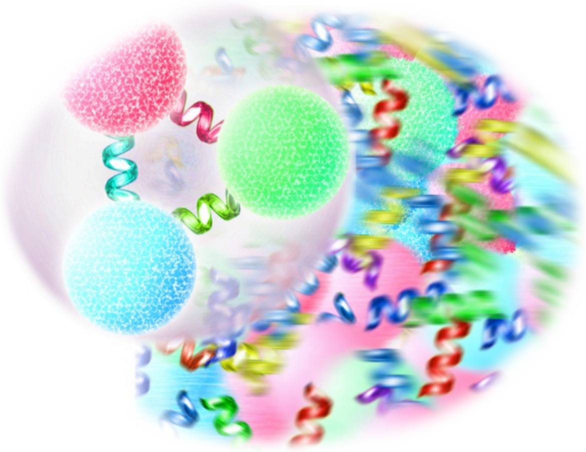

Unraveling the organization of the QCD tapestry

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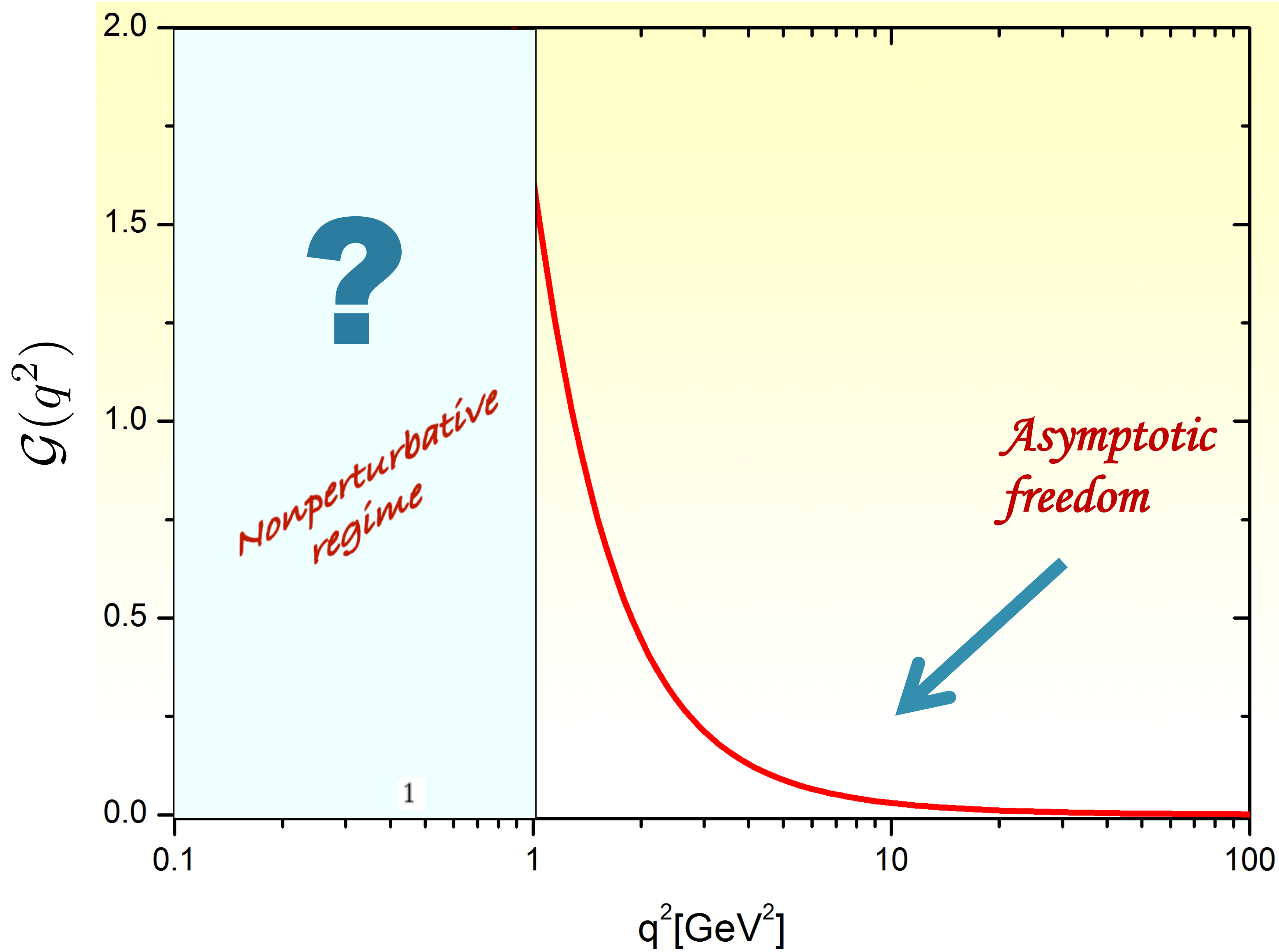
DISCRETE 2014, 2-6 DECEMBER 2014 KING'S COLLEGE, LONDON

Perturbative vs Non-perturbative



Off-shell QCD Green's functions

Green's functions:

Propagators and vertices

Although they are:

- Gauge-dependent
- Renormalization point (μ) and scheme-dependent

However

- They capture characteristic features of the underlying dynamics, both perturbative and non-perturbative.
- When appropriately combined they give rise to physical observables.

