

# Magnetized White Dwarfs Revisited



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# Outline

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Introduction

Equations of State

Weak Field Limit

Strong Field Limit

Structure Equations

Results

Weak Field Limit

Conclusions

Future Directions

# Introduction

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- Magnetic White Dwarfs observational surface field until  $10^9$  G.
- Schwinger magnetic field:

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- Studies considering isotropic pressure:
  - Weak Field Limit,  $B < B_c$  (Suh & Mathews, 2000)<sup>†</sup>

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- Studies considering isotropic pressure:
  - Weak Field Limit,  $B < B_c$  (Suh & Mathews, 2000)<sup>†</sup>
  - Strong Field Limit,  $B > B_c$  (Das & Mukhopadhyay 2012, 2013, 2014)<sup>‡</sup>

<sup>†</sup> Suh, I.-S. & Mathews, G. J. 2000, ApJ, 530, 949

<sup>‡</sup> Das, U. & Mukhopadhyay, B. 2012, IJMP D, 21, 1242001

Das, U. & Mukhopadhyay, B. 2013 PRL, 110, 071102

Das, U. & Mukhopadhyay, B. 2014 arXiv e-prints

# Introduction

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- Cylindrical metric (Manreza Paret et al. 2014)<sup>†</sup>:

$$ds^2 = -e^{2\Phi(r)} dt^2 + e^{2\Lambda(r)} dr^2 + r^2 d\phi^2 + e^{2\Psi(r)} dz^2$$

to include anisotropic pressures and anisotropic structure equations.

(See Daryel's talk tomorrow.)

<sup>†</sup>Manreza Paret, D., Horvath, J. E., & Pérez Martínez, A. 2014, arXiv:1407.2280, accepted in RA&A 2015

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- **Objectives:**

- Find *more treatable (semi-analytical)* EoS in each field limit.
- Solve structure equations in spherical and cylindrical metric to compare with previous works.
- Discuss scalar Virial theorem bound for magnetic field, inverse  $\beta$  decay, ... (Coelho et al. 2014)<sup>‡</sup>

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## Equations of State: Weak Field Limit

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Starting from thermodynamical potential at  $T = 0$ :

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Starting from thermodynamical potential at  $T = 0$ :

$B < B_c \rightarrow$  Euler-MacLaurin Expansion

$$P_{\parallel}(x, B) = \frac{m^4}{12\pi^2(\hbar c)^3} \Phi_{+}(x, B)$$

$$P_{\perp}(x, B) = \frac{m^4}{12\pi^2(\hbar c)^3} \Phi_{-}(x, B)$$

$$E(x, B) = \frac{m^3}{12\pi^2(\hbar c)^3} \left( \frac{1}{x} \left[ \frac{B}{B_c} \right]^2 + 4x^3 \right) \left( \frac{Am_N}{Z} + m\sqrt{x^2 + 1} \right) - \frac{m^4}{12\pi^2(\hbar c)^3} \Phi_{+}(x, B)$$

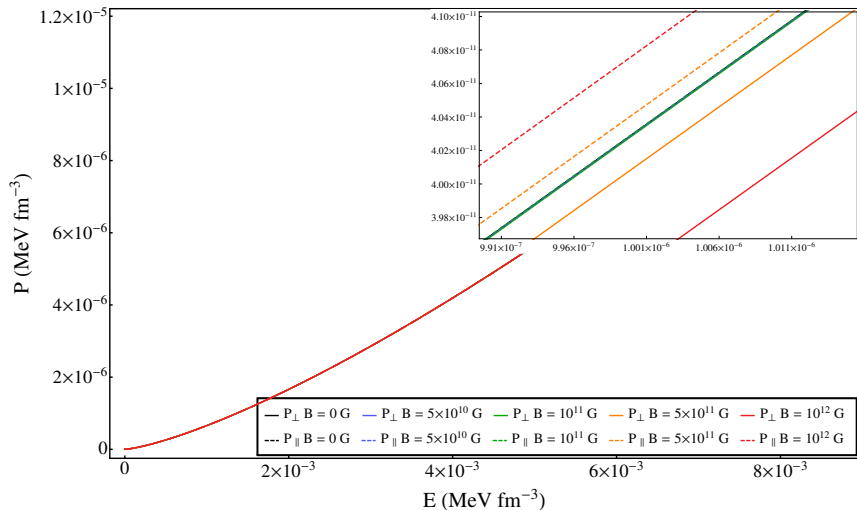
$$\Phi_{\pm}(x, B) = x\sqrt{x^2 + 1} \left( x^2 - \frac{3}{2} \right) + \frac{3}{2} \ln \left[ \sqrt{x^2 + 1} + x \right] \pm \left[ \frac{B}{B_c} \right]^2 \ln \left[ \sqrt{x^2 + 1} + x \right]$$

with:

$x$ : dimensionless Fermi momentum,  $A$ : atomic mass,  $Z$ : atomic number,

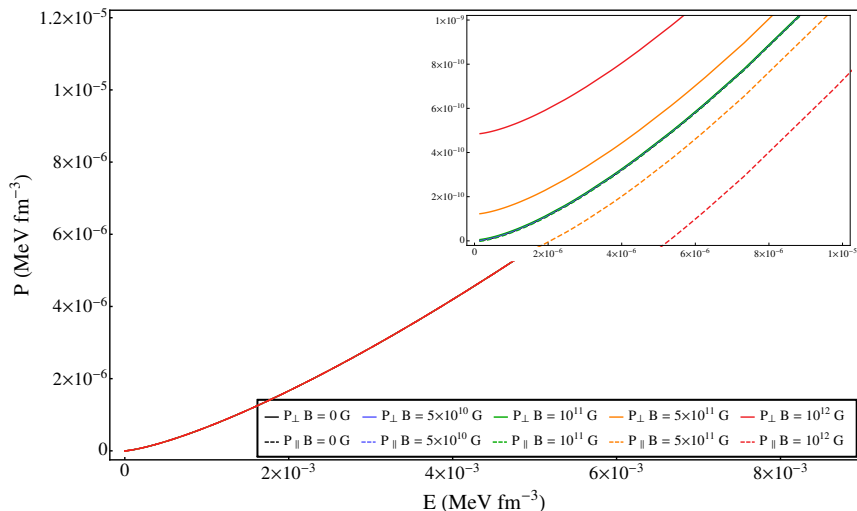
$m$ : electron mass, and  $m_N$ : nucleon mass.

# Equations of State: Weak Field Limit



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$$P_{\perp} = E = -P_{\parallel} = \frac{B^2}{8\pi}$$



# Equations of State: Strong Field Limit

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From Numeric EoS obtained in (Manreza Paret et al. 2014)<sup>†</sup>:

$$\text{Polytropic Parametrization: } P = K E^{\Gamma}$$

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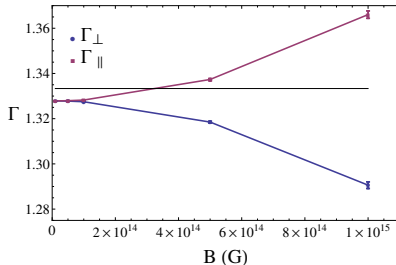
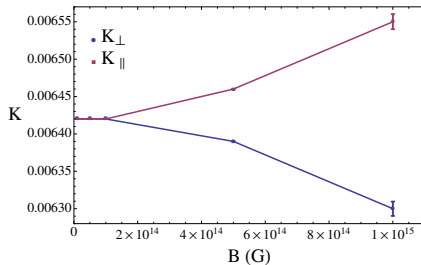
From Numeric EoS obtained in (Manreza Paret et al. 2014)<sup>†</sup>:

Polytropic Parametrization:  $P = K E^\Gamma$

$B$ (G)	$K_\perp ([\text{MeV fm}^{-3}]^{1-\Gamma_\perp})$	$K_\parallel ([\text{MeV fm}^{-3}]^{1-\Gamma_\parallel})$
$1 \times 10^{13}$	$0.00642 \pm 0.012 \times 10^{-5}$	$0.00642 \pm 0.013 \times 10^{-5}$
$5 \times 10^{13}$	$0.00642 \pm 0.012 \times 10^{-5}$	$0.00642 \pm 0.014 \times 10^{-5}$
$1 \times 10^{14}$	$0.00642 \pm 0.011 \times 10^{-5}$	$0.00642 \pm 0.020 \times 10^{-5}$
$5 \times 10^{14}$	$0.00639 \pm 0.024 \times 10^{-5}$	$0.00646 \pm 0.027 \times 10^{-5}$
$1 \times 10^{15}$	$0.00630 \pm 0.945 \times 10^{-5}$	$0.00655 \pm 1.000 \times 10^{-5}$
$B$ (G)	$\Gamma_\perp$	$\Gamma_\parallel$
$1 \times 10^{13}$	$1.32786 \pm 0.002 \times 10^{-2}$	$1.32787 \pm 0.002 \times 10^{-2}$
$5 \times 10^{13}$	$1.32777 \pm 0.002 \times 10^{-2}$	$1.32796 \pm 0.002 \times 10^{-2}$
$1 \times 10^{14}$	$1.32749 \pm 0.002 \times 10^{-2}$	$1.32824 \pm 0.003 \times 10^{-2}$
$5 \times 10^{14}$	$1.31850 \pm 0.037 \times 10^{-2}$	$1.33734 \pm 0.041 \times 10^{-2}$
$1 \times 10^{15}$	$1.29060 \pm 0.147 \times 10^{-2}$	$1.36604 \pm 0.164 \times 10^{-2}$

