Super-Chandrasekhar masses for magnetized White Dwarfs?

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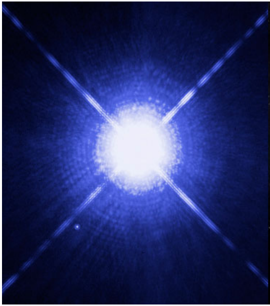
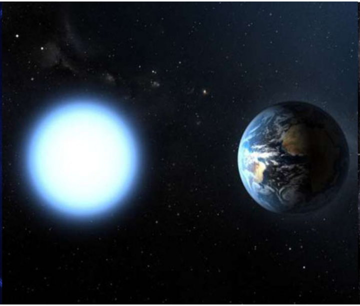
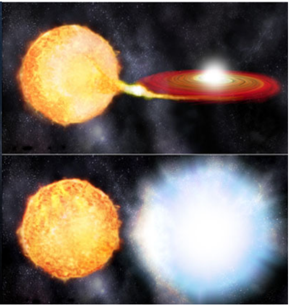
White Dwarf Stars.

WD

- $\mathrm{M} < 1.4\mathrm{M}_\odot \quad (0.6\mathrm{M}_\odot)$
- $\mathrm{R} \sim 6000\,\mathrm{km}$
- $\rho \sim 10^6\,\mathrm{g/cm^3}$
- $B \lesssim 10^9\,\mathrm{G}$

Type Ia supernovae.

Main characteristics

- Highly energetic events from the thermonuclear explosion of a WD.
- Luminosity curves with the presence of the sequence $^{56}Ni \rightarrow ^{56}Co \rightarrow ^{56}Fe$.
- Luminosity curves for different Ia are very similar $\Rightarrow$ Standard candles[†].

[†] M. S. Perlmutter. *et al*. Project. Measurements of Ω and Λ from 42 High-Redshift Supernovae. ApJ, 517:565-586 (1999). doi:10.1086/307221.

A. G. Riess. *et al*. Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. AJ, 116:1009-1038 (1998). doi:10.1086/300499.

Type Ia supernovae.

Explosion mechanism for SN Ia.

1. Single-degenerate model:
   - Binary system of stars that belongs to the main sequence.
   - Evolution of one of the stars into a WD.
   - Evolution of the other star into a red giant.
   - Accretion of mass from the red giant into the WD until the Chandrasekhar limit which trigger the supernova Ia explosion.
2. Double-degenerate model:
   - Binary system of two WD.
   - The two WD merge forming a system with mass greater than the Chandrasekhar limit in which carbon fusion is then ignited.

Overluminous Type Ia supernovae.

Recent observation have shown the existence of SN Ia with exceptionally higher luminosities:

- SNLS-03D3bb; (Howell et al. 2006, Nature, 443, 308)
- SN 2006gz (Hicken et al. 2007, ApJ, 669, L17)
- SN 2007if (Akerlof et al.2007, CBET, 1059, 1)
- SN 2009dc (Tanaka et al. 2009, arXiv:0908.2057)

The explanation of this overluminosities is based on the supposition that the progenitor WD has some extra fuel for nuclear fusion reactions. This will require a WD with $M_{WD} > M_{Ch}$, in order to justify the existence of this masses in WD there are several hypothesis:

1. Merge of two WD (Double-degenerate model).
2. Rotation.
3. Charged WD.
4. High magnetic fields.

Overluminous Type Ia supernovae.

Recently there has been a great activity around magnetized WD:

- Das, U., & Mukhopadhyay, B. 2012, Phys. Rev. D, 86, 042001
- Kundu, A., & Mukhopadhyay, B. 2012, Modern Physics Letters A, 27, 1250084
- Das, U., & Mukhopadhyay, B. 2012, IJMPD, 21, 1242001
- Das, U., & Mukhopadhyay, B. 2013, PRL, 110, 071102
- Coelho, J. G., Marinho, R. M., Malheiro, M. et al. 2014, ApJ, 794, 86.
- Nityananda, R., & Konar, S. 2014, Phys. Rev. D, 89, 103017
- Bera, P., & Bhattacharya, D. 2014, MNRAS, 445, 3951
- Liu, H., Zhang, X., & Wen, D. 2014, Phys. Rev. D, 89, 104043
- Das, U., & Mukhopadhyay, B. 2014, arXiv:1406.0948
- Nityananda, R., & Konar, S. 2015, Phys. Rev. D, 91, 028301
- D. Manreza, A. P. Martinez and J. E. Horvath, arXiv:1501.04619. Accepted in Research in Astronomy and Astrophysics (RAA)

Overluminous Type Ia supernovae.

