Has Saturation Found Its Smoking Gun?

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Theorem (Hinchcliffe’s Rule)

Any headline that ends in a question mark can be answered by the word NO.
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The Saturation Problem

Structure of Protons and Nuclei

Saturation regime: gluon self-interactions become important

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Saturation regime: gluon self-interactions become important
The Saturation Problem

Why $pA$?

Saturation regime is

$$Q^2 \lesssim Q^2_s = c A^{1/3} Q_0^2 \left( \frac{x_0}{x} \right)^\lambda$$

- Heavy ions (large $A$) make saturation more accessible
- Light projectiles (protons) prevent QGP and medium effects

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- Heavy ions (large $A$) make saturation more accessible
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Factorization

$k_T$ factorization
- Central rapidity $Y \sim 0$
- $x_p, x_g \sim 0.1$
- Projectile and target treated in same model

Hybrid model
- Forward rapidity $Y \sim 3$ to $6$
- $x_p \gg x_g \sim 10^{-3}$
- Projectile treated in parton model
- Target treated as color glass condensate
- Suitable for saturation regime
$k_T$ factorization

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Cross section in the hybrid formalism:

\[
\frac{d^3\sigma}{dYd^2p_{\perp}} = \sum_i \int \frac{dz}{z^2} \frac{dx}{x} x f_i(x, \mu) D_{h/i}(z, \mu) F \left( x, \frac{p_{\perp}}{z} \right) \mathcal{P}(\xi)(\ldots)
\]

- Parton distribution (initial state projectile)
- Gluon distribution (initial state target)
- Fragmentation function (final state)
- Kinematic factors
Calculating Inclusive Hadron Production

History of the pA Calculation

- Dumitru and Jalilian-Marian (2002)
- Dumitru, Hayashigaki, and Jalilian-Marian (2006)
- Fujii et al. (2011)
- Albacete et al. (2013)
- Rezaeian (2013)
- Stašto, Xiao, and Zaslavsky (2014)
- Kang, Vitev, and Xing (2014)
- Stašto, Xiao, Yuan, et al. (2014)
- Altinoluk et al. (2014)
- Watanabe et al. (2015)
Dumitru and Jalilian-Marian (2002)

First calculation of inclusive cross section
No numerical results
Dumitru, Hayashigaki, and Jalilian-Marian (2006)

Prefactor $K = 1.6$
Fujii et al. (2011)

Prefactor $K = 1.5$ for charged particles
$K = 0.5$ for neutral particles
Inelastic Diagrams

Leading:

Next-to-leading:
Albacete et al. (2013)

Prefactor $K = 1$ for charged hadrons
$K = 0.4$ for neutral hadrons
Calculating Inclusive Hadron Production » Next to Leading Order

Impact Parameter-Dependent CGC

Rezaeian (2013)

No yield predictions
Calculating Inclusive Hadron Production » Next to Leading Order

NLO Diagrams

Leading:

Next-to-leading:
includes virtual corrections

\[ K = 1 \]
Rapidity Correction

Kang, Vitev, and Xing (2014)

Rapidity correction (believed unphysical) (by us)
Staśto, Xiao, Yuan, et al. (2014)

BRAHMS $\eta = 3.2$

Primitive kinematical constraint

more on constraints: Beuf 2014.
Altinoluk et al. (2014)

\[
\frac{2(1 - \xi)\xi x g P^+}{k_{\perp}^2} > \tau
\]

No numerical results
Kinematical Constraint

Watanabe et al. (2015)

\[ p \rightarrow 1 - \xi \]

\[ A \rightarrow \]  

Kinematical constraint
First LHC numerical results
The Kinematical Constraint

Kinematical Constraint

Constraint:

\[ \xi \leq 1 - \frac{l^2}{x_p s} \]

figure adapted from Watanabe et al. 2015.
Constraint:

\[ \xi \leq 1 - \frac{l^2_{\perp}}{x ps} \]

then

\[ \int_0^{1-l^2_{\perp}/x ps} \frac{d\xi}{1-\xi} = \ln \frac{x ps}{l^2_{\perp}} = \ln \frac{1}{x g} + \ln \frac{k^2_{\perp}}{l^2_{\perp}} \]

Result:

\[ \frac{d^3\sigma}{dY d^2p_{\perp}} = \text{LO} + \text{NLO} + L_q + L_g \]
Challenges for Numerical Calculation

- Remove singularities
- Compute Fourier integrals
- Reduce numerical error
Eliminate delta functions and plus prescriptions

\[
\int_{\tau}^{1} dz \int_{z}^{1} d\xi \left[ \frac{F_s(z, \xi)}{(1 - \xi)_+} + F_n(z, \xi) + F_d(z, \xi) \delta(1 - \xi) \right]
\]

\[
= \int_{\tau}^{1} dz \int_{\tau}^{1} dy \frac{z - \tau}{z(1 - \tau)} \left[ \frac{F_s(z, \xi) - F_s(z, 1)}{1 - \xi} + F_n(z, \xi) \right]
\]

\[
+ \int_{\tau}^{1} dz \left[ F_s(z, 1) \ln \left( 1 - \frac{\tau}{z} \right) + F_d(z, 1) \right]
\]
Fourier integrals are highly imprecise

\[ \int d^2 \vec{r}_\perp S_Y^{(2)}(r_\perp) e^{i \vec{k}_\perp \cdot \vec{r}_\perp} (\ldots) \]

\[ \int d^2 \vec{s}_\perp S_Y^{(4)}(r_\perp, s_\perp, t_\perp) e^{i \vec{k}_\perp \cdot \vec{r}_\perp} (\ldots) \]

