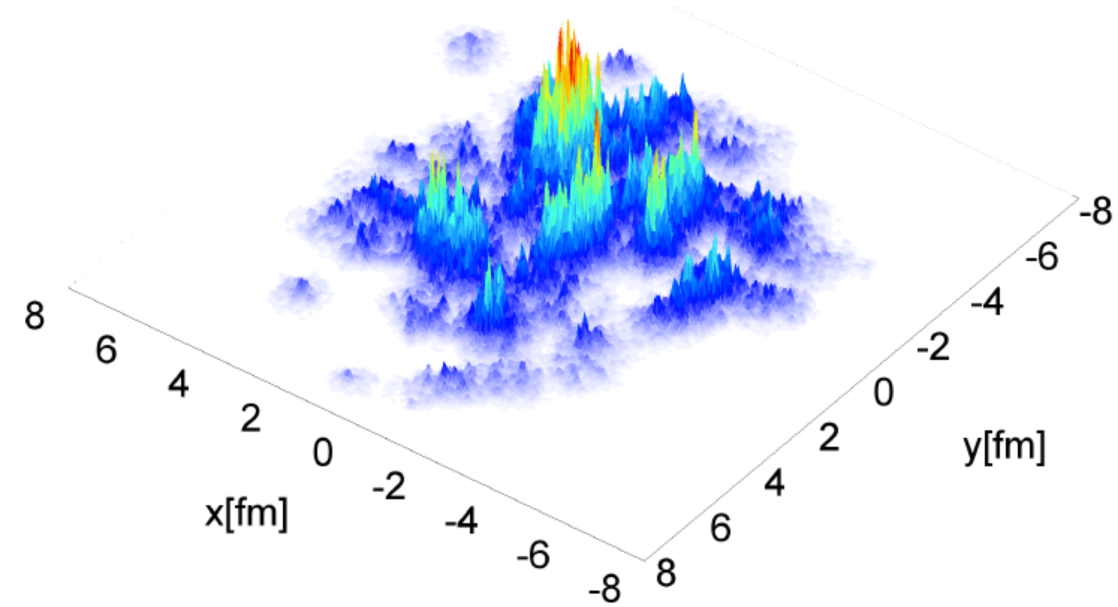
$$\partial_\mu F^{\mu\nu} - ig[A_\mu, F^{\mu\nu}] = 0$$
$$T^\mu{}_\nu(\tau = \tau_0) u^\nu = \epsilon\, u^\mu$$

Model : IP-Glasma I.C.

B. Schenke, P. Tribedy and R. Venugopalan (2012)

Classical YM dynamics with color sources in nuclei

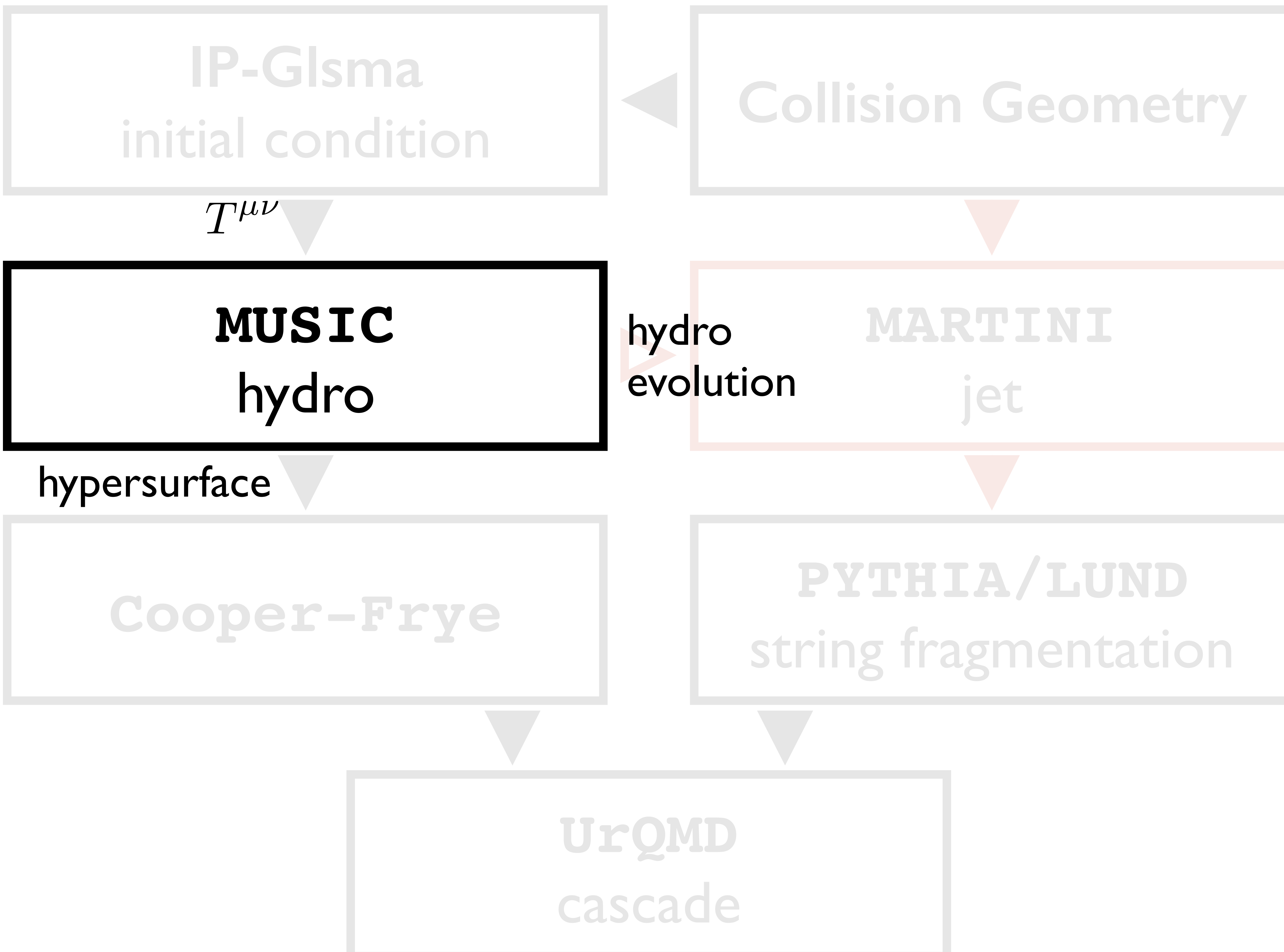




well describes v_n distribution

C. Gale, S. Jeon, B. Schenke, P. Tribedy and R. Venugopalan (2012)

Model Structure



Model : MUSIC hydro

B. Schenke, S. Jeon, and C. Gale (2010)

hydrodynamic equations of motion

Conservation equation $\partial_\mu T^{\mu\nu} = 0$

Decomposition $T^{\mu\nu} = \epsilon_0 u^\mu u^\nu - (P_0(\epsilon_0) + \Pi)\,\Delta^{\mu\nu} + \pi^{\mu\nu}$

EoS ($P_0(\epsilon_0)$) bulk (Π) shear ($\pi^{\mu\nu}$)

Local 3-metric $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

Local 3-gradient $\nabla^\mu = \Delta^{\mu\nu}\partial_\nu$

Model : MUSIC hydro

B. Schenke, S. Jeon, and C. Gale (2010)

equation of motion for viscous corrections

shear viscosity relaxation equation

$$\dot{\pi}^{\langle\mu\nu\rangle} = -\frac{\pi^{\mu\nu}}{\tau_\pi} + \frac{1}{\tau_\pi}\Big(2\eta\,\sigma^{\mu\nu} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\,\sigma^{\mu\nu}\Big)$$

expansion rate

$$\theta = \nabla_\mu u^\mu$$

shear tensor

$$\sigma^{\mu\nu} = \frac{1}{2}\left[\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{3}{2}\Delta^{\mu\nu}\left(\nabla_\alpha u^\alpha\right)\right] \equiv \nabla^{\langle\mu}u^{\nu\rangle}$$

Model : MUSIC hydro

B. Schenke, S. Jeon, and C. Gale (2010)

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shear

$$\frac{\eta}{s} = \mathrm{const}$$

Model : MUSIC hydro

B. Schenke, S. Jeon, and C. Gale (2010)

equation of motion for viscous corrections

shear viscosity relaxation equation

$$\dot{\pi}^{\langle\mu\nu\rangle} = -\frac{\pi^{\mu\nu}}{\tau_\pi} + \frac{1}{\tau_\pi}\Big(2\eta\,\sigma^{\mu\nu} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\,\sigma^{\mu\nu}\Big)$$

14-moment approximation in the small mass limit

G. Denicol, S. Jeon, and C. Gale (2014)

$$\frac{\eta}{\tau_\pi} = \frac{1}{5}(\epsilon_0 + P_0) \qquad \frac{\delta_{\pi\pi}}{\tau_\pi} = \frac{4}{3} \qquad \frac{\lambda_{\pi\Pi}}{\tau_\pi} = \frac{6}{5} \qquad \frac{\tau_{\pi\pi}}{\tau_\pi} = \frac{10}{7}$$