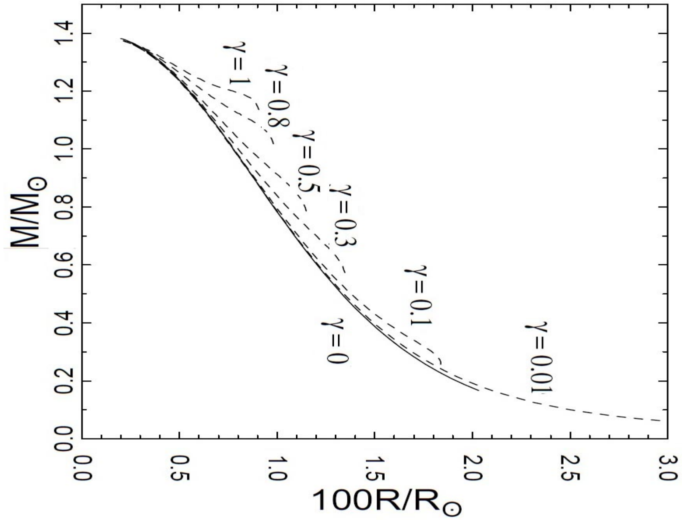


Figure: M-R diagram from: I.-S. Suh and G. J. Mathews. ApJ, 530:949-954 (2000). doi:10.1086/308403. $\gamma = B/B_e^c, B_e^c = 4.4 \times 10^{13}\mathbf{G}$

Overluminous Type Ia supernovae.

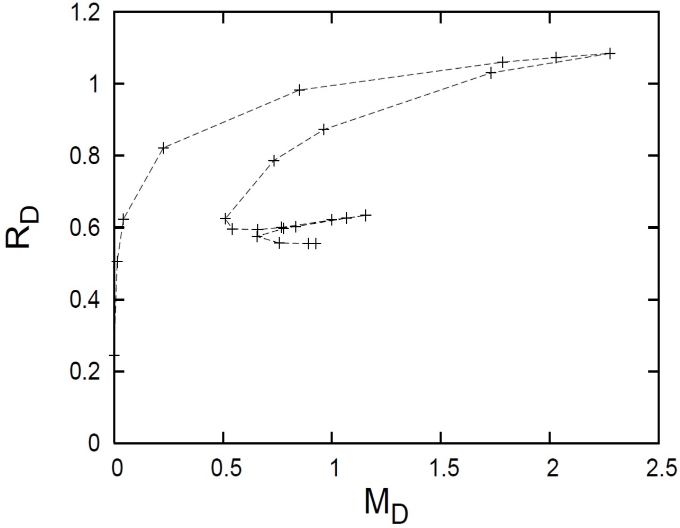


Figure: M-R diagram from: Das, U., & Mukhopadhyay, B. 2012, Phys. Rev. D, 86, 042001

Effect of the magnetic field in the EoS of WD matter.

Constant magnetic field in the x_3 direction.

- High magnetic fields have been detected in WD: $B_{\mathrm{WD}} \lesssim 10^{13}$ G.
- Effects of the magnetic field in a fermion gas:
  1. Quantization of the spectrum: $\varepsilon_l = \sqrt{p_3^2 + 2e_f B l + m_f^2}$.
  2. The density of states now becomes proportional to the field $2\int \frac{d^3p}{(2\pi)^3} \to \sum_l g(l) \frac{eB}{(2\pi)^2} \int dp_z$.
  3. A magnetic field breaks the rotational symmetry, giving rise to an anisotropy in the energy-momentum tensor†:

$$T^a{}_b = \begin{pmatrix} P_\perp & 0 & 0 & 0 \\ 0 & P_\perp & 0 & 0 \\ 0 & 0 & P_\parallel & 0 \\ 0 & 0 & 0 & E \end{pmatrix}$$

$$P_\perp = -\Omega - B\mathcal{M} + \frac{B^2}{8\pi}$$

$$P_\parallel = -\Omega - \frac{B^2}{8\pi}$$

$$E = \Omega + \mu N + m_N N \frac{A}{Z} + \frac{B^2}{8\pi}$$

- Effects in the structure equations $\Rightarrow$ modification of the WD M-R relation.

