Quantum Magnetic Collapse

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Introduction

The electron dynamics in a magnetic field has an anisotropic spectrum

$$E_{n,p_3} = c\sqrt{p_3^2 + 2eB(n+1/2) \pm eB + m^2c^2}.$$

where $n = 0, 1, ...$ are Landau quantum numbers.

$$Anisotropic\, dynamics \Rightarrow anisotroic\, thermodyamics$$

even more

$$Anisotropic\, dynamics \Rightarrow anisotropic\, photon\, propagation$$

The magnetic field breaks the symmetry of vacuum

Motivations

Motivations: Jets in galaxies and other astrophysical objects. They are long linear and magnetized structures. Does them have a connection with magnetized quantum gases? In a paper with Chaichian et al, (PRL, 84, 5261,2000) we have shown that in degenerate conditions of the electron gas, the pressure transverse to the magnetic field **B**, $p_\perp = -\Omega - B\mathcal{M}$, where $\mathcal{M}$ is the magnetization, may vanish. The remaining hadron pressure may not be able to compensate the gravitational pressure, leading to a transverse collapse. The effect is of pure quantum origin and it is easy to understand since if all electrons are confined to the LLL, the quantum average of their transverse momentum vanishes. The required magnetic field is of the order or greater than the Schwinger critical value $B_c = m^2c^3/e\hbar \sim 4.4 \times 10^13$ G.

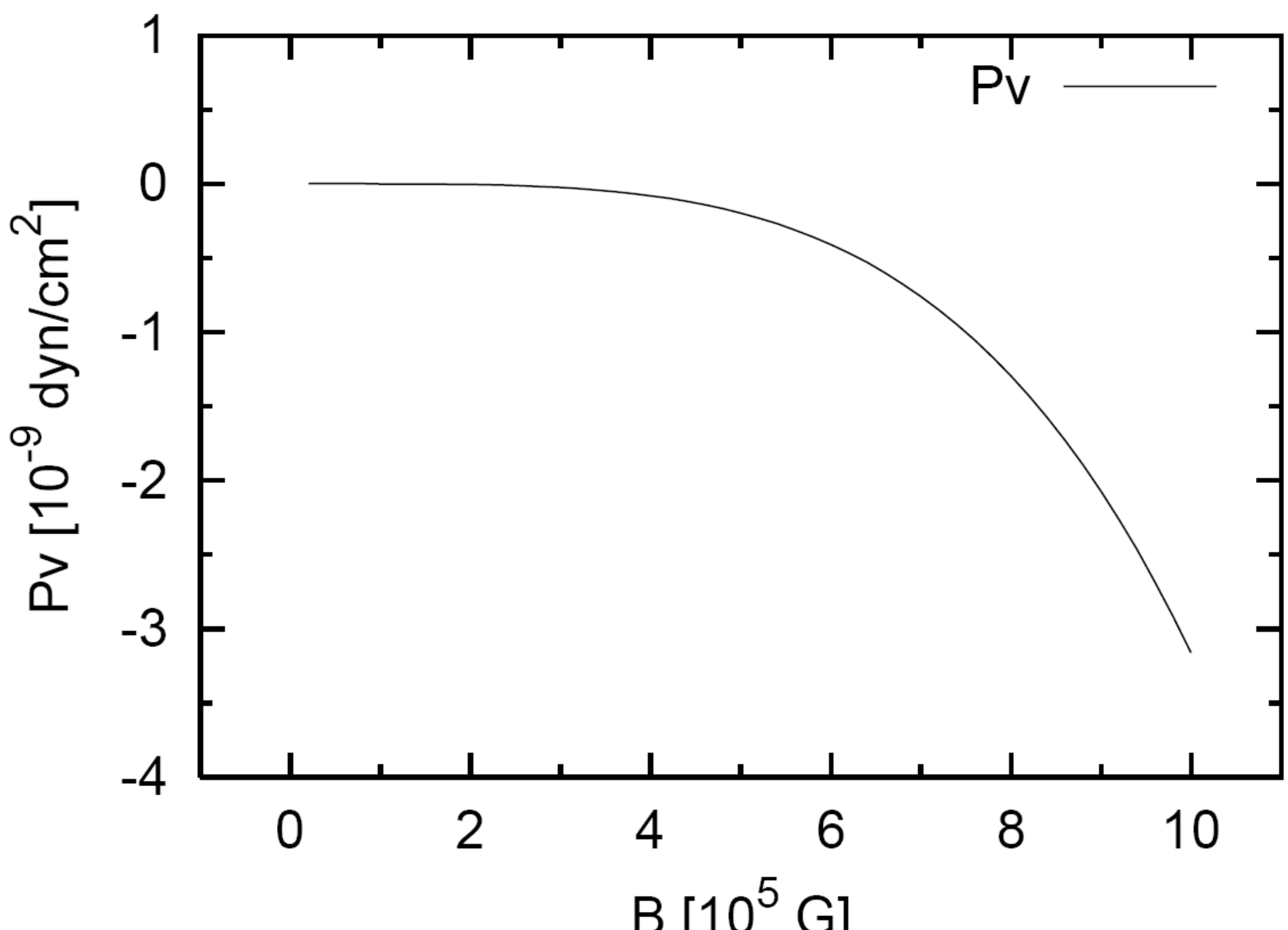
The approximate expression for the magnetization in that case in the degenerate limit is

