Has Saturation Found Its Smoking Gun?

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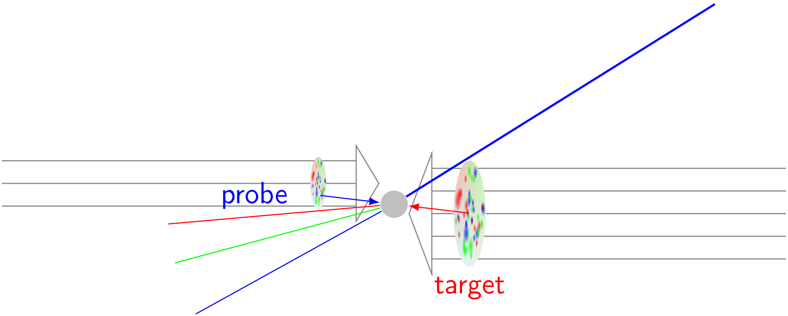
Réncontres du Vietnam 2015

Theorem (Hinchcliffe's Rule)

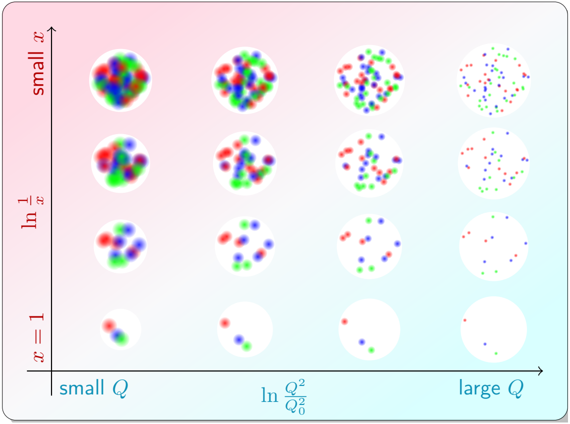
Any headline that ends in a question mark can be answered by the word ***NO***.

Wikipedia: Betteridge's law of headlines

Initial State



Structure of Protons and Nuclei



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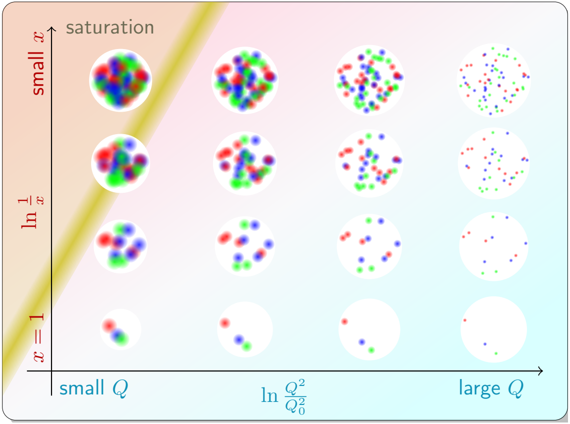
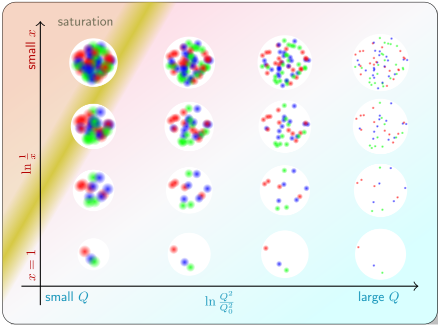


Saturation regime: gluon self-interactions become important

Why pA?

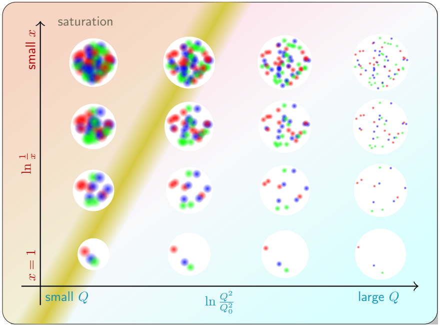


Saturation regime is

$$Q^2 \lesssim Q_s^2 = cA^{1/3} Q_0^2 \left(\frac{x_0}{x}\right)^\lambda$$

- Heavy ions (large A) make saturation more accessible
- Light projectiles (protons) prevent QGP and medium effects

Why $\mathrm{p}A$?



