Propagation of photons along the direction of the magnetic field.

Quantum Faraday Effect.

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STARS2015-SMFNS2015

Outline

1. Introduction, Faraday rotation angle.

2. Propagation of photons along the direction of the magnetic field

    . Quantum Faraday Effect, initial results
    . Solution of the dispersion equation near the thresholds.

3. Diluted gas: magnetosphere of neutron stars.
    . Quantum Faraday Effect.
    . Solution of the dispersion equation near the thresholds.

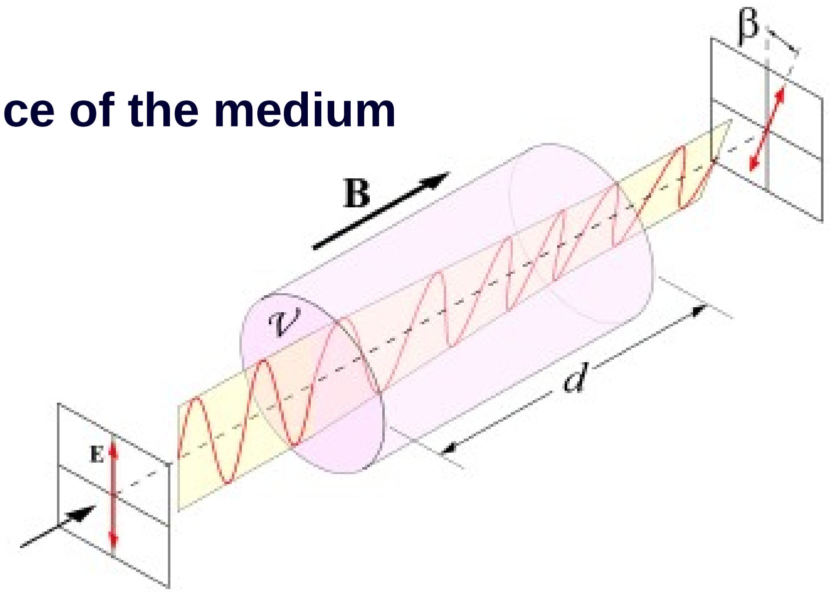
4. Summary.

5. Work in progress.

Faraday Effect, our initial motivation

It is well known that an electromagnetic wave propagating through a medium in an ambient magnetic field suffers Faraday rotation.

Classical explanation of the phenomenon: **birefringence of the medium**



Astrophysical applications!!!!!!

. Measurements of the magnetic field in the interstellar medium.

. Particle density in the ionosphere

. Difference between the amount of matter and antimatter

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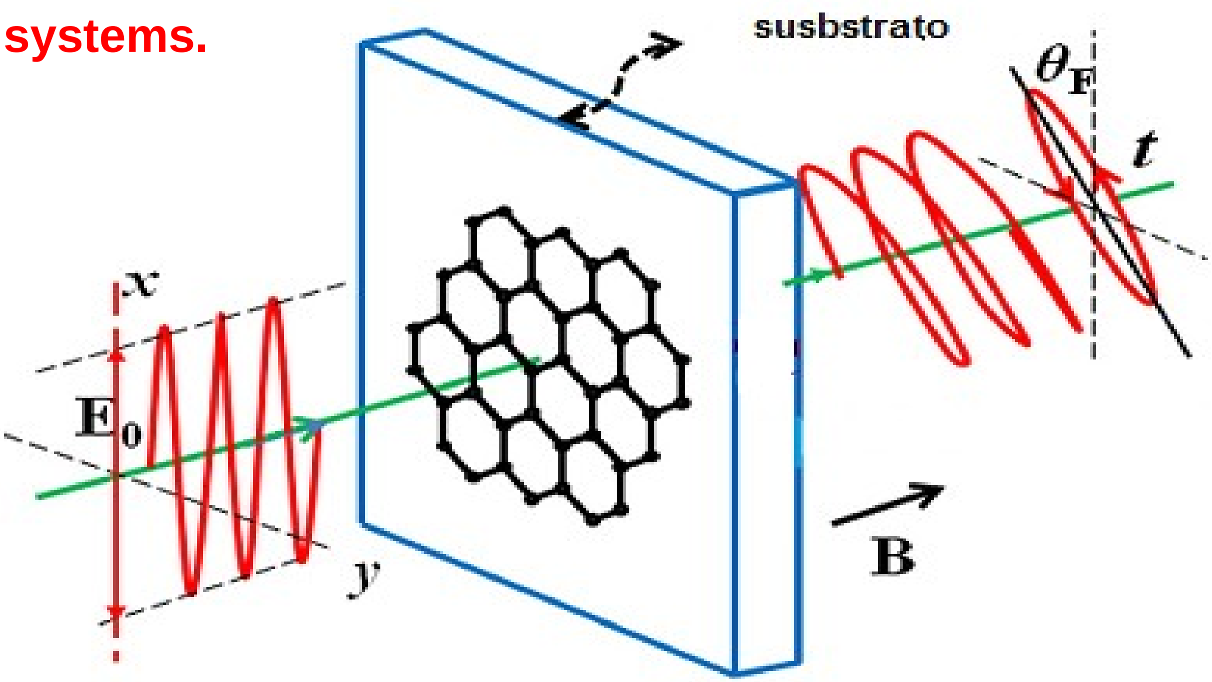
Faraday rotation in graphene-like systems.