Easiest solution: transform to momentum space

\[ F(k_\perp) = \frac{1}{(2\pi)^2} \int\int d^2 \vec{r}_\perp S_Y^{(2)}(r_\perp) e^{i \vec{k}_\perp \cdot \vec{r}_\perp} = \frac{1}{2\pi} \int_0^\infty dr_\perp S_Y^{(2)}(r_\perp) J_0(k_\perp r_\perp) \]

and compute \( F \) directly
Fourier integrals are highly imprecise

\[ \int d^2 \vec{r}_\perp S_Y^{(2)}(r_\perp) e^{i \vec{k}_\perp \cdot \vec{r}_\perp} (\ldots) \]

\[ \int d^2 \vec{s}_\perp S_Y^{(4)}(r_\perp, s_\perp, t_\perp) e^{i \vec{k}_\perp \cdot \vec{r}_\perp} (\ldots) \]

Alternate solution: algorithms for direct evaluation of multidimensional Fourier integrals (not explored)
\[
\int \frac{d^2 x_\perp}{(2\pi)^2} S(x_\perp) \ln \frac{c_0^2}{x_\perp^2 \mu^2} e^{-i k_\perp \cdot x_\perp}
= \frac{1}{\pi} \int \frac{d^2 l_\perp}{l_\perp^2} \left[ F(k_\perp + l_\perp) - J_0 \left( \frac{c_0}{\mu} l_\perp \right) F(k_\perp) \right]
\]

\[
\int \frac{d^2 r_\perp}{(2\pi)^2} S(r_\perp) \left( \ln \frac{r_\perp^2 k_\perp^2}{c_0^2} \right)^2 e^{-i k_\perp \cdot r_\perp}
= \frac{2}{\pi} \int \frac{d^2 l_\perp}{l_\perp^2} \ln \frac{k_\perp^2}{l_\perp^2} \left[ \theta(k_\perp - l_\perp) F(k_\perp) - F(k_\perp + l_\perp) \right]
\]
Numerical Adaptation

Remaining Evaluation Errors

- Inaccuracy of Fourier integrals
- Monte Carlo statistical error
- Cancellation of large terms

Multiple runs to improve statistics
Numerical Adaptation

Remaining Evaluation Errors

- Inaccuracy of Fourier integrals
- Monte Carlo statistical error
- Cancellation of large terms

Two parallel implementations of selected parts:
- Mathematica, for rapid prototyping
- C++, for execution speed

$\alpha_s N_c / (\pi^2 k_{\perp}^4)$
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Results with Kinematical Correction

RHIC Results

New terms improve matching at low $p_\perp$

data: Arsene et al. 2004; Adams et al. 2006.
plots: Staśto, Xiao, and Zaslavsky 2014; Watanabe et al. 2015.
Results with Kinematical Correction

RHIC Results

New terms improve matching at low $p_{\perp}$

data: Arsene et al. 2004; Adams et al. 2006.
plots: Staśto, Xiao, and Zaslavsky 2014; Watanabe et al. 2015.
rcBK calculation matches neatly up to \( p_\perp \approx 6 \text{ GeV} \)

data: Milov 2014.
plots: Watanabe et al. 2015.
Higher rapidity alters low-$p_{\perp}$ result

GBW

rcBK $\Lambda_{QCD}^2 = 0.01$

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Higher rapidity alters low-$p_{\perp}$ result
Importance of Higher Rapidity

Higher rapidity alters low-$p_\perp$ result

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\[ \frac{d^5\sigma_{pA \rightarrow hX}}{dY d^2p_\perp d^2b_\perp} = \int \frac{dz dx}{z^2} q(x, Q_f^2) D_{q/h}(z, Q_f^2) \frac{d^5\sigma^{tot}_{qA}}{dY_q d^2q_\perp d^2b_\perp} \]

**Latest result**

- First *complete* numerical implementation of the NLO $pA \rightarrow h + X$ cross section (no, really this time, we promise)
- First numerical results at LHC parameters

Potentially sensitive probe of small-$x$ gluon distribution
Conclusion

Outlook

Future work:

1. Investigate hotel bar
2. Investigate higher order corrections or resummation
3. Use data to tune models of gluon distribution

Critical step

More forward-rapidity data from LHC experiments
Section 7

Supplemental Slides
Derivation of the Kinematical Constraint

\[ p \left( x_P^+ , 0 , 0_{\perp} \right) \rightarrow A \left( 0 , x_g P^- , k_{\perp} + l_{\perp} \right) \rightarrow \times \left( \xi x_P^+ , k^- , k_{\perp} \right) \]

\[ (1 - \xi) x_P^+ , l^- , l_{\perp} \]

\[ x_g P^- = \frac{l_{\perp}^2}{2(1 - \xi)x_P^+} + \frac{k_{\perp}^2}{2\xi x_P^+} \leq P^- \]

\[ x_g \leq 1 \]

\[ \xi \leq 1 - \frac{l_{\perp}^2}{x_P s} \]

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figure adapted from Watanabe et al. 2015.
The Beam Direction Problem

- $y < 0$
- $y = 0$ (LHC)
- $y > 0$

$p, d \rightarrow A$

forward hadron production

figure adapted from Watanabe et al. 2015.
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data: Milov 2014; Abelev et al. 2013.
plots: Watanabe et al. 2015.
LHC Predictions for Run II

GBW

rcBK $\Lambda_{QCD}^2 = 0.01$

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