Crucial pieces for completing the QCD puzzle

The nonperturbative QCD problems

◎ The Green's functions are crucial for exploring the outstanding nonperturbative problems of QCD:

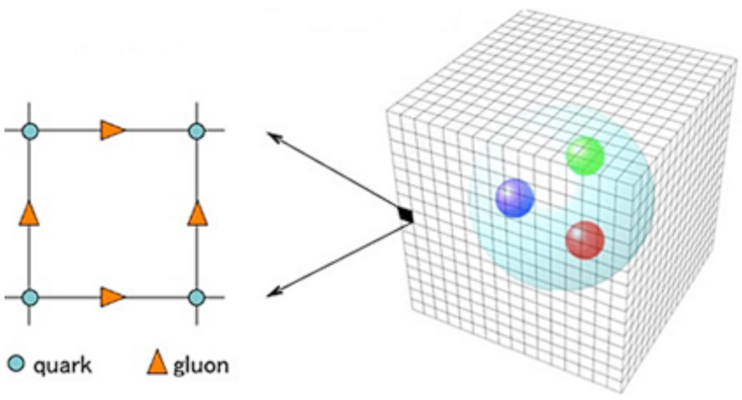
Bound states

Mass generation

Confinement

Non-pertubative tools

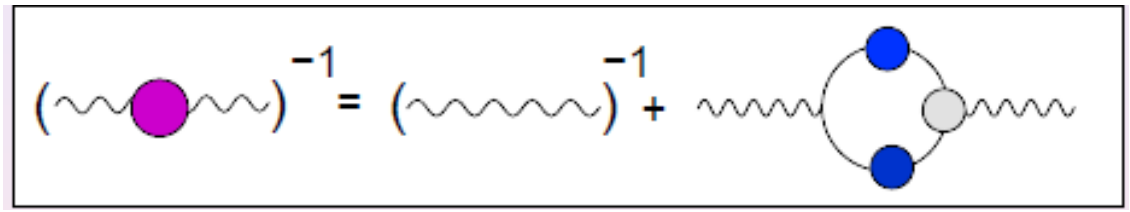
- Non-perturbative physics requires special tools.
- For QCD we have:
- **Lattice simulations**



- Space-time is discretized;
- Several symmetries compromised (eventually recovered);
- The precision depends on the lattice spacing parameter and volume;
- Hard to deal with a large disparity of physical scales.

Schwinger-Dyson equations

- Insightful computational framework
- Equations of motion for off-shell Green's functions.
- Derived formally from the generating functional.



- Infinite system of coupled non-linear integral equations
- Inherently non-perturbative, but at the same time captures the perturbative behavior → It accommodates the full range of physical momenta.

Difficulties with SDEs

- The need for truncations is evident

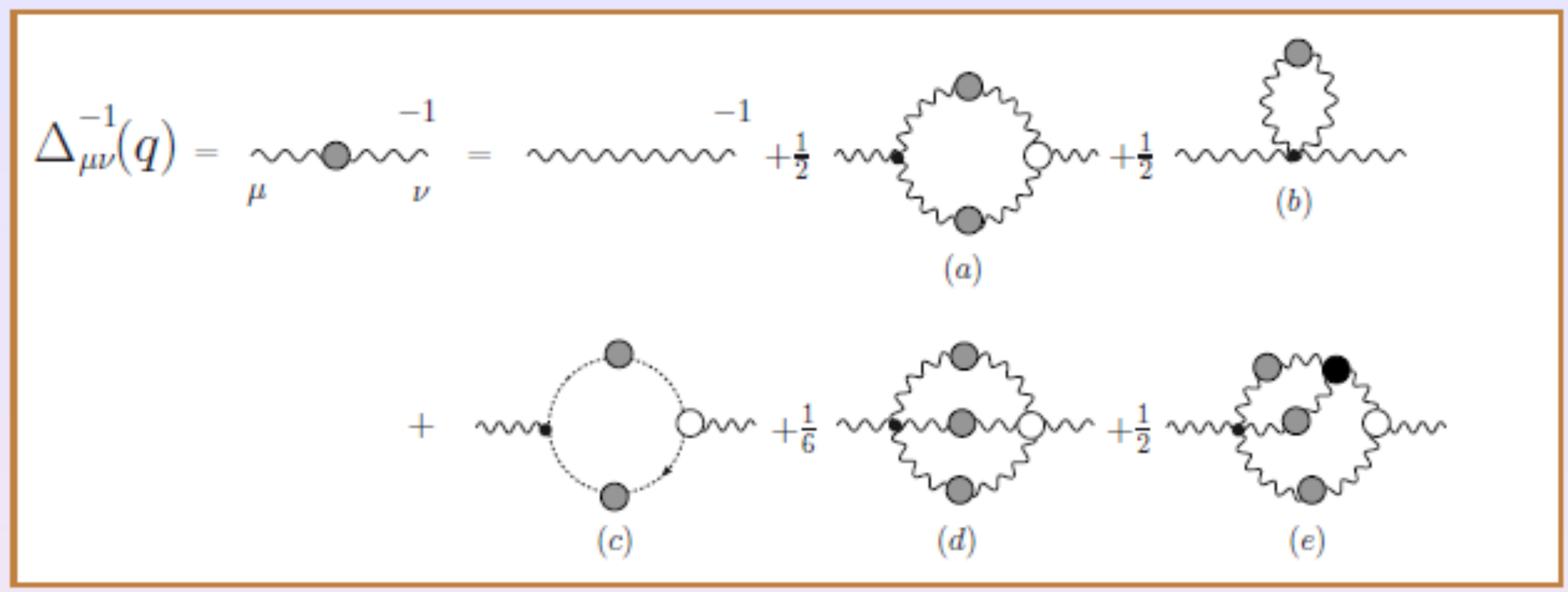
  - ✓ No obvious expansion parameter, so, no formal way of estimating the size of the omitted terms. However, it seems that the "projection" of higher Green's functions on the lower ones is "small".

