Simulations of Accretion Discs in Pseudo-Complex General Relativity

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Content

- An introduction to an extension of General Relativity (GR)
- GR with dark energy
- Circular orbits and galactic black holes
- Model of Page-Thorne for accretion discs: Calculations within GR and pc-GR
- Discussion of results and possible observations

Which algebraic extension of GR makes sense?

$$X = X_1 + IX_2 \quad , \quad I^2 = 1$$ „pseudo-compleja"

There were several attempts to extend algebraicaly GR, e.g., to complex variables and/or metrics (Einstein 1950, Mantz 2008,….). However, as shown in

P.F.Kelly and R.B. Mann, "Ghost properties of algebraically extended theories of gravitation", *Classical and Quantum Gravity* **3** (1986), 705

Only the pseudo-complex extension do not produce Non-physical states.

Mathematics:

Pseudo-complex variables

$$X = X_1 + IX_2 \quad , \quad I^2 = 1$$

Alternative:

$$\sigma_\pm = \frac{1}{2}(1 \pm I) \quad \rightarrow \quad X = X_+\sigma_+ + X_-\sigma_-$$

$$\sigma_\pm{}^2 = \sigma_\pm$$

Properties

$$\sigma_+ + \sigma_- = \frac{1}{2}(1+I) + \frac{1}{2}(1-I) = 1$$

$$\sigma_+ - \sigma_- = \frac{1}{2}(1+I) - \frac{1}{2}(1-I) = I$$

$$\rightarrow \quad X = X_1(\sigma_+ + \sigma_-) + (\sigma_+ - \sigma_-)X_2 = (X_1 + X_2)\sigma_+ + (X_1 - X_2)\sigma_-$$

$$= X_+\sigma_+ + X_-\sigma_-$$

Zero diviso

$$\sigma_+\sigma_- = \frac{1}{2}(1+I)\frac{1}{2}(1-I) = \frac{1}{4}(1-I^2) = 0$$

$$\rightarrow \quad (\lambda_1\sigma_+)(\lambda_2\sigma_-) = 0$$

→ NO inverse

The numbers form a ring and not a field.

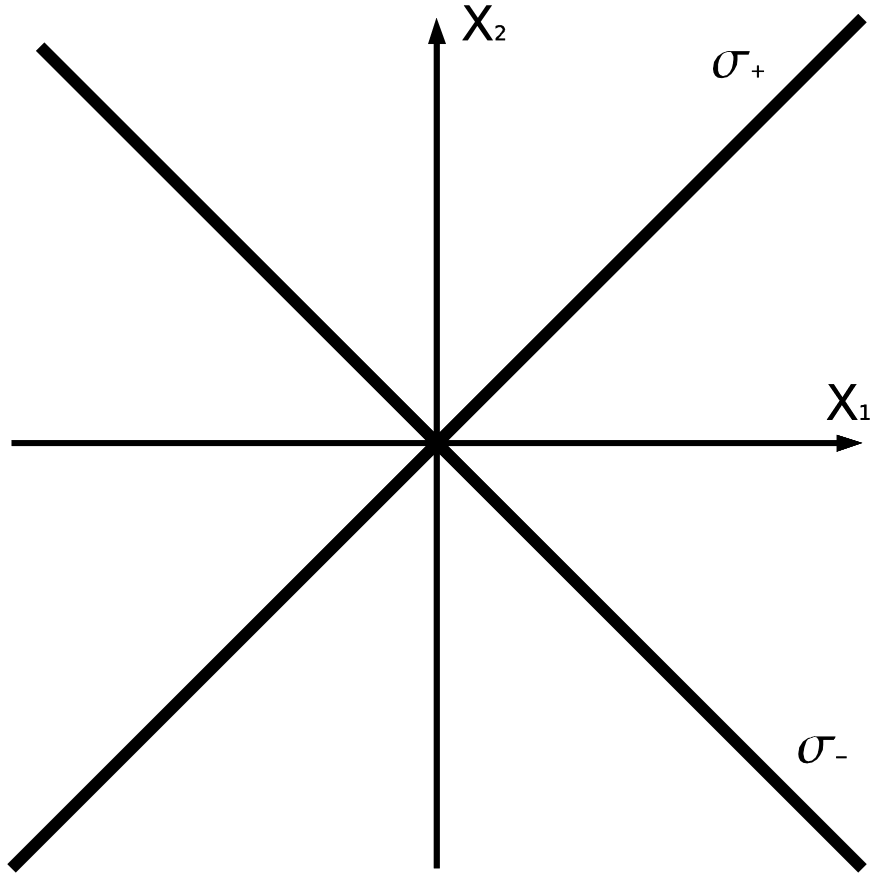
Summary:

$$\sigma_+ = \frac{1}{2}(1+I) \quad , \qquad \sigma_- = \frac{1}{2}(1-I) \quad ,$$

$$x_+ = x_R + x_I \quad , \qquad x_- = x_R - x_I \quad ,$$

$$1 = \sigma_+ + \sigma_- \quad , \qquad I = \sigma_+ - \sigma_- \quad ,$$

$$x_R = \frac{1}{2}(x_+ + x_-) \quad , \quad x_R = \frac{1}{2}(x_+ - x_-)$$



Diagonal lines → Zero divisor

All calculations are “nearly” the same as
In normal calculus.

One has to be carefull crossing the lines of
the zero divisor.

Metric

- Definition of the metric, without torsión

$$g_{\mu\nu} = g^{+}_{\mu\nu}\sigma_{+} + g^{-}_{\mu\nu}\sigma_{-} \quad , \quad g_{\mu\nu} = g_{\nu\mu}$$

* Line element

$$\begin{aligned} d\omega^2 &= g_{\mu\nu} DX^{\mu} DX^{\nu} \\ &= g^{+}_{\mu\nu} DX^{\mu}_{+} DX^{\nu}_{+}\sigma_{+} + g^{-}_{\mu\nu} DX^{\mu}_{-} DX^{\nu}_{-}\sigma_{-} \\ &= [g^{0}_{\mu\nu}\left(dx^{\mu}dx^{\nu} + dy^{\mu}dy^{\nu}\right) + 2h_{\mu\nu}dx^{\mu}dy^{\nu}] \\ &\quad + I\,[h_{\mu\nu}\left(dx^{\mu}dx^{\nu} + dy^{\mu}dy^{\nu}\right) + 2g^{0}_{\mu\nu}dx^{\mu}dy^{\nu}] \end{aligned}$$

→REAL

Ecuaciones de Einstein

$$L = \sqrt{-g}\,R$$

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = -\frac{8\pi\kappa}{c^2} T_{\Lambda}^{\mu\nu}$$

This suggests that the right hand side is related
To an energy-momentum tensor.
.

for $$\left| y^{\mu} \right| << 1 \rightarrow g_{\mu\nu}(X) \approx g_{\mu\nu}(x)$$

„Possible“ origin of $T_{\mu\nu}$:

Quantum fluctuations (Casimir effect)
[M. Visser, Phys. Rev D **54** (1996), 5103; ibid **54** (1996), 5116; ibid **57** (1997), 936
y C. Barceló et al., Phys. Rev. D **77** (2008), 044032]

Semi-classical approach:
Advantage: Can determine the dark energy distribution.
Disadvantage: Can not determine back-reaction (at least VERY difficult).

Our approach:
Advantage: Can calculate the back-reaction → No event horizon
Disadvantage: We can not deduce the dark energz distribution by
First principles → Models!

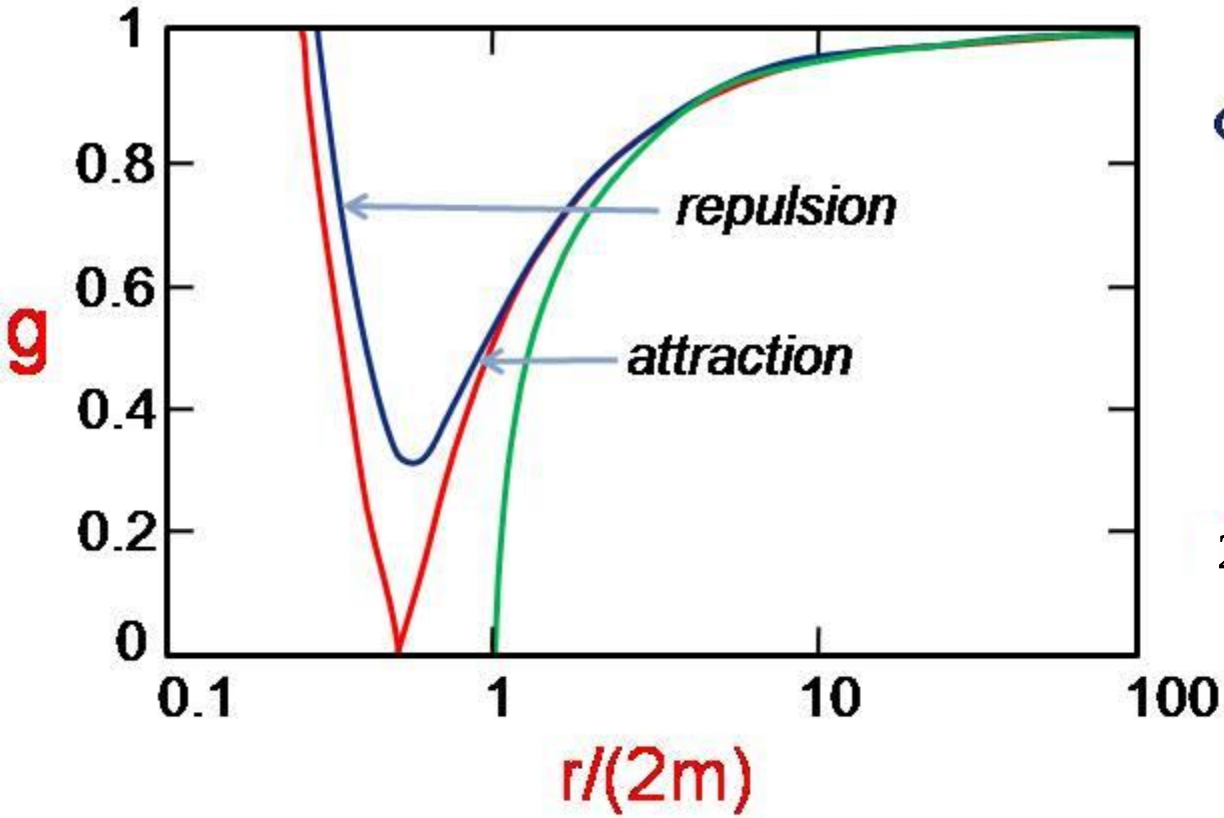
İboth procedures are compementary!

PRINCIPLE:
Mass not only curves the space
BUT
Also changes vacuum properties

→ No event horizon!

The Red Shift

The Red Shift g factor: $d\tau \approx \sqrt{g_{00}^0}\,dt = \sqrt{\left(1 - \frac{2m}{r} + \frac{B}{2r^3}\right)}dt \equiv gdt$



The metric component of the time must be positive

$$g_{00}^0 > 0 \Rightarrow B > \frac{64}{27}m^3$$

Antigravitation below $2/3$ of the Schwarzschild radius!