second-order transport coefficients

Model : MUSIC hydro

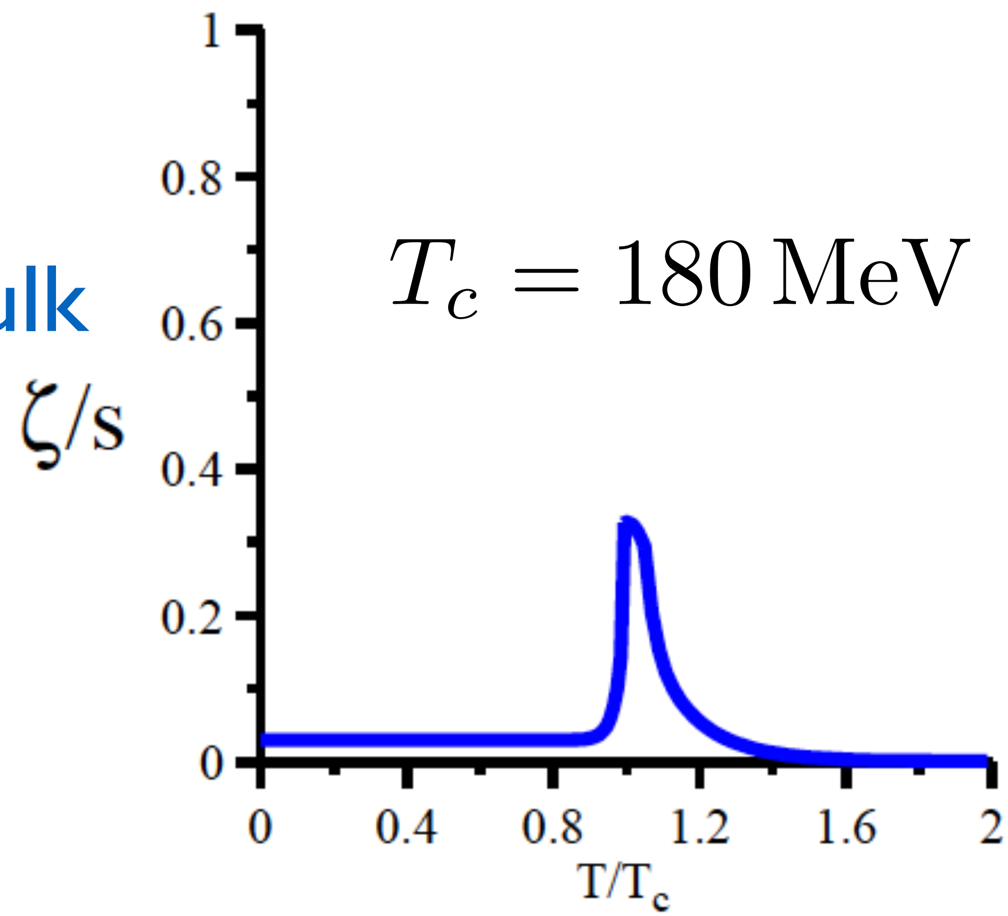
B. Schenke, S. Jeon, and C. Gale (2010)

equation of motion for viscous corrections

bulk viscosity relaxation equation

$$\dot{\Pi} = -\frac{\Pi}{\tau_\Pi} + \frac{1}{\tau_\Pi}\left(-\zeta\,\theta - \delta_{\Pi\Pi}\Pi\,\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}\right)$$

bulk



$T_c = 180\,\mathrm{MeV}$

F. Karsch, D. Kharzeev and K. Tuchin (2008)

J. Noronha-Hostler, J. Noronha and C. Greiner (2009)

Model : MUSIC hydro

B. Schenke, S. Jeon, and C. Gale (2010)

equation of motion for viscous corrections

bulk viscosity relaxation equation

$$\dot{\Pi} = -\frac{\Pi}{\tau_\Pi} + \frac{1}{\tau_\Pi}\left(-\zeta\,\theta - \delta_{\Pi\Pi}\Pi\,\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}\right)$$

14-moment approximation in the small mass limit

G. Denicol, S. Jeon, and C. Gale (2014)

$$\frac{\zeta}{\tau_\Pi} = 15\left(\frac{1}{3} - c_s^2\right)^2 (\epsilon_0 + P_0)$$

$$\frac{\delta_{\Pi\Pi}}{\tau_\Pi} = \frac{2}{3} \qquad \frac{\lambda_{\Pi\pi}}{\tau_\Pi} = \frac{8}{5}\left(\frac{1}{3} - c_s^2\right)$$

second-order
transport coefficients

Model : Equation of state

P. Huovinen, and P. Petreczky (2010)

Equation of state : hadron gas + lattice data

Only those included in UrQMD

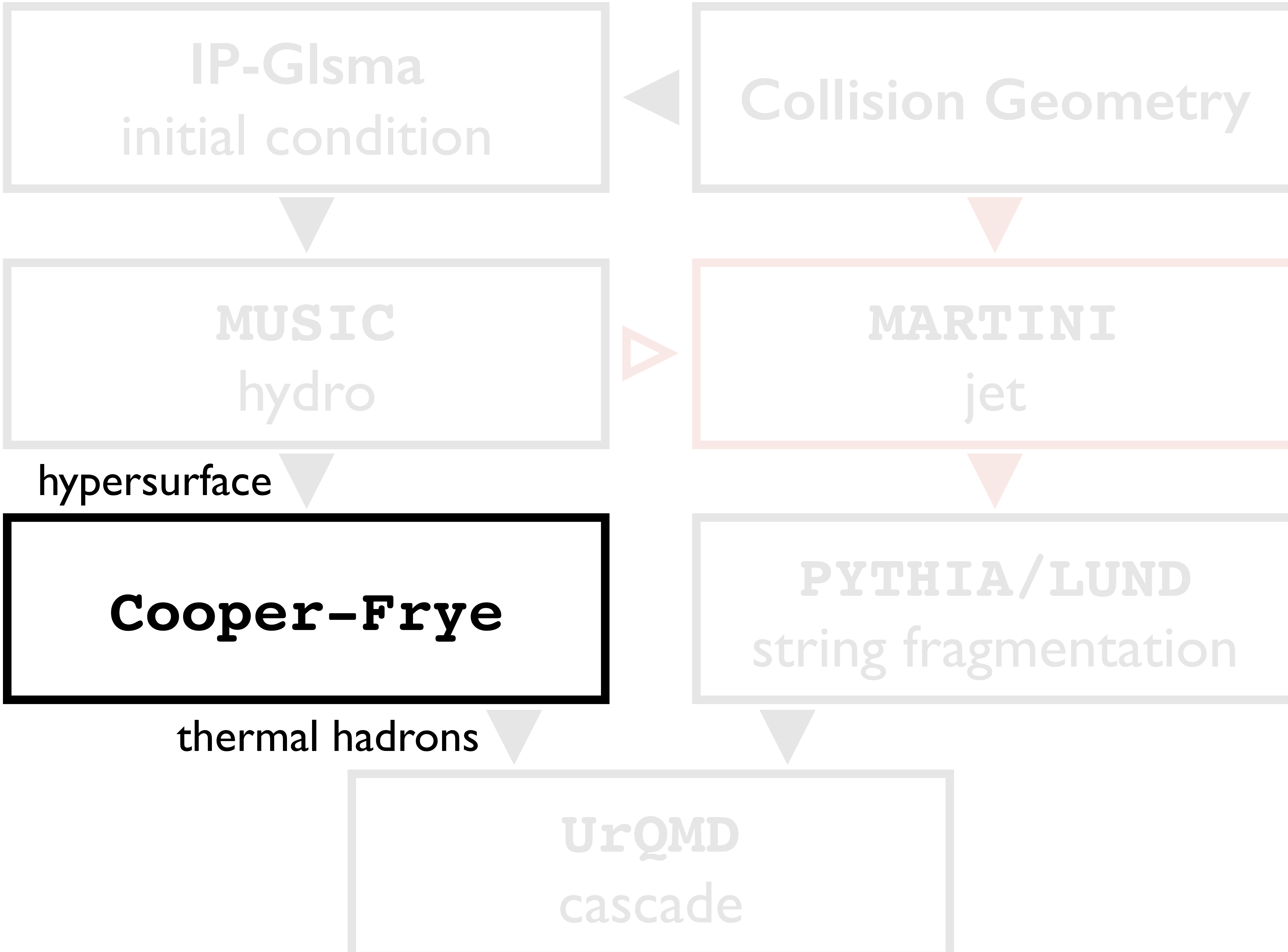
Cross over phase transition around T = 180 MeV

Initial condition + Hydro equation + EoS

Hydrodynamic evolution

Up to the isothermal hypersurface at $T_{\rm sw} = 145\,{\rm MeV}$
to switch from hydrodynamics to transport

Model Structure



Model : Cooper-Frye sampling

F. Cooper and G. Frye (1974)

sampling particles according to the Cooper-Frye formula

$$\left.\frac{dN}{d^3\mathbf{p}}\right|_{\text{1-cell}} = \left[f_0(x,\mathbf{p}) + \delta f_{\text{shear}}(x,\mathbf{p}) + \delta f_{\text{bulk}}(x,\mathbf{p})\right] \frac{p^\mu \Delta^3 \Sigma_\mu}{E_{\mathbf{p}}}$$

$$f_0(x,\mathbf{p}) = \frac{1}{\exp\left[(p\cdot u)/T\right] \mp 1}$$

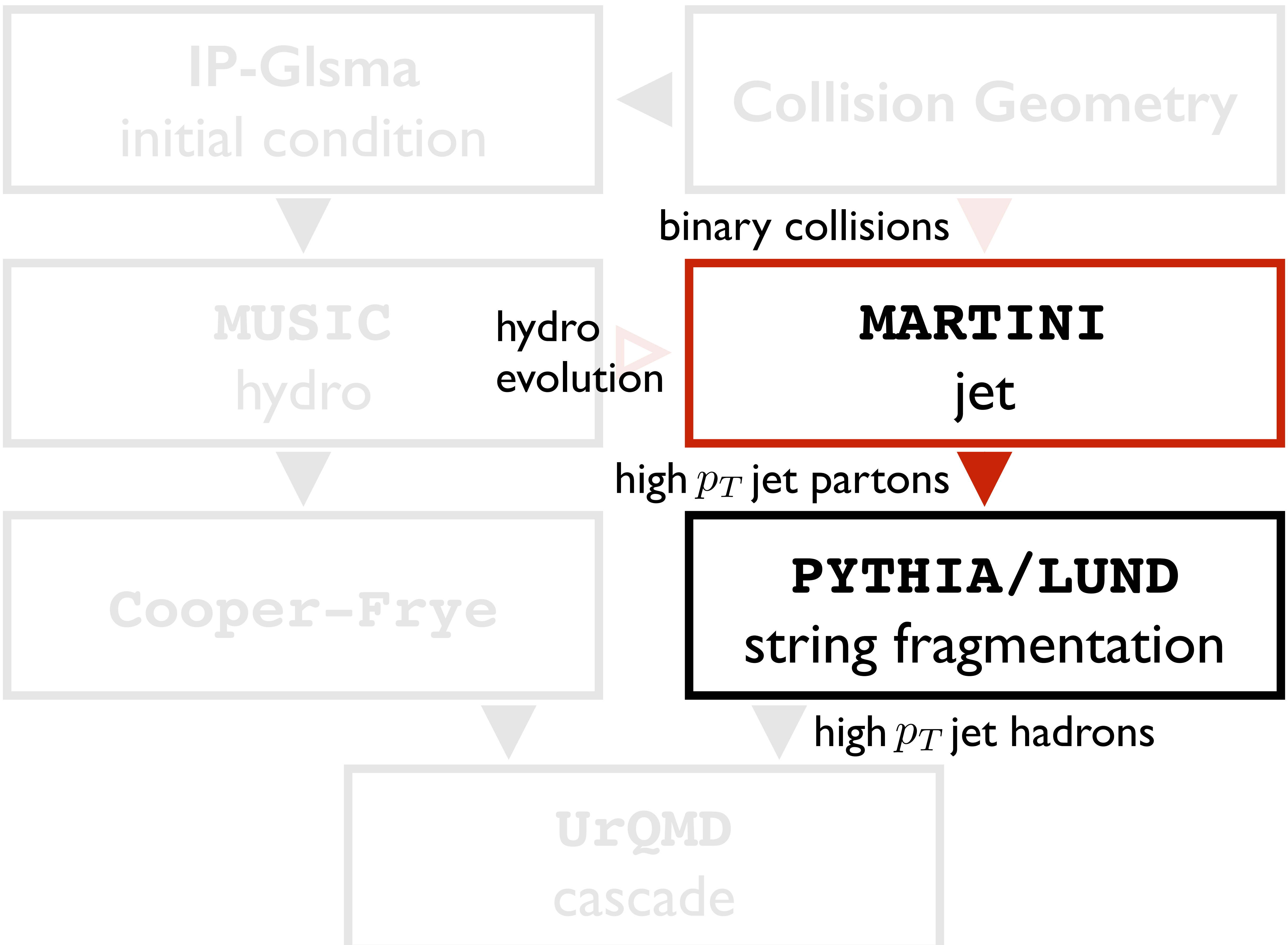
$$\delta f_{\text{shear}}(x,\mathbf{p}) = f_0(1 \pm f_0)\frac{p^\mu p^\nu \pi^{\mu\nu}}{2T^2(\epsilon_0 + P_0)}$$