J. Appl. Phys. 113, 17B529 (2013)

Faraday rotation has been reported in graphene, we use QFT, to describe the astrophysical background and also, graphene-like systems.

Faraday Effect, our initial motivation

Main result for graphene-like systems: **Faraday angle** → **Hall Conductivity**

Our aim: To test if in the 3D plasma the same result is obtained

Faraday Effect and Hall Conductivity

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Propagation of photons along the direction of an external magnetic field

Faraday Effect in a 3D electron-positron gas

2D limit, graphene-like systems

We obtain that the Faraday angle is proportional, in the first order approximation, to the Hall conductivity. ***L. Cruz, A. Pérez, H. Pérez, E. Rodríguez PRA 88, 052126 (2013)***

Now we are interested in to use our model for a possible astrophysical scenario, then we continue with the calculation in the 3D electron-positron gas.

Polarization tensor, theoretical overview.

Maxwell equation: Photons propagating along the direction of the external magnetic field

$$[k^2 g_{\mu\nu} + \Pi_{\mu\nu}(k \mid A_\mu^{ext})] = 0$$

→ Polarization tensor, Interaction with the medium

The diagonalization of the polarization tensor leads to the equation:

$$\prod_{\mu\nu} C_\nu^{(i)} = \kappa^{(i)} C_\mu^{(i)}$$

⇒ Having three non vanishing eigenvalues and three eigenvectors.

For the particular case of propagation along the external field B the second mode is a pure longitudinal wave and the transverse modes are:

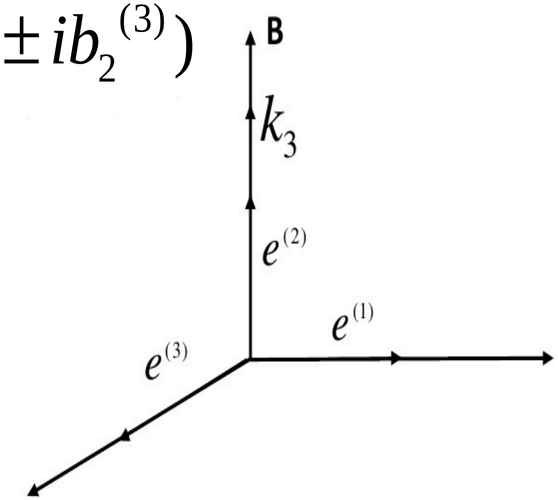
$$C_\mu^{\prime(1,3)} = R(C_\mu^{(3)} \pm i C_\mu^{(1)})$$

⇒ This describes two circularly polarized waves in the plane orthogonal to B having different eigenvalues.