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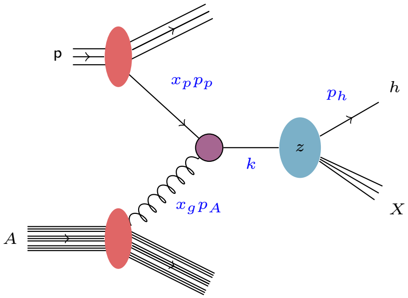
Factorization



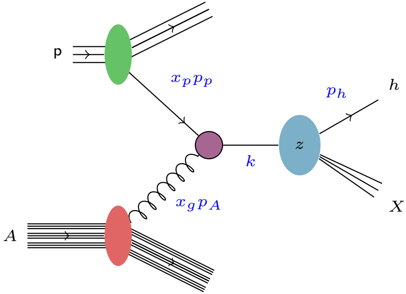
k_T factorization

- Central rapidity $Y \sim 0$
- $x_p, x_g \sim 0.1$
- Projectile and target treated in same model

Hybrid model

- Forward rapidity $Y \sim 3$ to 6
- $x_p \gg x_g \sim 10^{-3}$
- Projectile treated in parton model
- Target treated as color glass condensate
- Suitable for saturation regime

Factorization



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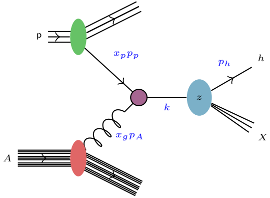
- Suitable for saturation regime

Hybrid Model

Cross section in the hybrid formalism:

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}Y\,\mathrm{d}^2\vec{p_\perp}} = \sum_i \int \frac{\mathrm{d}z}{z^2}\frac{\mathrm{d}x}{x} x f_i(x,\mu) D_{h/i}(z,\mu) F\left(x, \frac{p_\perp}{z}\right) \mathcal{P}(\xi)(\ldots)$$

- Parton distribution (initial state projectile)
- Gluon distribution (initial state target)
- Fragmentation function (final state)
- Kinematic factors



History of the pA Calculation

- Dumitru and Jalilian-Marian (2002)
- Dumitru, Hayashigaki, and Jalilian-Marian (2006)
- Fujii et al. (2011)
- Albacete et al. (2013)
- Rezaeian (2013)
- Staśto, Xiao, and Zaslavsky (2014)
- Kang, Vitev, and Xing (2014)
- Staśto, Xiao, Yuan, et al. (2014)
- Altinoluk et al. (2014)
- Watanabe et al. (2015)

Gluon dist

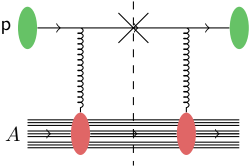
- GBW
- MV/AAMQS
- LO BK
- rcBK
- b-CGC
- NLO BK

Cross section

- LO
- inel NLO
- other NLO
- rapidity NLO
- splitting NLO

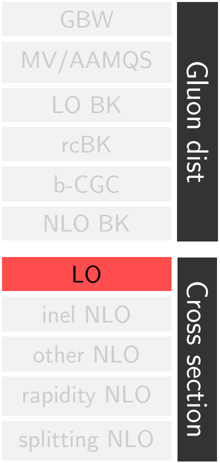
First Calculation

Dumitru and Jalilian-Marian (2002)



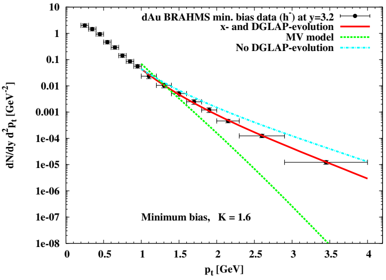
First calculation of inclusive cross section

No numerical results



First Numerical Results

Dumitru, Hayashigaki, and Jalilian-Marian (2006)



Prefactor $K = 1.6$

Gluon dist

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Cross section

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- rapidity NLO
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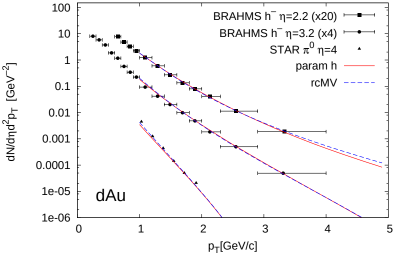
Incorporating rcBK