P. Bozek (2010)

$$\delta f_{\text{bulk}}(x,\mathbf{p}) = -f_0(1 \pm f_0)\frac{C_{\text{bulk}}\Pi}{T}\left[c_s^2(p\cdot u) - \frac{(-p^\mu p^\nu \Delta_{\mu\nu})}{3(p\cdot u)}\right]$$

$$\frac{1}{C_{\text{bulk}}} = \frac{1}{3T}\sum_n m_n^2 \int \frac{d^3\mathbf{k}}{(2\pi)^3 E_{\mathbf{k}}} f_{n,0}(1 \pm f_{n,0})\left(c_s^2 E_{\mathbf{k}} - \frac{|\mathbf{k}|^2}{3E_{\mathbf{k}}}\right)$$

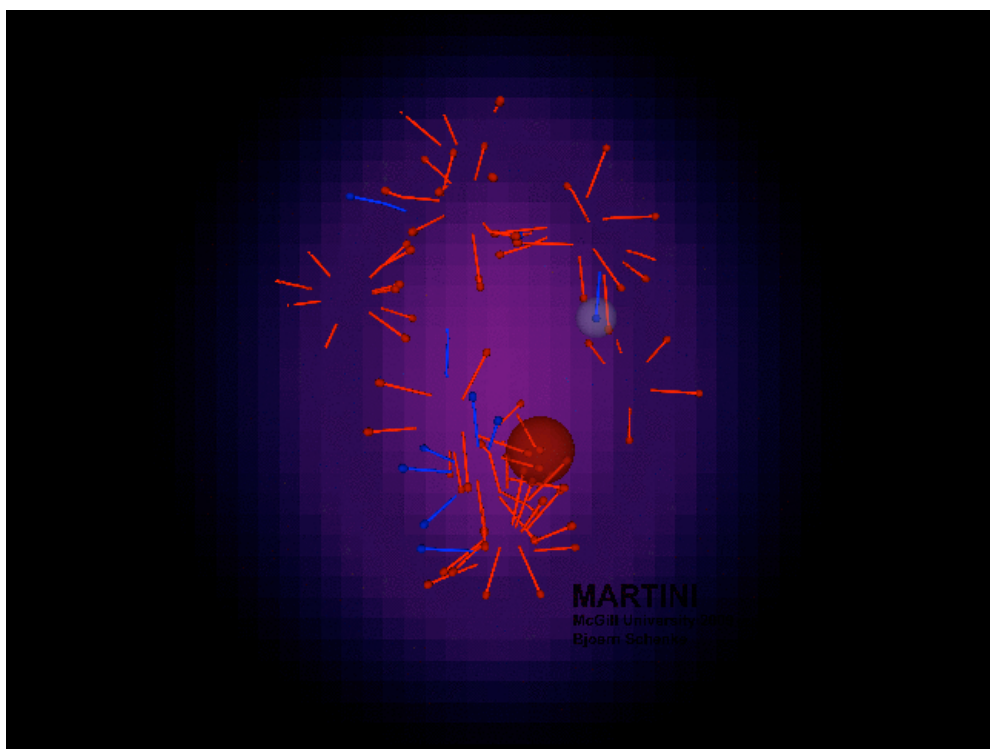
Model Structure



Model : MARTINI jets

Modular Algorithm for Relativistic Treatment of heavy IoN Interaction

B. Schenke, C. Gale and S. Jeon (2010)



Hard process at the position of binary collision (PYTHIA)

↓

Energy loss
- Radiation (AMY)
- Collision (with thermal partons)

↓

Fragmentation into hadrons (PYTHIA / LUND string model)

Model : MARTINI jets

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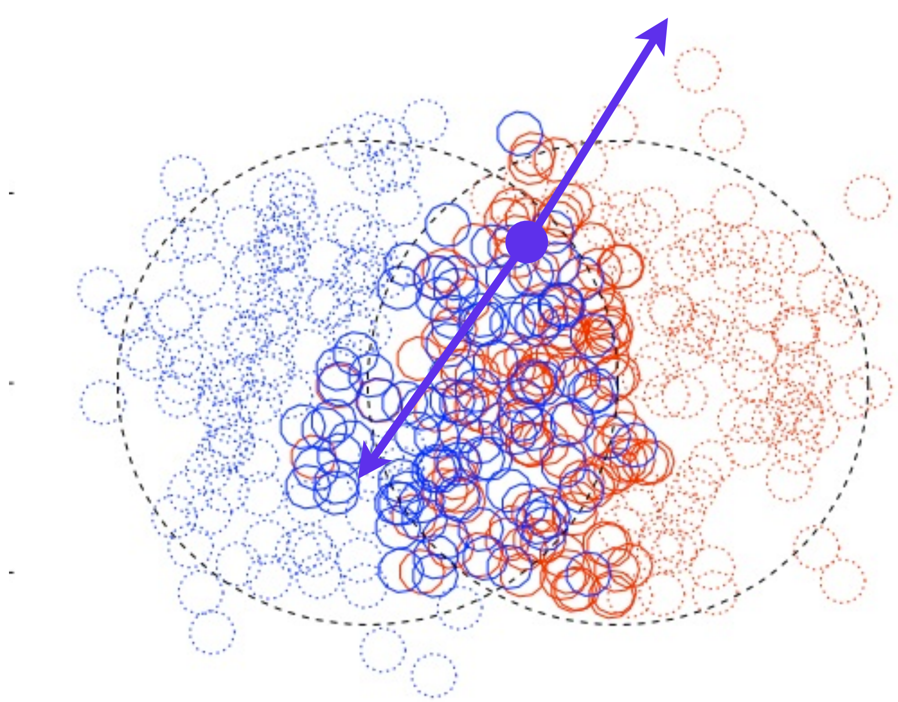

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Model : MARTINI jets

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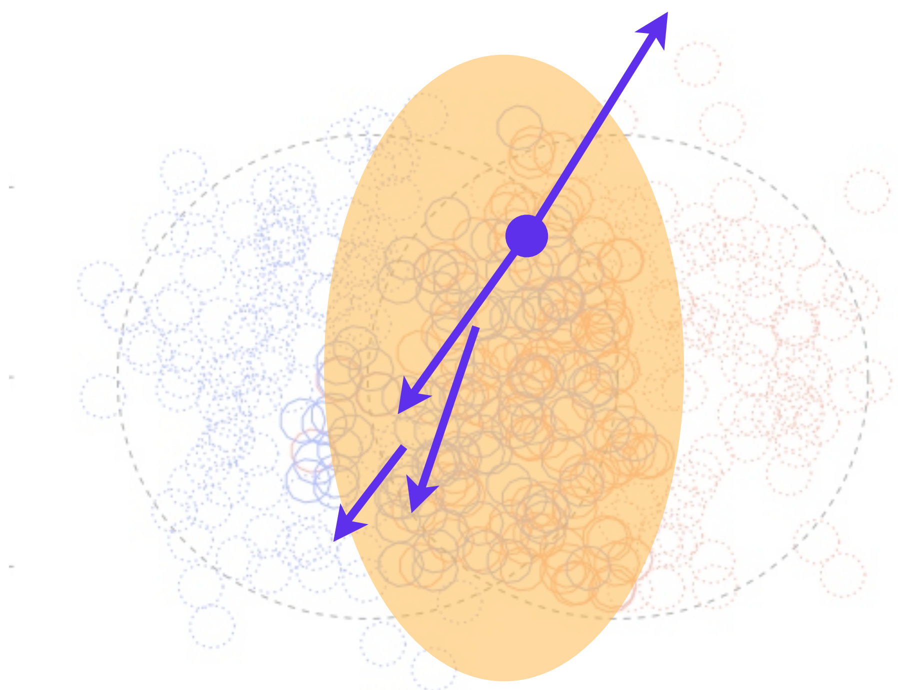

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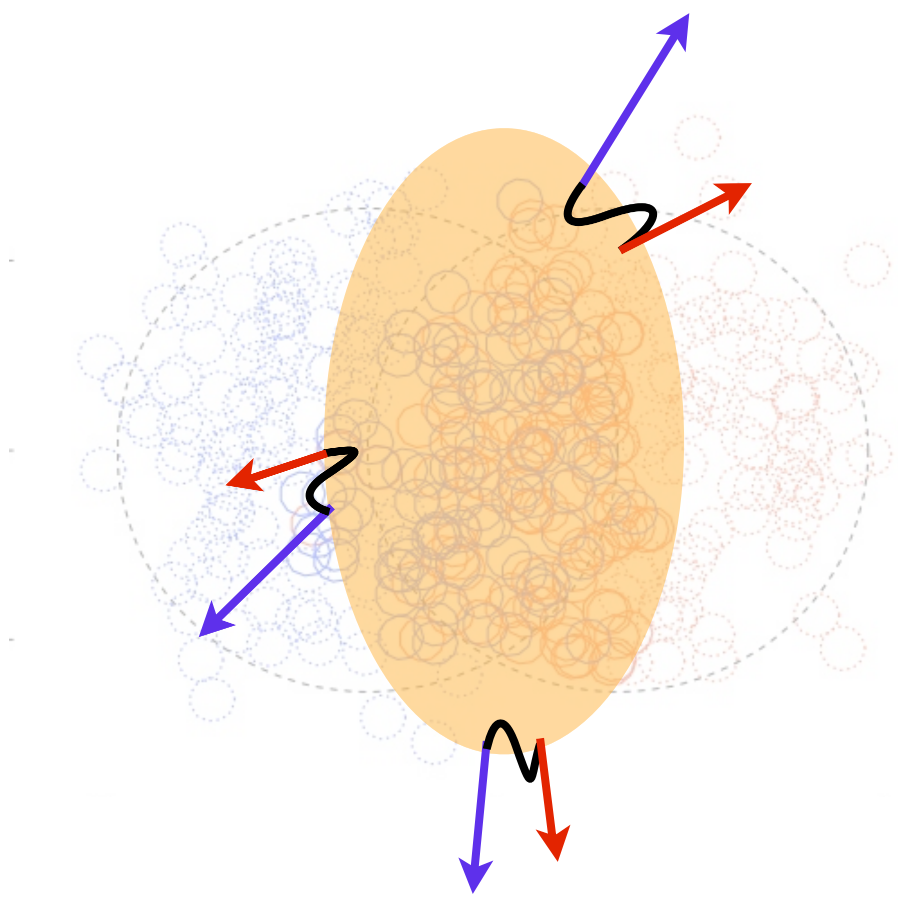