Circular orbit of a particle, around a great mass

(T. Schönenbach et al., MNRAS **430** (2013), 2999, DOI: 10.1093/mnras/stt108)

Lagrangian: $$L = g_{00}c^2\dot{t}^2 + g_{11}\dot{r}^2 + g_{22}\dot{\vartheta}^2 + g_{33}\dot{\varphi}^2 + 2g_{03}c\dot{t}\dot{\varphi} = \frac{ds^2}{ds^2} = 1$$

→ geodesic

s is the curve parameter. After variation (prime=derivative in r):

$$\frac{d}{ds}\left(2g_{11}\dot{r}\right) = g'_{00}c^2\dot{t}^2 + g'_{11}\dot{r}^2 + g'_{22}\dot{\vartheta}^2 + g'_{33}\dot{\varphi}^2 + 2g'_{03}c\dot{t}\dot{\varphi}$$

Circular motion: $\dot{r} = 0$, $\vartheta = \frac{\pi}{2}$ → $0 = g'_{00}(r_0)c^2\dot{t}^2 + g'_{33}(r_0)\omega^2\dot{t}^2 + 2g'_{03}(r_0)\omega c\dot{t}^2$

Solve for frequency:

$$\omega_\pm = c\frac{-g'_{03} \pm \sqrt{\left(g'_{03}\right)^2 - g'_{00}g'_{33}}}{g'_{33}}$$

→ with

$$\omega_\pm = c\frac{-ah(r) \pm \sqrt{2rh(r)}}{-2r + -a^2h(r)} = \frac{c\sqrt{h(r)}}{-a\sqrt{h(r)} \mp \sqrt{2r}} ,$$

$$a = \frac{-\kappa J}{mc^3} \qquad h(r) = \frac{2m}{r^2} - \frac{3B}{2r^4} .$$

Or:

$$\omega_\pm = \frac{c}{-a \mp \sqrt{\frac{2r}{h(r)}}}$$

$$\omega_{\pm} = c\frac{-ah(r) \pm \sqrt{2rh(r)}}{-2r + -a^2h(r)} = \frac{c\sqrt{h(r)}}{-a\sqrt{h(r)} \mp \sqrt{2r}},$$

(¡a<0!)

with

$$h(r) = \frac{2m}{r^2} - \frac{3B}{2r^4}.$$

Negative sign and r large: ω_->0 → PROGRADE

Positive and r large: ω_+<0 → RETROGRADE

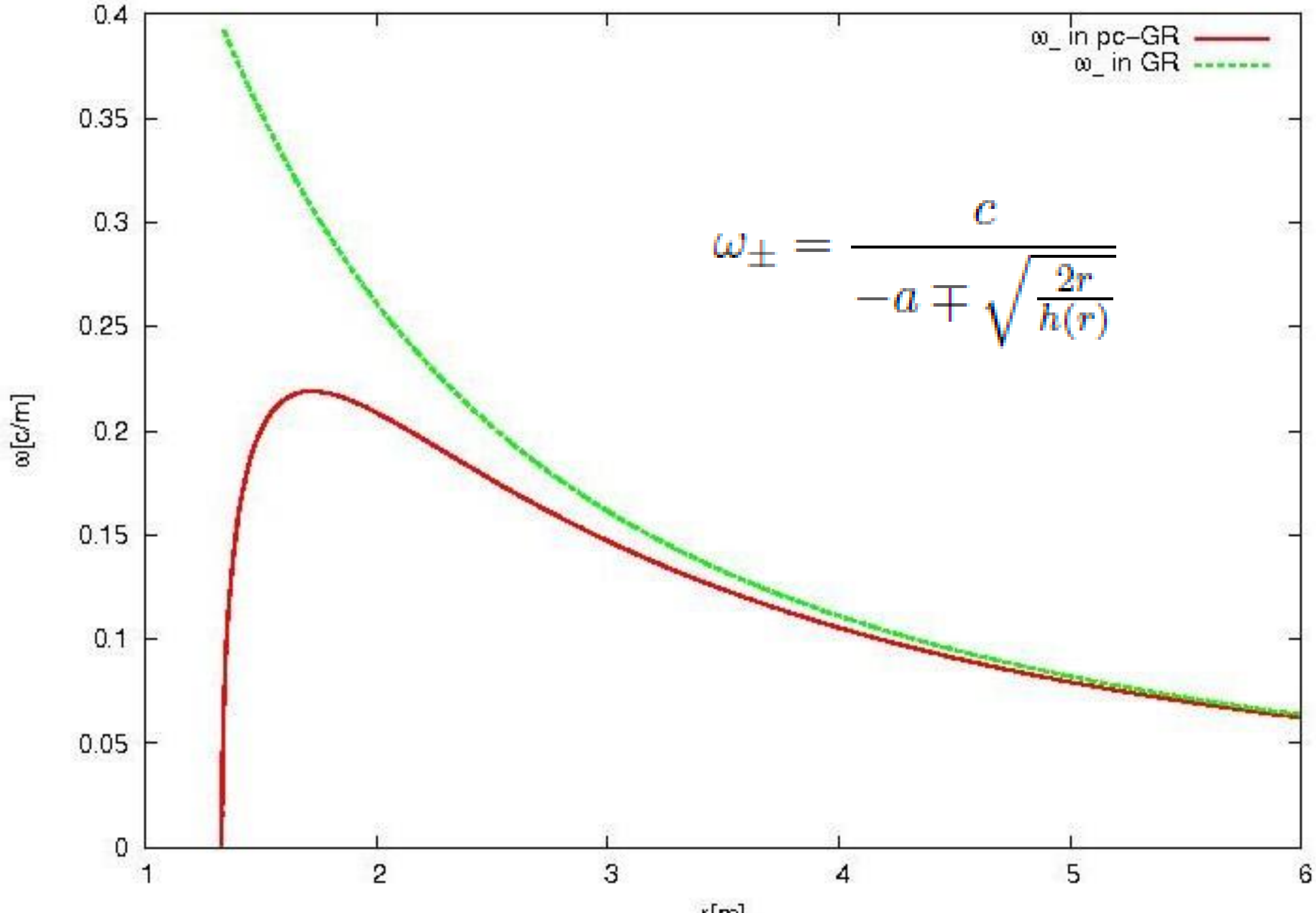
→For small distances r: $2r < a^2h(r),$

In this case bth frequencies are positive.

→ Frame dragging.

Circular orbital and redshift:

a=-0.995m



0.219 for 4Millon solar masses = 9.4 minutes

Redshift

$$d\tau^2 = g_{00} dt^2 \quad , \quad \tau_0 = \sqrt{g_{00}} t_{obs} \quad \rightarrow \quad \nu_{obs} = \sqrt{g_{00}} \nu_0$$

Redshift:

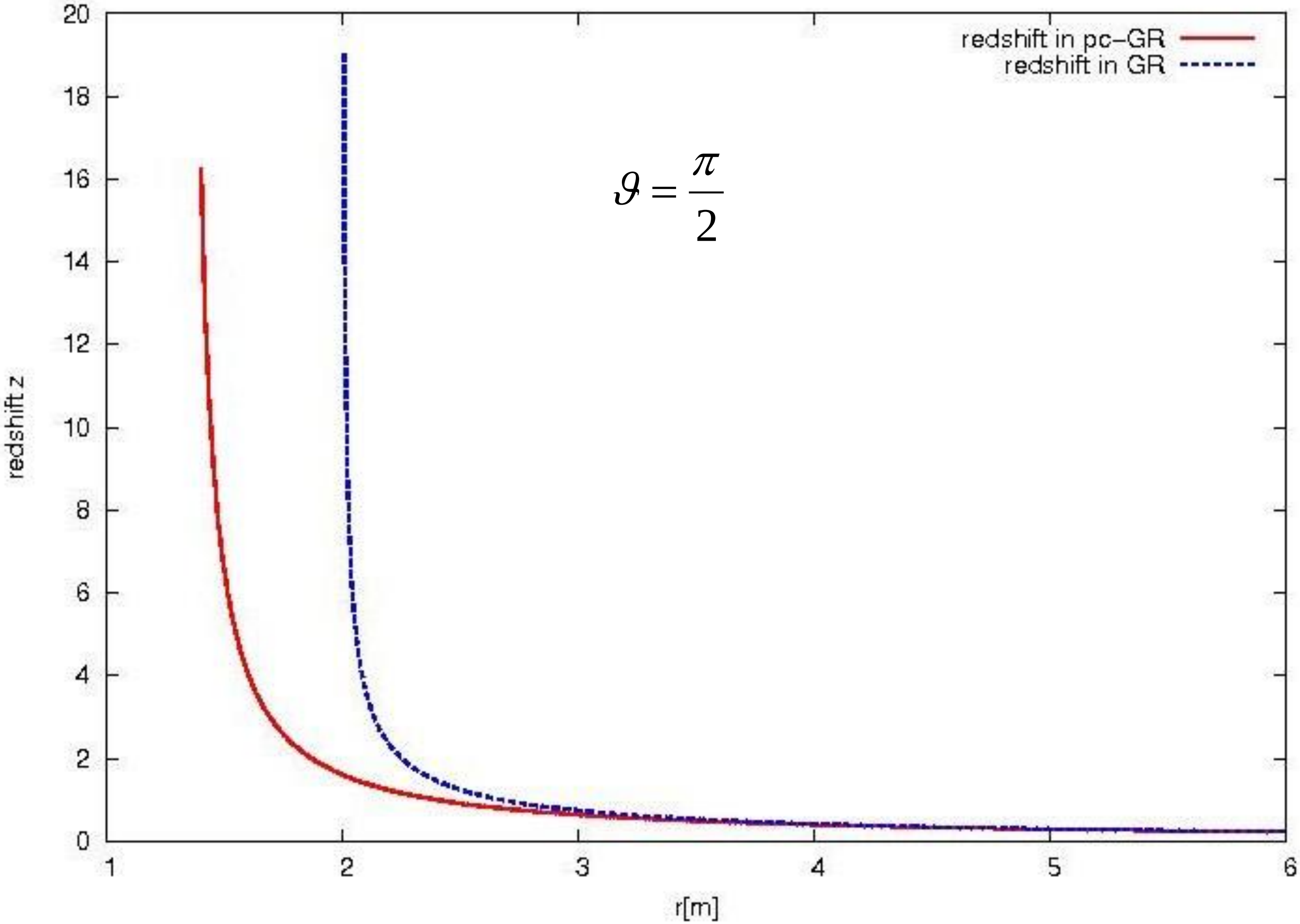
$$z := \frac{\nu_0 - \nu_{obs}}{\nu_{obs}} = \frac{1}{\sqrt{g_{00}}} - 1$$

Kerr:

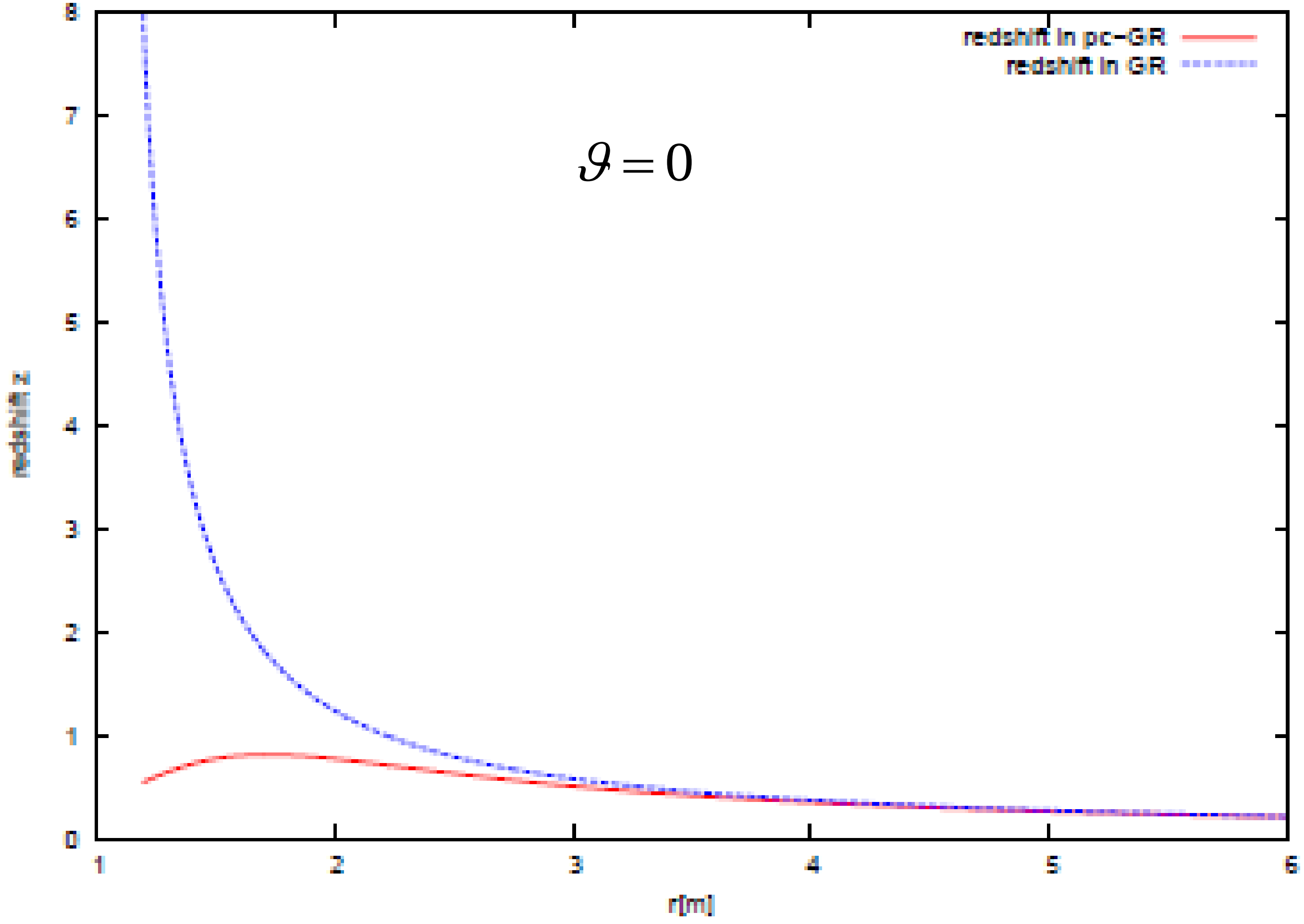
$$z = \frac{\sqrt{r^2 + a^2 \cos^2(\vartheta)}}{\sqrt{r^2 - 2mr + a^2 \cos^2(\vartheta) + \frac{B}{2r}}} - 1$$