Fujii et al. (2011)



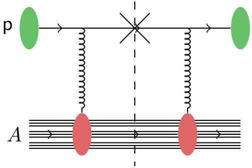
Prefactor $K = 1.5$ for charged particles
$K = 0.5$ for neutral particles

GBW
MV/AAMQS
LO BK
rcBK
b-CGC
NLO BK
Gluon dist

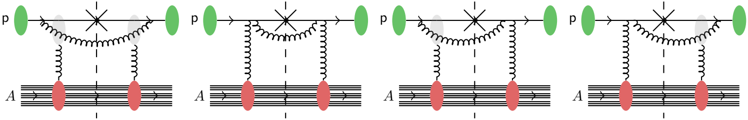
LO
inel NLO
other NLO
rapidity NLO
splitting NLO
Cross section

Inelastic Diagrams

Leading:

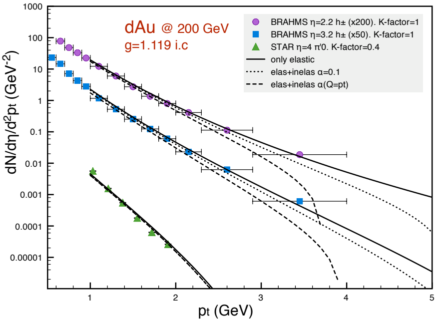


Next-to-leading:



Inelastic NLO Terms

Albacete et al. (2013)



Prefactor $K = 1$ for charged hadrons
$K = 0.4$ for neutral hadrons

GBW
MV/AAMQS
LO BK
rcBK
b-CGC
NLO BK

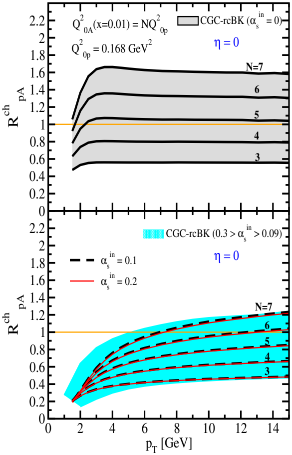
Gluon dist

LO
inel NLO
other NLO
rapidity NLO
splitting NLO

Cross section

Impact Parameter-Dependent CGC

Rezaeian (2013)



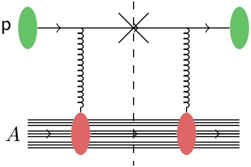
Gluon dist:
- GBW
- MV/AAMQS
- LO BK
- **rcBK**
- **b-CGC**
- NLO BK

Cross section:
- **LO**
- inel NLO
- other NLO
- rapidity NLO
- splitting NLO

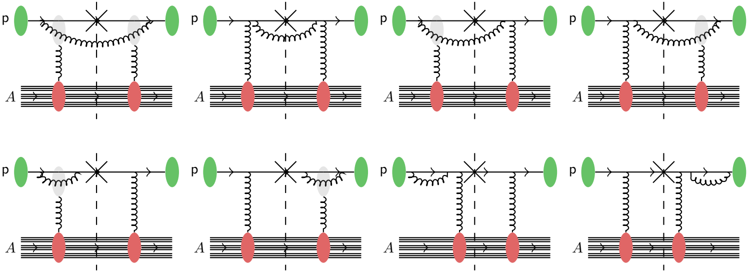
No yield predictions

NLO Diagrams

Leading:



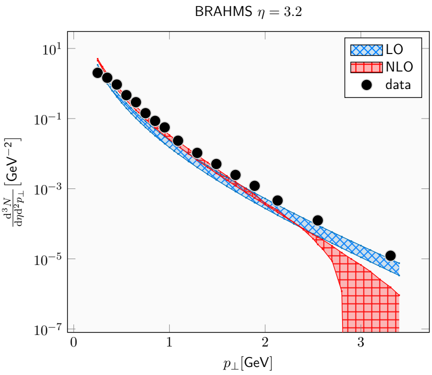
Next-to-leading:



Chirilli, Xiao, and Yuan 2012

NLO Numerical Result

Staśto, Xiao, and Zaslavsky (2014)