  - ✓ Casual truncation interferes with the symmetries encoded in the form of the SDEs

$$q^{\mu}\Pi_{\mu\nu}(q) = 0$$

- Self-consistent **truncation scheme** must be used.

The complete SDE for the gluon propagator



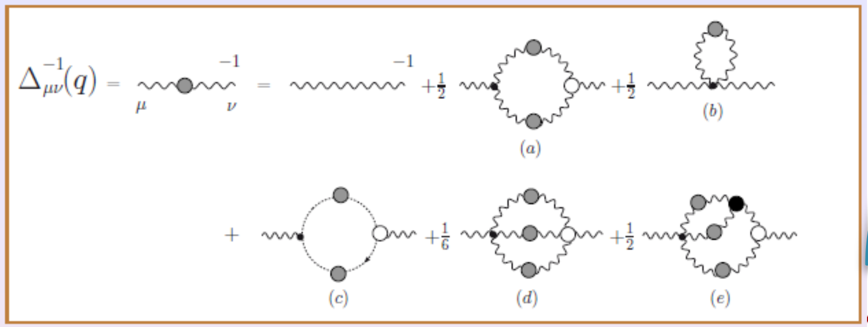
◎ Retaining only (a) and (b) is not correct even at one loop

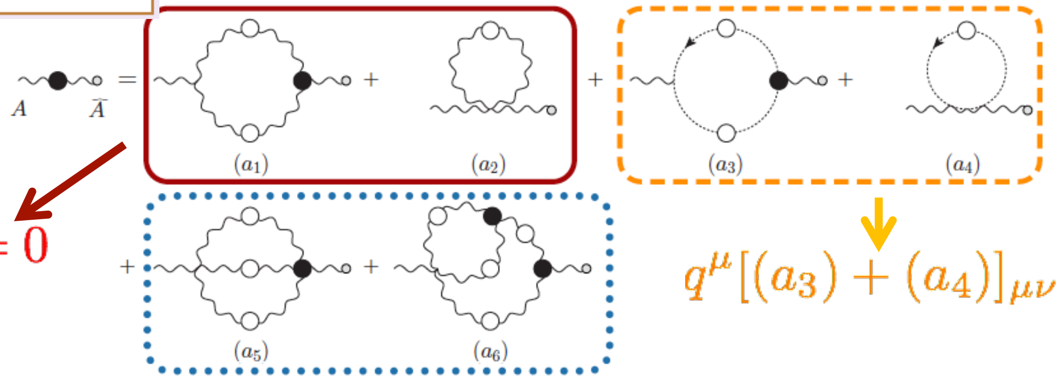
$$q^{\mu}\Pi_{\mu\nu}(q)|_{(a)+(b)} \neq 0$$

◎ Adding (c) is not sufficient for a full analysis → beyond one loop

$$q^{\mu}\Pi_{\mu\nu}(q)|_{(a)+(b)+(c)} \neq 0$$

Pinch Technique – Background Field Method





$$q^{\mu}[(a_1)+(a_2)]_{\mu\nu}=0$$

$$q^{\mu}[(a_3)+(a_4)]_{\mu\nu}=0$$

$$q^{\mu}[(a_5)+(a_6)]_{\mu\nu}=0$$

- **Transversality** is enforced separately for gluon and ghost loops, and order by order in the “dressed-loop” expansion!