†M. Chaichian, *et all*. Physical Review Letters, 84:5261 (2000).

A. Pérez Martínez, *et all*. European Physical Journal C, 29, (2003).

Effect of the magnetic field in the EoS of WD matter.

In the limit $T \to 0$ we have

$$\Omega = -\frac{eB}{4\pi^2} \sum_{l=0}^{l_{max}} g(l) \left\{ \mu p^l - (\varepsilon^0)^2 \ln\left[\frac{\mu + p^l}{\varepsilon^0}\right] \right\},$$

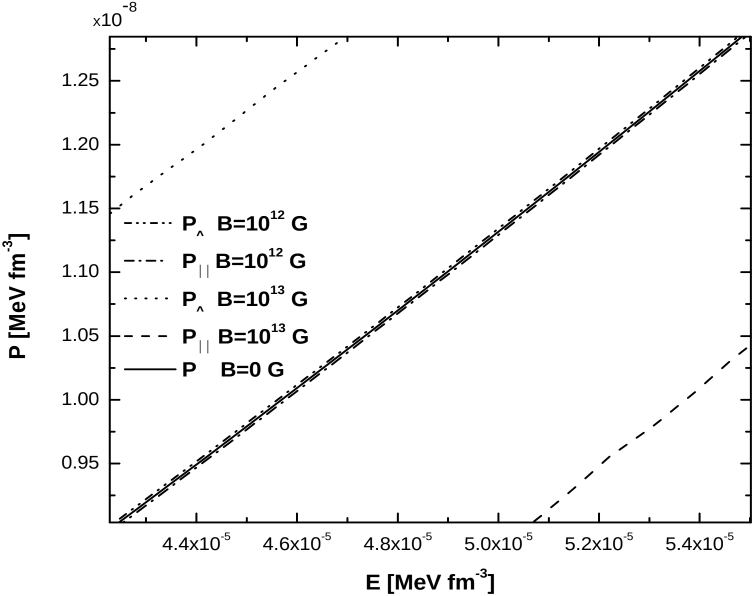
where

$$g(l) = 2 - \delta_{l0},$$

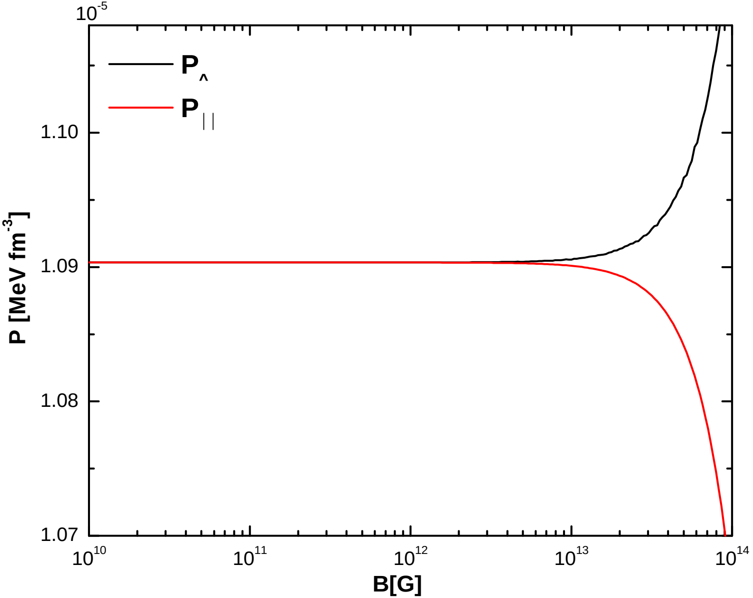
$$p^l = \sqrt{\mu^2 - (\varepsilon^0)^2},\ \varepsilon^0 = \sqrt{m^2 + 2eBl},$$

$$l_{max} = I[\frac{(\mu^2 - m^2)}{2eB}].$$

Effect of the magnetic field in the EoS of WD matter.



Effect of the magnetic field in the EoS of WD matter.