Gluon dist
GBW
MV/AAMQS
LO BK
rcBK
b-CGC
NLO BK

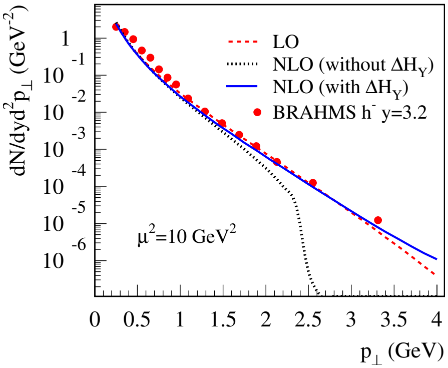
Cross section
LO
inel NLO
other NLO
rapidity NLO
splitting NLO

Includes virtual corrections

$K = 1$

Rapidity Correction

Kang, Vitev, and Xing (2014)



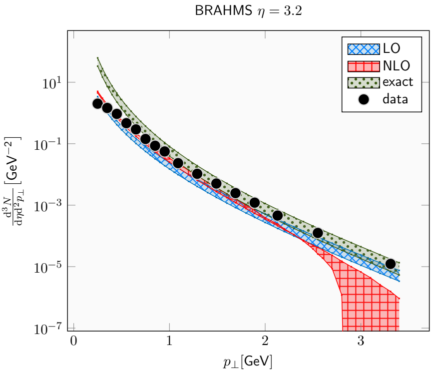
Gluon dist: GBW, MV/AAMQS, LO BK, rcBK, b-CGC, NLO BK

Cross section: LO, inel NLO, other NLO, rapidity NLO, splitting NLO

Rapidity correction (believed unphysical) (by us)

Matching to Collinear

Staśto, Xiao, Yuan, et al. (2014)



Gluon dist
GBW
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Cross section
LO
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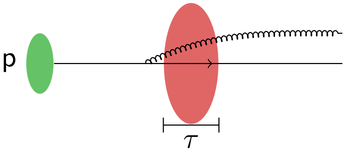
Primitive kinematical constraint

more on constraints: Beuf 2014.

Ioffe Time

Altinoluk et al. (2014)



$$\frac{2(1-\xi)\xi x_g P^+}{k_\perp^2} > \tau$$

No numerical results

GBW
MV/AAMQS
LO BK
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b-CGC
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Gluon dist
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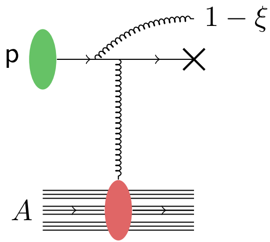
other NLO
rapidity NLO
splitting NLO
Cross section

Kinematical Constraint

Watanabe et al. (2015)



Kinematical constraint
First LHC numerical results

Gluon dist

- GBW
- MV/AAMQS
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- NLO BK

Cross section

- LO
- inel NLO
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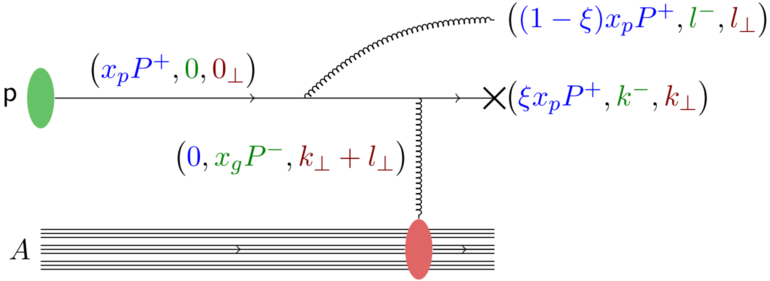
Kinematical Constraint



Constraint:

$$\xi \leq 1 - \frac{l_\perp^2}{x_p s}$$

figure adapted from Watanabe et al. 2015.

Extra Terms

Constraint:

$$\xi \leq 1 - \frac{l_\perp^2}{x_p s}$$

then

$$\int_0^{1-l_\perp^2/x_p s} \frac{\mathrm{d}\xi}{1-\xi} = \ln\frac{x_p s}{l_\perp^2} = \ln\frac{1}{x_g} + \underbrace{\ln\frac{k_\perp^2}{l_\perp^2}}_{\text{missed}}$$

Result:

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}Y\,\mathrm{d}^2p_\perp} = \mathsf{LO} + \mathsf{NLO} + L_q + L_g$$

Watanabe et al. 2015.