- Vertices satisfy Abelian like Ward-Takahashi identities

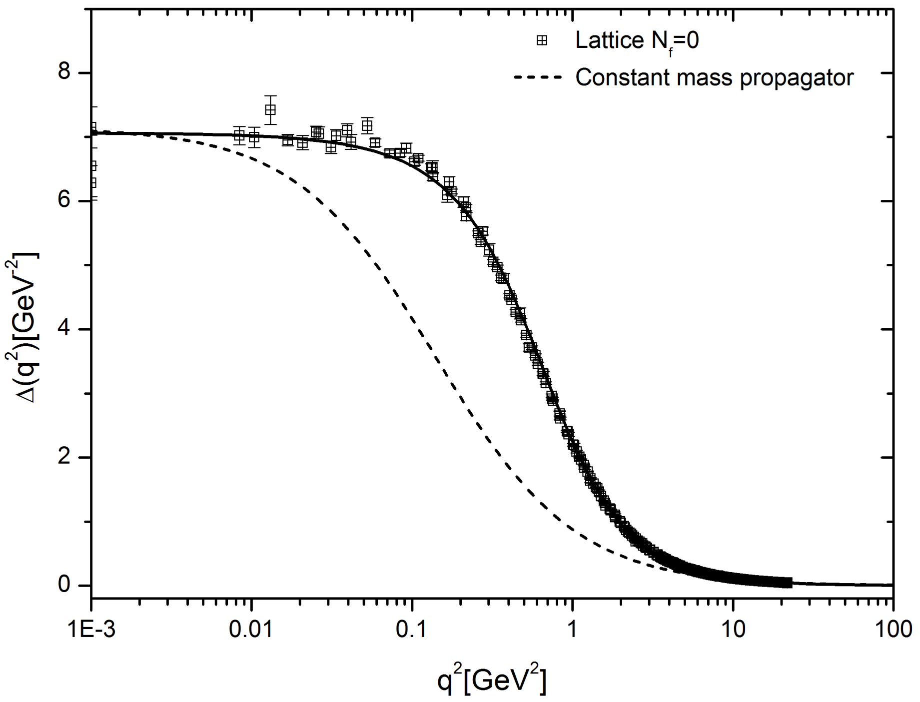
$$q^{\alpha}\widehat{\Gamma}_{\alpha\mu\nu}(q,r,p)=\Delta^{-1}_{\mu\nu}(p)-\Delta^{-1}_{\mu\nu}(r)$$

A.C. Aguilar and J.P. ,JHEP 0612, 012 (2006)
D. Binosi and J. P. . , Phys.Rev. D 77, 061702 (2008); JHEP 0811:063,2008.

Mass generation

Lattice data

- Lattice establishes that the gluon propagator (**in the Landau gauge**) saturates in the deep IR



- The gluon propagator can be parametrized (Euclidean space) by

$$\Delta^{-1}(q^2) = q^2 J(q^2) + m^2(q^2)$$

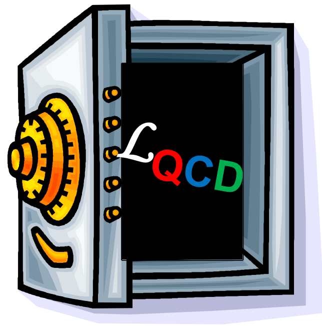
- Saturation is explained through gluon mass generation

J. M. Cornwall, Phys. Rev. D26, 1453 (1982)

$$\Delta^{-1}(0) = m^2(0)$$

I.L.Bogolubsky, et al , PoS LAT2007, 290 (2007)
A. C. Aguilar, D. Binosi and J. P., Phys.Rev. D78 (2008) 025010

Gluon mass generation



- The dynamical gluon mass should be generated without modifying the QCD lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a + \frac{1}{2\xi} (\partial^\mu A_\mu^a)^2 + \partial^\mu \bar{c}^a \partial_\mu c^a + g f^{abc} (\partial^\mu \bar{c}^a) A_\mu^b c^c$$

where the gluonic field strength tensor

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$$

- A mass term ($m^2 A_\mu^2$) is forbidden by gauge invariance.

- The mechanism should not generate quadratic divergences → to renormalize them away you must add a mass term.

Schwinger Mechanism

- Dyson resummation:

J.S. Schwinger, Phys. Rev.125, 397 (1962);
Phys.Rev.128, 2425 (1962).

$$\Delta(q^2) = \frac{1}{q^2[1+\Pi(q^2)]}$$

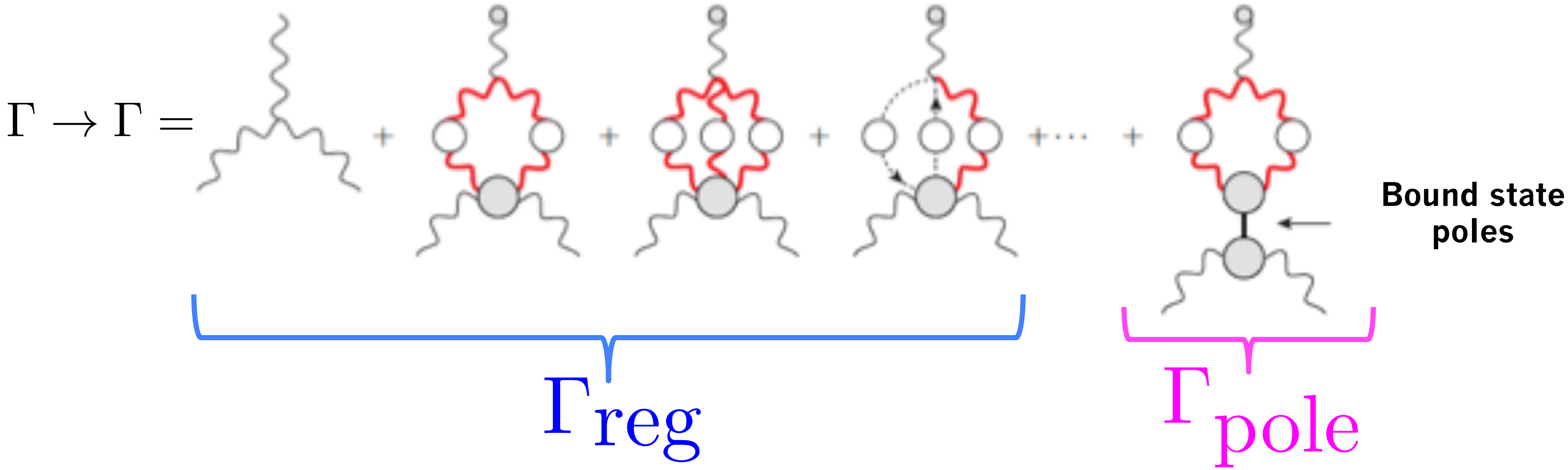
- If the vaccum polarization $\Pi(q^2)$ has a pole with positive residue m^2 i.e.