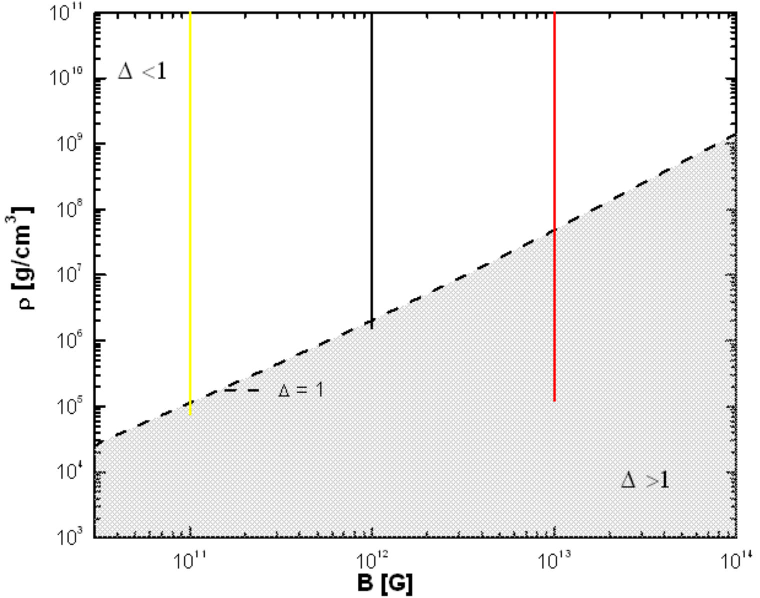
Effect of the magnetic field in the EoS of WD matter.

To quantify the anisotropy we have defined the splitting coefficient as

$$\Delta = \frac{|P_\perp - P_\parallel|}{P(B \to 0)}.$$

We will use as a criterion that the border separating the isotropic and the anisotropic regions is just $\Delta \simeq \mathcal{O}(1)$, so that by solving the equation $\Delta(\mu_e, B) = 1$, one can distinguish the anisotropic region from the isotropic one.

Effect of the magnetic field in the EoS of WD matter.



TOV equation

To find the static structure of a relativistic spherical star we have to solve the well-known TOV equations.

$$G^{\mu}{}_{\nu} \equiv R^{\mu}{}_{\nu} - \frac{1}{2} R g^{\mu}{}_{\nu} = 8\pi G \mathcal{T}^{\mu}{}_{\nu}, \tag{1}$$

$(\mu, \nu = 0, 1, 2, 3)$, using the Schwarzschild metric

$$ds^2 = -e^{2\nu} dt^2 + e^{2\lambda} dr^2 + r^2 d\Omega^2, \quad d\Omega^2 = d\theta^2 + \sin^2\phi d\phi^2, \tag{2}$$

and the energy-momentum tensor

$$\mathcal{T}^{\mu}{}_{\nu} = (\epsilon + P) u^{\mu} u_{\nu} + P g^{\mu}{}_{\nu}, \tag{3}$$

Tolman–Openheimer–Volkof (TOV):

$$\begin{aligned} \frac{dM}{dr} &= 4\pi G \epsilon, \\ \frac{dP}{dr} &= -G \frac{(\epsilon + P)(M + 4\pi P r^3)}{r^2 - 2rM}, \end{aligned}$$

and boundary conditions $P(R) = 0$, $M(0) = 0$.

TOV equation

Results: WD M-R,

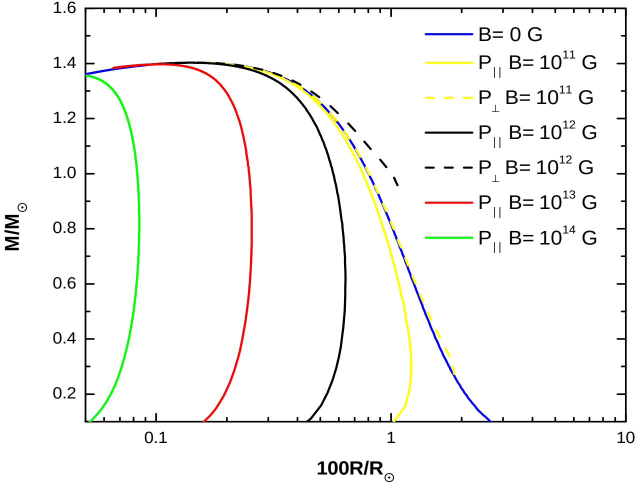


Figure: M-R configurations obtained for magnetized WD.

Structure equations in cylindrical symmetry.