Hard process at the position of binary collision (PYTHIA)

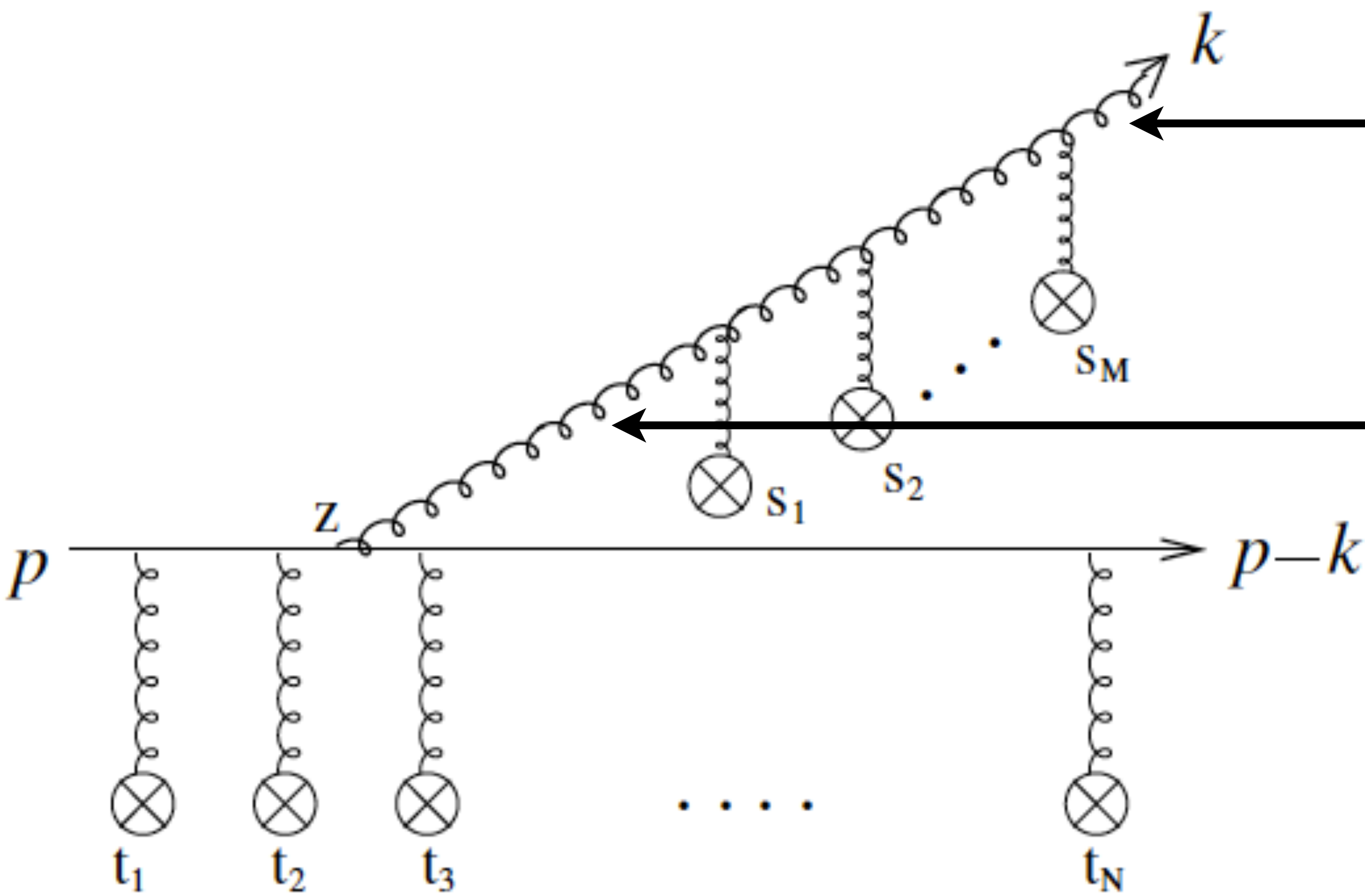
Energy loss
- Radiation (AMY)
- Collision (with thermal partons)

Fragmentation into hadrons (PYTHIA / LUND string model)

Model : jet energy loss

Radiative energy loss (AMY)

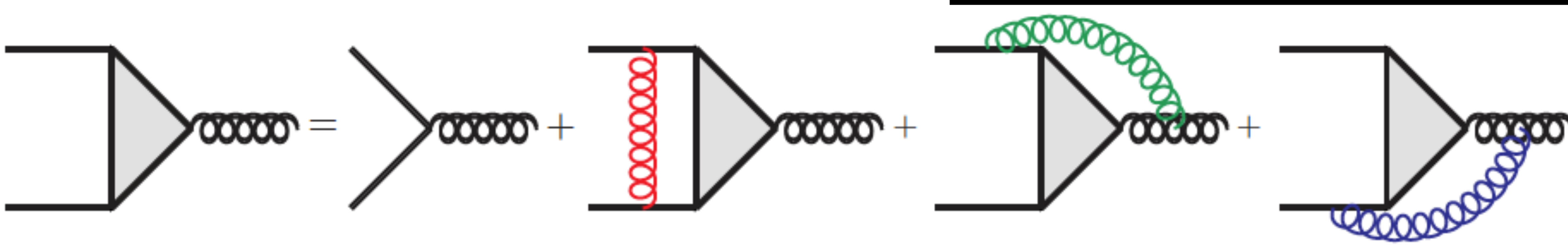
P. Arnold, G. Moore and L. Yaffe (2002)



Collinear emission

Nearly on-shell intermediate propagator

Infinite number of diagrams
> Integral equations



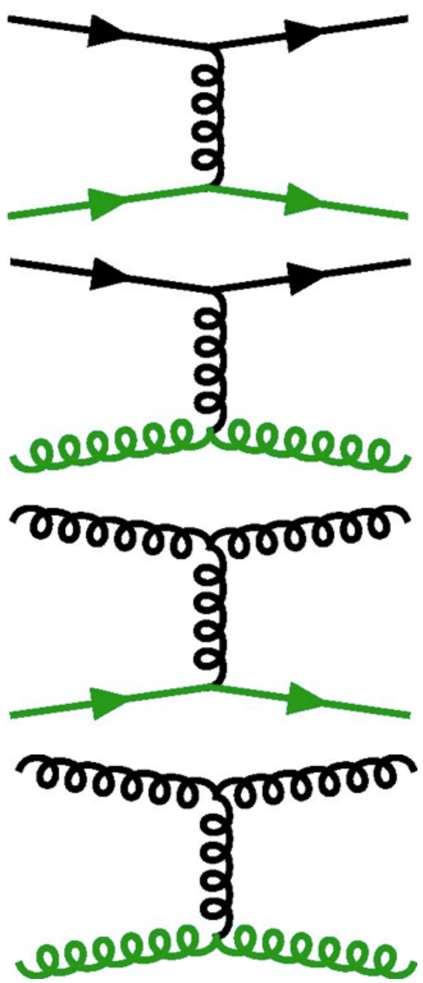
figures by G-Y. Qin

Model : jet energy loss

Collisional energy loss (soft approximation)

B. Schenke, C. Gale and G-Y. Qin (2009)

G-Y. Qin et al. (2008)



$$\left.\frac{dE}{dt}\right|_{qq} = \frac{2}{9} n_f \pi \alpha_s^2 T^2 \left[\ln \frac{ET}{m_g^2} + c_f \frac{23}{12} + c_s\right]$$