Challenges for Numerical Calculation

- Remove singularities
- Compute Fourier integrals
- Reduce numerical error

Removing Singularities

Eliminate delta functions and plus prescriptions

$$\int_\tau^1 \mathrm{d}z \int_{\frac{\tau}{z}}^1 \mathrm{d}\xi \left[\frac{F_s(z,\xi)}{(1-\xi)_+} + F_n(z,\xi) + F_d(z,\xi)\delta(1-\xi)\right]$$
$$= \int_\tau^1 \mathrm{d}z \int_\tau^1 \mathrm{d}y \frac{z-\tau}{z(1-\tau)} \left[\frac{F_s(z,\xi) - F_s(z,1)}{1-\xi} + F_n(z,\xi)\right]$$
$$+ \int_\tau^1 \mathrm{d}z \left[F_s(z,1)\ln\left(1-\frac{\tau}{z}\right) + F_d(z,1)\right]$$

$$\delta^2(\vec{r}_\perp) \int \frac{\mathrm{d}^2\vec{r}'_\perp}{r'^2_\perp} e^{i\vec{k}_\perp\cdot\vec{r}'_\perp} - \frac{1}{r^2_\perp} e^{-i\xi'\vec{k}_\perp\cdot\vec{r}_\perp}$$
$$= \frac{1}{4\pi}\int \mathrm{d}^2\vec{k}'_\perp e^{-i\vec{k}'_\perp\cdot\vec{r}_\perp} \ln\frac{(\vec{k}'_\perp - \xi'\vec{k}_\perp)^2}{k^2_\perp}$$

Fourier Integrals

Fourier integrals are highly imprecise

$$\int \mathrm{d}^2\vec{r}_\perp S_Y^{(2)}(r_\perp) e^{i\vec{k}_\perp\cdot\vec{r}_\perp}(\ldots)$$

$$\int \mathrm{d}^2\vec{s}_\perp S_Y^{(4)}(r_\perp, s_\perp, t_\perp) e^{i\vec{k}_\perp\cdot\vec{r}_\perp}(\ldots)$$

Easiest solution: transform to momentum space

$$\begin{aligned} F(k_\perp) &= \frac{1}{(2\pi)^2}\iint \mathrm{d}^2\vec{r}_\perp S_Y^{(2)}(r_\perp) e^{i\vec{k}_\perp\cdot\vec{r}_\perp} \\ &= \frac{1}{2\pi}\int_0^\infty \mathrm{d}r_\perp S_Y^{(2)}(r_\perp) J_0(k_\perp r_\perp) \end{aligned}$$

and compute F directly

Fourier Integrals

Fourier integrals are highly imprecise

$$\int \mathrm{d}^2\vec{r}_\perp S_Y^{(2)}(r_\perp)e^{i\vec{k}_\perp\cdot\vec{r}_\perp}(\ldots)$$

$$\int \mathrm{d}^2\vec{s}_\perp S_Y^{(4)}(r_\perp, s_\perp, t_\perp)e^{i\vec{k}_\perp\cdot\vec{r}_\perp}(\ldots)$$

Alternate solution: algorithms for direct evaluation of multidimensional Fourier integrals (not explored)

New Fourier Transforms

$$\int \frac{\mathrm{d}^2 x_\perp}{(2\pi)^2} S(x_\perp) \ln \frac{c_0^2}{x_\perp^2 \mu^2} e^{-ik_\perp \cdot x_\perp}$$
$$= \frac{1}{\pi} \int \frac{\mathrm{d}^2 \vec{l_\perp}}{l_\perp^2} \left[F(\vec{k_\perp} + \vec{l_\perp}) - J_0\left(\frac{c_0}{\mu} l_\perp\right) F(k_\perp) \right]$$

$$\int \frac{\mathrm{d}^2 r_\perp}{(2\pi)^2} S(r_\perp) \left(\ln \frac{r_\perp^2 k_\perp^2}{c_0^2}\right)^2 e^{-ik_\perp \cdot r_\perp}$$
$$= \frac{2}{\pi} \int \frac{\mathrm{d}^2 \vec{l_\perp}}{l_\perp^2} \ln \frac{k_\perp^2}{l_\perp^2} \left[\theta(k_\perp - l_\perp) F(k_\perp) - F(\vec{k_\perp} + \vec{l_\perp}) \right]$$

Watanabe et al. 2015.