Schwarzschild:

$$z = \frac{1}{\sqrt{1 - \frac{2m}{r} + \frac{B}{2r^3}}} - 1$$



redshift in pc-GR
redshift in GR
$\vartheta = \frac{\pi}{2}$
redshift z
r[m]
0
2
4
6
8
10
12
14
16
18
20
1
2
3
4
5
6



redshift in pc−GR
redshift in GR
$\vartheta = 0$
redshift z
r[m]
0
1
2
3
4
5
6
7
8

Fe K and QPO emission radii are not consistent in GR

the relativistic Fe K line fitting, results in inner radii r_{in} of about 2 R_G

the QPO frequencies, if associated with Keplerian, Vertical, and Radial motion, would place the QPOs at 10-50 R_G, and not within 2 R_G

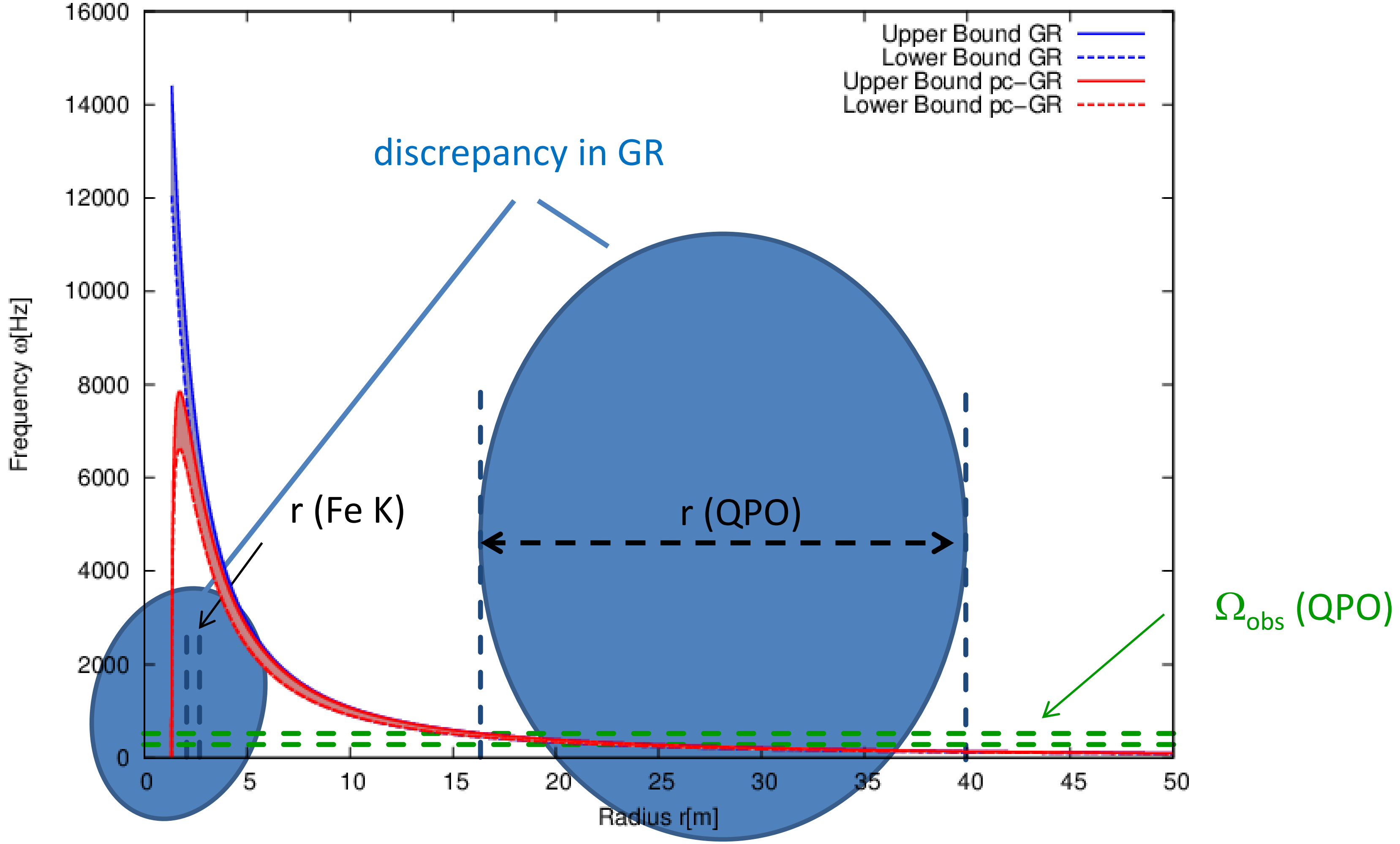
as these systems are not thought to have truncated disks at these high accretion rates, it is rather puzzling to have this discrepancy between QPO frequency and the inner radius

very little has been done beyond phenomenological description and this is all speculative and there is no clear model to explain this

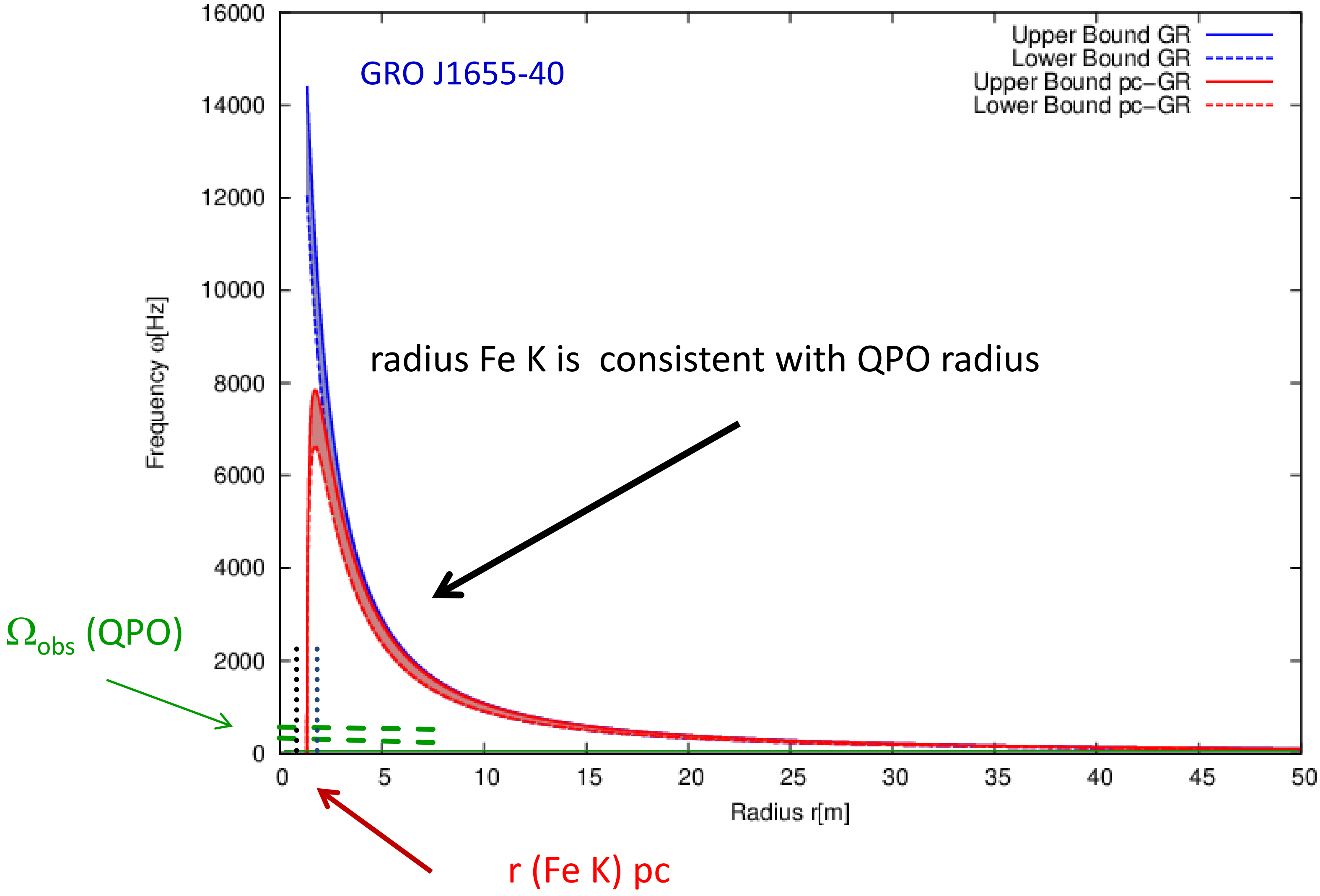
the pc-theory might offer an solution, as it provides QPO frequencies that are consistent with Fe K emission

GRO J1655-40

$M = 6.30 \pm 0.5$
$a = 0.92 \pm 0.2$



Galactic binaries in the pc-theory



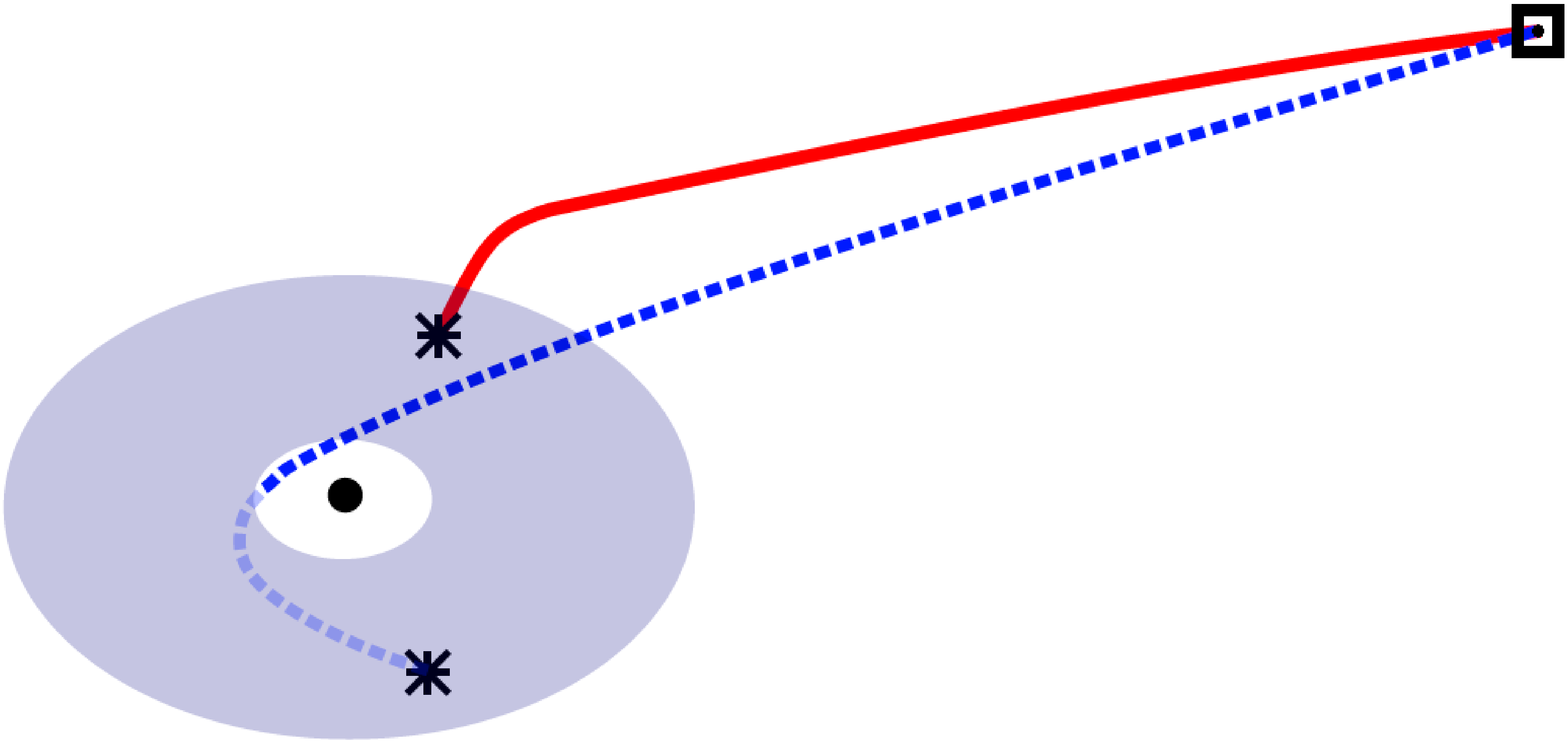
Theory

(Page and Thorne)

