CKM and PMNS mixing matrices from discrete subgroups of SU(2)

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Sneak Preview!

PMNS mixing angles from 2T, 2O, 2I

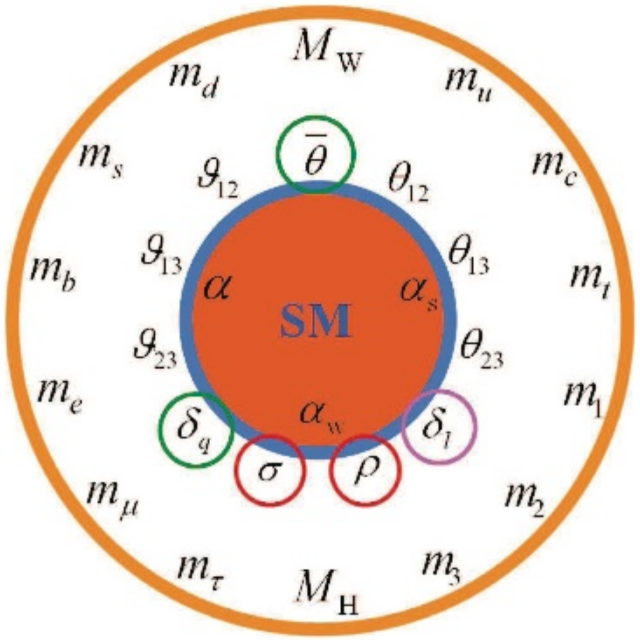
- Predicted: $\theta_{12} = 34.29^\circ$, $\theta_{23} = -42.85^\circ$, $\theta_{13} = -8.56^\circ$
- Empirical: $\theta_{12} = \pm 34.47^\circ$, $\theta_{23} = \pm(38\text{-}45^\circ)$, $\theta_{13} = \pm 8.73^\circ$

$0^\circ \leq \delta \leq \pm 14.8^\circ$ from V_{e3}

Potter, F., CKM and PMNS mixing matrices from discrete subgroups of SU(2), 2014, *Progress in Physics*, 10(1), pp.1-5.

Standard Model

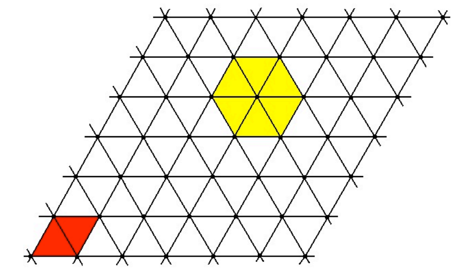
- Very successful theory (since 1970s)
- Local gauge group $SU(2)_L \times U(1)_Y \times SU(3)_C$
- Incomplete: 28 parameters!
- Assumes space is continuous



Fritzsch-Xing "pizza"

1411.2713v2

Discrete space-time



- Discrete space at Planck scale
- $R^4 = C^2$ lattice of mathematical nodes
- Nodes have no measurable physical properties
- Properties 'emerge' for particle collection of nodes
- Particles have discrete rotational symmetries

Flavor states (ν_e, e)

- 2 weak isospin states in each lepton family
- basis states ± 1/2 from SU(2)
- unit quaternion $q = a + b\mathbf{i} + c\mathbf{j} + d\mathbf{k}$ a,b,c,d real
- $i^2 = j^2 = k^2 = ijk = -1$ [W.R. Hamilton on Brougham Bridge 1843]
- Pauli matrices: $i\sigma_x = k$, $i\sigma_y = j$, $i\sigma_z = i$

$$q = \begin{vmatrix} a + bi & c + di \\ -c + di & a - bi \end{vmatrix}$$

Geometrical approach

- Discrete (finite) quaternion groups: binary, double
- $2T$, $2O$, $2I$, $2D_{2n}$, $2C_n$, C_n (n odd) [Duval; Conway & Smith]
- 2T: (24) 1,1',1",2,2',2",3
- 2O: (48) 1,1',2,2',2",3,3',4
- 2I: (120) 1,2,2',3,3',4,4',5,6
- contain a 3-D volume; regular 3-D polyhedrons

2T, 2O, 2I for 3 families

- N_i from group relation to j-invariant / syzygies [F. Klein 1884]

		N_i	Mass Mev/c^2	Mass Mev/c^2	Family
2T	[3,3,2]	1	0.511	~0	ν_e, e
2O	[4,3,2]	108	105.7	~0	ν_μ, μ
2I	[5,3,2]	1728	1776.8	~0	ν_τ, τ

- j-invariant of elliptic modular functions, Möbius trans
- partition function of QFT based on Monster Group!