Quantum Faraday effect

Quantum Faraday Effect

The electric polarization vectors can then be written as $e^{(1,3)} = (b_1^{(1)} \pm i b_2^{(3)})$



Corresponding to the eigenvalues $\kappa^{(1,3)} = t \pm \sqrt{-r^2}$

Dispersion equation $$k_{\pm} = \sqrt{\omega^2 + \kappa^{(1,3)}}$$

Faraday Angle $$\theta_F = \frac{1}{2}(k_+ - k_-)L$$

$$r(k_3, \omega, B, T, \mu)$$

$$t(k_3, \omega, B, T, \mu)$$

Solution of the dispersion equation in the long wave limit

$$r(k_3 \approx 0, \omega, B, T, \mu)$$

$$t(k_3 \approx 0, \omega, B, T, \mu)$$

Faraday angle, degenerate limit

Degenerate limit **μ>>T**

Neutron stars $T_F \approx 10^{12} K \quad T \approx 10^8 K$

$$n_e(\varepsilon_{p,n}) = \theta(\mu - \varepsilon_{p,n}) \qquad n_p(\varepsilon_{p,n}) \to 0$$

$$\frac{\theta_F}{L} \approx \frac{1}{2c}\frac{\operatorname{Im} r}{\omega} = \frac{1}{2c}\sigma^H_{3D}$$

$$r = \frac{i\omega e^3 B}{2\pi^2}\sum_{n,n'} F^{(3)}_{n,n'}(0)\int_{-\infty}^{\infty} dp_3 \frac{(k_\parallel^{\;2} + 2eB(n+n'))}{|Q|^2}(n_e(\varepsilon_{p,n}) - n_p(\varepsilon_{p,n}))$$

$$|Q|^2 = [2p_3k_3 + k_\parallel^{\;2} + 2eB(n'-n)]^2 - 4\omega^2\varepsilon^2_{p,n}$$

$$\frac{1}{s-\omega-i\varepsilon} = VP\frac{1}{s-\omega} + i\pi\delta(s-\omega)$$

Far from the threshold we
only have the contribution of the **PV.**

Faraday angle, degenerate limit

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Singularities

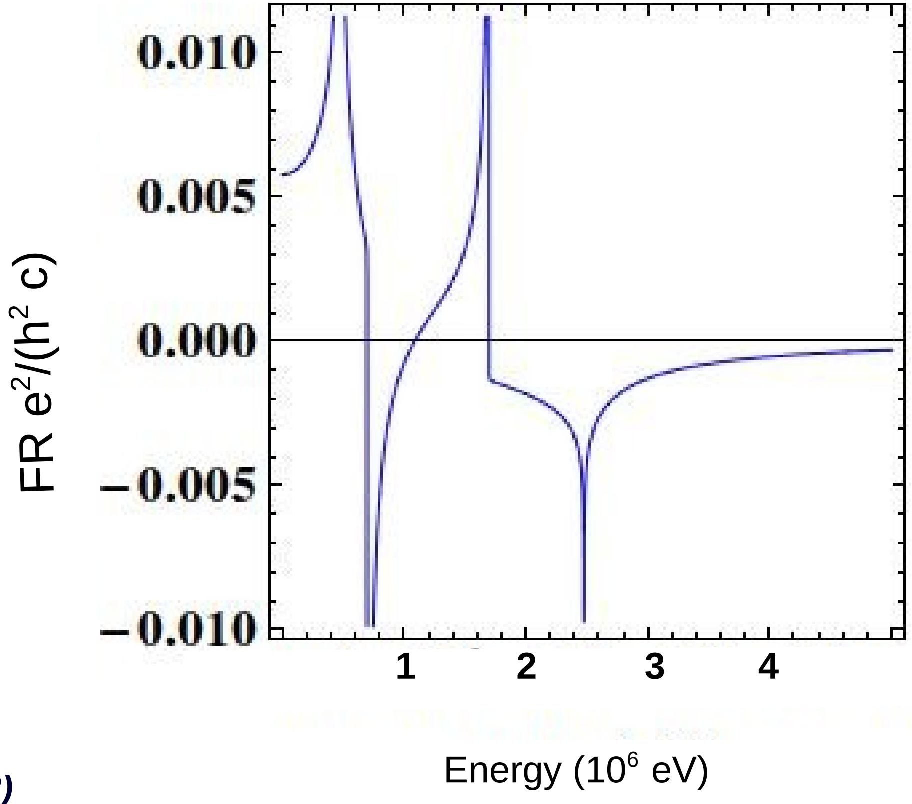
$$\sigma_H^{3D} = -\frac{e^3 B}{2\pi^2} \sum_{n,n'}^{n_\mu, n'_\mu} F^{(3)}_{n,n'}(0) \frac{k_\parallel^2 + 2eB(n+n')}{2\omega\Lambda} \left(\ln \left| \frac{p_F - p_3^{(2)}}{p_F + p_3^{(2)}} \right| + \ln \left| \frac{p_F + p_3^{(1)}}{p_F - p_3^{(1)}} \right| \right)$$

$$\Lambda = (2eB(n'-n) - \omega^2)^2 - 4\omega^2 \varepsilon_{0n}^2$$

Conductivity and Faraday angle threshold, related to photon absorption process.