- A THIN accretion disk, infinitely extended
- Emission of energy only through photons
- Internal transport through shears
- RAYTRACING

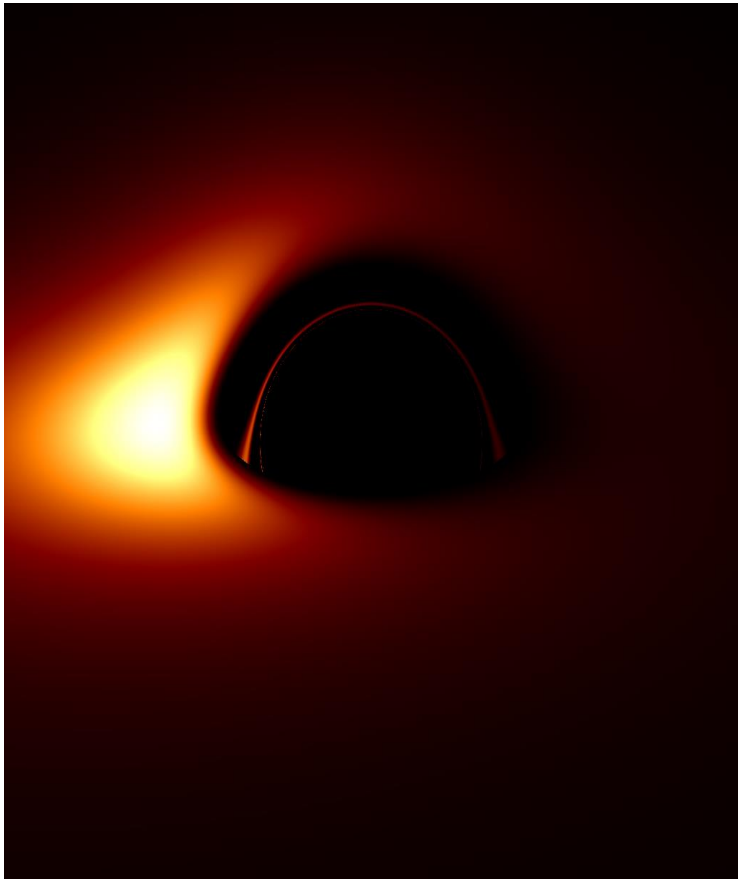
D. N. Page and K. S. Thorne, The Astrophysical Journal **191** (1974), 499.

Calculation of paths: Hamilton-Jacobi formalism.

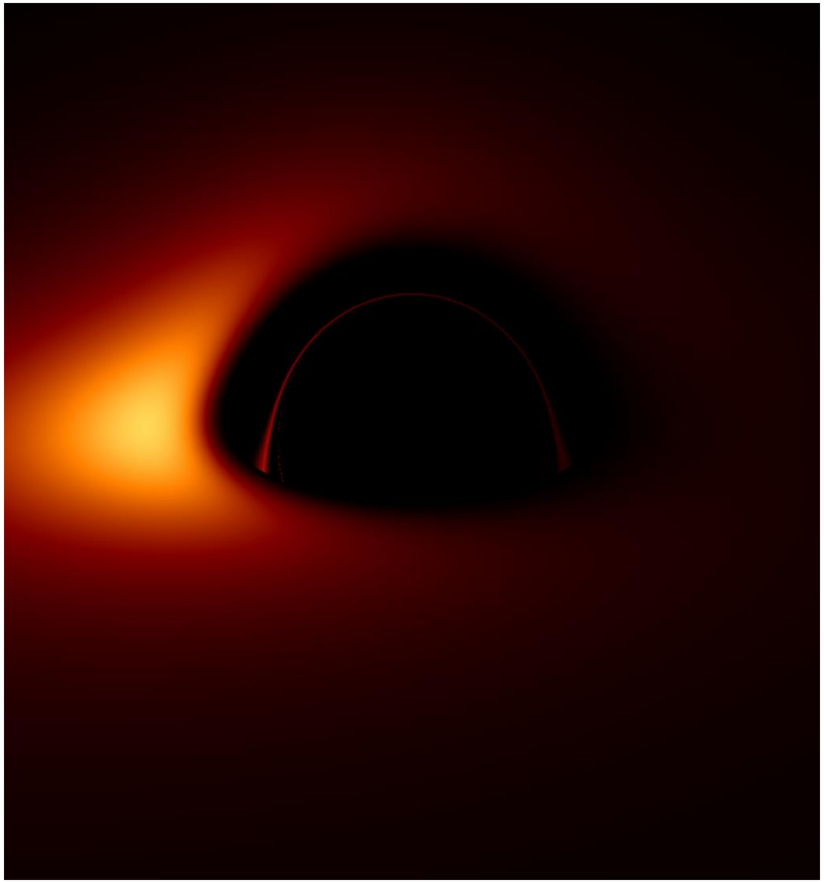


a=0, 70 degrees

pc-GR:

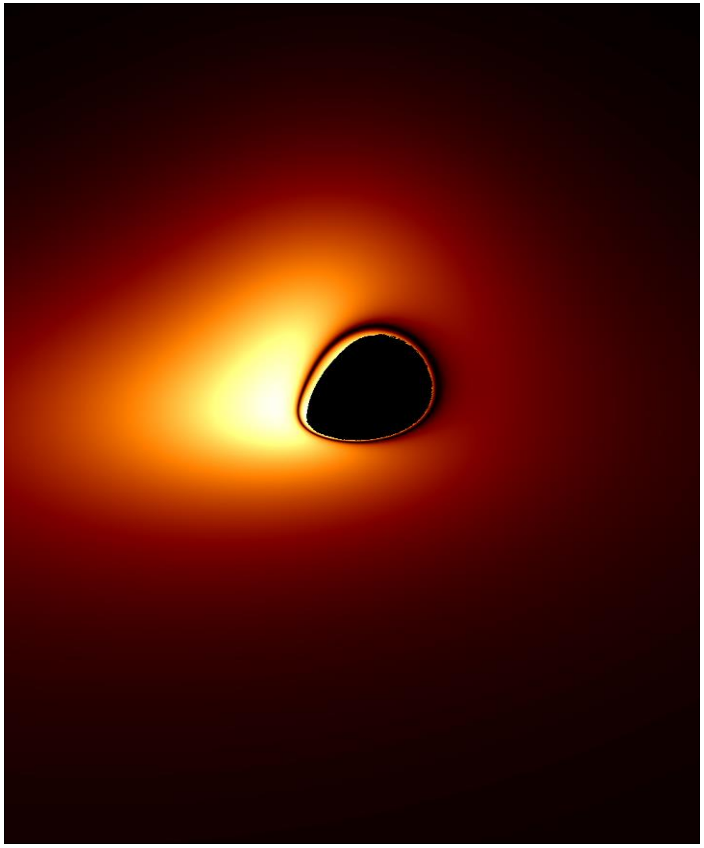


GR:

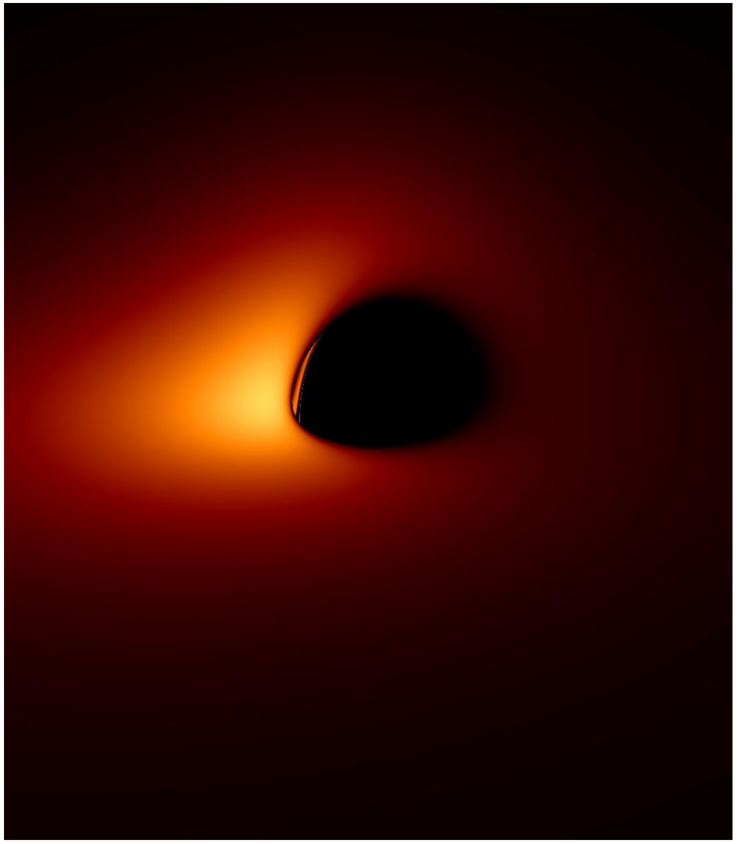


a=0.9, 70 degrees

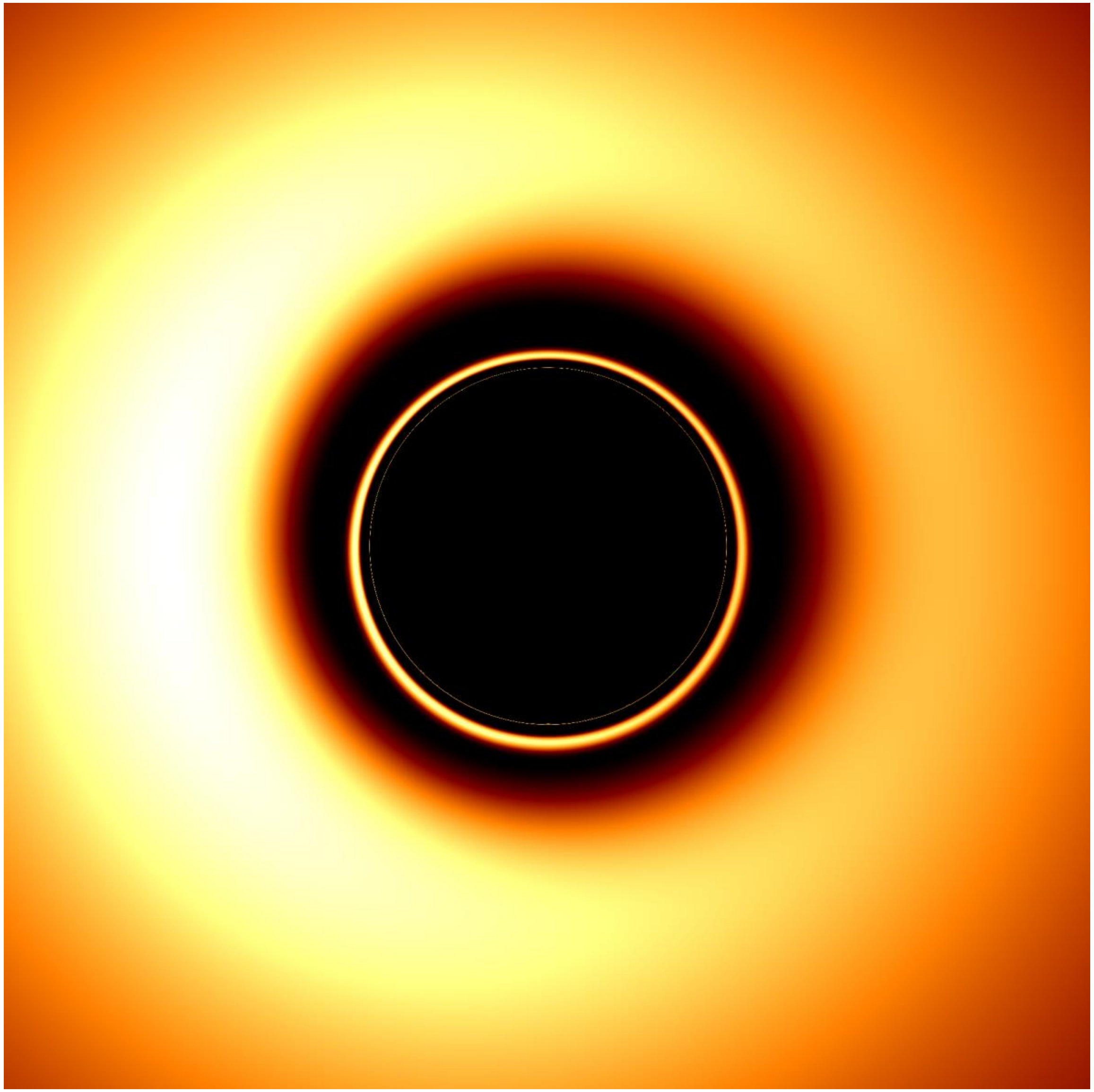
pc-GR:



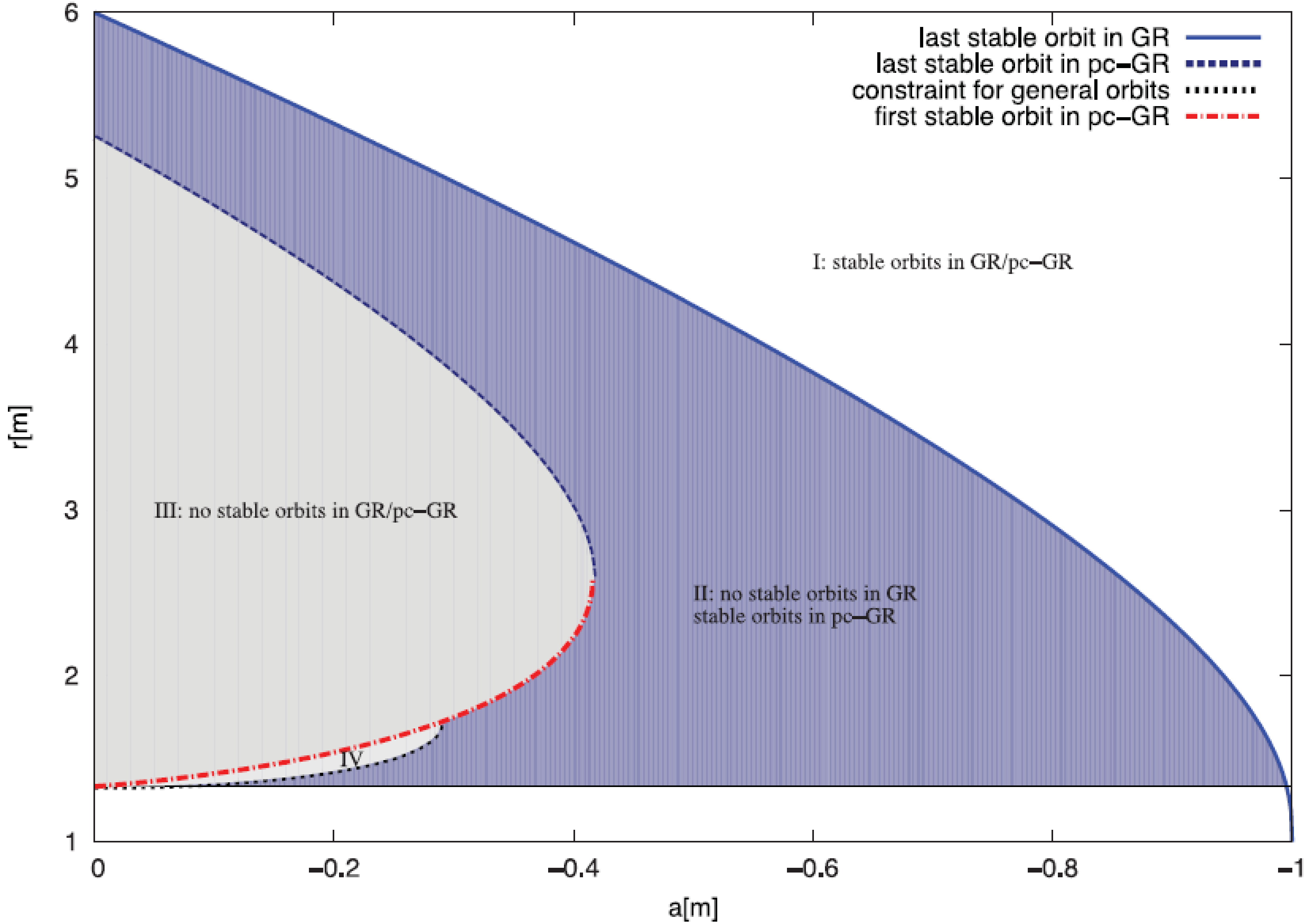
GR:



Due to a maximal ω appears a dark ring!

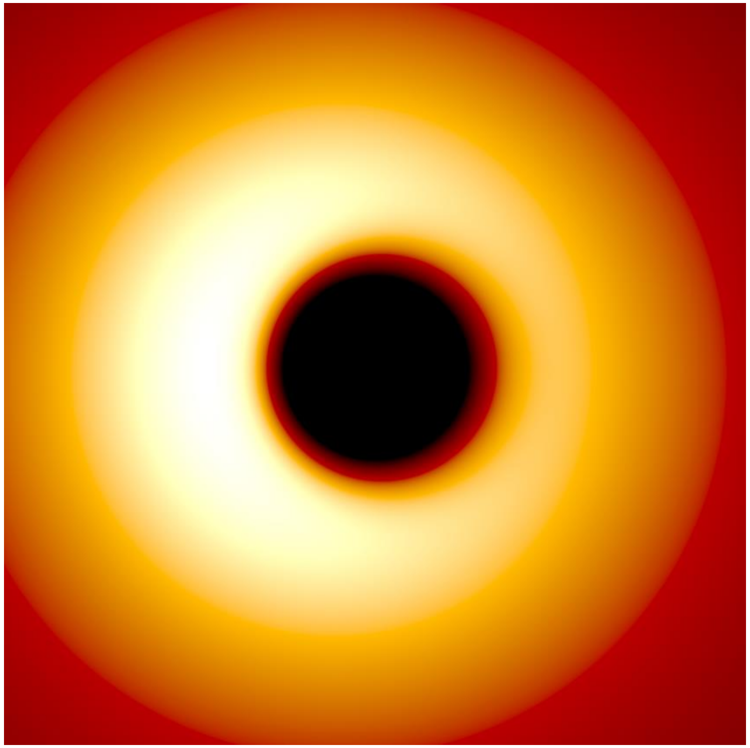


a=0: Observe the Einstein rings of first and second order!

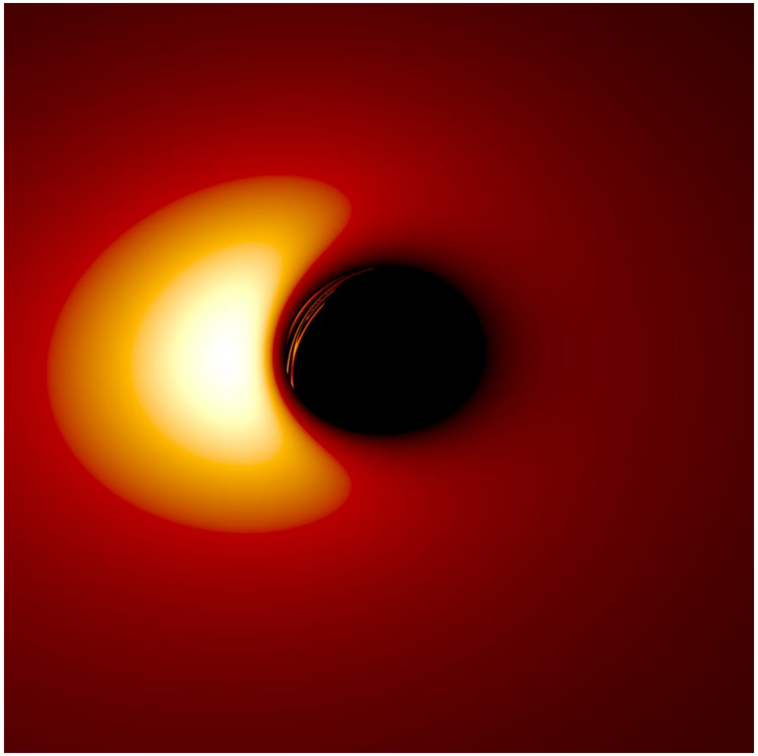


last stable orbit in GR
last stable orbit in pc−GR
constraint for general orbits
first stable orbit in pc−GR
I: stable orbits in GR/pc−GR
II: no stable orbits in GR
stable orbits in pc−GR
III: no stable orbits in GR/pc−GR
IV
r[m]
a[m]
6
5
4
3
2
1
0
−0.2
−0.4
−0.6
−0.8
−1