$$\mathcal{M} = \frac{e}{4\pi^2\hbar^2c^2}\left(\mu\sqrt{\mu^2 - m^2c^4} - m^2 \ln \frac{\mu + \sqrt{\mu^2 - m^2c^4}}{m}\right) \quad (1)$$

The magnetization is a function of the chemical potential μ, which is a function of the density N (see below)

Magnetized vacuum pressure

We can show also, independently, that there is a negative (Casimir-like) pressure in magnetized vacuum in the direction orthogonal to the field $\mathbf{B}$ as shown in (HPR, IJMPA 21, 3761, 2006). It acts as a compression



Vacuum is compressed inward in the direction orthogonal to **B**, as it may happen in an electron gas in a magnetic field, in which the pressures are $p_3 = -\Omega$ parallel to **B**, and $p_\perp = p_3 - B\mathcal{M}$. If $\mathcal{M} > 0$ (paramagnetic) and $p_3 < B\mathcal{M}$, then $p_\perp < 0$, the electron gas is compressed orthogonal to **B**. This happens if the magnetic field increases, for instance, in a body with axial symmetry. If the electron pressure contributes significatively to the total pressure of the gas, if it vanishes, the effect is an axial compression of the whole body, confining the electron gas to a long ”cigar” shape body.

- Such compression is related to the following facts: the quantity $S_B = c\hbar/eB$ can be considered as the quantum of area corresponding to a flux quantum for a field intensity $\mathbf{B}$. Thus, by increasing B, S_B decreases. As a consequence, the spread of the electron and positron wave functions decreases exponentially with B in the direction orthogonal to the field since they depend on the transverse coordinates as $e^{-\xi^2}$ where $\xi^2 = x_\perp^2/S_B$.
- We could imagine, for instance, that for fields near the critical value, the spread of the wave function is concentrated in an area of order of a squared Compton waveleangth. As the motion of electrons and positrons along the magnetic field is free, from a macroscopic point of view they have essentially a one-dimensional behavior.

Magnetized Fermi gas

Let us assume a Fermi gas of charged particles at a very high density N and strong magnetic field B such that the system is confined to the lowest Landau level (LLL)

$$E_{n,p_3} = c\sqrt{p_3^2 + m^2c^2}.$$

Let us name $E_F = \sqrt{\mu^2 - m^2c^4}$ the longitudinal Fermi energy, We can write N as

$$N = \frac{eB}{2\pi\hbar^2c^2}E_F.$$

- If N grows suddenly faster than B then the longitudinal density, proportional to E_F, would increase with N up to large values. Obviously, the same happens for the Fermi momentum, $p_f = E_F/c$. If N grows μ grows also. For instance if $\mu > mc^2$, and μ grows $K > 1$ times, we have the speeds $v = K/\sqrt{K^2+1}c$. For instance if $K = 5$, the increase of momentum leads to $v \sim 0.98c$. Thus fermions would move at speeds close to c.
- We conclude that if the degenerate electron gas is compressed suddenly, the outcome could be a jet of quasi-luminal particles moving parallel to B. The increase of N could be the consequence of the sudden compression of the outer shells of a star if the transverse pressure exerted by the electrons in the inner core decreases and even vanishes. This happens when the magnetic field reaches its critical value, $B_c = m^2c^3/e\hbar$.