The cylindrically symmetric metric reads:

$$ds^2 = -e^{2\Phi}dt^2 + e^{2\Lambda}dr^2 + r^2 d\phi^2 + e^{2\Psi}dz^2, \quad (4)$$

where $\Phi, \Lambda, \Psi = f(r \equiv r_\perp)$ and the energy-momentum tensor is:

$$\mathcal{T}^{\mu}{}_{\nu} = diag(E, P_\perp, P_\perp, P_\parallel), \quad (5)$$

From the Einstein field equations in natural units and using the energy momentum conservation $(\mathcal{T}^{\mu}{}_{\nu;\mu})$ we obtain:

$$\begin{aligned}
P'_\perp &= -\Phi'(E + P_\perp) - \Psi'(P_\perp - P_\parallel), \\
4\pi e^{2\Lambda}(E + P_\parallel + 2P_\perp) &= \Phi'' + \Phi'(\Psi' + \Phi' - \Lambda') + \frac{\Phi'}{r}, \\
4\pi e^{2\Lambda}(E + P_\parallel - 2P_\perp) &= -\Psi'' - \Psi'(\Psi' + \Phi' - \Lambda') - \frac{\Psi'}{r}, \\
4\pi e^{2\Lambda}(P_\parallel - E) &= \frac{1}{r}(\Psi' + \Phi' - \Lambda').
\end{aligned}$$

This, together with the EoS $E \to f(P_\perp)$, $P_\parallel \to f(E)$ is a system of differential equations in the variables $P_\perp$, $P_\parallel$, E, Φ, Λ, Ψ.

Structure equations in cylindrical symmetry.

In order to compute the mass of the star we use the Tolman generalization:

$$M_T = \int \sqrt{-g}(T_0^0 - T_1^1 - T_2^2 - T_3^3)dV \tag{7}$$

in our case we have:

$$\begin{aligned} M_T &= \int re^{\Phi+\Psi+\Lambda}(E - 2P_\perp - P_\parallel)dV & (8)\\ &= \int_0^{2\pi}\int_{-R_\parallel}^{R_\parallel}\int_0^{R_\perp} re^{\Phi+\Psi+\Lambda}(E - 2P_\perp - P_\parallel)d\phi\, dz\, dr & (9)\\ &= 4\pi R_\parallel \int_0^{R_\perp} re^{\Phi+\Psi+\Lambda}(E - 2P_\perp - P_\parallel)dr & (10)\end{aligned}$$

in this way we can compute the mass per unit parallel radius($R_\parallel$)

$$\boxed{\frac{M_T}{R_\parallel} = 4\pi \int_0^{R_\perp} re^{\Phi+\Psi+\Lambda}(\epsilon - 2P_\perp - P_\parallel)dr} \tag{11}$$

Structure equations in cylindrical symmetry.

Results: WD M-R

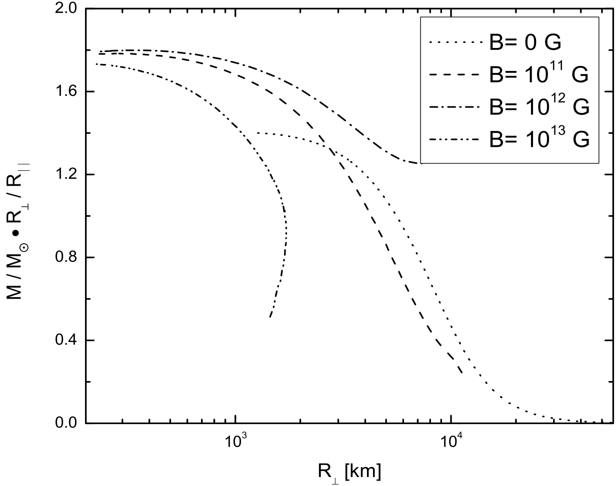


Figure: M-R configurations obtained using cylindrical symmetry.

Conclusions.

1 WD, spherical symmetry:

- For any value of the magnetic field we have not obtained $M_{WD} > M_{Ch} = 1.44 M_{\odot}$.
- There are not stable configurations when we use $P_{\perp}$ in the TOV equations for $B > 10^{13}$ G.

2 WD, cylindrical symmetry:

- We obtained a maximum magnetic field $B \sim 10^{13}$ G beyond which there are not stable equilibrium configurations. This bound for the value of the magnetic field is close (but slightly lower) to the one obtained based on the scalar virial theorem.
- Although in our model we can not compute the total mass due to the assumption that all the variables depend only on the perpendicular (equatorial) radius and not on the z-direction, the study rules out that the magnetic field could be the reason of the existence of super-Chandrasekhar masses for magnetized WD.

Muchas Gracias