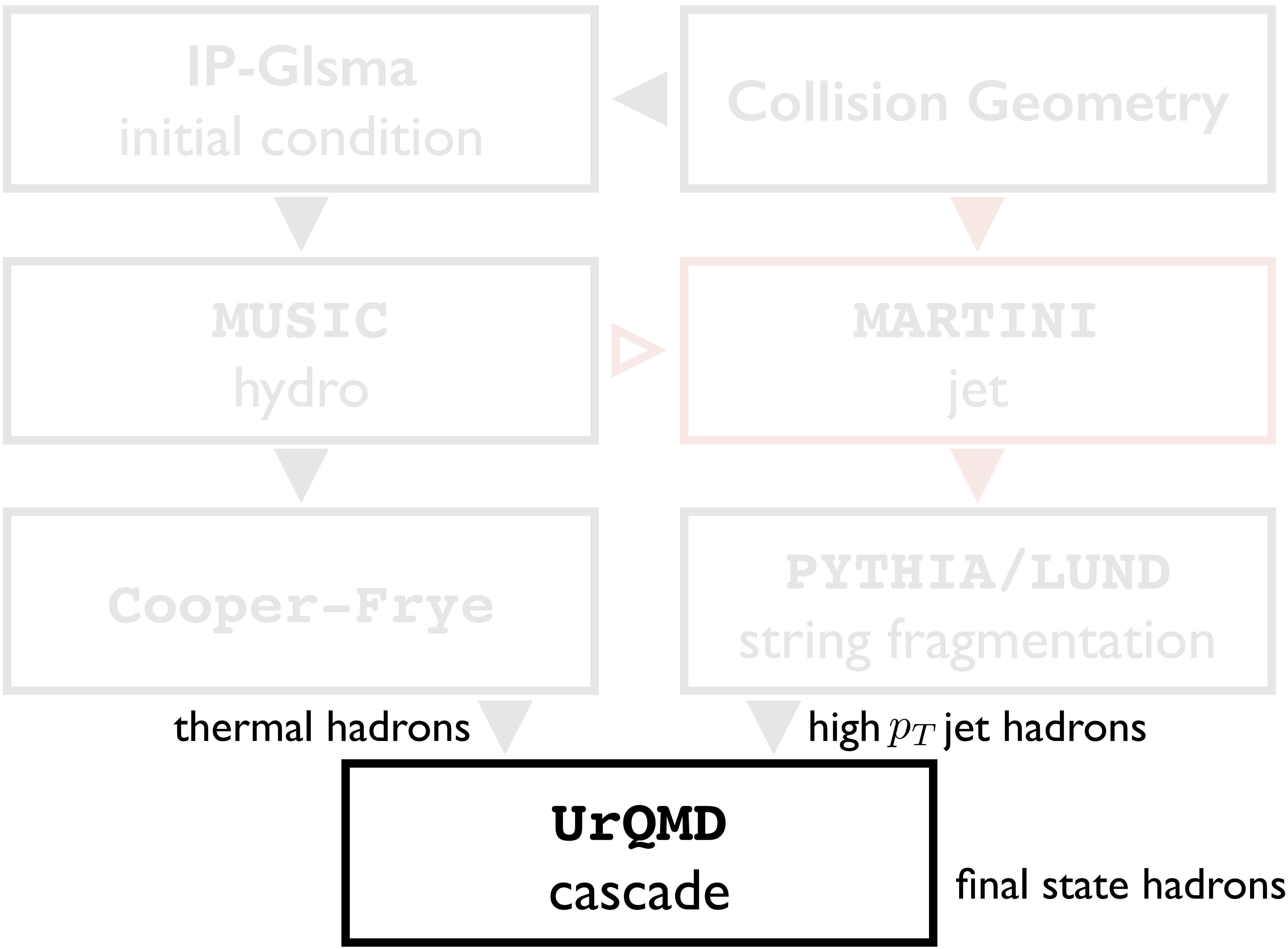
$$\left.\frac{dE}{dt}\right|_{qg} = \frac{4}{3} \pi \alpha_s^2 T^2 \left[\ln \frac{ET}{m_g^2} + c_b \frac{13}{6} + c_s\right]$$

$$\left.\frac{dE}{dt}\right|_{gq} = \frac{1}{2} n_f \pi \alpha_s^2 T^2 \left[\ln \frac{ET}{m_g^2} + c_f \frac{13}{6} + c_s\right]$$

$$\left.\frac{dE}{dt}\right|_{gg} = 3 \pi \alpha_s^2 T^2 \left[\ln \frac{ET}{m_g^2} + c_b \frac{131}{48} + c_s\right]$$

Hard part ready for transport calculations

Model Structure



Model : UrQMD cascade

Ultra-relativistic Quantum Molecular Dynamics

S. A. Bass *et al.* (1998)

Stochastic implementation of transport theory

$$p^\mu \frac{\partial}{\partial x^\mu} f_i(x,p) = \mathcal{C}_i[f]$$

What are there? : 55 baryons + 32 mesons with masses up to 2.25 GeV

Cross sections : based on experimental data
Jet-hadron interaction by PYTHIA

Keeps track of particle trajectories

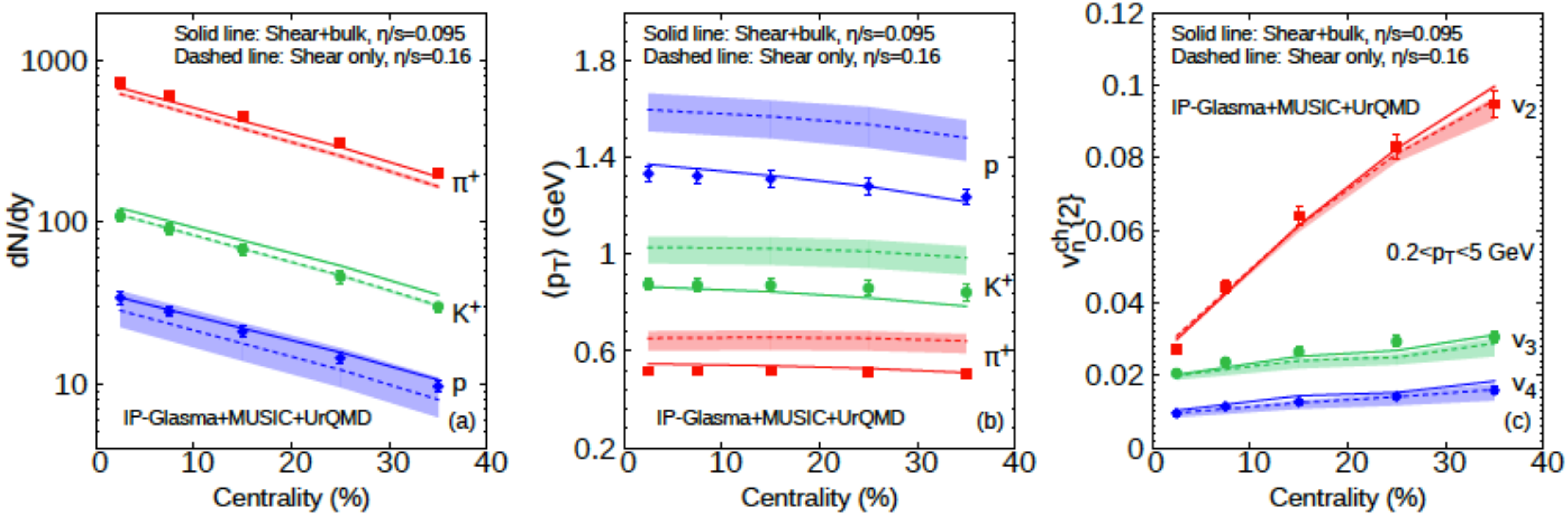
Results

- Determination of hydro background
- Extension to higher p_T range

Determination of hydro background

arXiv:1502.01675

S. Ryu, J-F, Paquet, G. Denicol, C. Shen, B. Schenke, S. Jeon and C. Gale



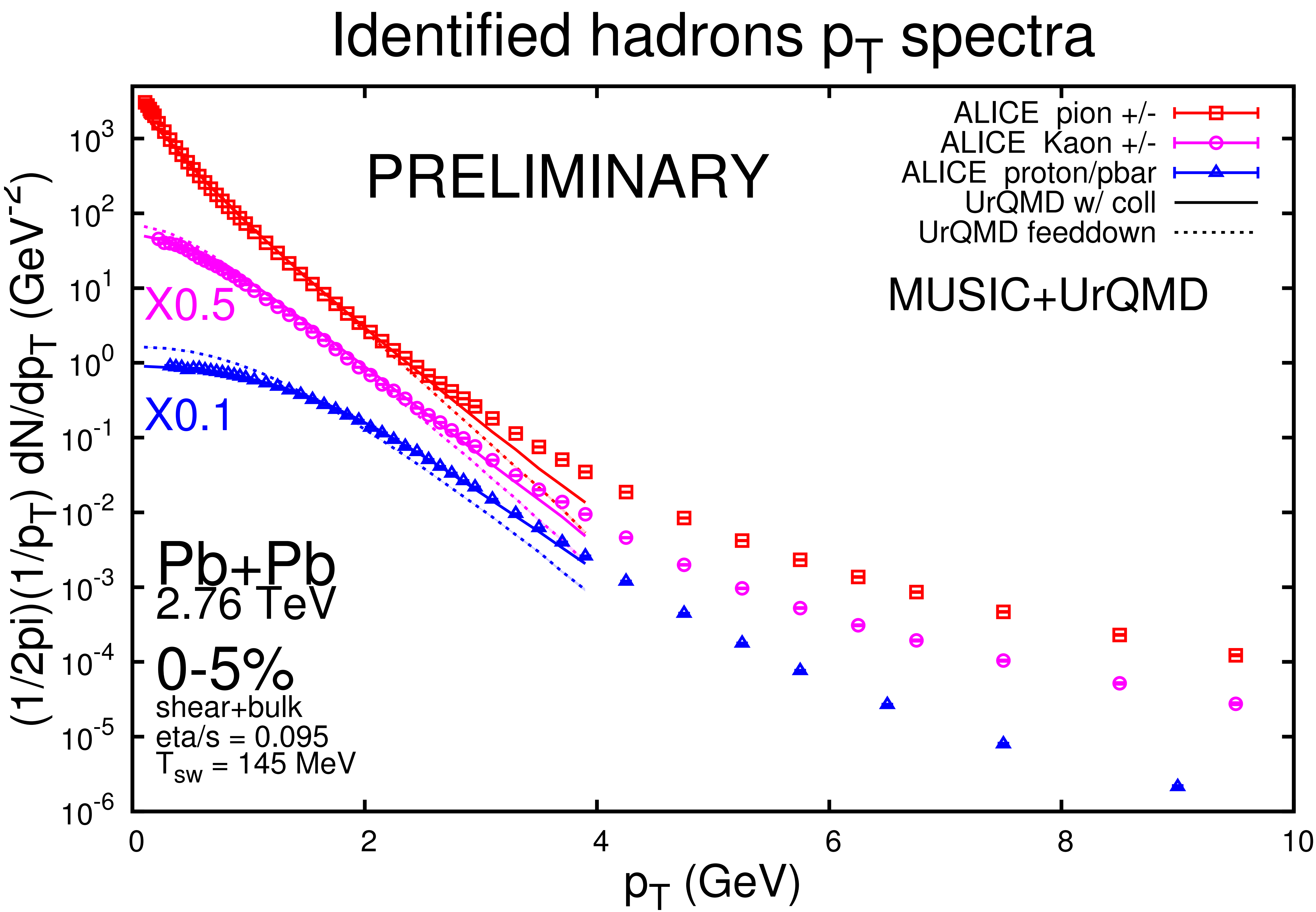
Parameters are tuned to fit **multiplicity, mean** p_T and **integrated flow coefficients** v_n.

The **bulk viscosity is crucial** to get those observables.