$$B = 10^{14} G,\ n_\mu = 0$$

$$\mu = 1 MeV$$



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Singularities

$$\sigma_H^{3D} = -\frac{e^3 B}{2\pi^2}\sum_{n,n'}^{n_\mu ,n'_\mu} F^{(3)}_{n,n'}(0)\,\frac{k_\|^2+2eB(n+n')}{2\omega\Lambda}\left(\ln\left|\frac{p_F - p_3^{(2)}}{p_F + p_3^{(2)}}\right| + \ln\left|\frac{p_F + p_3^{(1)}}{p_F - p_3^{(1)}}\right|\right)$$

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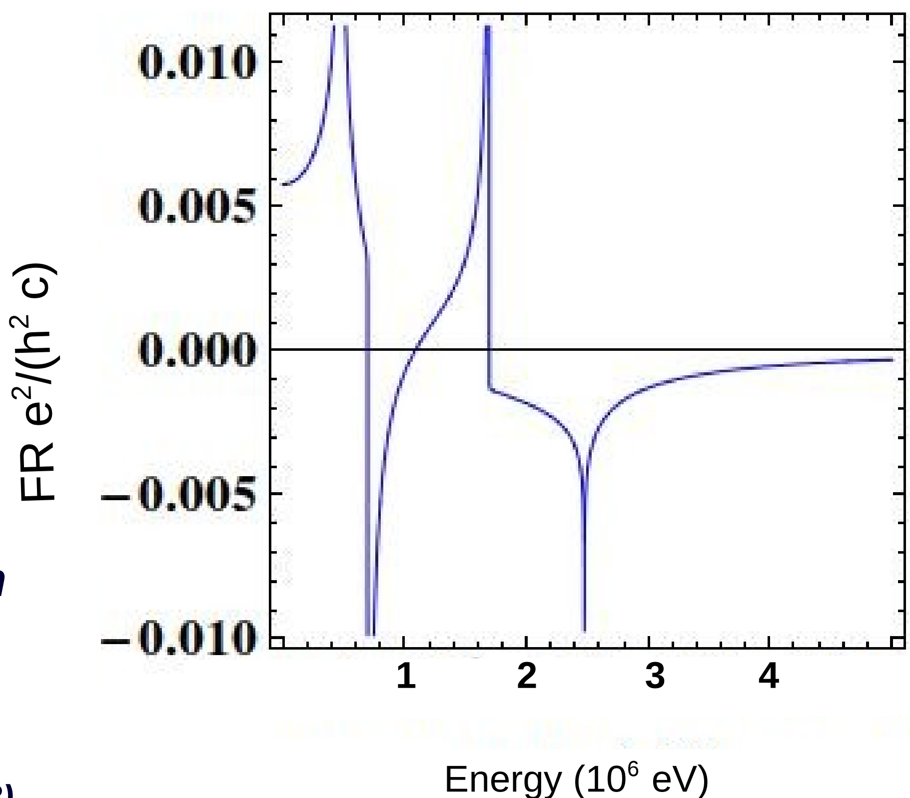

In order to solve the singularities the dispersion equation should be solved near the threshold

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Solution of the dispersion equation near the threshold