a=0.4, 10 degrees

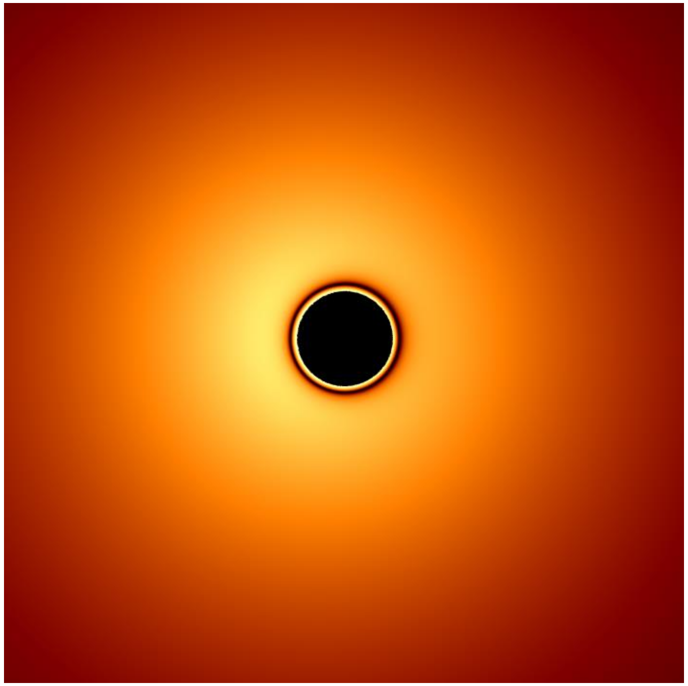


a=0.4, 45 degrees

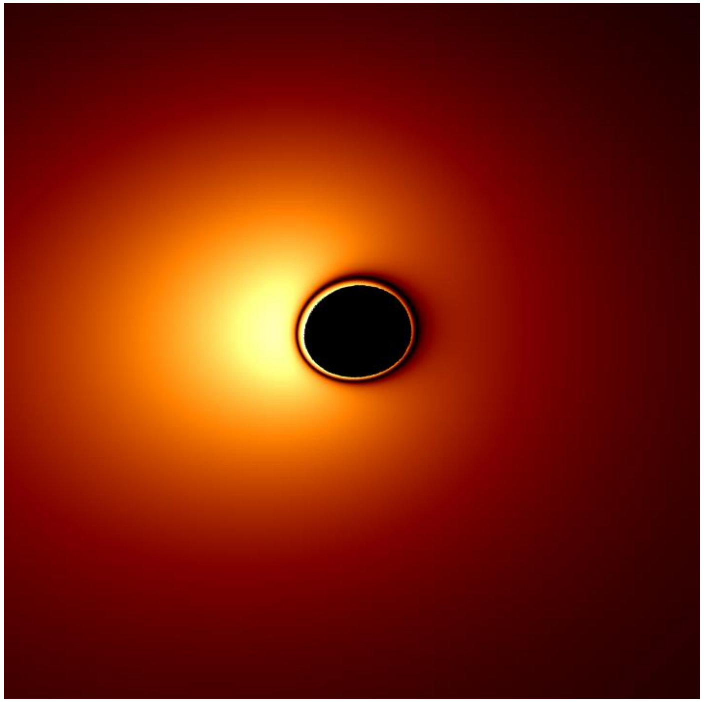


pc-GR

a=0.5, 10 degrees



a=0.5, 45 degrees

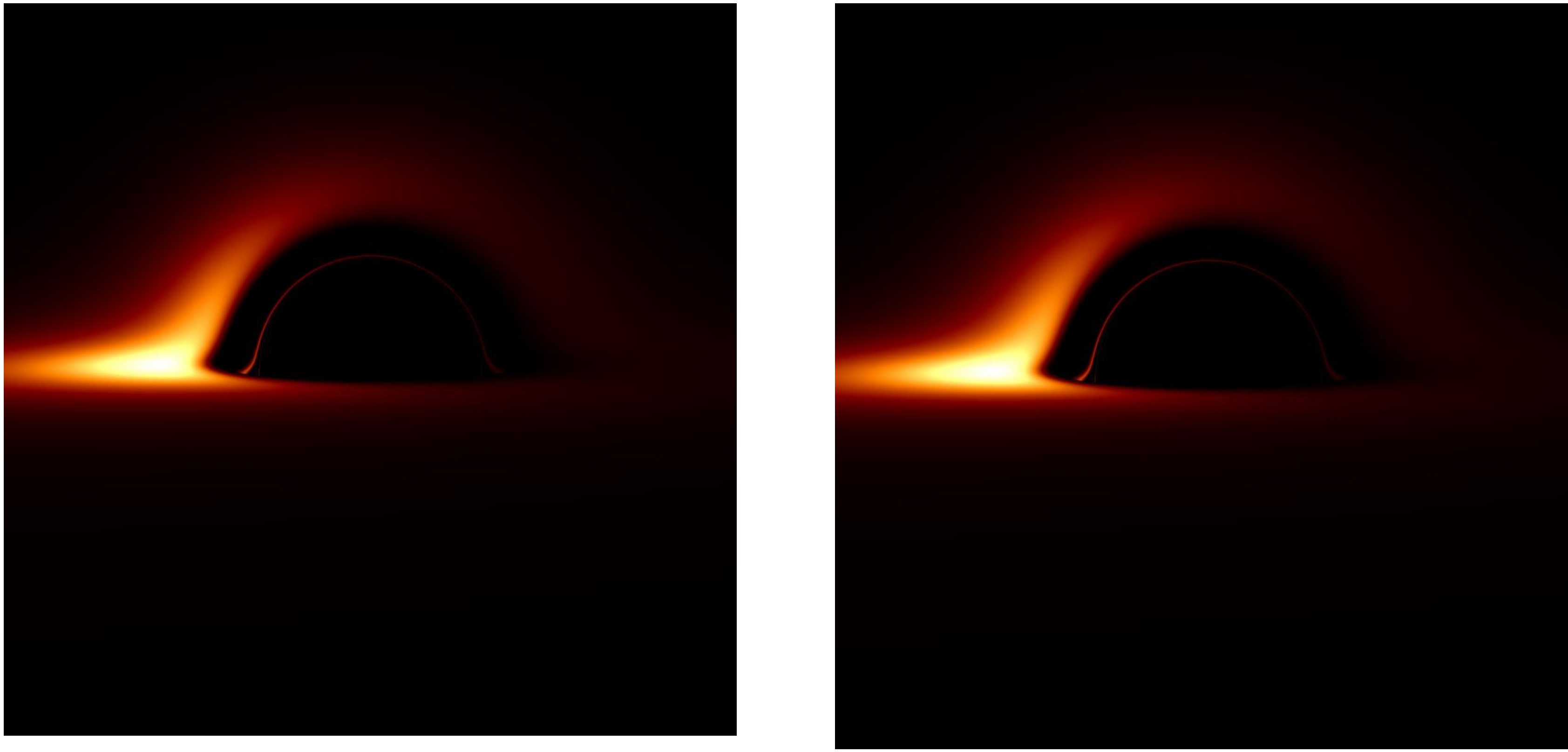


pc-GR

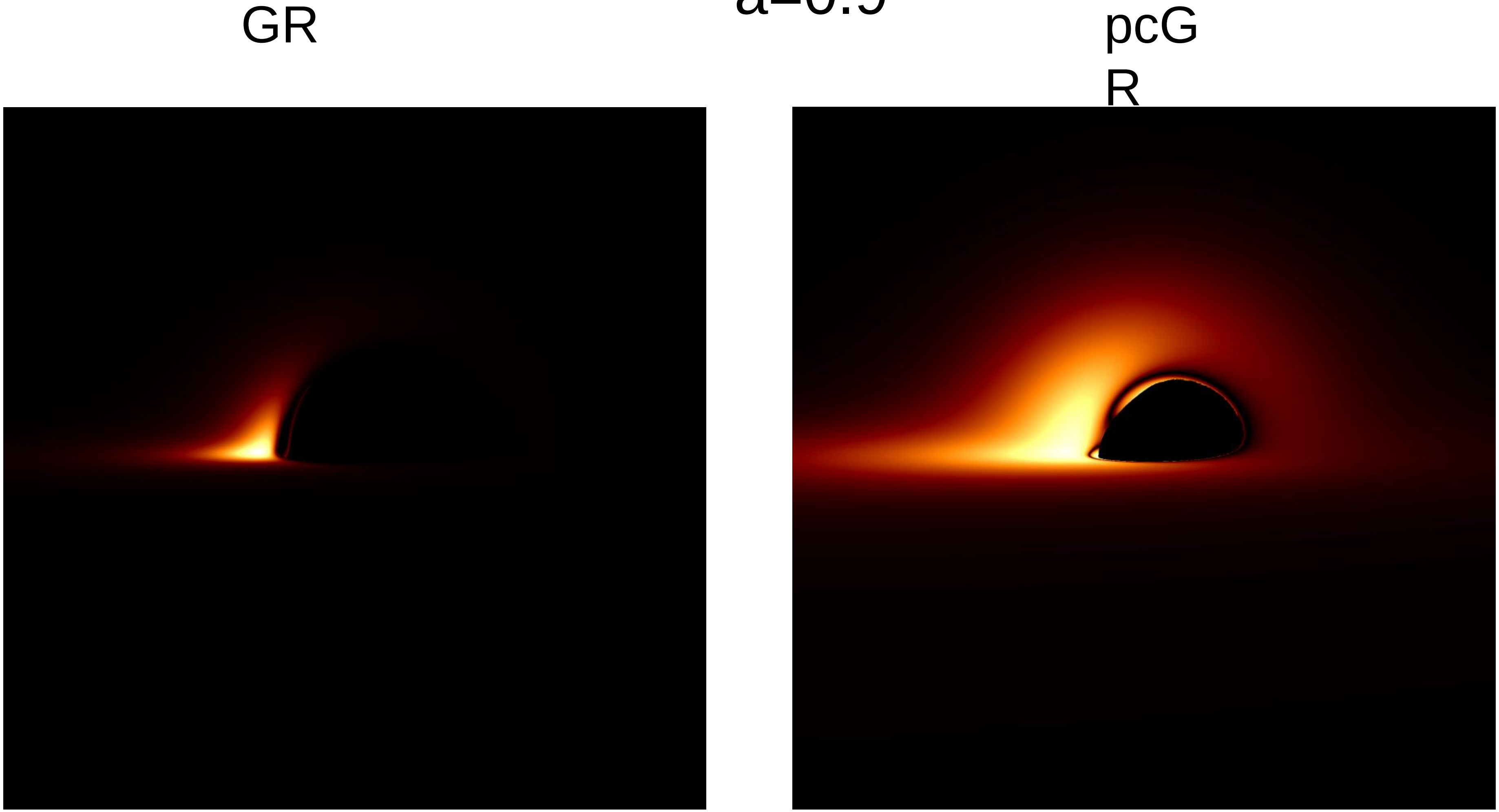
a=0

GR

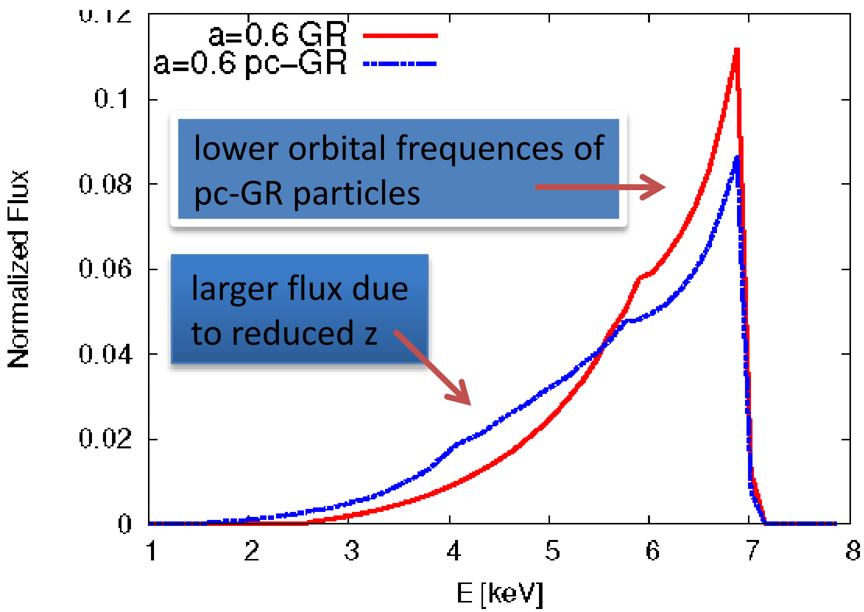
pcG
R



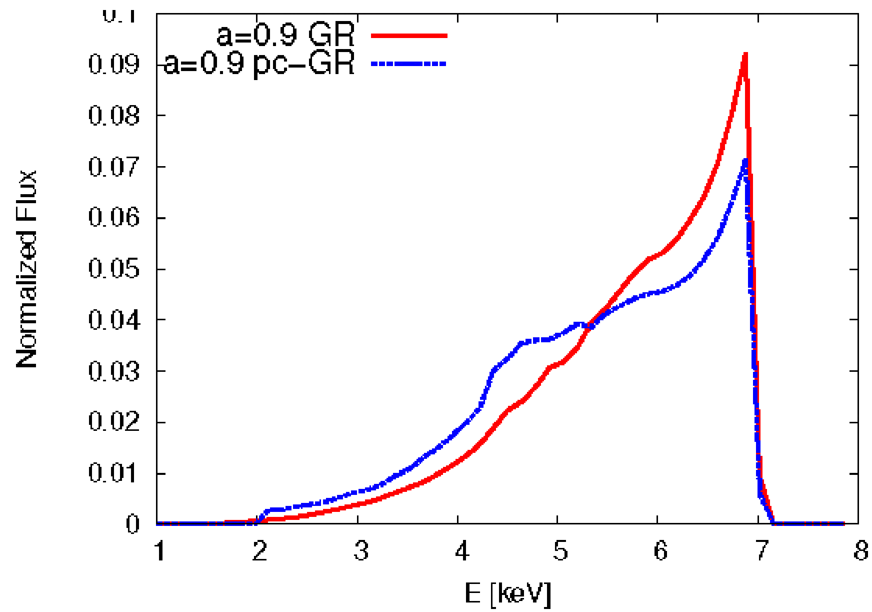
a=0.9



GC relativistic Fe K line results



(c) $a = 0.6$

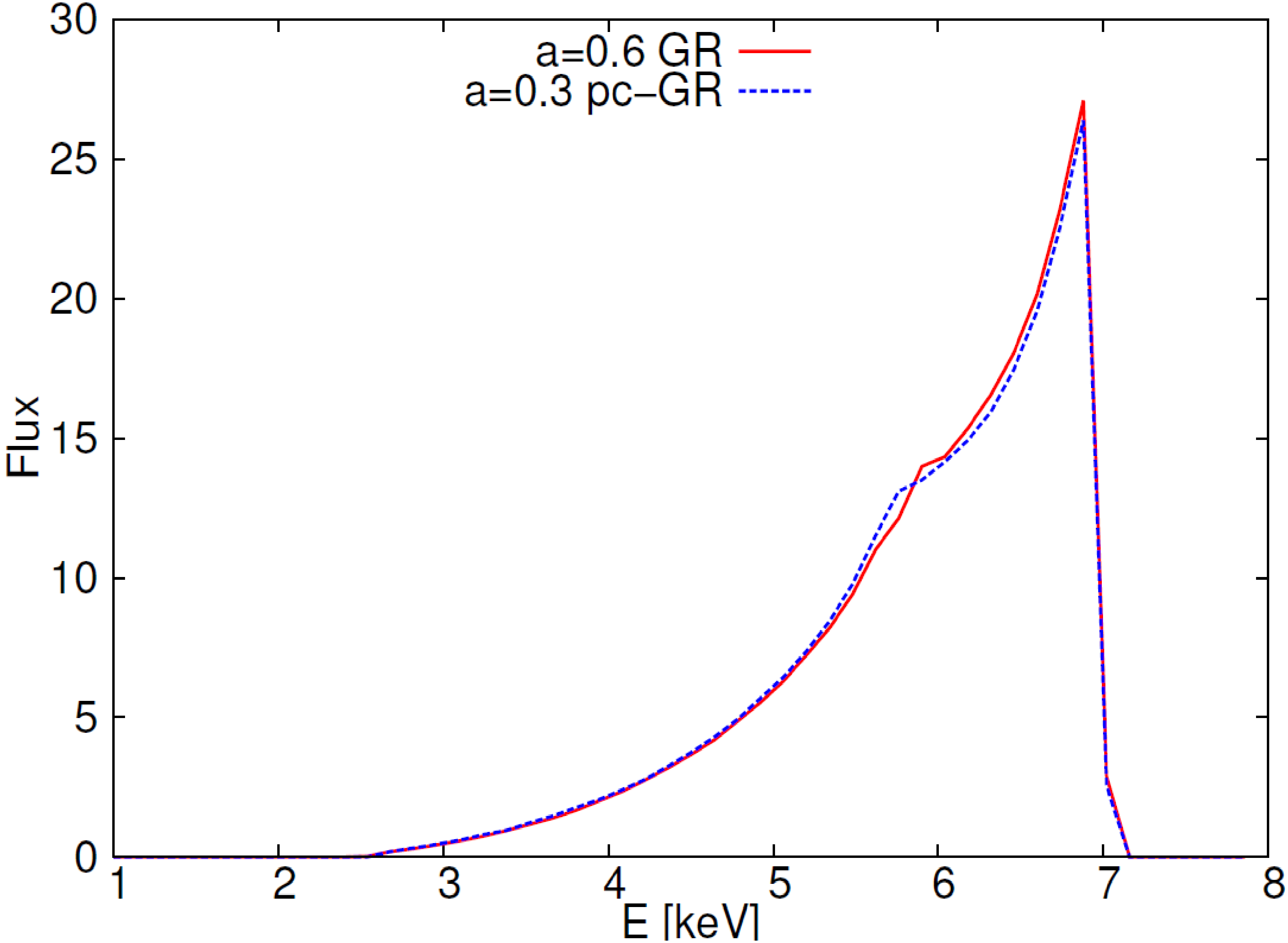


(d) $a = 0.9$

As pc-GR discs are brighter, the integrated line flux is larger in pc-GR