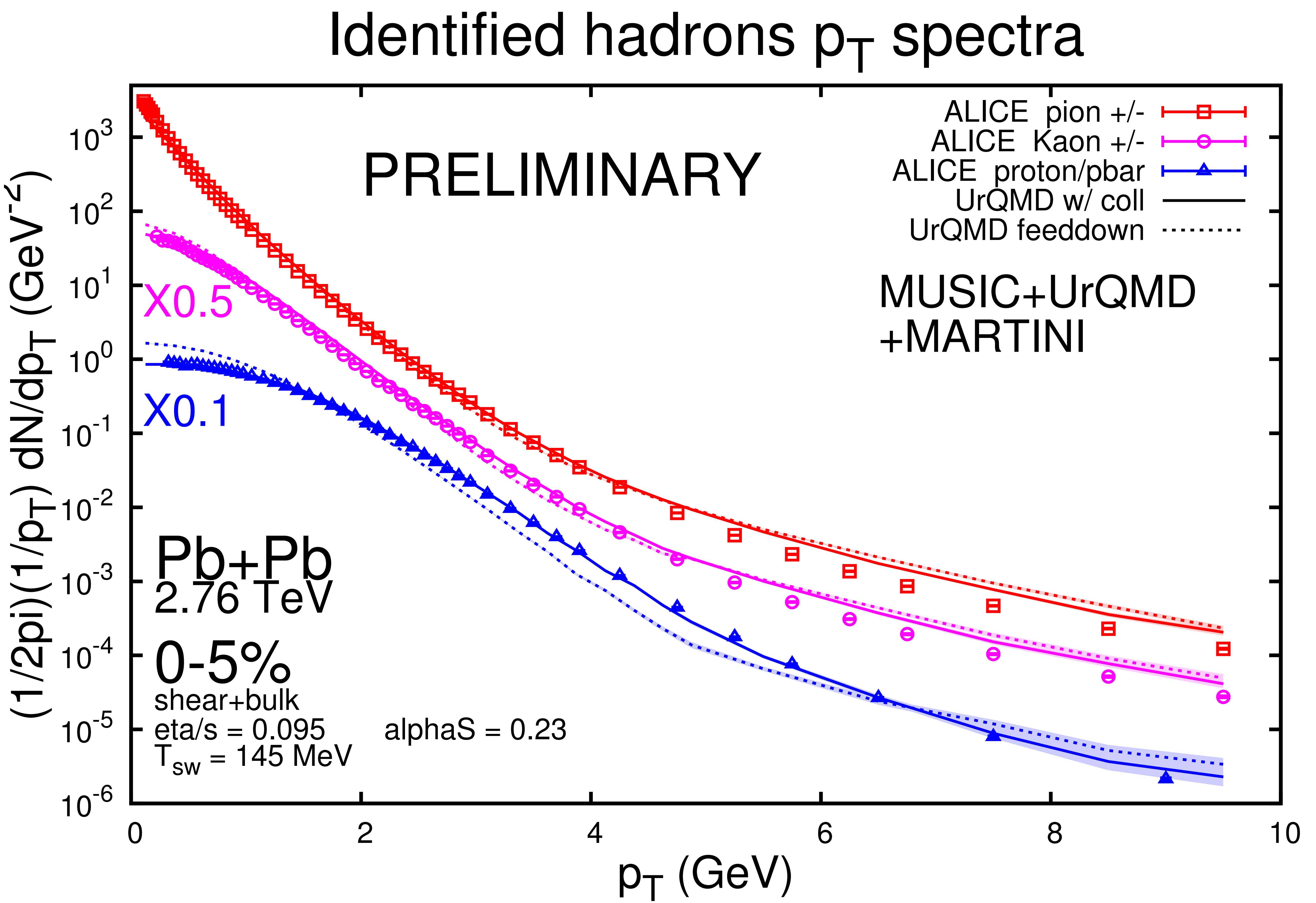
The **shear viscosity** $\frac{\eta}{s} = 0.095$ is favored.

Results

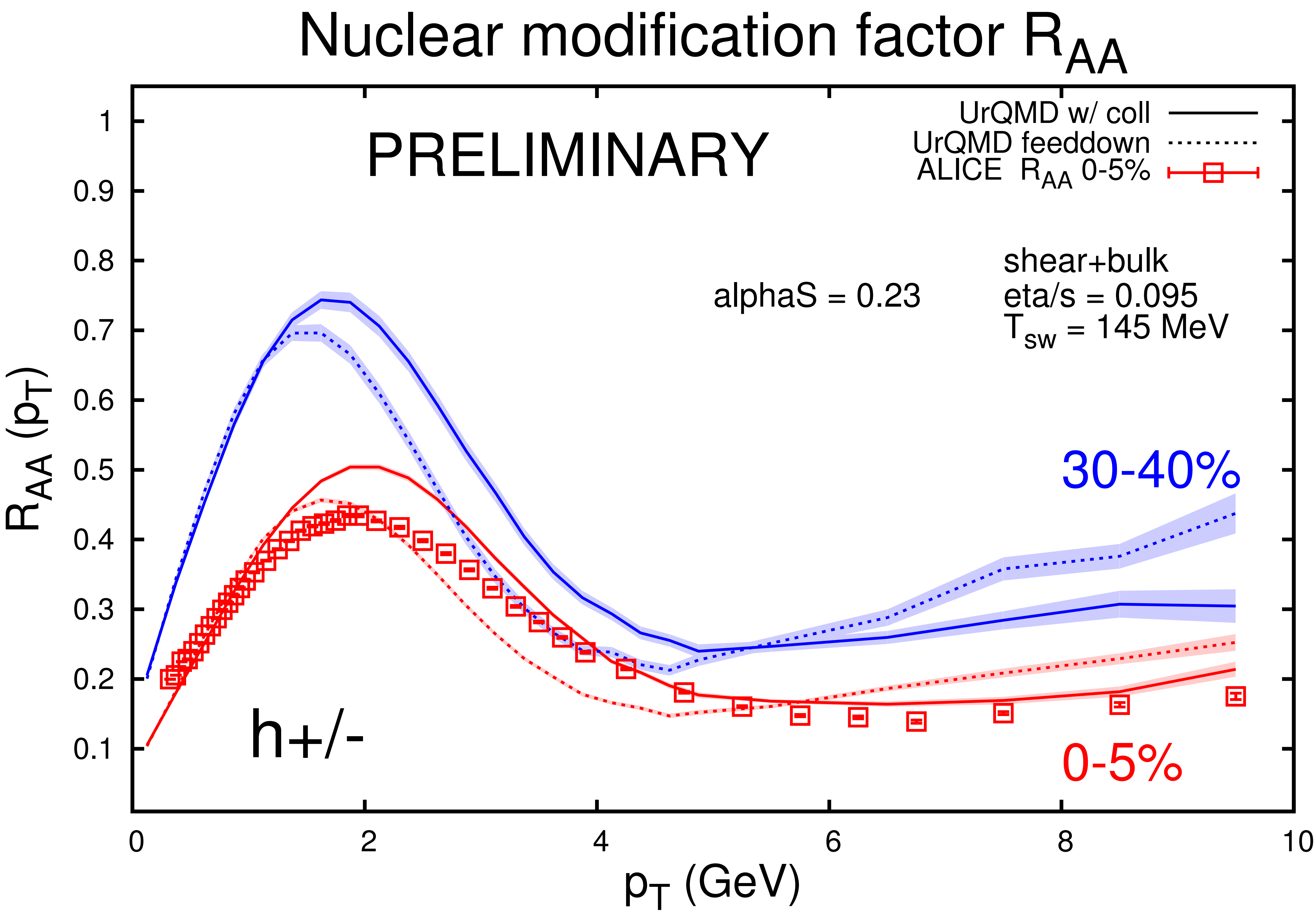
- Determination of hydro background
- Extension to higher p_T range



Identified hadrons p_T spectra
PRELIMINARY
ALICE pion +/-
ALICE Kaon +/-
ALICE proton/pbar
UrQMD w/ coll
UrQMD feeddown
MUSIC+UrQMD
X0.5
X0.1
Pb+Pb
2.76 TeV
0-5%
shear+bulk
eta/s = 0.095
T_{sw} = 145 MeV
$(1/2pi)(1/p_T)\ dN/dp_T\ (GeV^{-2})$
p_T (GeV)
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
2
4
6
8
10