Magnetized electron-photon jet gas

Let us consider a high density, such that $\mu > mc^2$ and a magnetic field of order of the critical Schwinger field $B_c = m^2c^3/e\hbar$. The magnetization ca be written

$$\mathcal{M} = \frac{e}{4\pi^2\hbar^2c^2}\mu\sqrt{\mu^2 - m^2c^4} \tag{2}$$

The condition for self-magnetization is

$$B_c = 4\pi\mathcal{M} \tag{3}$$

Let us call $\mu = Amc^2$. We have an equation $A\sqrt{A^2 - 1} = 1/4\alpha$ leading to $A \sim \sqrt{1/4\alpha} \sim 34.25$, and the critical electron density is of order $N = \frac{eB}{4pi^2}\frac{\sqrt{A^2-1}}{\hbar^2c^2}mc^2 \sim 10^{31}$. Larger electron densities would lead to supercritical fields $B \gg B_c$.

et us now consider photons that are created inside the star, whose magnetic field we assume to grow with axial symmetry towards its axis.

- Photons moving in a region of increasing magnetic field are decreased in its momentum orthogonal to B, up to the extreme case of critical field in which they are confined to move parallel to B.

- The final picture of electrons and photons is a magnetized jet: a one dimensional expanding object composed by electrons, photons and even light atoms pushed by them.

Let us see the details

For an incoming photon, in the low frequency limit $\omega < 2m$, low magnetic field case, $B < B_c$

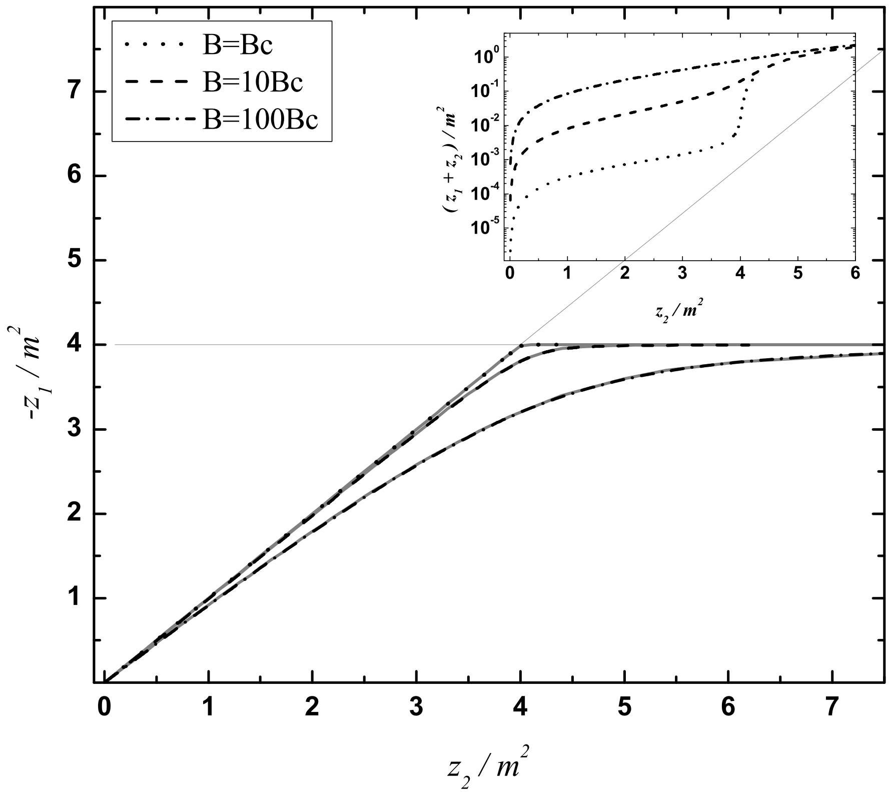
$$\omega^{i2} - k_{\parallel}^2 = k_{\perp}^2 \left(1 - \frac{C^i \alpha b^2}{45\pi}\right), \qquad C^i = 7, 4 \qquad i = 2, 3.$$

- This Eq. must be interpreted as the dispersion equation in presence of the magnetic field for an incoming photon which initially, far from the magnetized region, satisfied the usual light cone equation $\omega_0^2 = k_{\parallel}^2 + k_{\perp}^2$. In other words, the dispersion equation before the magnetic field was switched on.
- The effect of the magnetic field is to decrease the incoming transverse momentum squared by a factor $g(B)_{k_\perp}^{(i)} = 1 - f(B)^{(i)}/k_{\perp}^2 < 1$, to the effective value

$$k_{eff\perp}^2 = k_{\perp}^2 g(B)^{(i)} < k_{\perp}^2.$$

- Thus, as stated previously, the transverse momentum is not conserved in the magnetic field, and $k_{eff\perp}$ is the effective transverse momentum measured by an observer located in the region where the magnetic field is $\mathbf{B}$.

