Chiral Magnetic Effect in QED induced by longitudinal photons.

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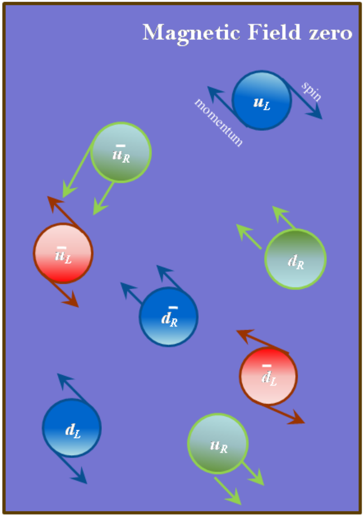

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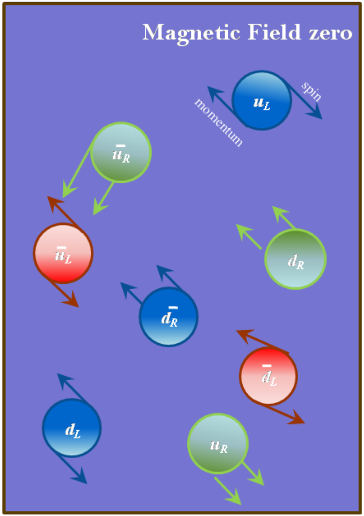

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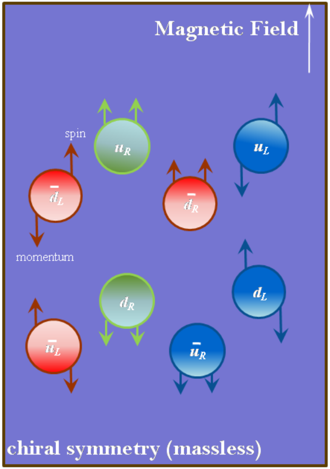
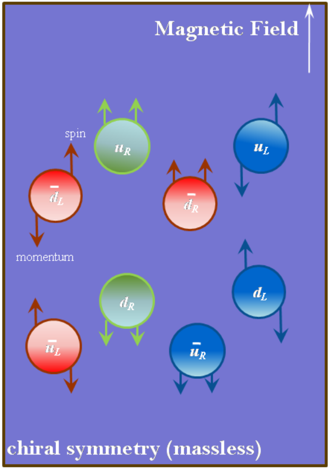


- A magnetic field will align the spins, depending on their electric charge.
- The momenta of the quarks align along the magnetic field. A quark with right-handed helicity will have momentum opposite to a left-handed one.
- !!!In this way the magnetic field can distinguish between right and left!!!



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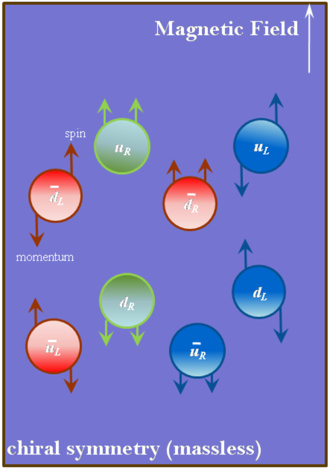

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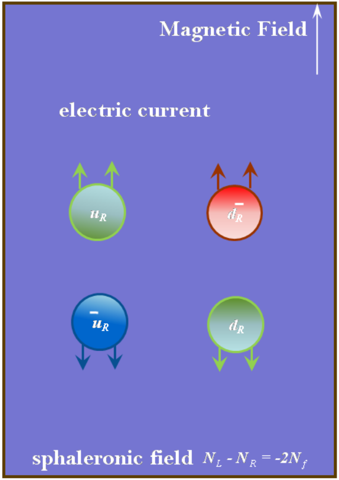
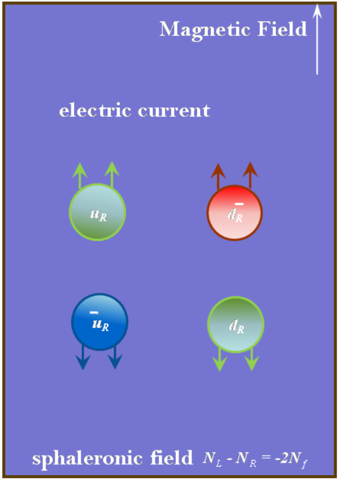


- What does a magnetic field do with chirality (generated by topological charge)?
- Positively charged particles move parallel to the magnetic field.
- Negatively charged particles move antiparallel to the magnetic field.
- !!An electric current is created along the magnetic field!!⇔ The Chiral Magnetic Effect in QCD.



