Discrete symmetries in the heterotic-string landscape

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with Alon Faraggi and Doron Gepner.

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Introduction

Motivation - Why heterotic string models?

- In heterotic string theory $(2,0)$ models play a prominent role.
- They are supersymmetric and they are in a sense "minimal" since supersymmetry requires (at least) $(2,0)$ superconformal symmetry. Banks et al '88
- They naturally allow for $SO(10)$ unification, which is well motivated by the standard model data.
- Some of the most realistic heterotic string models including orbifolds and free fermionic models are of this type.

Motivation - Why landscape symmetries?

- The landscape of heterotic $(2, 0)$ models is not fully classified.
- Any symmetries provide hints and possible tools for a better understanding.
- **Mirror symmetry** is a famous example of a discrete symmetry in this landscape with far reaching implications.
  Candelas et al '90, Greene and Plesser '90, Blumenhagen et al '96
- The **spinor-vector duality** and its extension discussed here is another discrete symmetry of the same type.
  Faraggi et al '07, Angelantonj et al '10, Faraggi et al '11, Athanasopoulos et al '14

Heterotic string: notation and conventions

A general state in a heterotic model is of the form:

$$\Phi_L \otimes \Phi_R$$

where

$$\Phi_L = (w_L)(h_L, Q_L), \quad \Phi_R = (w)(h, Q)(p).$$

- w_L is an $SO(2)$ weight (o, v, s, c)
- w is an $SO(10)$ weight (o, v, s, c)
- p an E_8 weight.

Heterotic string: notation and conventions

The appearance of the $SO(10)$ and E_8 weights is because of the *bosonic string map*.
Englert et al '86, Schellekens '87, Gepner '88, Lerche et al '89

This provides an elegant way to make compatible the following statements:

i) The only generic way to achieve modular invariance in a non-free CFT is to have a left-right symmetric spectrum.

ii) In the heterotic string the left and right sectors are treated differently.

We can then construct heterotic models from type II models using the bosonic string map that effectively replaces on the right-moving sector

$$w_{SO(2)} \rightarrow (w_{SO(10)}, w_{E_8})$$

This preserves the modular invariance of the theory.

Defining the concepts

Models with $N = (2,0)$ superconformal symmetry

By definition a CFT is said to have $N = 2$ world-sheet supersymmetry if it includes four fields:

$$T(z), G^{\pm}(z), J(z)$$

that satisfy the algebra:

$$\begin{aligned}
[L_m, L_n] &= (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0} \ , \\
\left[L_m, G^{\pm}_{n\pm a}\right] &= (\frac{m}{2} - n \mp a)G^{\pm}_{m+n\pm a} \ , \\
[L_m, J_n] &= -nJ_{m+n} \ , \\
&\vdots
\end{aligned}$$

where $G^{\pm}(e^{2\pi i}z) = -e^{\mp 2\pi i a}G^{\pm}(z)$.

Motivation from spinor-vector duality

- $(2,0)$ models appear after the right $N=2$ superconformal symmetry of (2,2) models breaks.
- $N=2$ SCS on the bosonic side is equivalent to E_6 gauge symnmetry.
- When E_6 is broken down to $SO(10)$ the representations decompose as:

$$\begin{aligned} \mathbf{27} &= \mathbf{16}+\mathbf{10}+\mathbf{1} \\ \overline{\mathbf{27}} &= \overline{\mathbf{16}}+\mathbf{10}+\mathbf{1} \end{aligned}$$

Motivation from spinor-vector duality

- If the breaking is implemented through a $\mathbb{Z}_2$ or $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold we naturally end up with two models: One with massless spinors and one with massless vectors and singlets. (Spinor-vector duality)
  Faraggi, Kounnas, Rizos '07
  Angelantonj, Faraggi, Tsulaia '10
  Faraggi, Florakis, Mohaupt, Tsulaia '11
- This agrees with our intuition from anomaly cancellation, but for generic breakings it is only true per sector.
- Can we generalize this result for the entire spectrum for generic models? (Ans: Not with any known string construction.)

The spectral flow

The $N = 2$ algebra is invariant under the following transformation which is known as the *spectral flow*:

$$\begin{aligned} L_n^{\eta} &= L_n + \eta J_n + \frac{c}{6}\eta^2 \delta_{n,0} \ , \\ G_{n\pm a}^{\eta\pm} &= G_{n\pm(a+\eta)}^{\eta\pm} \ , \\ J_n^{\eta} &= J_n + \frac{c}{3}\eta\delta_{n,0}. \end{aligned}$$

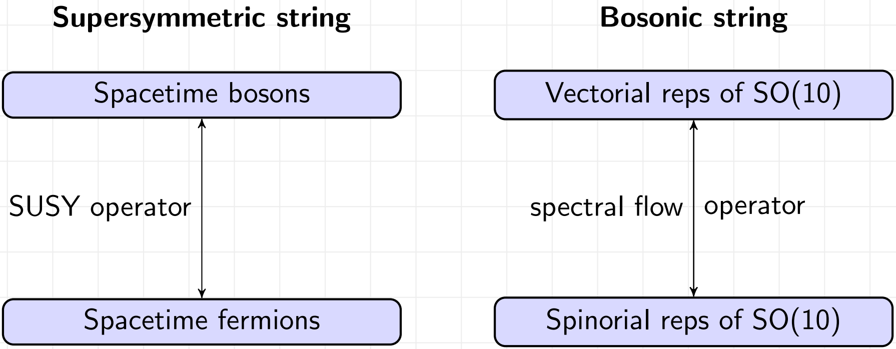
This implies the existence of a *spectral flow operator* U_η that acts on states in the following way:

$$U_\eta|h, Q\rangle = |h_\eta, Q_\eta\rangle = \left|h - \eta Q + \frac{\eta^2 c}{6}, Q - \frac{c\eta}{3}\right\rangle.$$

Schwimmer and Seiberg '87

The spectral flow

Every supersymmetric theory has an operator that relates bosons to fermions. On the bosonic sector of the heterotic string this operator relates vectorial and spinorial representations of $SO(10)$:



Defining the tools

Simple currents Schellekens and Yankielowicz '89, '90

- The simple current method provides a way to construct new modular invariants from a given one.
- This generic CFT construction generalizes both the orbifold and the free fermionic construction.