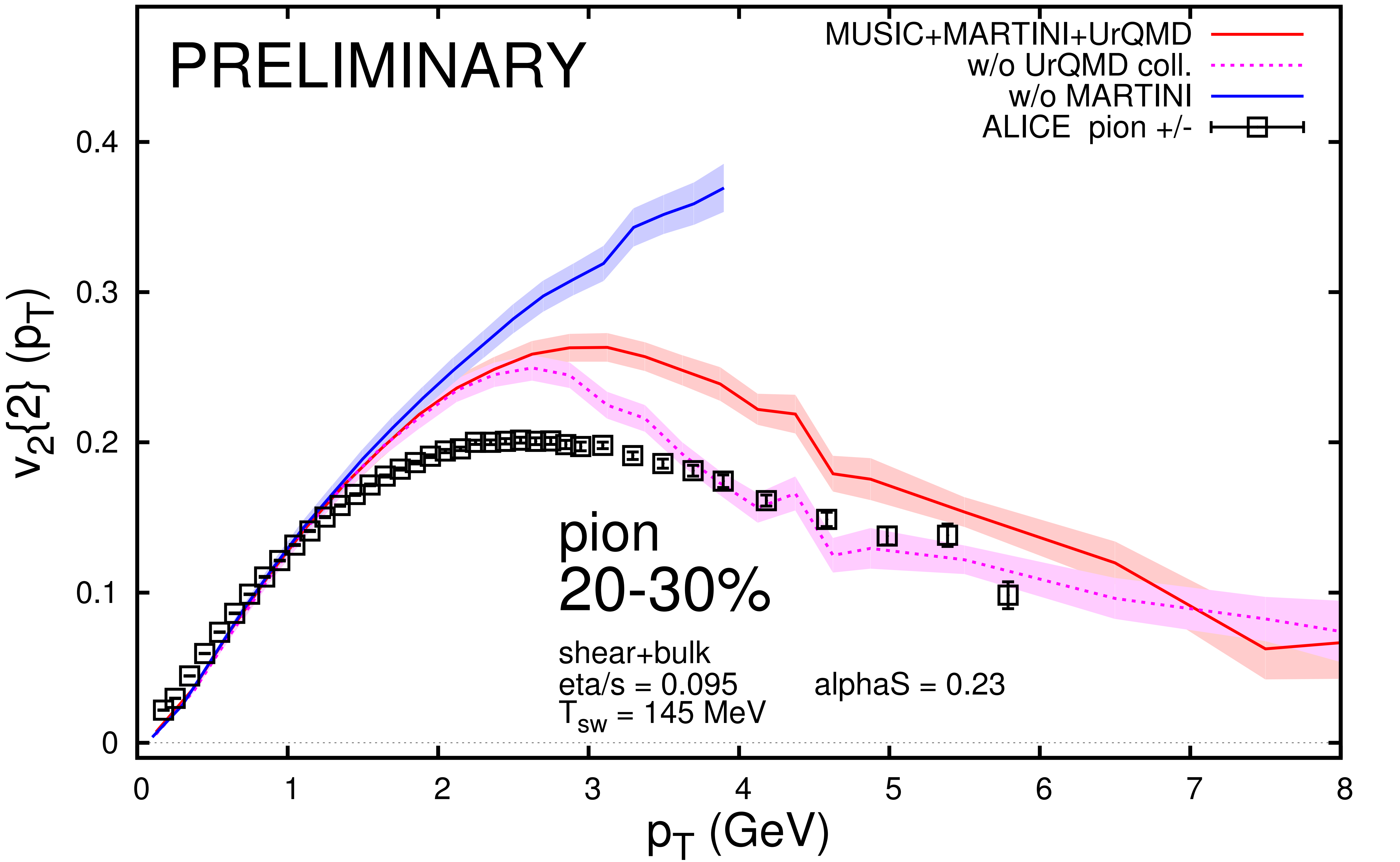


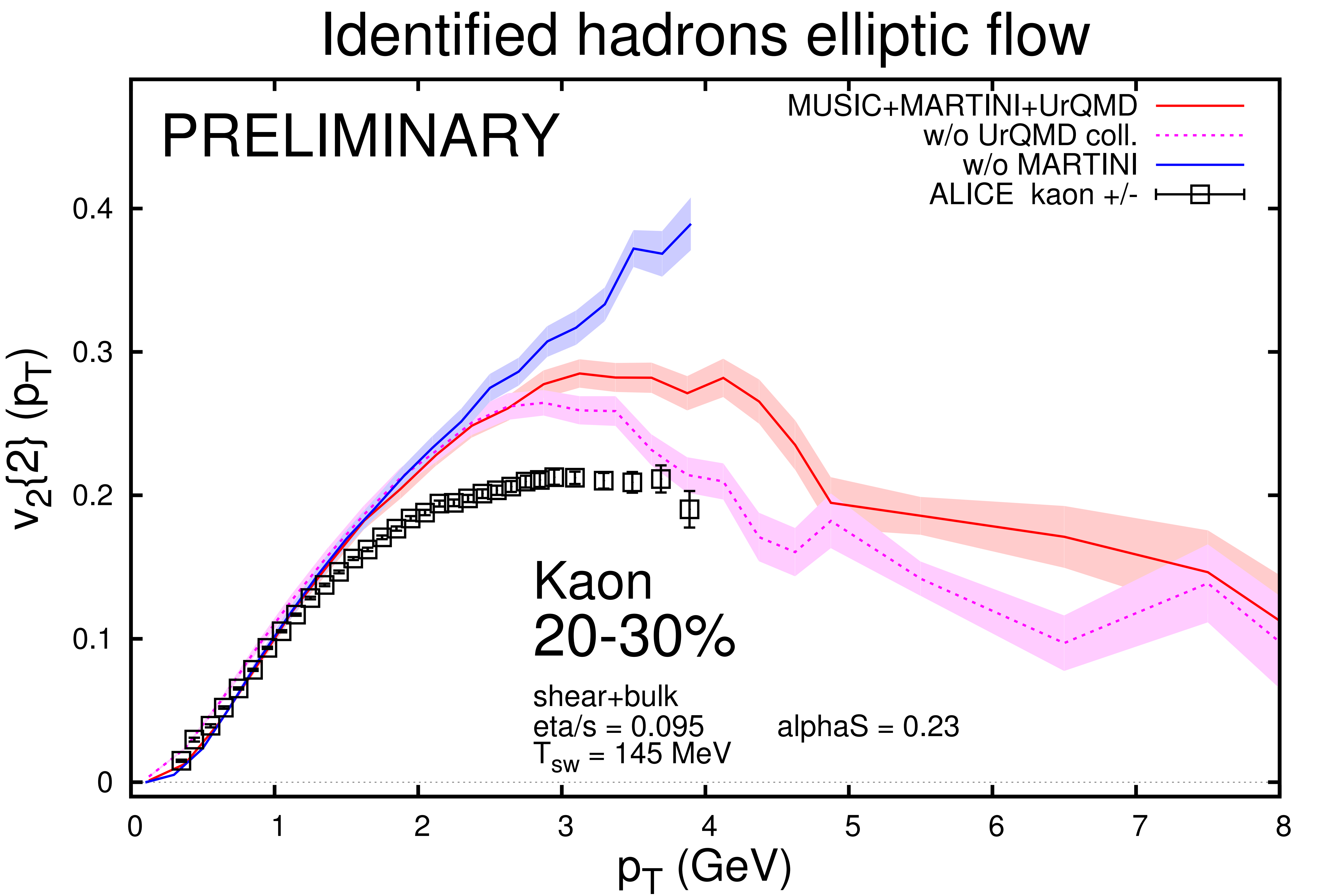
Identified hadrons pT spectra
PRELIMINARY
ALICE pion +/-
ALICE Kaon +/-
ALICE proton/pbar
UrQMD w/ coll
UrQMD feeddown
MUSIC+UrQMD
+MARTINI
X0.5
X0.1
Pb+Pb
2.76 TeV
0-5%
shear+bulk
eta/s = 0.095
alphaS = 0.23
Tsw = 145 MeV
(1/2pi)(1/pT) dN/dpT (GeV-2)
pT (GeV)



Nuclear modification factor R_{AA}
PRELIMINARY
UrQMD w/ coll
UrQMD feeddown
ALICE R_{AA} 0-5%
shear+bulk
alphaS = 0.23
eta/s = 0.095
T_{sw} = 145 MeV
30-40%
0-5%
h+/-
R_{AA} (p_T)
p_T (GeV)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
2
4
6
8
10