Flavors from basis states

- Two degenerate basis states $|1>$ and $|2>$ $E = E_0$
- Linear superposition -> flavor states

$$|I> = (|1> - |2>)/\sqrt{2} \qquad E_I = E_0 + A_i$$

$$|II> = (|1> + |2>)/\sqrt{2} \qquad E_{II} = E_0 - A_i$$

	$2A_i$
ν_e, e	~0.511
ν_μ, μ	~105.7
ν_τ, τ	~1776.8

The A_i could be a function of the local environment!

Quaternion generators?

- $R_s = i \times U_s$ for [p,q,2];
- $U_1 = j$, $U_3 = i$
- $U_2 = -i\mathrm{Cos}\,\pi/q - j\mathrm{Cos}\,\pi/p + k\mathrm{Sin}\,\pi/h$ [H.S.M. Coxeter]

[p,q,2]	h	U_2
[3,3,2]	4	$-i/2 - j/2 + k/\sqrt{2}$
[4,3,2]	6	$-i/2 - j/\sqrt{2} + k/2$
[5,3,2]	10	$-i/2 - \phi j/2 + \phi^{-1}k/2$

$\phi = (\sqrt{5} + 1)/2$, the golden ratio

Make combined U_2's = k

- 3 equations for 3 unknown k components
- angles from arccosines of factors
- q rotations in R^3 are double angle

	factor	Angle°	Angle°/2
[3,3,2]	-0.2645	105.34°	52.67°
[4,3,2]	0.8012	36.76°	18.38°
[5,3,2]	-0.5367	122.46°	61.23°

- Predicted: $\theta_{12} = 34.29^\circ$, $\theta_{23} = -42.85^\circ$, $\theta_{13} = -8.56^\circ$

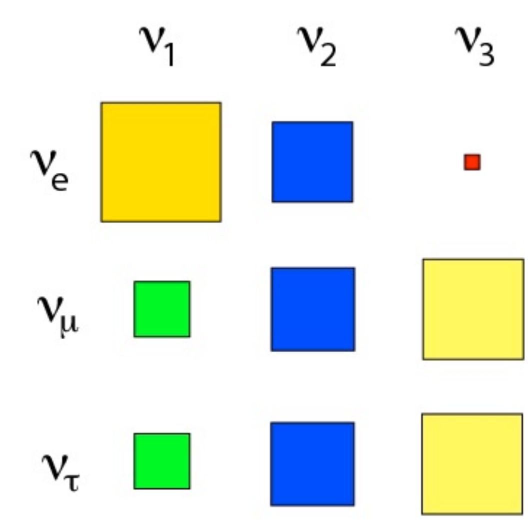
PMNS comparisons

- Predicted: θ_{12} = 34.29°, θ_{23} = -42.85°, θ_{13} = -8.56°
- Empirical: θ_{12} = ±34.47°, θ_{23} = ~ ±42°?, θ_{13} = ±8.73°

Predicted
$$\begin{bmatrix} 0.817 & 0.557 & -0.149e^{-i\delta} \\ -0.413-0.084e^{i\delta} & 0.605-0.057e^{i\delta} & -0.673 \\ -0.383+0.090e^{i\delta} & 0.562+0.061e^{i\delta} & 0.725 \end{bmatrix}$$

Empirical
$$\begin{bmatrix} 0.822 & 0.547 & -0.150+0.038i \\ -0.356+0.0198i & 0.704+0.0131i & 0.614 \\ 0.442+0.0248i & -0.452+0.0166i & 0.774 \end{bmatrix}$$

0° ≤ δ ≤ ±14.8° from V_{e3}



What is learned?