$$\Pi(q^2) = m^2/q^2$$

- Then $\Delta^{-1}(q^2) = q^2 + m^2$ $\Rightarrow$ $\Delta^{-1}(0) = m^2$

The vector meson becomes massive even though it is massless at the level of the fundamental Lagrangian

◎ Mass generation requires special type of nonperturbative vertices



◎ Triggers the Schwinger mechanism;

◎ Satisfies its own Bethe-Salpeter equation;

◎ Dynamically realized → Composite "would be" Goldstone bosons;

◎ Longitudinally coupled → decouple from the S-matrix.

R. Jackiw and K. Johnson, Phys. Rev. D8, 2386 (1973)
J. M. Cornwall and R.E. Norton, Phys. Rev. D8, 3338 (1973)
E. Eichten and F. Feinberg, Phys. Rev. D10, 3254 (1974)
E.C.Poggio, E.Tomboulis and S.H.Tye, Phys.Rev.D 11, 2839 (1975)

Gluon mass equation

- We can derive **a system** which describes the **dynamical gluon mass**

$$J(q^2) = 1 + \int_k \mathcal{K}_1(k, q, m^2, \Delta)$$

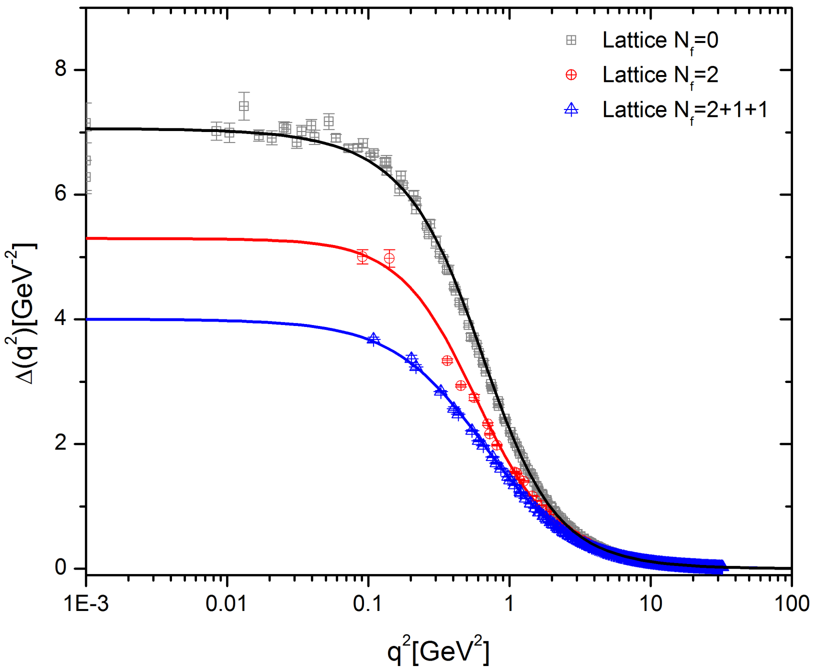
$$m(q^2) = \int_k \mathcal{K}_2(k, q, m^2, \Delta)$$