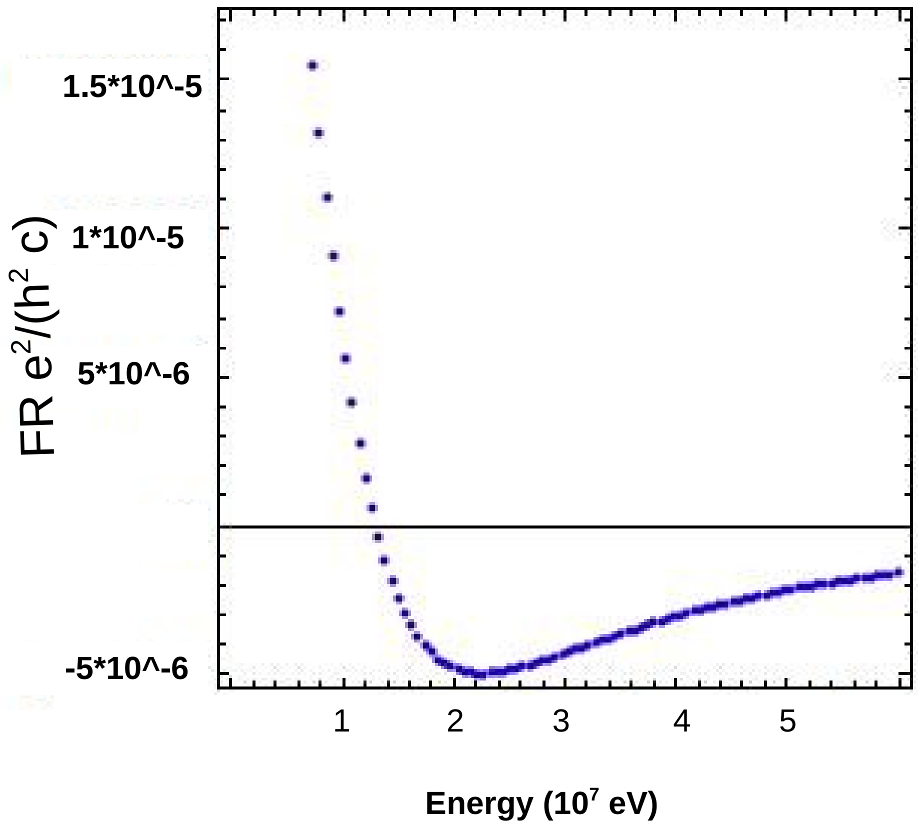
Near the threshold we have the contribution of the imaginary part

$$\frac{1}{s-\omega-i\varepsilon} = VP\frac{1}{s-\omega} + i\pi\delta(s-\omega)$$

ω = ω +i Γ

Absorption

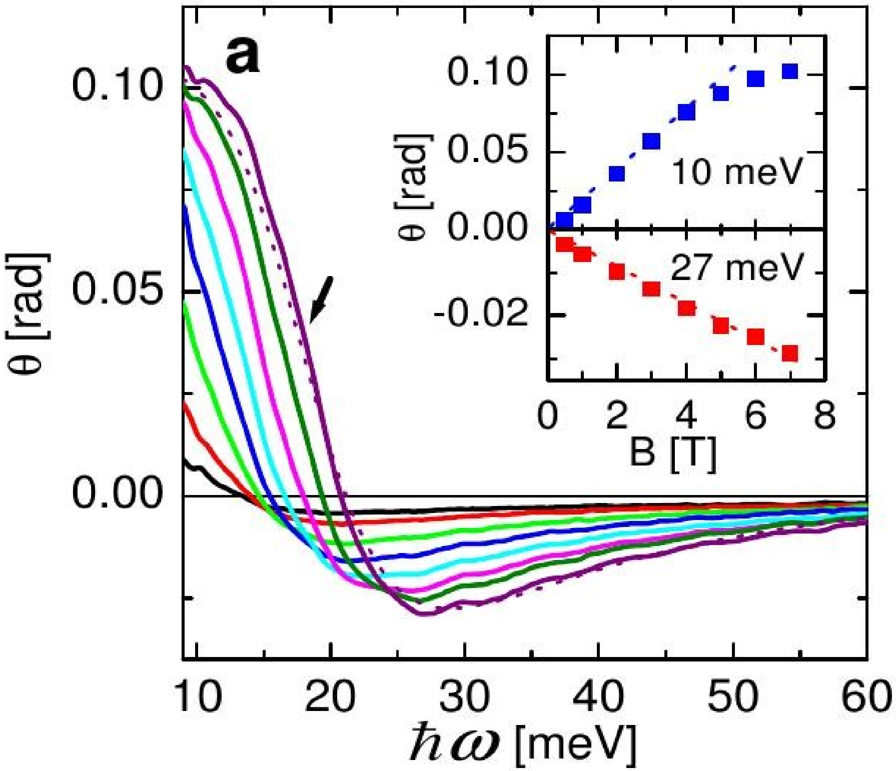
Finite Faraday angle!!!!!!