- PMNS angles from U_2 generator projections to k
- Leptons represent 3 discrete binary groups in R^3
- [3,3,2], [4,3,2], [5,3,2] for e, μ, τ families [Z^0 decays]
- Mass values related to j-invariant [Monster Group]
- Normal mass hierarchy for ν's: $\theta_{23} = -42.85^\circ$
- Space could be discrete at Planck scale
- 3 lepton families only; regular polyhedrons in R^3

Rotation groups in R^4

- $R^4 = C^2$
- Rotations simultaneous in two orthogonal planes
- Discrete groups identified as $(L/L_k; R/R_k)$
- L, R = discrete groups of quaternions
- SO(4) = SO(3) x SO(3): SU(2) x SU(2)
- Two sets of generators: like i,j,k and i,j,k
- [3,3,3], [4,3,3], [3,4,3], [5,3,3]: regular 4-D polytopes

Quark discrete symmetries

- Quark states defined in $R^4 = C^2$; reg. 4-D polytopes

Group	Family	N_i	u mass GeV/c^2	my 1992	d mass GeV/c^2	my 1992
[3,3,3]	u, d	1	0.03		0.04	
[4,3,3]	c, s	1	1.3	[1.5]	0.1	
[3,4,3]	t, b	108	173.3	{162}	4.2	[5.0]
[5,3,3]	t', b'	1728	2773?	{2600}	67?	{80}

Geometrical Basis for the Standard Model, *Int'l J. Theor. Phy*. 33, 279-305.

Quark generators

- $U_1 = j, \quad U_3 = i$
- Projections to planes are regular polygons $2C_n$
- $U_2 = i \exp[2\pi i/h]$ [H.S.M. Coxeter]

Group	Family	U_2	Angle°	Angle θ°
[3,3,3]	u, d	$i \exp[2\pi i/5]$	72	81.504
[4,3,3]	c, s	$i \exp[2\pi i/8]$	45	50.940
[3,4,3]	t, b	$i \exp[2\pi i/12]$	30	33.960
[5,3,3]	t', b'	$i \exp[2\pi i/30]$	12	13.584
			159°	179.988°

Angle Diff.
30.564° (15.282°)
20.376° (10.188°)

Q_L mixing

- $V = U_L D^{\dagger}_L$
- Bi-quaternion case - Bogoliubov mixing [Jourjine 1307.2694]
- $U_L = W^u_{14,23}\, W^u_{12,34}$, $D_L = W^d_{14,23} W^d_{12,34}$
- $V_{CKM4} = W^u_{14,23}\, W^u_{12,34}(W^d_{14,23} W^d_{12,34})^{\dagger}$
- 4 isospin cases: (0,0), (1/2,0), (0,1/2), (1/2,1/2)
- Equal, simultaneous, isospin 1/2 rotations

CKM4 math

- SU(2) matrix blocks mixing 1,2 and 3,4

$$W^{u}_{12,34} W^{d\dagger}_{12,34} = \begin{bmatrix} x_1 & y_1 & 0 & 0 \\ z_1 & w_1 & 0 & 0 \\ 0 & 0 & x_2 & y_2 \\ 0 & 0 & z_2 & w_2 \end{bmatrix}$$

$$\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} \cos\theta\, e^{i\alpha} & -\sin\theta\, e^{i\beta} \\ \sin\theta\, e^{i\gamma} & \cos\theta\, e^{i\delta} \end{bmatrix}$$

$$W^{u,d}_{14,23} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

- $V_{CKM4} = W^{u}_{14,23}\, W^{u}_{12,34} (W^{d}_{14,23} W^{d}_{12,34})^{\dagger}$