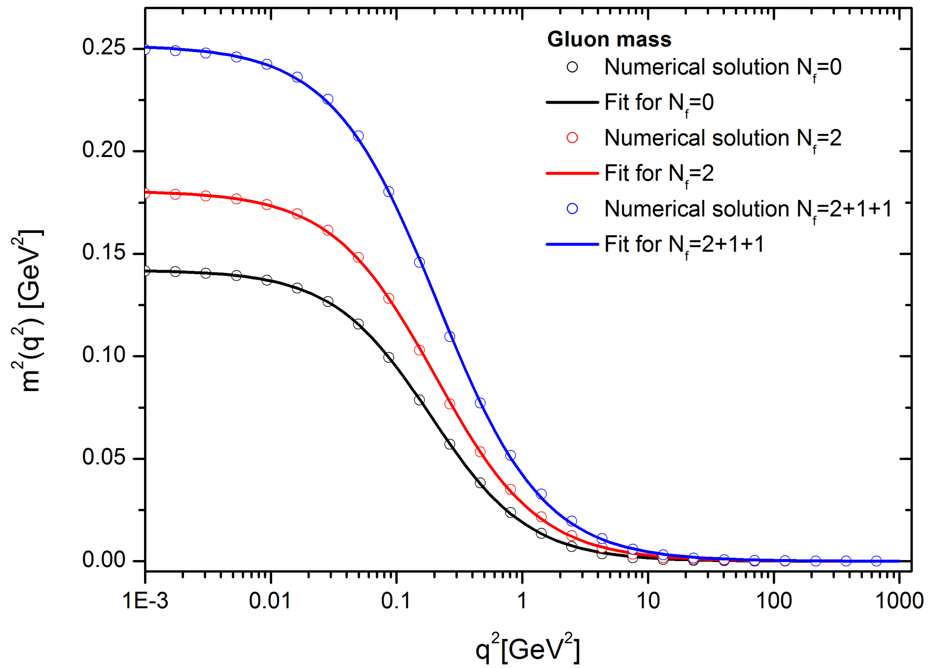
- In the limit $q^2 \to 0$ $\mathcal{K}_2(q^2, m^2, \Delta) \neq 0$

because of the inclusion of the massless poles.

A. C. Aguilar, D. Binosi and J. P., Phys. Rev. D 84, 085026 (2011)
D. Binosi, D. Ibanez and J.P., Phys. Rev. D86, 085033 (2012)

Gluon Propagator and its running mass





N_f	m_0 (MeV)
0	375
2	425
2+1+1	500

I.L.Bogolubsky, et al, PoS LAT2007, 290 (2007)
A. Ayala, et. al Phys. Rev. D86 (2012) 074512
A. Bashir, A. Raya and J. Rodriguez-Quintero, Phys. Rev. D88, 054003 (2013)

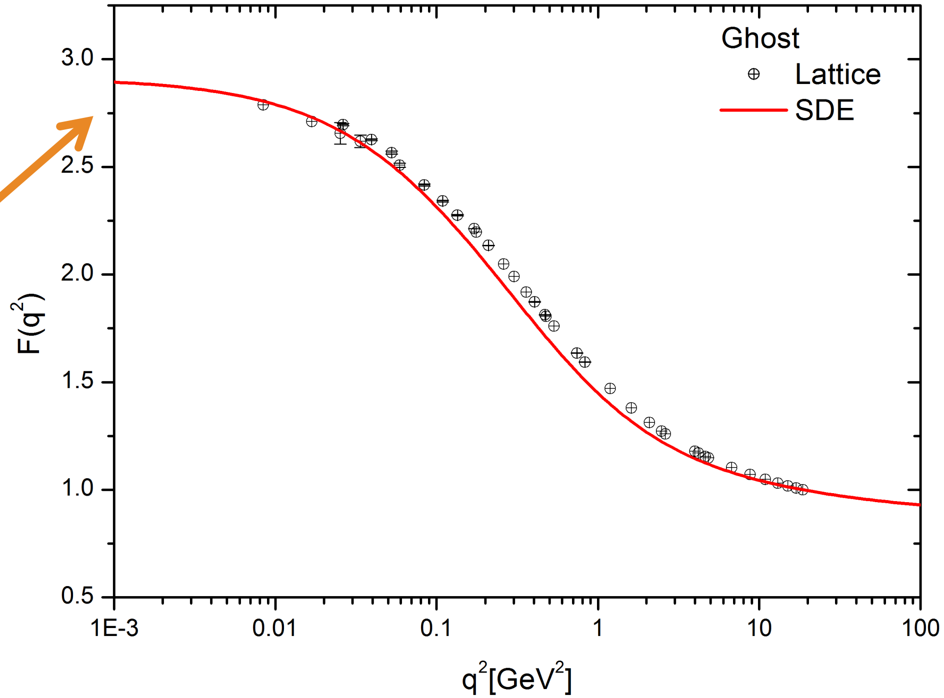
A. C. Aguilar, D. Binosi and J. P. Phys.Rev. D88 (2013) 074010 ; D89 (2014) 085032
A. C. Aguilar and J. P. Phys. Rev. D89 (2014) 085032

Ghost Sector

Ghost Sector

- The finiteness of the dressing of the gluon propagator, $F(p^2)$, is a consequence of the massiveness of the gluon propagator.