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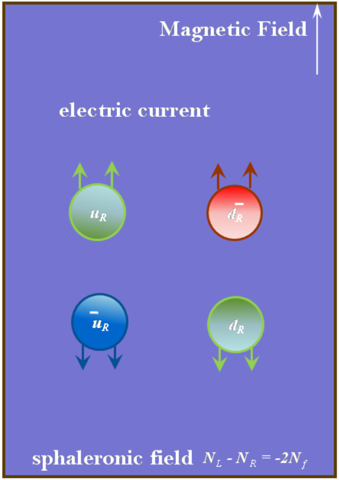

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Objectives.

- Demonstrate the existence of the chiral magnetic effect in an electron-positron magnetized gas in QED. *A pseudo-vector Ohm current is induced by the electric field related to the longitudinal photons in QED propagating parallel to the external magnetic field* ***B*** *and separating opposite charges of the same heliticity*.
- Obtain an anomaly relation in a medium of massive particles in presence of an external magnetic field, that bears some analogy to the Adler-Bell-Jackiw. *The effect is interesting in connection to the QCD chiral magnetic case reported in current literature*.

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- Study the process associated to the chiral symmetry in a charged medium in presence of an external field **B**.

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- Study the process associated to the chiral symmetry in a charged medium in presence of an external field **B**.

The equation of Schwinger-Dyson for the photon:

$$[k^2 g_{\mu\nu} - \Pi_{\mu\nu}(k|A_\mu^{ext})]a^\nu(k) = 0$$

- a_μ (*radiation field*) is a small perturbation added to A_μ^{ext} (*external field*)
- the total external electromagnetic field is $A_\mu^{ext} + a_\mu$ (*the electric field of the wave* $E \ll B$).

The second term takes into account the interaction of the radiation with the external field through the virtual pairs (e^+, e^-) and the excitation of particles in electron-positron plasma.

The quantum corrections are given for the tensor of polarization $\Pi_{\mu\nu}$.

In the external constant field B, electrons and positrons move in bound states characterized by energy levels given by:

$$\varepsilon_{n_l,p_3} = \sqrt{p_3^2 + m^2 + |e|B(2n_l + 1 - sgn(e)s_3)}$$

- $s_3 = \pm 1$ are spin eigenvalues along x_3.
- $n_l = 0, 1, ...$ are the Landau quantum numbers. These are two-fold spin degenerate, except for the ground state ε_0 in which $n_l = 0$, and for electrons is $s_3 = -1$ and for positrons $s_3 = 1$.
- Quantum states are also degenerate with regard to the orbit's center coordinates.

For a system in equilibrium at temperature T and chemical potential μ.

Net density of charged particles in the ground state (LLL)

$$N_0 = \frac{eB}{2\pi^2}\int_{-\infty}^{0} dp_3(n_R^e - n_L^p) + \int_0^{\infty} dp_3(n_L^e - n_R^p)$$

Magnetization in the LLL

$$\mathcal{M}_0 = \frac{e}{4\pi^2}\int_{-\infty}^{0} \frac{p_3^2 dp_3}{\varepsilon_0}(n_R^e + n_L^p) + \int_0^{\infty} \frac{p_3^2 dp_3}{\varepsilon_0}(n_L^e + n_R^p)$$

where $n^{e,p} = [1 + e^{(\varepsilon_0 \mp \mu)/T}]^{-1}$ (T is given in energy units)are the electron and positron densities.

- !!!!both terms in the sum of the right side are equal!!!! $\Rightarrow$!!! equal densities of L,R helicities in the state of equilibrium!!!

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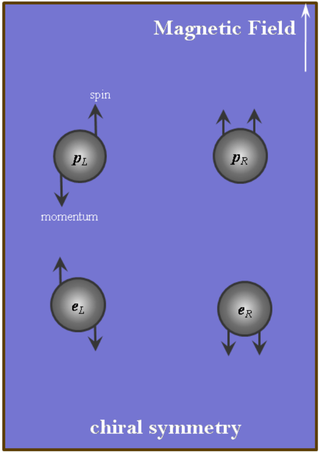

- The magnetic moments are aligned along **B**, showing a paramagnetic behavior.
- left-electrons and right-positrons move parallel to the magnetic field.
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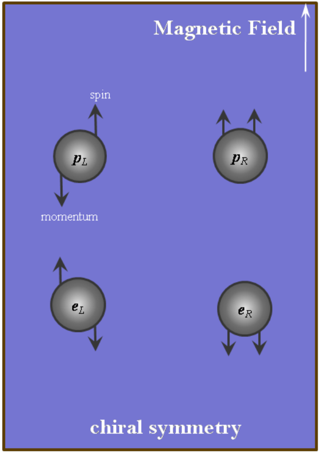
- Notice that to higher Landau quantum numbers contribute both to paramagnetic and diamagnetic terms.