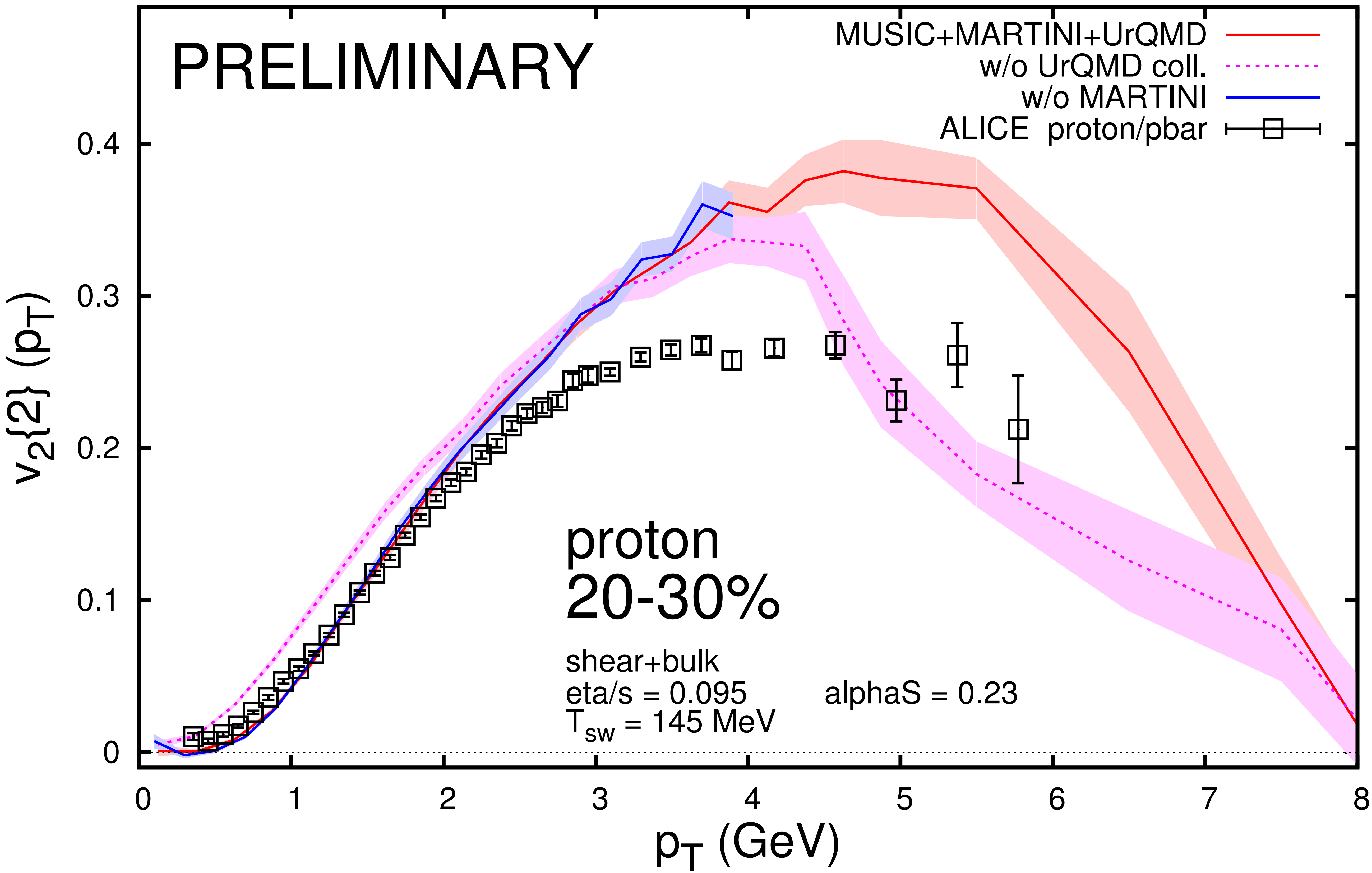
Identified hadrons elliptic flow

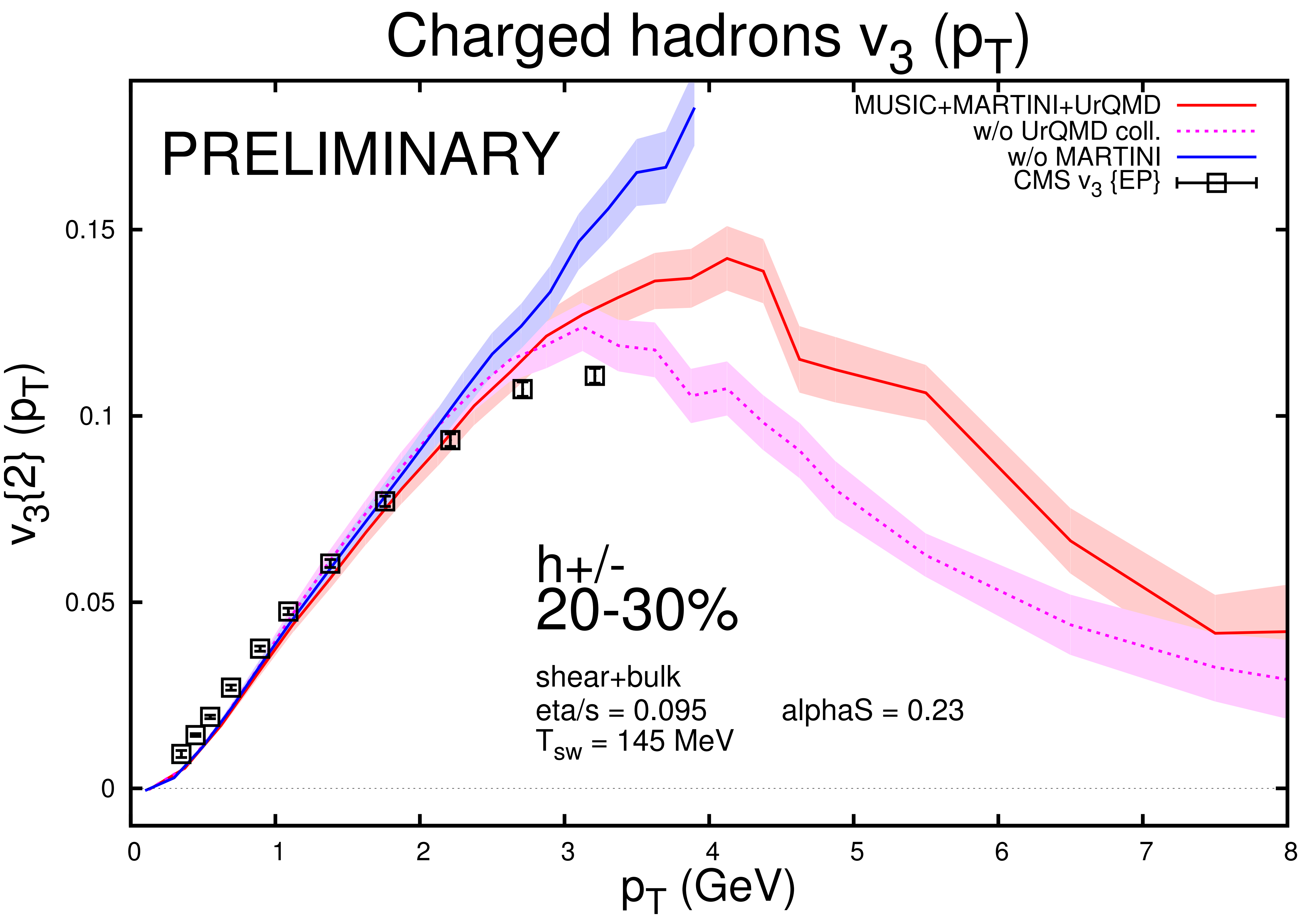


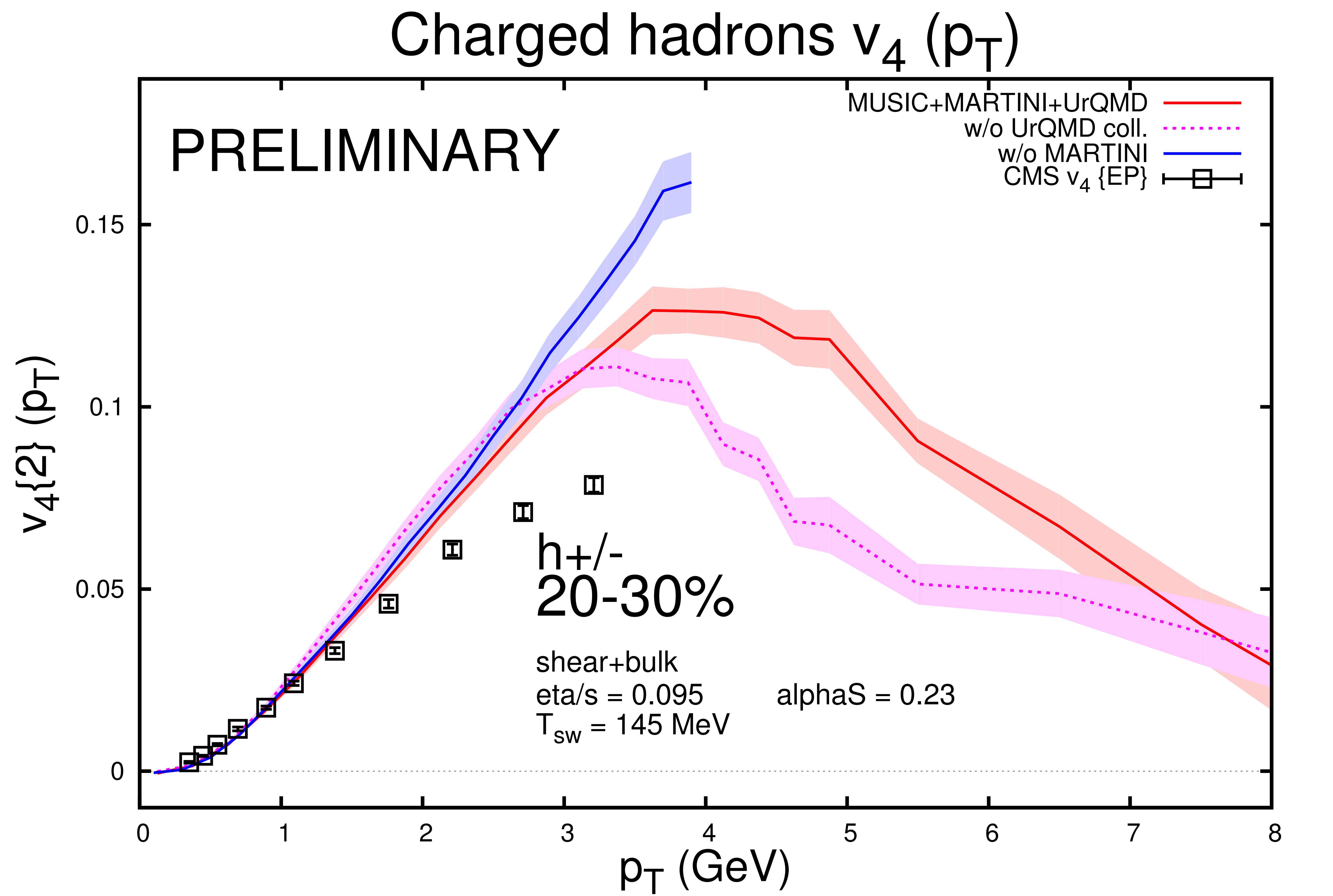


Identified hadrons elliptic flow
PRELIMINARY
MUSIC+MARTINI+UrQMD
w/o UrQMD coll.
w/o MARTINI
ALICE kaon +/-
Kaon
20-30%
shear+bulk
eta/s = 0.095
alphaS = 0.23
T_sw = 145 MeV
v_2{2} (p_T)
p_T (GeV)
0
0.1
0.2
0.3
0.4
0
1
2
3
4
5
6
7
8

Identified hadrons elliptic flow




Charged hadrons v_3 (p_T)
PRELIMINARY
MUSIC+MARTINI+UrQMD
w/o UrQMD coll.
w/o MARTINI
CMS v_3 {EP}
h+/-
20-30%
shear+bulk
eta/s = 0.095
alphaS = 0.23
T_{sw} = 145 MeV
$v_3\{2\}$ (p_T)
p_T (GeV)
0
0.05
0.1
0.15
0
1
2
3
4
5
6
7
8



Charged hadrons v_4 (p_T)
PRELIMINARY
MUSIC+MARTINI+UrQMD
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Conclusions

- For the first time, we developed an event generator with
  * sub-nucleonic fluctuation
  * viscous hydrodynamics with both of shear and bulk
  * energy loss of high p_T particles
  * hadronic afterburner in the late stage
  to describe heavy ion collisions.
- Both low p_T and intermediate p_T observables are **well reproduced**.
- Effect of hadronic re-scattering is **significant**.
  > must be properly taken into account.

Outlook

- pA collisions
  * nuclear modification R_{pA}
  * anisotropic flows v_n
  at low and intermediate p_T range
- Medium response to jet energy loss
- Jet cone size dependence
- etc …

Backup Slides

Model : Cooper-Frye sampling

F. Cooper and G. Frye (1974)

sampling particles according to the Cooper-Frye formula

1. sample number of particles based on Poisson distribution

$$\bar{N}|_{\text{1-cell}} = \begin{cases} [n_0(x) + \delta n_{\text{bulk}}(x)]\, u^\mu \Delta\Sigma_\mu & \text{if} \quad u^\mu \Delta\Sigma_\mu \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$n_0(x) = d \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, f_0(\mathbf{k})$$

$$\delta n_{\text{bulk}}(x) = d \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, \delta f_{\text{bulk}}(\mathbf{k})$$

2. sample momentum of each particles
according to the Cooper-Frye formula shown in the main slide

Model : jet energy loss

Radiative energy loss (AMY)

P. Arnold, G. Moore and L. Yaffe (2002)

$$\frac{d\Gamma}{dk}(p,k) = \frac{C_s g^2}{16\pi p^7} \frac{e^{k/T}}{e^{k/T} \mp 1} \frac{e^{(p-k)/T}}{e^{(p-k)/T} \mp 1} \left\{ \begin{array}{ll} \frac{1+(1-x)^2}{x^3(1-x)^2} & q \to qg \\ N_f \frac{x^2+(1-x)^2}{x^2(1-x)^2} & g \to q\bar{q} \\ \frac{1+x^4+(1-x)^4}{x^3(1-x)^3} & g \to gg \end{array} \right\}$$

$$\times \int \frac{d^2\mathbf{h}}{(2\pi)^2} 2\mathbf{h} \cdot \mathrm{Re}\, \mathbf{F}(\mathbf{h},p,k)$$

$$2\mathbf{h} = i\,\delta E(\mathbf{h},p,k)\mathbf{F}(\mathbf{h},p,k) + g_s^2 \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} \frac{m_D^2}{\mathbf{q}_\perp^2(\mathbf{q}_\perp^2 + m_D^2)}$$
$$\times \{ (C_s - C_A/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h} - k\,\mathbf{q}_\perp)] + (C_A/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h} + p\,\mathbf{q}_\perp)]$$
$$+ (C_A/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h} - (p-k)\,\mathbf{q}_\perp)] \}$$

$$\delta E(\mathbf{h},p,k) = \frac{\mathbf{h}^2}{2pk(p-k)} + \frac{m_k^2}{2k} + \frac{m_{(p-k)}^2}{2(p-k)} - \frac{m_p^2}{2p} \qquad \mathbf{h} \equiv (\mathbf{k} \times \mathbf{p}) \times \mathbf{e}_{||}$$