Finite result



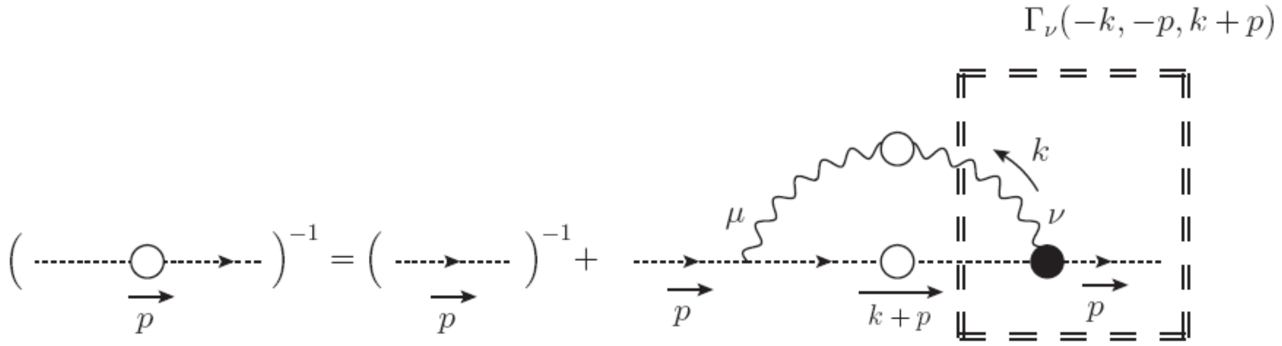
$$D(q^2) = \frac{F(q^2)}{q^2}$$

Gluon mass

$$F^{-1}(q^2) = 1 + \frac{C_{\rm A} g^2}{24\pi^2} \ln\left(\frac{q^2 + \rho m^2(q^2)}{\mu^2}\right)$$

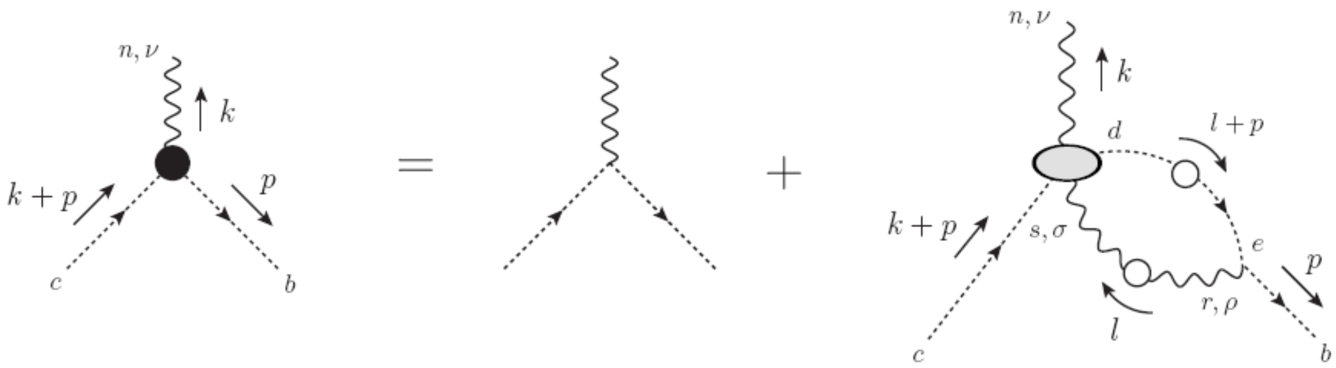
A. C. Aguilar, D. Binosi and J. P., Phys.Rev. D78 (2008) 025010
P. Boucaud, et al JHEP 0806, 099 (2008)