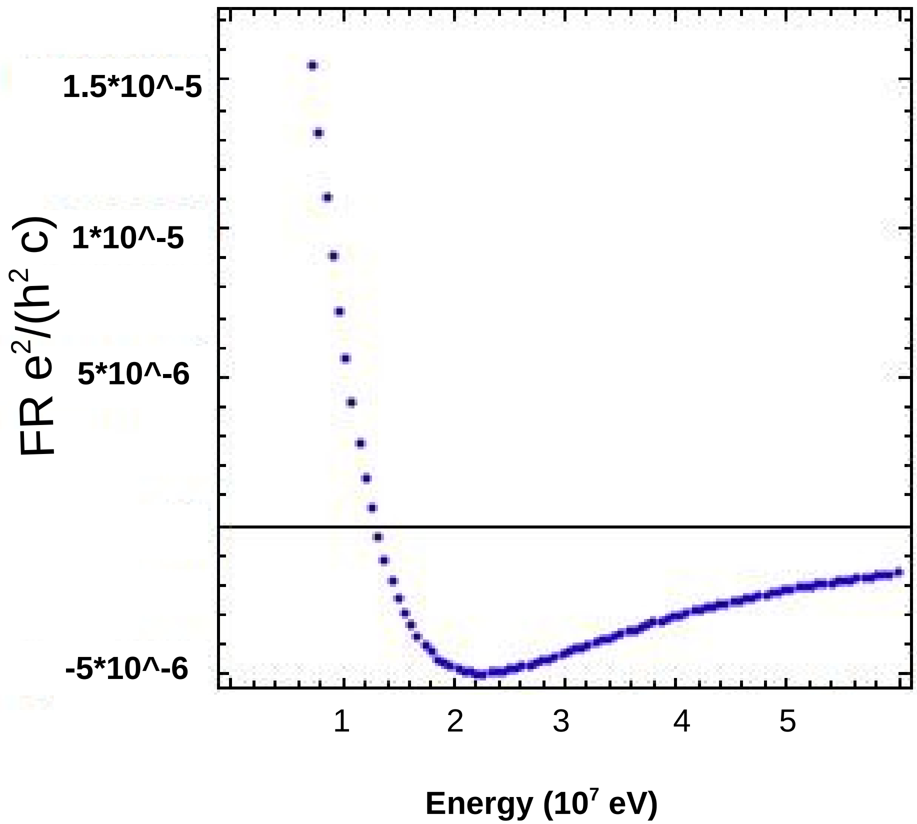


$$\mu = 1MeV \quad B = 10^{14}G,\ n_\mu = 0$$

Solution of the dispersion equation near the threshold

To test our results





Graphene plate

3D plasma

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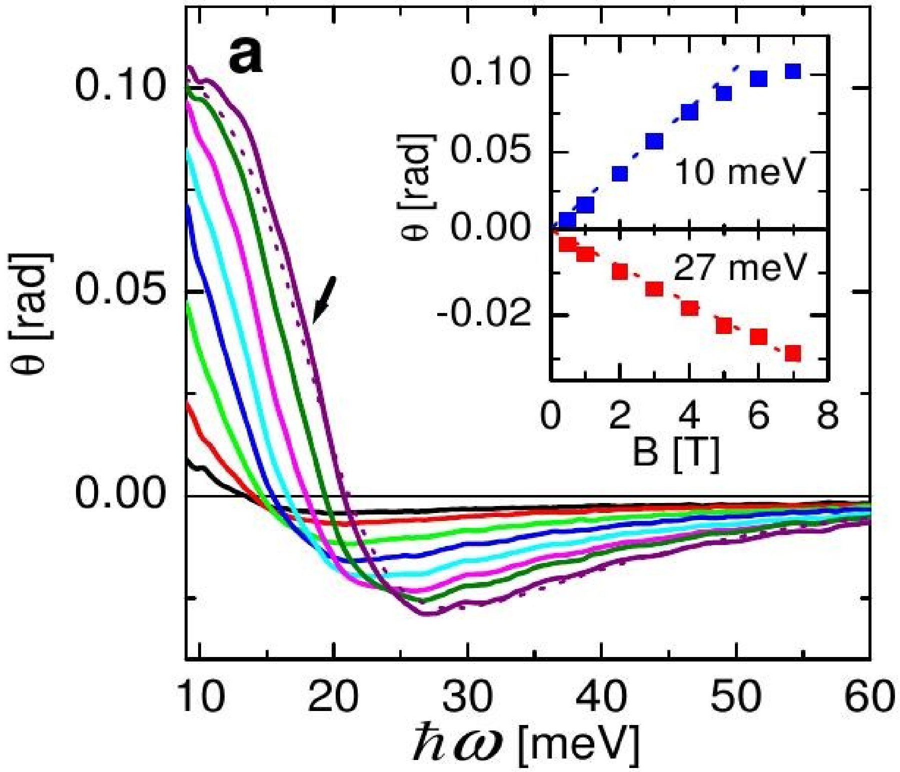