Remaining Evaluation Errors

- Inaccuracy of Fourier integrals
- Monte Carlo statistical error
- Cancellation of large terms

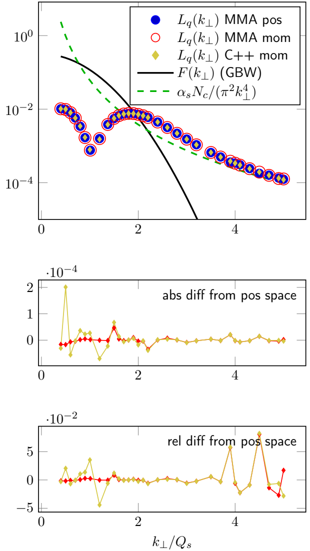
Multiple runs to improve statistics

Remaining Evaluation Errors

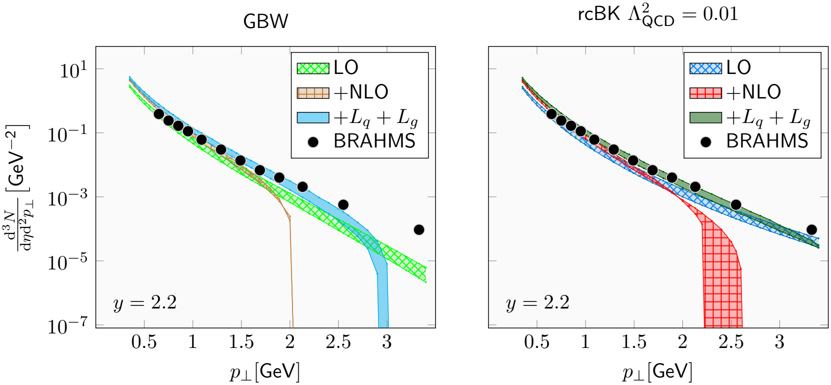
- Inaccuracy of Fourier integrals
- Monte Carlo statistical error
- Cancellation of large terms

Two parallel implementations of selected parts:

- Mathematica, for rapid prototyping
- C++, for execution speed



RHIC Results

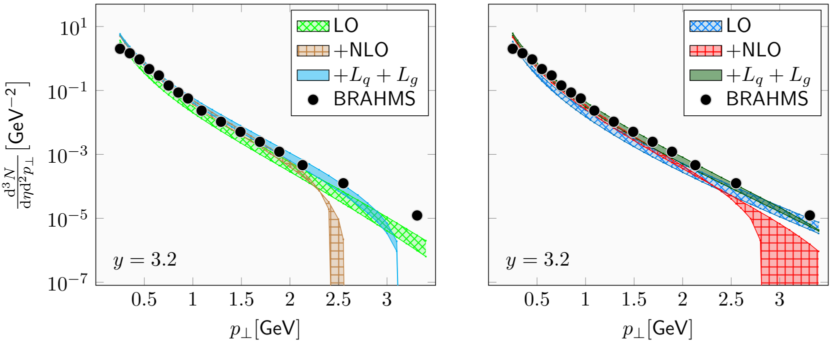


New terms improve matching at low $p_\perp$

data: Arsene et al. 2004; Adams et al. 2006.

plots: Staśto, Xiao, and Zaslavsky 2014; Watanabe et al. 2015.

RHIC Results



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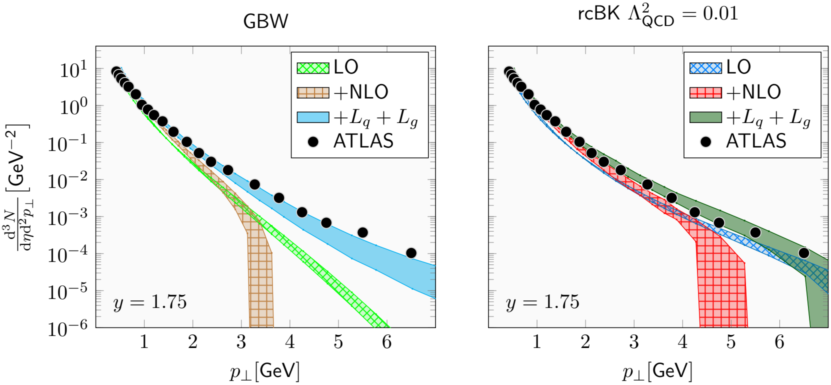
LHC Results