The line profiles are clearly different from standard GR

This offers a second robust measurements to test pc-GR versus GR



a=0.6 GR
a=0.3 pc−GR
Flux
E [keV]
0
5
10
15
20
25
30
1
2
3
4
5
6
7
8

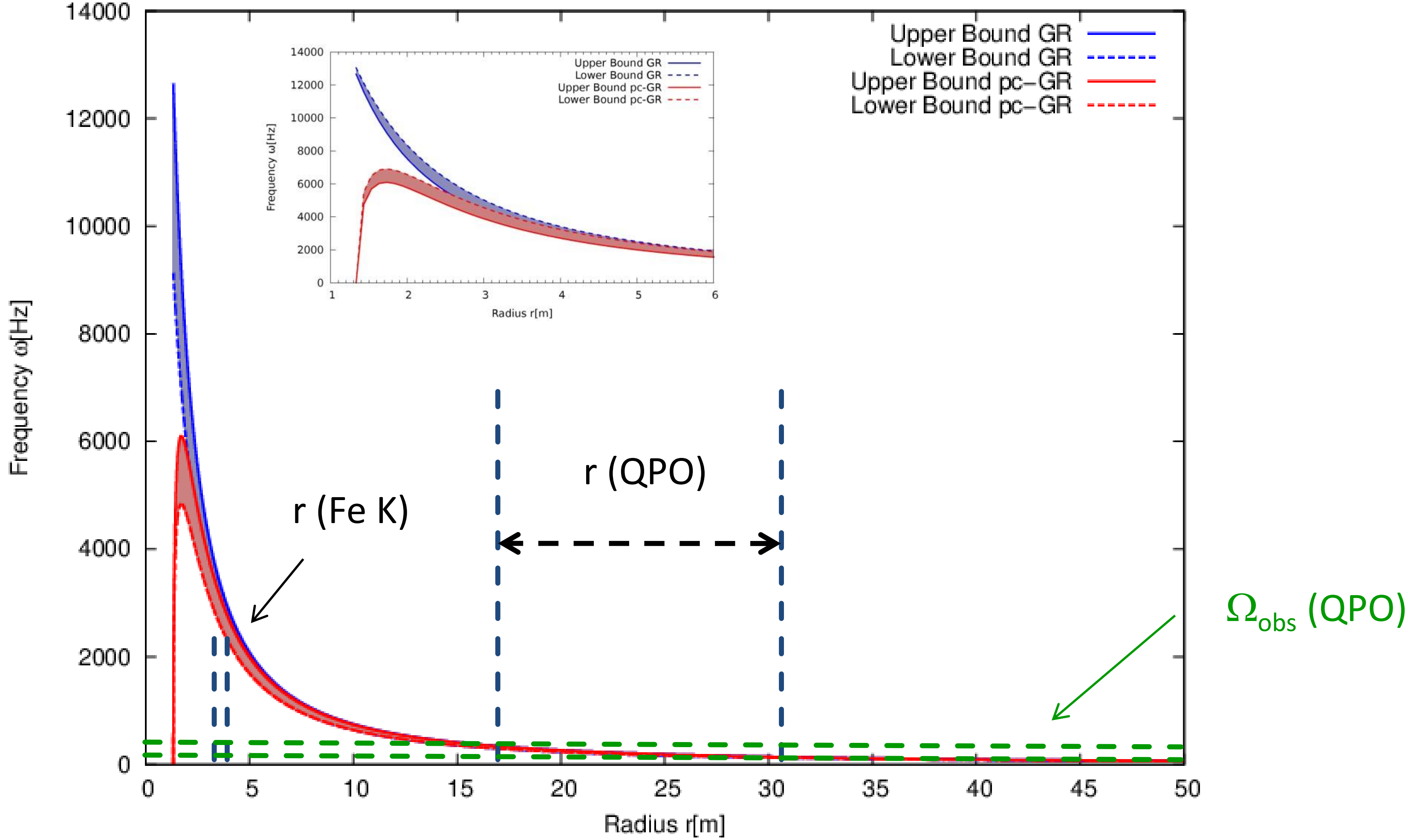
Conclusions

•Only a pseudo-complex algebraic extension has no unphysical particles:
Mass not only curves space-time but also changes vacuum properties nearby → No event horizon
* Predictions with clear deviations to standart GR
* Aplications: Particles in a circular orbit, accretion discs, raytracing→ simulations of accretion discs.