Definition of simple currents

Def: For a unitary RCFT, with highest weight representations $[\phi_i]$ and fusion algebra

$$[\phi_i] \times [\phi_j] = \sum_k N_{ij}^k [\phi_k], \quad N_{ij}^k \in \mathbb{N}_0,$$

a highest weight representation β is called a *simple current* if its fusion with any other highest weight takes the form

$$[\beta] \times [\phi_i] = [\phi_{\beta(i)}].$$

It is customary in many constructions to use additive notation for the states:

$$\beta + \phi_i = \phi_{\beta(i)}$$

and we adopt this from now on.

Simple current modular invariants

For any model with a given partition function:

$$Z[\tau, \bar{\tau}] = \sum_{i,j} \chi_i(\tau) M_{ij} \chi_j(\bar{\tau}),$$

we can construct a new one with

$$Z[\tau, \bar{\tau}] = \sum \chi_i(\tau) M_{ik} M_{kj}(\beta) \chi_j(\bar{\tau}),$$

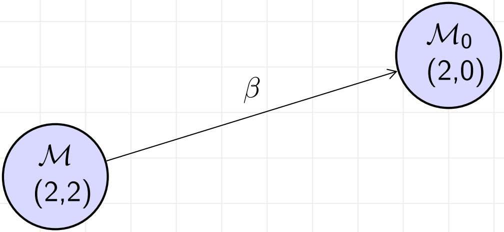
where

$$M_{kj}(\beta) = \frac{1}{N} \sum_{n=1}^{N_J} \delta(\Phi_k, \Phi_j + n\beta) \cdot \delta_{\mathbb{Z}}(Q_\beta(\Phi_k) + \frac{n}{2} Q_\beta(\beta))$$

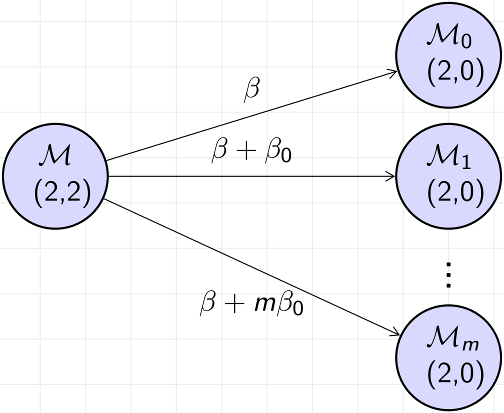
is called a *simple current modular invariant*. In the new model, some states have been projected out and some new states have appeared as well.

Describing the idea

The basic idea



The basic idea



The (2,2) model $\mathcal{M}$ is naturally associated with an entire family of (2,0) models. We studied how the models within each family are related.

Some useful results

It can be proven that:

- The untwisted sector of all the models (w.r.t. to the simple current defining the model) is the same.
- The models will in general have a different number of twisted sectors and are therefore not identical.
- Massless states in the $n-$th twisted sector of the $m-$th model satisfy:

$$Q_\beta(\Phi_L) + \frac{n}{2} Q_\beta(\beta) + mn Q_{\beta_0}(\beta) \in \mathbb{Z},$$

$$n\Big(h(\beta) + \frac{1}{2} Q_\beta(\beta)\Big) \in \mathbb{Z}.$$

Back where it all started from

- Whenever the spectral flow operator is of order 2, we only get two (2,0) models in a family. This is particularly common in many free-fermionic models. This suggests a connection with the spinor-vector duality...

- A direct check shows that indeed the previous construction reproduces the spinor-vector duality whenever the spectral flow operator is of order 2.

- We have therefore given a way to generalize the spinor-vector duality to theories with arbitrary internal Rational CFT!

Summary and outlook

1. The space of $(2,0)$ models is of great interest because of the requirement of spacetime SUSY and the acommodation of $SO(10)$ unification.
2. The spinor-vector duality is a very interesting feature of some classes of free fermionic models.
3. Generalizing the idea to arbitrary rational CFTs we discovered that it is the spectral flow operator that in reality induces an entire family of models.
4. We hope that studying the underlying symmetries of the heterotic-string landscape will prove useful in the long term goal of classifying completely all the $(2,0)$ models.

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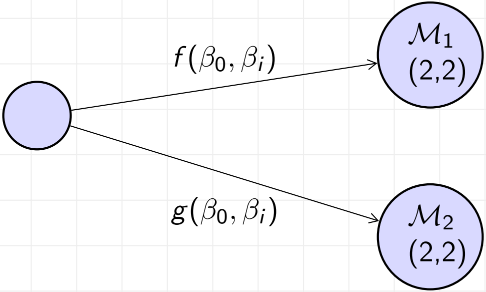
Thank you very much!

The mirror symmetry case Greene and Plesser '90



There are two physically equivalent ways to define $(2,2)$ models.