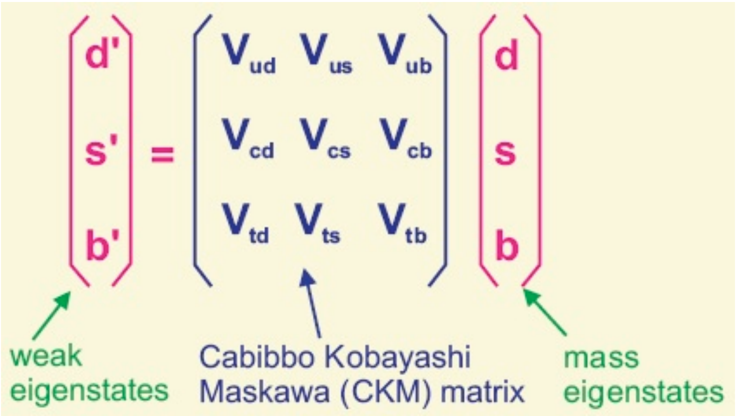
$$V_{CKM4} = \frac{1}{2} \begin{bmatrix} x_1 + x_2 & y_1 + y_2 & x_1 - x_2 & y_1 - y_2 \\ z_1 + z_2 & w_1 + w_2 & z_1 - z_2 & w_1 - w_2 \\ x_1 - x_2 & y_1 - y_2 & x_1 + x_2 & y_1 + y_2 \\ z_1 - z_2 & w_1 - w_2 & z_1 + z_2 & w_1 + w_2 \end{bmatrix}.$$

CKM4 & CKM

- Half-angles: $\theta_1 = 15.282^\circ$, $\theta_2 = 10.188^\circ$
- Ignoring up to 8 phases!

$$V_{CKM4} = \begin{bmatrix} 0.9744 & 0.2203 & 0.0098 & 0.0433 \\ 0.2203 & 0.9744 & 0.0433 & 0.0098 \\ 0.0098 & 0.0433 & 0.9744 & 0.2203 \\ 0.0433 & 0.0098 & 0.2203 & 0.9744 \end{bmatrix}$$

$$V_{CKM} = \begin{bmatrix} 0.9745 & 0.2246 & 0.0036 \\ 0.2244 & 0.9736 & 0.0415 \\ 0.0088 & 0.0407 & 0.9991 \end{bmatrix}$$



Tau decays: $V_{us} = 0.2204 \pm 0.0014$ [arXiv:1411.4526]

Take 3x3 submatrix, impose unitarity, $V_{tb} = 0.999$

Does not agree if only 3 quark groups!

What is learned?

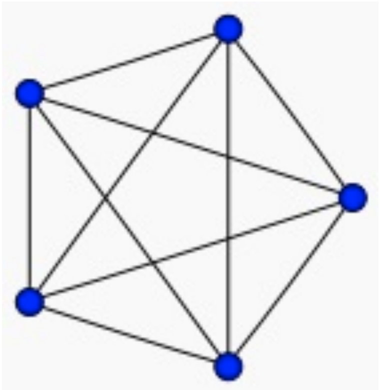
- CKM values derive from U_2 generators
- [3,3,3], [4,3,3], [3,4,3], [5,3,3]: regular 4-D polytopes
- Quark states defined in R^4
- 4 quark families predicted: where is 4th family?
- Mass values from j-invariant constants N_i
- Space discrete at Planck scale
- End of "Russian doll" hierarchy at lattice nodes

Origin of quark color?

- 4-D property: i.e., 4-D rotations in R^4
- 3 pairs of rotation planes: [wx,yz], [xy,zw], [yw,xz]
- Call them *red, green, blue*: exact symmetry
- Can use 4x4 real matrices (shows SU(3) symmetry)
- 3-D leptons do not experience 4-D color
- get 8 gluons from matrix products: QCD
- Hadrons are 3-D: intersections of 4-D quarks

Why are there leptons?

- Question by C. Jarlskog at DISCRETE 2008
- QCD is self-contained: world by itself!
- ANS: In graph theory, Kuratowski's theorem
- Graph is planar iff does not contain K_5 or $K_{3,3}$
- {3,3,3} for up/down quarks is K_5 [subgraph {3,3,2} for e]
- All other quarks must decay - no decays by gluon!
- Need EW interaction



EW interaction

- $SU(2)_L \times U(1)_Y$ is $SU'(2) = SU(2) \times I$ [Altmann]
- Chiral: $q_1 q_2$ produces LH doublets, RH singlets
- Need 2I x 2I' for operators: 4 generators [Baez]
- Leads to W^+, W^-, Z^0, γ
- Anti-particles: R^4 and R'^4 from Bott periodicity 4n

The Bigger Picture!