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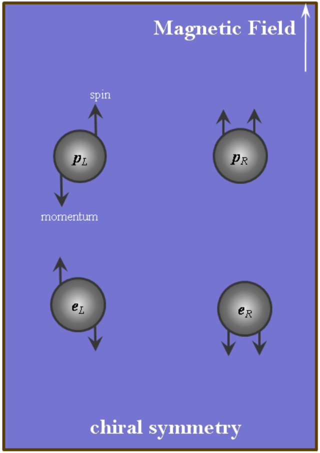

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If we will make here the fundamental assumption that (with $\mu \gtrsim m$): $\boxed{eB \gg m^2, \mu, T^2}$ $\Rightarrow$ LLL dominant.

The diagonalization of the photon self-energy tensor leads to the equation:

$$\Pi_{\mu\nu} b^{\nu(i)} = \eta_i b_\mu^{(i)}$$

having three non-vanishing eigenvalues η_i and three eigenvectors $b_\mu^{(i)}$ for $i = 1, 2, 3$, corresponding to three photon propagation modes.

For each mode it is obtained a dispersion law

$$k^2 = \eta_i(k_3, k_\perp, \omega, B)$$

$k^2 = k_3^2 + k_\perp^2 - \omega^2$, k_3 and $k_\perp$ are respectively the components of the photon four-momentum in directions $\|$ and $\perp$ to **B**, and ω its energy.

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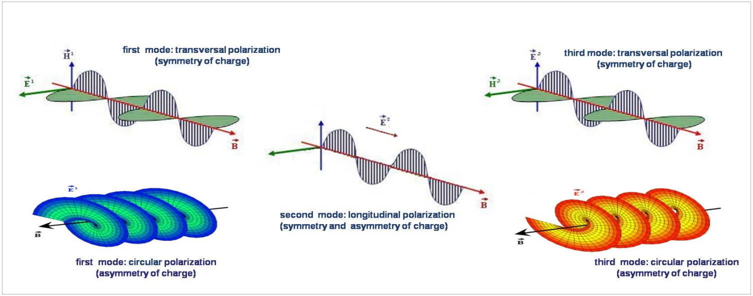
In a charged medium, for propagation along the field **B**, in addition to the two transverse modes, there is a longitudinally polarized eigenmode independent of the charge-symmetry.

Propagation parallel to the magnetic field



The longitudinal mode is a pseudo-vector

$$b_\mu^{(2)}(k) = R\tilde{F}_{\mu\nu}k^\nu$$

- R is an arbitrary constant
- $\tilde{F}_{\mu\nu}$ is the dual of the electromagnetic field tensor $F_{\mu\nu}$

The electric polarization vector associated to longitudinal mode

$$\mathbf{E}^{(2)} = R(k_3^2 - \omega^2)\mathbf{B}$$

!!!! For QED in a charged $e^\pm$ medium, the longitudinal mode along **B** is not on the light cone, that is $k_3^2 - \omega^2 \neq 0$!!!!

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The electromagnetic current as a function of $A_\mu^{ext} + a_\mu$ depends on the two relativistic invariants:

$$\mathfrak{F} = \frac{1}{4} F^{\mu\nu} F_{\mu\nu} = \frac{1}{2}(B^2 - E^2) \simeq \frac{1}{2} B^2, \quad E \ll B$$

$$\mathfrak{G} = \frac{1}{4} \tilde{F}^{\mu\nu} F_{\mu\nu} = \mathbf{B} \cdot \mathbf{E}$$

Notice that for the case of propagation along **B**, the pseudo-scalar $\mathfrak{G} \neq 0$ only for the longitudinal mode.