APENDICE

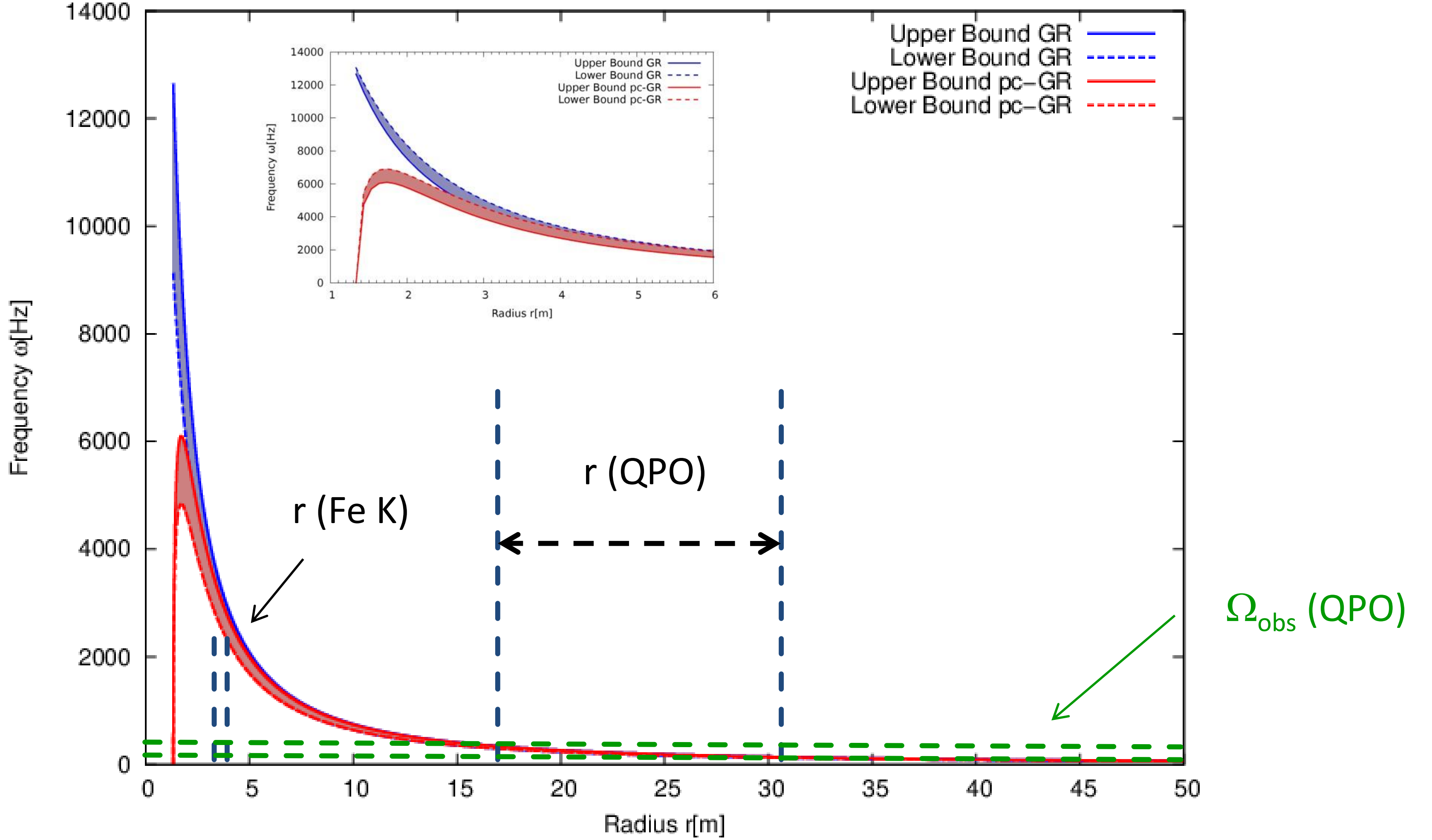
XTE J1550-564

$$M = 9.10 \pm 0.61$$
$$a = 0.50 \pm 0.15$$



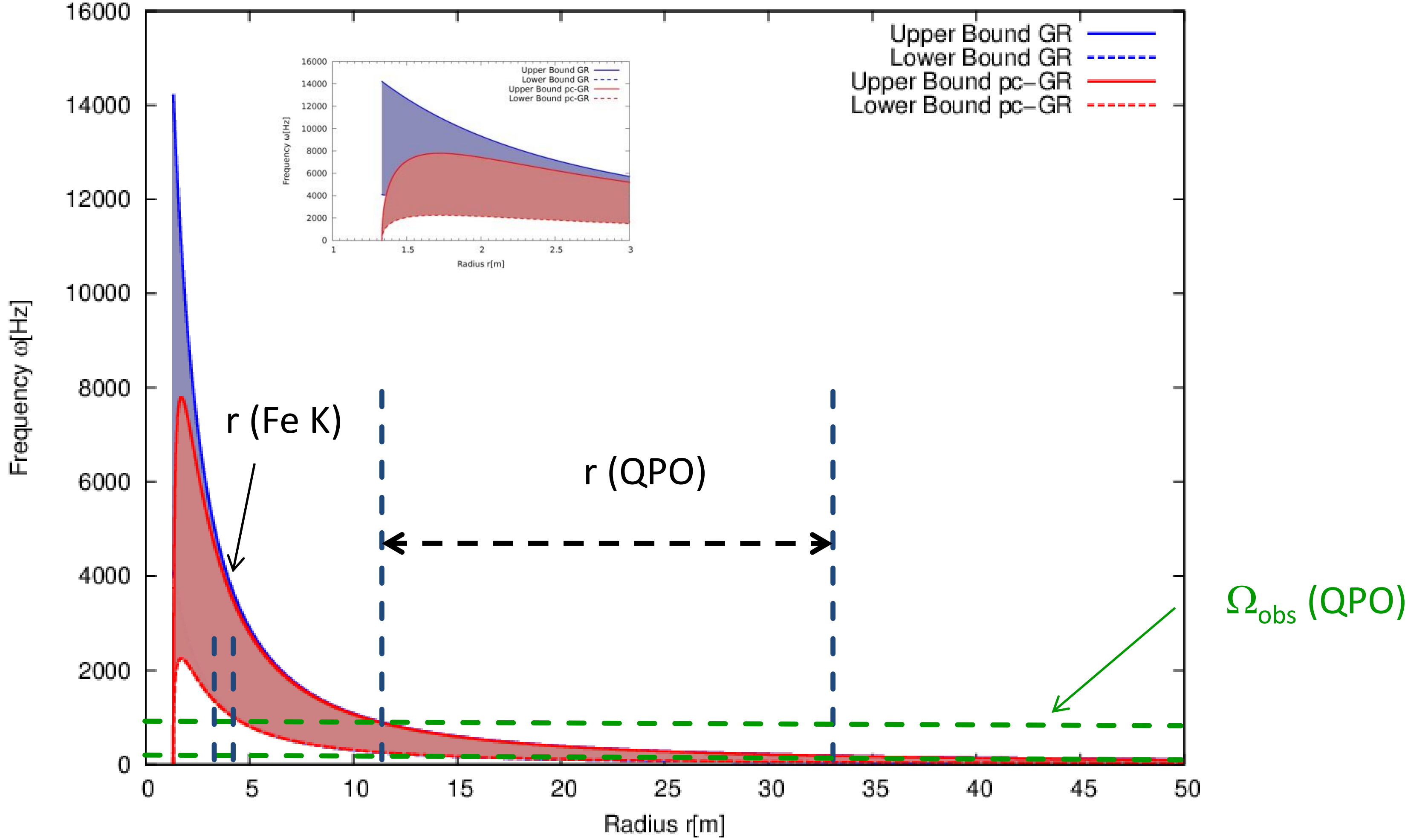
XTE J1550-564

$$M = 9.10 \pm 0.61$$
$$a = 0.50 \pm 0.15$$



J1752-223

$$M = 9.80 \pm 0.90$$
$$a = 0.52 \pm 0.09$$



Métrica inversa:

$$
\begin{aligned}
g^{00} &= -\frac{(r^2+a^2)^2-a^2\Delta\sin^2\vartheta}{\Sigma\Delta} \quad , \\
g^{11} &= \frac{\Delta}{\Sigma} \quad , \\
g^{22} &= \frac{1}{\Sigma} \quad , \\
g^{33} &= \frac{\Delta-a^2\sin^2\vartheta}{\Sigma\Delta\sin^2\vartheta} \quad , \\
g^{03} &= -\frac{a\psi}{\Sigma\Delta} \quad . \quad .
\end{aligned}
$$

Momentos lineales:

$$
\begin{aligned}
\text{-E\_t} = \quad p_t &= g_{00}\dot{t} + g_{03}\dot{\varphi} = -\left(1-\frac{\psi}{\Sigma}\right)\dot{t} - a\frac{\psi}{\Sigma}\sin^2\vartheta\dot{\varphi} \\
p_r &= g_{11}\dot{r} = \frac{\Sigma}{\Delta}\dot{r} \\
p_\vartheta &= g_{22}\dot{\vartheta} = \Sigma\dot{\vartheta} \\
\text{L\_z} = \quad p_\varphi &= g_{30}\dot{t} + g_{33}\dot{\varphi} = -a\frac{\psi}{\Delta}\sin^2\vartheta\dot{t} + \left[(r^2+a^2) + a^2\frac{\psi}{\Sigma}\sin^2\vartheta\right]\sin^2\vartheta\dot{\varphi}
\end{aligned}
$$

Una cuarta constante de movimiento se obtiene, pidiendo
Separabilidad de la ecuación de Hamilton-Jacobi (constante de Carter):

Ecuación H-J:
$$\frac{\partial S}{\partial \lambda} = \frac{1}{2} g^{\alpha\beta} \left(\frac{\partial S}{\partial x^\alpha} \frac{\partial S}{\partial x^\beta} \right)$$

Acción:
$$S = -\frac{1}{2}\zeta\lambda - Et + L_z\varphi + S_\vartheta + S_r$$

Sustituir todo:

$$2\frac{\partial S}{\partial \lambda} = g^{00}\left(\tfrac{\partial S}{\partial t}\right)^2 + g^{11}\left(\tfrac{\partial S}{\partial r}\right)^2 + g^{22}\left(\tfrac{\partial S}{\partial \vartheta}\right)^2 + g^{33}\left(\tfrac{\partial S}{\partial \varphi}\right)^2 + 2g^{03}\tfrac{\partial S}{\partial t}\tfrac{\partial S}{\partial \varphi}$$
$$\Leftrightarrow \quad -\zeta = g^{00}E^2 + g^{33}L_z^2 - 2g^{03}EL_z + g^{11}\left(\tfrac{\partial S}{\partial r}\right)^2 + g^{22}\left(\tfrac{\partial S}{\partial \vartheta}\right)^2 \quad (5.82)$$

Sustitución de los elementos de matiz de g:

$$\begin{aligned}
-\zeta &= -\tfrac{(r^2+a^2)^2-a^2\Delta\sin^2\vartheta}{\Sigma\Delta}E^2+\tfrac{\Delta}{\Sigma}\left(\tfrac{\partial S}{\partial r}\right)^2+\tfrac{1}{\Sigma}\left(\tfrac{\partial S}{\partial\vartheta}\right)^2\\
&\quad+\tfrac{\Delta-a^2\sin^2\vartheta}{\Sigma\Delta\sin^2\vartheta}L_z^2+2\tfrac{a\psi}{\Sigma\Delta}EL_z\\
\Leftrightarrow\quad -\zeta\Sigma &= \Delta\left(\tfrac{\partial S}{\partial r}\right)^2+\left(\tfrac{\partial S}{\partial\vartheta}\right)^2\\
&\quad-\tfrac{(r^2+a^2)^2}{\Delta}E^2+a^2\sin^2\vartheta E^2+\tfrac{1}{\sin^2\vartheta}L_z^2-\tfrac{a^2}{\Delta}L_z^2+\tfrac{2a\psi}{\Delta}EL_z\\
\Leftrightarrow\quad -\zeta\Sigma &= \Delta\left(\tfrac{\partial S}{\partial r}\right)^2+\left(\tfrac{\partial S}{\partial\vartheta}\right)^2\\
&\quad-\tfrac{1}{\Delta}\left((r^2+a^2)E-aL_z\right)^2-\tfrac{2}{\Delta}(r^2+a^2)aEL_z\\
&\quad+\tfrac{1}{\sin^2\vartheta}\left(-L_z+a\sin^2\vartheta E\right)^2+2aEL_z+\tfrac{2a\psi}{\Delta}EL_z \qquad (5.83)
\end{aligned}$$

• • • resultado final:

$$\begin{aligned}
-\zeta\Sigma &= \Delta\left(\tfrac{dS_r}{dr}\right)^2+\left(\tfrac{dS_\vartheta}{d\vartheta}\right)^2\\
&\quad-\tfrac{1}{\Delta}\left((r^2+a^2)E-aL_z\right)^2+\tfrac{1}{\sin^2\vartheta}\left(-L_z+a\sin^2\vartheta E\right)^2
\end{aligned}$$

Usando:

$$\begin{aligned}
\frac{1}{\sin^2\vartheta}\left(-L_z+a\sin^2\vartheta E\right)^2 &= \tfrac{L_z^2}{\sin^2\vartheta}+a^2\sin^2\vartheta E^2-2aL_zE\\
&= \tfrac{L_z^2}{\sin^2\vartheta}+a^2E^2-a^2\cos^2\vartheta E^2-2aL_zE+L_z^2-L_z^2\\
&= \tfrac{L_z^2-L_z^2\sin^2\vartheta}{\sin^2\vartheta}-a^2\cos^2\vartheta E^2+(aE+L_z)^2\\
&= \left(\tfrac{L_z^2}{\sin^2\vartheta}-a^2E^2\right)\cos^2\vartheta+(aE-L_z)^2 \quad ,
\end{aligned}$$

Obtenemos finalmente:

$$-\zeta r^2 - \Delta \left(\frac{dS_r}{dr}\right)^2 + \frac{1}{\Delta}\left((r^2+a^2)E - aL_z\right)^2 - (aE - L_z)^2$$
$$= \left(\zeta a^2 + \frac{L_z^2}{\sin^2\vartheta} - a^2E^2\right)\cos^2\vartheta + \left(\frac{dS_\vartheta}{d\vartheta}\right)^2 \quad .$$

Ambos lados dependen de variables diferentes, asi que deben ser constante: Constante de Carter

→ $$\left(\frac{dS_r}{dr}\right)^2 = \frac{R}{\Delta^2} \qquad \text{y} \qquad \left(\frac{dS_\vartheta}{d\vartheta}\right)^2 = \theta$$

con

$$R := \left[(r^2+a^2)E - aL_z\right]^2 - \Delta\left[\mathcal{C} + (aE - L_z)^2 + \zeta r^2\right]$$

$$\theta := Q - \left(a^2(\zeta - E^2) + \frac{L_z^2}{\sin^2\vartheta}\right)\cos^2\vartheta$$

Ecuaciones de movimiento: $\dot{x}^{\mu} = g^{\mu\alpha} p_{\alpha} = g^{\mu\alpha} \frac{\partial S}{\partial x^{\alpha}}$

O:

$$\begin{aligned}
\dot{x}^0 = \dot{t} \quad &= \tfrac{1}{\Sigma\Delta} \left\{ \left[(r^2 + a^2)^2 + a^2 \Delta \sin^2 \vartheta \right] E - a\psi L_z \right\} \\
\dot{x}^1 = \dot{r} \quad &= \pm \tfrac{\sqrt{R}}{\Sigma} \\
\dot{x}^2 = \dot{\vartheta} \quad &= \pm \tfrac{\sqrt{\vartheta}}{\Sigma} \\
\dot{x}^3 = \dot{\varphi} \quad &= \tfrac{1}{\Sigma\Delta} \left[\left(\tfrac{\Delta}{\sin^2 \vartheta} - a^2 \right) L_z + a\psi E \right] \quad .
\end{aligned}$$

...

Obteniendo las ecuaciones geodésicas, el uso del principio de Hamilton nos lleva a la ecuación
(ϑ =Θ)

$$\int_{r_{\mathrm{em}}}^{r_{\mathrm{obs}}} \frac{\mathrm{d}r}{\sqrt{R}} = \int_{\vartheta_{\mathrm{em}}}^{\vartheta_{\mathrm{obs}}} \frac{\mathrm{d}\vartheta}{\sqrt{\vartheta}}$$

FLUX: $$F = \frac{\dot{M}_0}{4\pi\sqrt{-g}} f$$

con $$f = -\omega_{|r}(E - \omega L_z)^{-2} \int\limits_{r_{\omega_{\max}}}^{r} (E - \omega L_z) L_{z|r} \mathrm{d}r$$

La integración es desde el punto con la derivada de ω igual a 0 al punto r:

→Para r>ω-max se integra hacia afuera (transporte del flujo hacia afuera)

Para r<ω-max se integra hacia adentro (transporte del flujo hacia adentro)

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