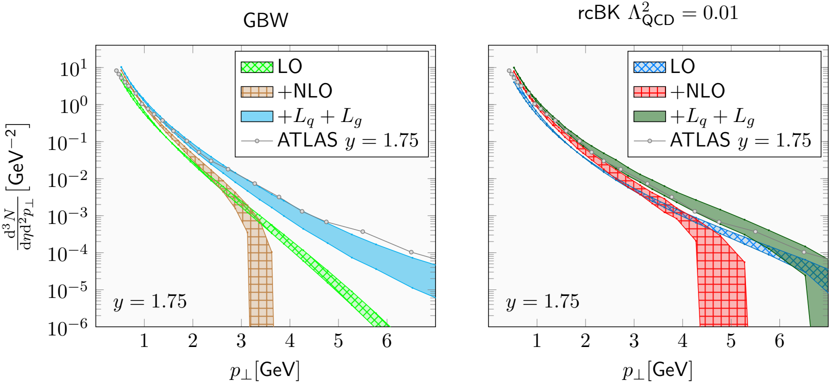


rcBK calculation matches neatly up to $p_\perp \approx 6$ GeV

data: Milov 2014.

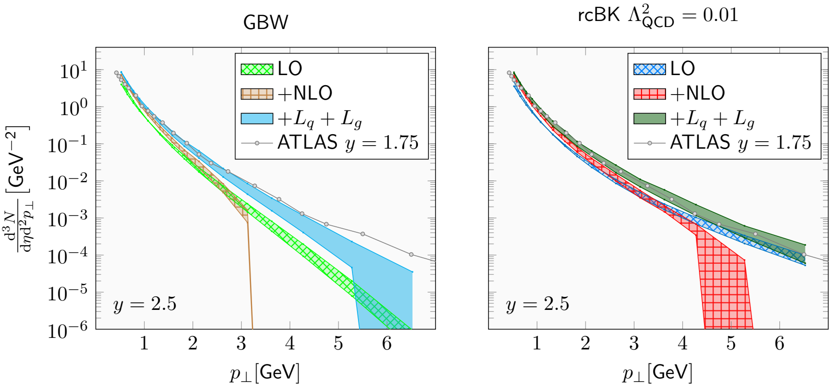
plots: Watanabe et al. 2015.

Importance of Higher Rapidity



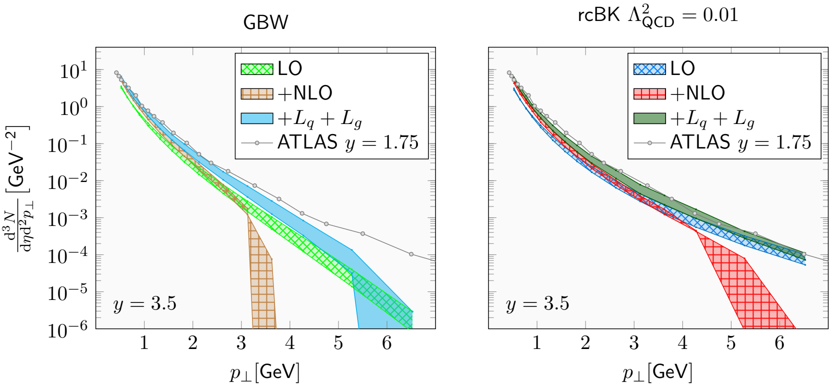
Higher rapidity alters low-$p_\perp$ result

Importance of Higher Rapidity



Higher rapidity alters low-$p_\perp$ result

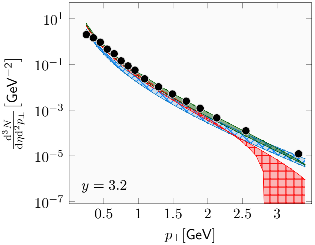
Importance of Higher Rapidity



Higher rapidity alters low-$p_\perp$ result

Summary

$$\frac{d^5\sigma^{pA\to hX}}{dY d^2p_\perp d^2b_\perp} = \int \frac{dz dx}{z^2} q(x, Q_f^2) D_{q/h}(z, Q_f^2) \frac{d^5\sigma_{qA}^{\text{tot}}}{dY_q d^2q_\perp d^2b_\perp} \longrightarrow$$