An expansion in functional series gives:

$$j_\mu(A_\mu^{ext} + a_\mu) = j_\mu(A_\mu^{ext}) + (\delta j_\mu / \delta A_\nu^{ext}) a_\nu + \ldots$$

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Its linear term in a_ν is:

$$j_i = \Pi_{i\nu} a_\nu = Y_{ij} E_j, \quad \nu = 1, 2, 3, 4, \quad i, j = 1, 2, 3$$

- $E_j = i(\omega a_j - k_j a_0)$ is the electric field, with $a_4 = i a_0$ and $k_4 = i\omega$.
- $j_\mu(A_\mu^{ext}) = \rho \delta_{\mu 4}$, where ρ is the charge density.
- a_μ is in general a linear combination of the eigenmodes $b_\mu^{(i)}$. Below we particularize to the case in which a_μ is the eigenvector $a_\mu = b_\mu^{(2)}$, for which $(\mathbf{E} \parallel \mathbf{B})$.

The complex conductivity tensor or admittivity is:

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We must observe that from the linear approximation of j_i, and the eigenvalue equation one gets also:

$$j_i = \Pi_{i\nu} b^{\nu(2)} = s b_i^{(2)}$$

$s = b_\nu^{(2)} \Pi^\nu_\rho b^{\rho(2)}$ is the eigenvalue of the photon self-energy tensor corresponding to the longitudinal mode.

- !!!!As $b_\nu^{(2)}$ is a pseudo-vector !!!! $\Rightarrow$!!!For propagation along B the current j_ν is also a pseudo-vector!!!
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Using the two-dimensional transversality condition $\boxed{\Pi_{\mu\nu}k_\nu = 0}$, we obtain:

The non-conservation of the two-dimensional axial current:

$$k_\mu j_A^\mu = \frac{z_1}{k_4} j_3 \neq 0$$

Here $z_1 = k_3^2 - \omega^2$, we are interested only in the region of real frequency $k_3^2 > z_1$ and momentum.

- This puts in evidence the role of the electric field, characterizing the longitudinal pseudo-vector mode, in the breaking of the chiral symmetry in both the C-symmetric and non-symmetric cases.

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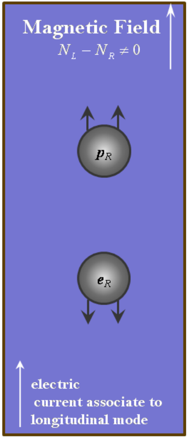
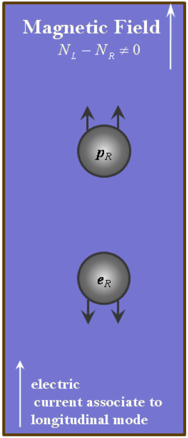


- The longitudinal mode produces an electric current along **B**, in which opposite charged particles, moving in opposite directions, have the same helicity.
- !!!Thus, a chiral magnetic effect is produced in the frame of QED!!!



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The current density can be written in the general form as:

$$j_i = \sigma^0_{ij} E_j + (E \times S)_i$$

$$\sigma^0_{ij} = Im[\Pi^s_{ij}]/\omega, \quad S_i = \frac{1}{2}\epsilon^{ijk}\sigma^H_{jk}$$

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- ϵ^{ijk} is the third rank antisymmetric unit tensor
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Here we will only work with the Ohm current associated to the longitudinal mode.

For the specific case of propagation along **B**.

The current density associated to the longitudinal mode can be expressed in the form:

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Here we will only work with the Ohm current associated to the longitudinal mode.

For the specific case of propagation along **B**.

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Scalar s in the one-loop approximation

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The term ΔH accounts for the pair creation (only in the region $z_1 < -4\varepsilon_{n,0}^2$).

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Anomaly relation in a medium of massive particles

$$k_\mu j_A^\mu = a[m\mathbb{A}(m) + \mathbb{C}(m)]\frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B}$$

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In the $m \to 0$ limit, it is obtained: $\mathbb{A}(m \to 0) \to 0$ and $\mathbb{C}(m \to 0) \neq 0$.

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Conclusions:

1. We conclude that as a consequence of general properties of propagation of an electromagnetic wave parallel to a constant strong magnetic field in a dense medium, a chiral magnetic effect exists in QED, manifested in the induction of an electric pseudo-vector current, separating charges of the same heliticity along the external field **B** in analogy to the QCD chiral magnetic case.
2. The chiral conductivity was calculated in the general case of finite temperature and density , and for massive fermions. which might be relevant for astrophysical applications.
3. The chiral magnetic effect we have discussed in QED in a medium can be extended to quark-antiquark electromagnetic interactions and be combined with QCD effects.

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