- Telescoping up to larger spaces R^8, R^{10}, R^{24}

Potter, F. , 2006, **Unification of Interactions in Discrete Spacetime**,
Progress in Physics, Vol. 1 (January), pp. 3-9.
http://www.ptep-online.com/index_files/2006/PP-04-01.PDF

R^4 connects to R^8

- Icosian: $q = (e_1+e_2\sqrt{5})+(e_3+e_4\sqrt{5})i +(e_5+e_6\sqrt{5})j +(e_7+e_8\sqrt{5})k$ [octonion]
- e_j rational number; only one e_j in pair non-zero
- 2I: 120 icosians - D_8 lattice in R^8 [red squares]
- 2I': 120 icosians - D'_8 lattice in R^8 [black squares] [reciprocal]
- $D_8 + D'_8 = E_8$ lattice [checkerboard] [3 E8 lattices = Leech]
- Discrete Weyl E_8 symmetry group - not E_8

Lorentz transformation

- Use Penrose's "heavenly sphere" for discrete (1,3)-D
- Sphere is tesselated by triangles in discrete space
- Same discrete quaternion groups as before!
- Another E_8 lattice with discrete Weyl E_8 symmetry

Unique result!

- Weyl E_8 x Weyl E_8 = ‘discrete’ PSL(2,O) [octonions]
- 2x2 matrices with octonion entries
- Know PSL(2,O) = SO(9,1)
- Weyl E_8 x Weyl E_8 = “Weyl” SO(9,1) [8D+8D=10D]!
- 10-D discrete spacetime → 8-D $_{\text{discrete}}$ + 8-D $_{\text{discrete}}$
- 4-D discrete internal space + 4-D discrete spacetime
- Discrete -> UNIQUE! not 10^{500} possibilities

Summary

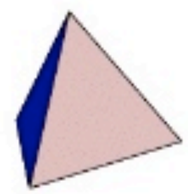
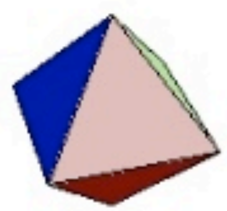
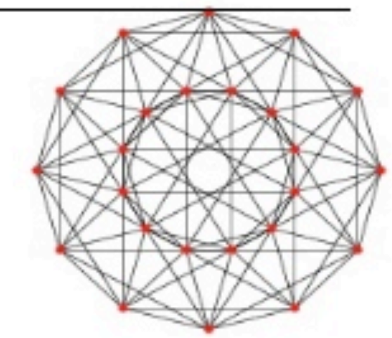
- Leptons: 3 families: [3,3,2], [4,3,2], [5,3,2] in R^3
- Quarks: 4 families: [3,3,3],[4,3,3],[3,4,3],[5,3,3] in R^4
- CKM4 & PMNS from U_2 generators acting together
- SM is an excellent approximation to Nature
- Discrete symmetries at Planck scale
- Jarlskog value 10^{13} times if 4th quark family -> BAU [Hou]

Predictions

- No multiverse, no landscape, no supersymmetry, etc.
- Monster Group dictates all of physics via j-invariant
- Cannot rule out sterile neutrino!
- Weyl E_8 x Weyl E_8 = "Weyl" SO(9,1) [8D+8D=10D]!
- leptons/quarks -> atoms -> molecules -> humans
- WE ARE MATHEMATICS!
- AMAZING MATHEMATICAL UNIVERSE

[NOT the Tegmark kind!!]

Copied from DISCRETE ‘08 presentation:

Invariant	Leptons		3-D	Quarks		4-D
{1/4}				u d	<3,3,3>	
1	ν_e e	<3,3,2>		c s	<4,3,3>	
108	ν_μ μ	<4,3,2>		t b	<3,4,3>	
1728	ν_τ τ	<5,3,2>		t' b'	<5,3,3>