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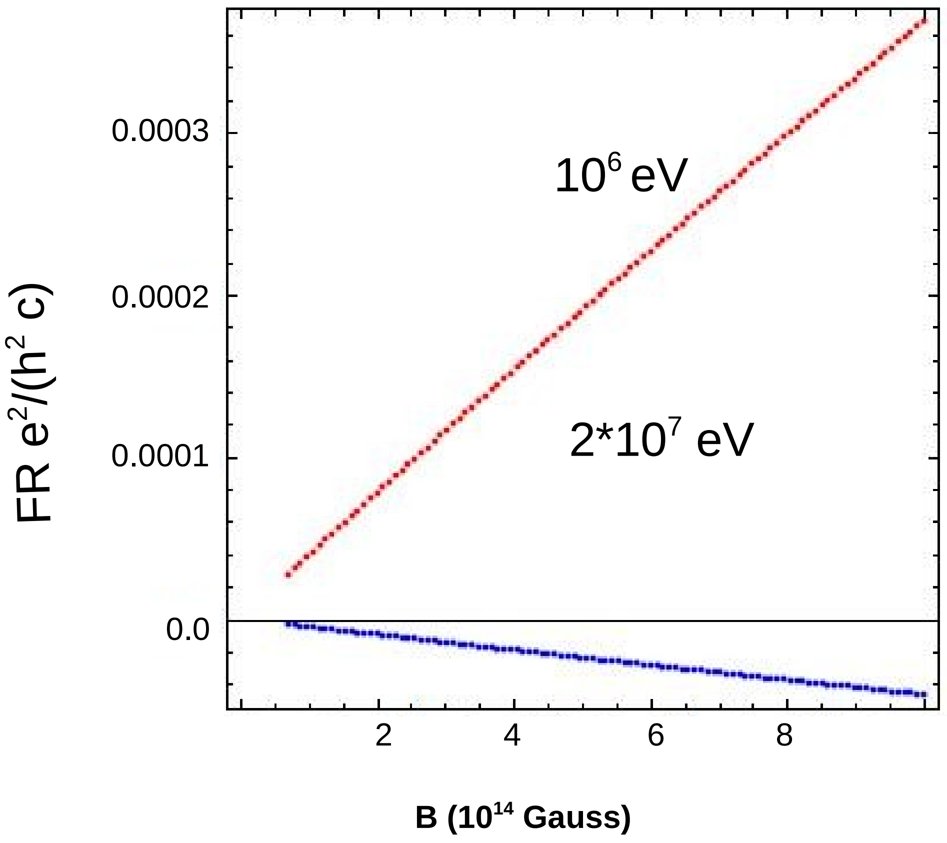


3D plasma

Diluted gas limit

In the magnetosphere of neutron stars the distribution of particle follow a Boltzmann distribution:

$$n_{e,p} = \frac{1}{1 + e^{(\varepsilon_e \mp \mu)\beta}} = e^{-(\varepsilon_e \mp \mu)\beta}$$

Main approximations:

$\mu \ll T$ Relativistic gas

Strong field limit, LLL

Dispersion equation, Faraday angle !!!!!

$$k_{\pm} = \sqrt{\omega^2 + \kappa^{(1,3)}} \longrightarrow \kappa^{(1,3)} = t \pm \sqrt{-r^2}$$

$$\theta_F = \frac{1}{2}(k_+ - k_-)L$$

Our aim now is to solve the dispersion equation and to find the Faraday angle in the diluted gas approximation.