Latest result

- First *complete* numerical implementation of the NLO $pA \to h + X$ cross section (no, really this time, we promise)
- First numerical results at LHC parameters

Potentially sensitive probe of small-x gluon distribution

Outlook

Future work:

1. Investigate hotel bar
2. Investigate higher order corrections or resummation
3. Use data to tune models of gluon distribution

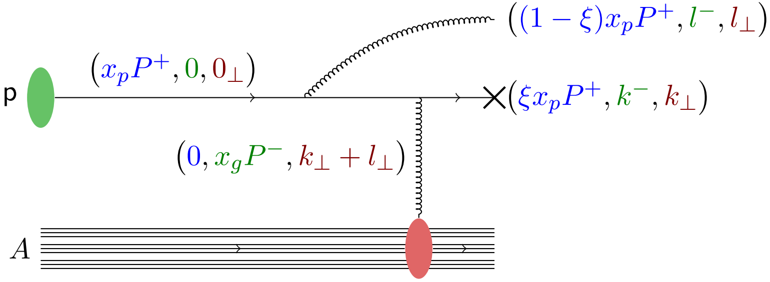
Critical step

More forward-rapidity data from LHC experiments

Section 7

Supplemental Slides

Derivation of the Kinematical Constraint



$$x_gP^- = \frac{l_\perp^2}{2(1-\xi)x_pP^+} + \frac{k_\perp^2}{2\xi x_pP^+} \leq P^-$$

$$x_g \leq 1$$

$$\xi \leq 1 - \frac{l_\perp^2}{x_ps}$$

figure adapted from Watanabe et al. 2015.

The Beam Direction Problem

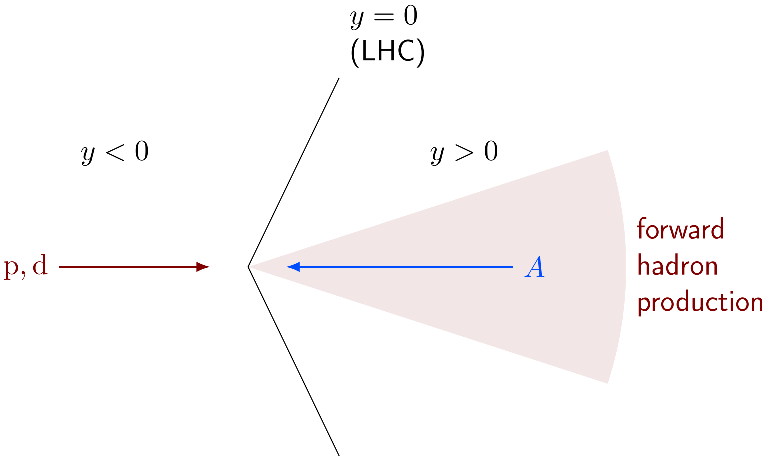
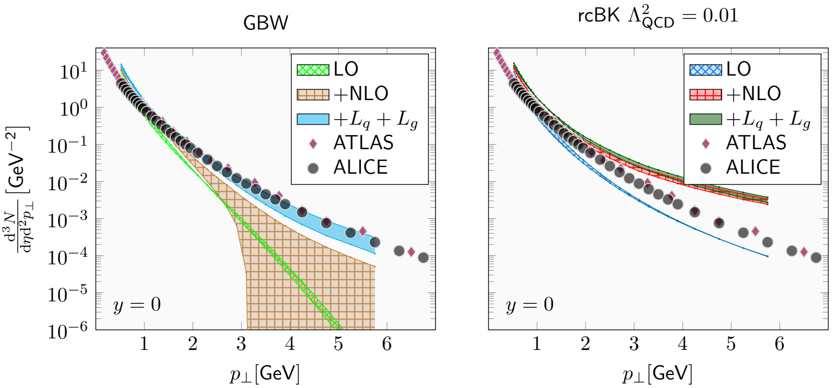


figure adapted from Watanabe et al. 2015.

LHC Results at Central Rapidity



data: Milov 2014; Abelev et al. 2013.

plots: Watanabe et al. 2015.

LHC Predictions for Run II