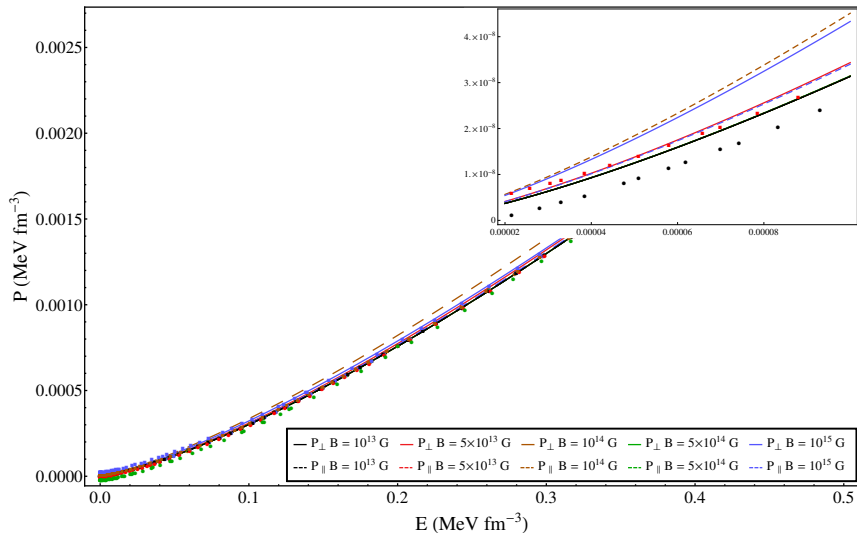
# Equations of State: Strong Field Limit

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- Anisotropy becomes important for higher values of  $B$ .
- Parameter's uncertainty increases with  $B$ .

# Equations of State: Strong Field Limit





# Structure Equations

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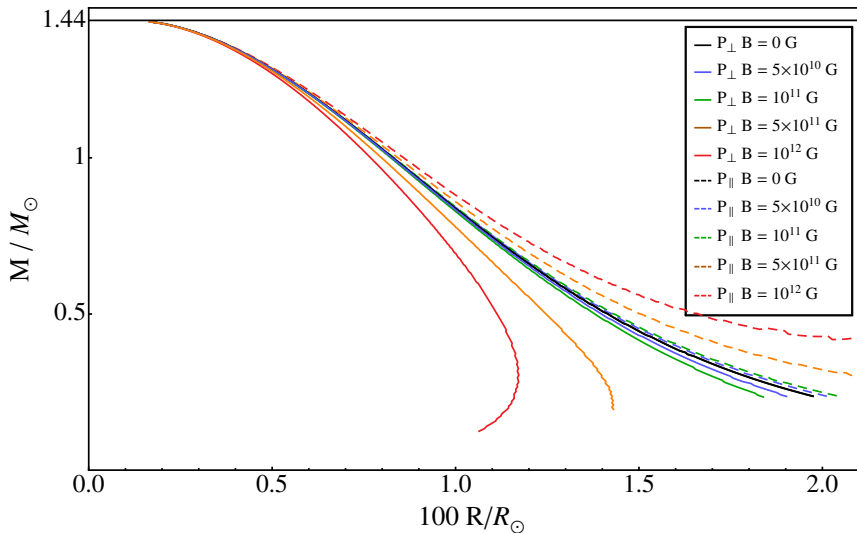
- Spherical Symmetry: TOV equations

$$\begin{aligned}\frac{dM}{dr} &= 4\pi G E \\ \frac{dP}{dr} &= -G \frac{(E + P)(M + 4\pi r^3)}{r^2 - 2rM}\end{aligned}$$

Boundary Conditions:  $P(R) = 0, M(0) = 0$ .

- Solve for  $P_{\parallel}$  and  $P_{\perp}$  independently.

## Results: Weak Field Limit



# Conclusions

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- EoS: For  $B < B_c$  pressure and energy density become:

$$P(x, B) = P(x, 0) \pm [B/B_c]^2 f(x)$$

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- For  $B \gtrsim B_c$  anisotropic effects must be considered.
- Chandrasekhar Mass limit holds. (More details in Daryel's talk.)

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# Future Directions

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- Solve anisotropic structure equations from (Manreza Paret et al. 2014).
- Given our *more treatable (semi-analytical)* EoS, analyze:
  - (a) inverse  $\beta$  decay and pycnonuclear fusion reactions,
  - (b) macroscopic consequences of anisotropic structure equations solutions,
  - (c) bound on the magnetic field imposed by the scalar Virial theorem.(Coelho et al. 2014)<sup>‡</sup>

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