Faraday angle

$$\theta_l = \frac{4e^3 B e^{\mu\beta} K(m\beta)}{8} \frac{w}{w^2 - (eB)^2}$$

Resonant behavior, related to photon absorption

In some pulsars surface:

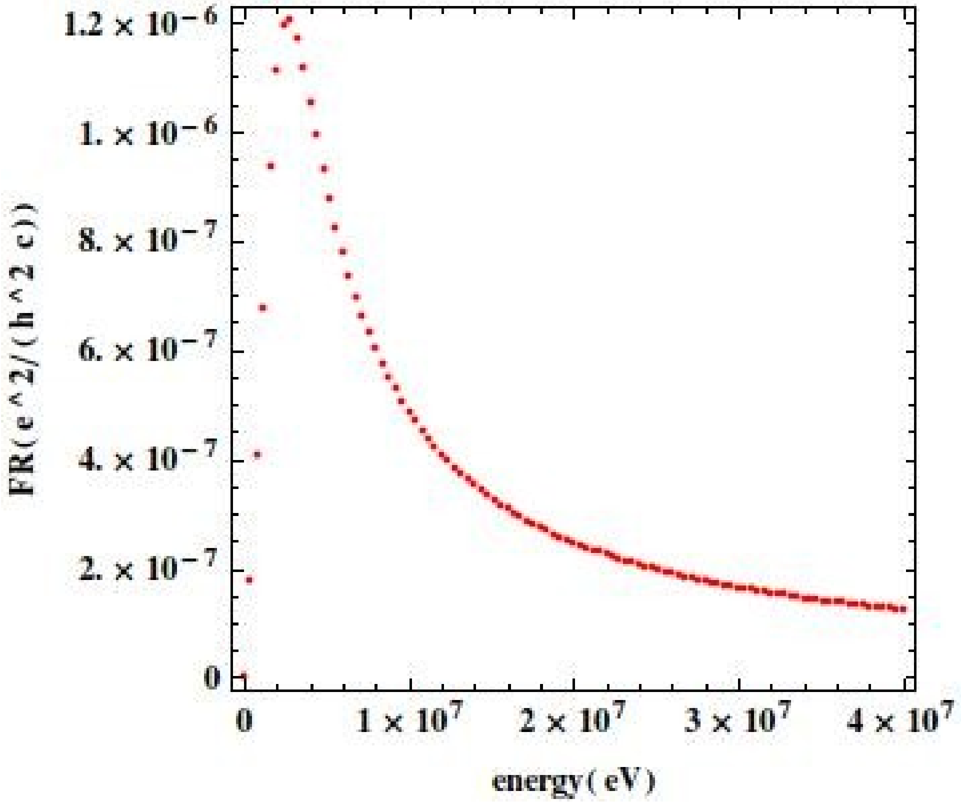
- ω ~10^2 eV
- B~ 10^12 G

eB>>ω^2

But some magnetars radiates in the X and Gamma spectrum

eB~ω^2

Solution of the dispersion equation near the threshold , **absorption processes.**



Summary.

From the formalism of QFT the FR angle, in a 3D relativistic plasma and in a 2D massless relativistic fermion gas could be studied, a relation between the FR angle and the Hall conductivity is obtained.

The solution of the dispersion equation in the long wave ($k_3 \to 0$), degenerate and strong field limit was considered near the threshold.

A finite value for the Faraday angle was obtained. The effect is enhanced due to the cyclotron resonance. **The change in the sign of the angle could be used as a way to measure the energy of the incoming radiation..... and the magnetic field....**

We moved forward to the description of a more realistic astrophysical context, and studied the solution of the dispersion equation and the Faraday angle in the diluted gas limit.

A resonant Faraday angle was obtained, and the corrections near the thresholds were considered.

Work in progress and other results.

We are working in the weak field limit in order to reproduce the semi-classical approximation.

We are considering a power law decay for the magnetic field and the particle density to estimate the FR angle,

$$B, \mu \sim 1/r^3 .$$

Main goal: to propose a model which could be used with a real FR measurement in an astrophysical context.