- The initial photon energy decreased from $\omega_0 \to \omega = \sqrt{k_{\parallel}^2 + k_{eff\perp}^2}$. For propagation orthogonal to $\mathbf{B}$, it is $\omega = \omega_0\sqrt{g(B)^{(i)}}$, since $\omega_0 = k_\perp$. The non conservation of momentum leads to the decrease of the photon energy, which is red-shifted for incoming photons. As the field intensity increases the quantity $g(B)$ decreases.
- The role of the separation between Landau levels of virtual pairs becomes more and more significant as one approaches the first threshold of resonance, which is the quasi-pair region, where $B \lesssim B_c$.



- For frequencies $\omega \simeq 2m$ and $k_{\parallel} < \omega$, the dispersion equation for the second mode may be written (Shabad, Ann. Phys. 90, 166, 1975), if the polarization operator is expressed as a sum over Landau levels n, n' of the virtual electron-positron pairs, in terms of the dominant term $n = n' = 0$, as $z_1 + z_2 = \frac{2\alpha eBme^{-z_2/2eB}}{\sqrt{z_1 + 4m^2}}$.
- Notice that its limit for $\mathbf{k} \to \mathbf{0}$ is $\omega \neq 0$. Actually, it describes a massive vector boson particle closely related to the electron-positron pair.[1]
- This Eq. has solutions found by Shabad as those of a cubic equation. One can estimate its behavior very near $z_1 = -4m^2$, by assuming $z_1 = -4m^2 + \epsilon$ and $z_2 = 4m^2 - \epsilon$, where ϵ is a small quantity. One can obtain the solution approximately as $z_1 = -4m^2 + (2\alpha eBme^{-z_2/2eB})^{2/3}$. This means approximately

$$\omega^2 = \sqrt{k_{\parallel}^2 + 4m^2 - (2\alpha eBme^{-2m^2/eB})^{2/3}}.$$

- Thus, the transverse momentum of the original photon is trapped by the magnetized medium, the resulting quasi-particle being deviated to move along the field as a vector boson of mass $\omega_t = \sqrt{4m^2 - m^2(2\alpha be^{-2/b})^{2/3}}$.

[1] This is not in contradiction with the gauge invariance property of the photon self energy.

Conclusions

1. Our approach is approximate. A more complete discussion would be made by following the method of Shabad (1975).
2. This quasi-pair is obviously paramagnetic, as can be checked easily. It differs totally from lower frequency photons originally propagating parallel to $\mathbf{B}$. For slightly larger energies such that $z_1 = -4m^2 - \epsilon$, and b of order unity, that is $B \sim B_c$, they decay in observable electron-positron pairs, and the polarized vacuum becomes absorptive.
3. Thus, near the critical field B_c our problem bears some analogy to light passing near a black hole: for $r \simeq r_G$, the light is deviated enough to be absorbed by the black hole. Among other differences in both cases, it must be remarked that the gravitational field in black holes is usually centrally symmetric, whereas our magnetic field is axially symmetric.

4 Notice that the massive photon bears an effective mass $m_{eff} = \sqrt{4m^2 - m^2(2\alpha b e^{-2/b})^{2/3}}$ which is obtained in first order of perturbation theory. For frequencies larger than the threshold of pair creation, we expect a similar spectrum. A more accurate calculation would demand to sum over powers of α.

5 We may, however, estimate the spectrum of massive vector boson particles (positronium) moving along B and having a magnetic moment which is the sum of the magnetic moments of both particles $\mu = e\hbar/mc$. We may easily show that for particles moving parallel to B we would have a spectrum of form $E = \sqrt{k_3^2 + 4m^2 - 2m\mu B}$. This means that for magnetic fields of order $2B_c$, the magnetic field could be maintained self-consistently by magnetized positronium. Although positronium has a very small lifetime, in an electron-positron medium along the jet, Pauli's principle would preserve it with some stability.

6 All this requires further analysis, however, it looks an area which deserves attention.