The ghost SDE

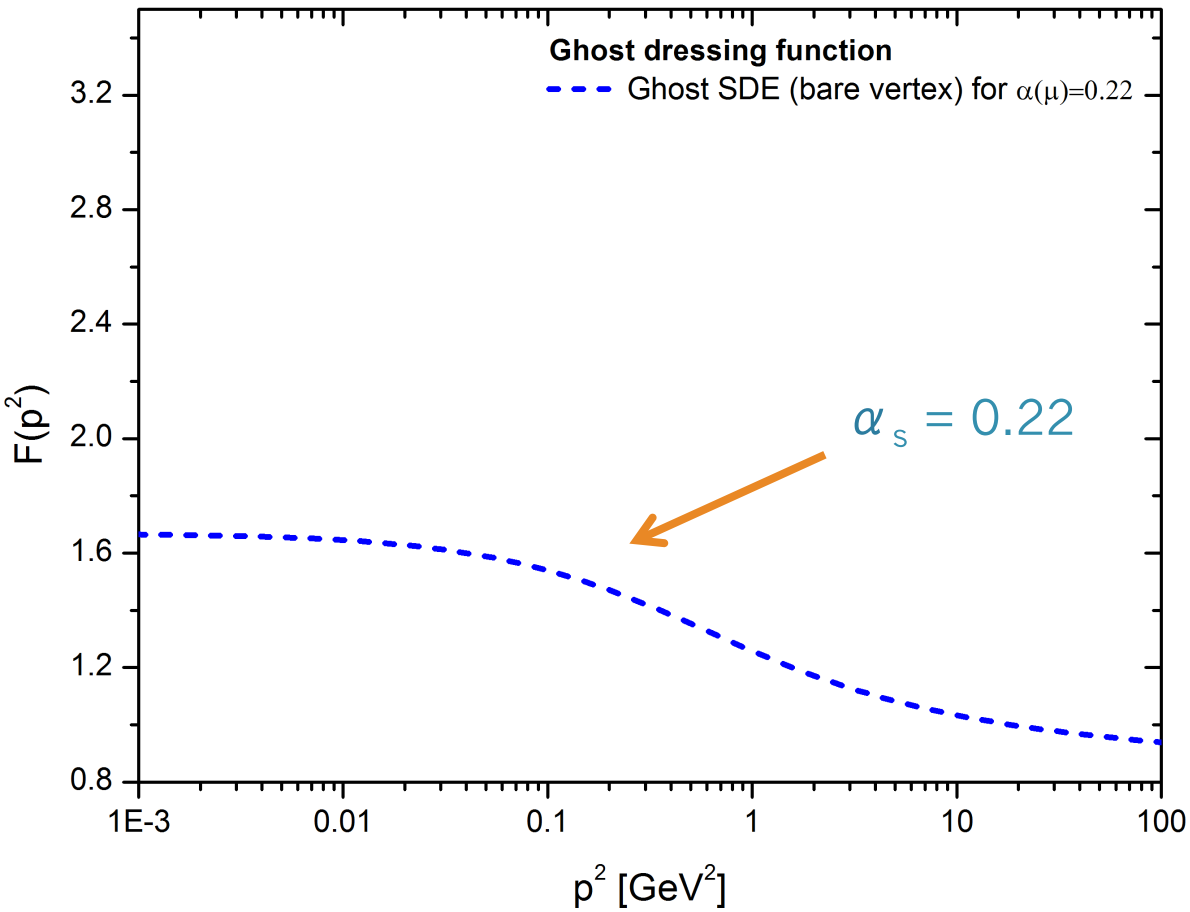


$$F^{-1}(p^2) = 1 + ig^2 C_{\rm A}\frac{1}{p^2}\int_k \Gamma^{[0]}_{\mu}(k,-k-p,p)\Delta^{\mu\nu}(k)\Gamma_\nu(-k,-p,k+p)D(k+p)\,,$$

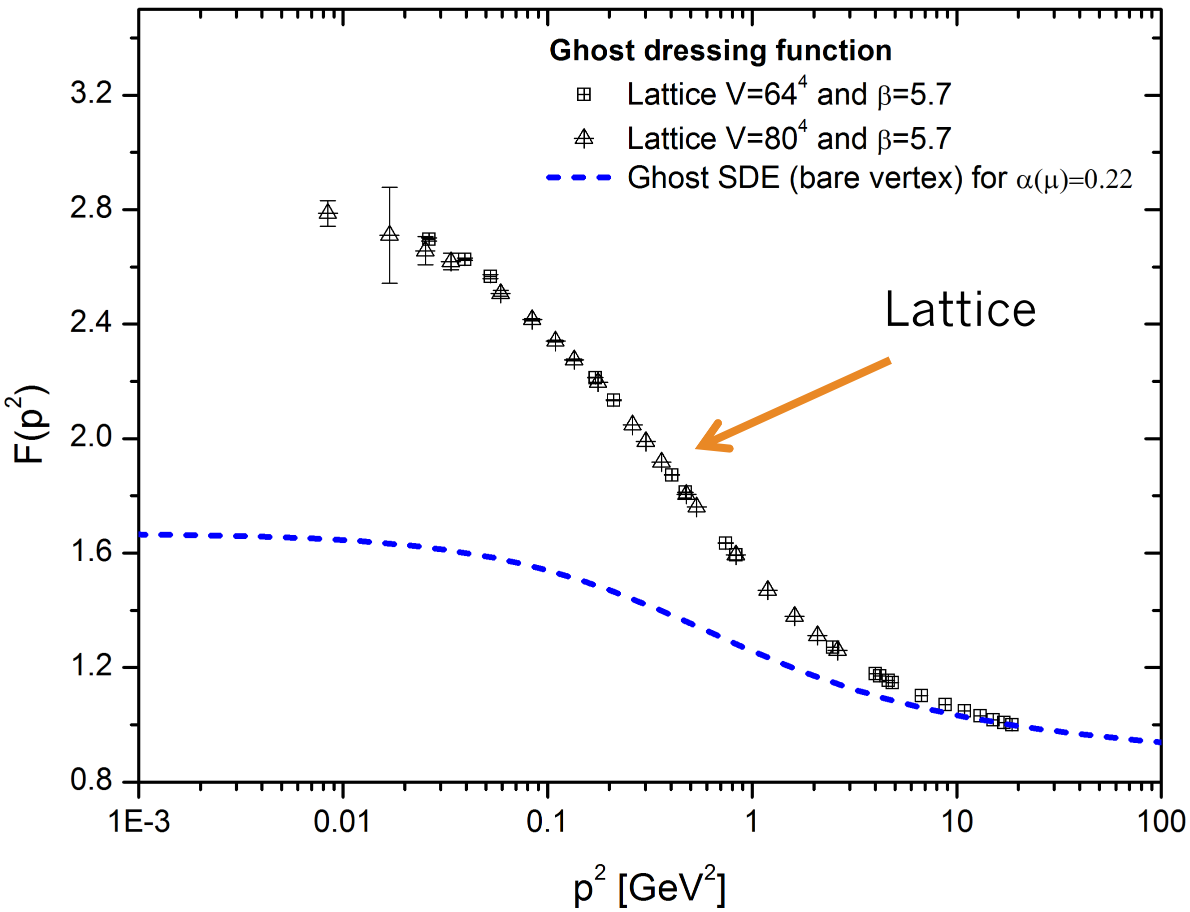
- The SDE for the ghost-gluon vertex is represented by



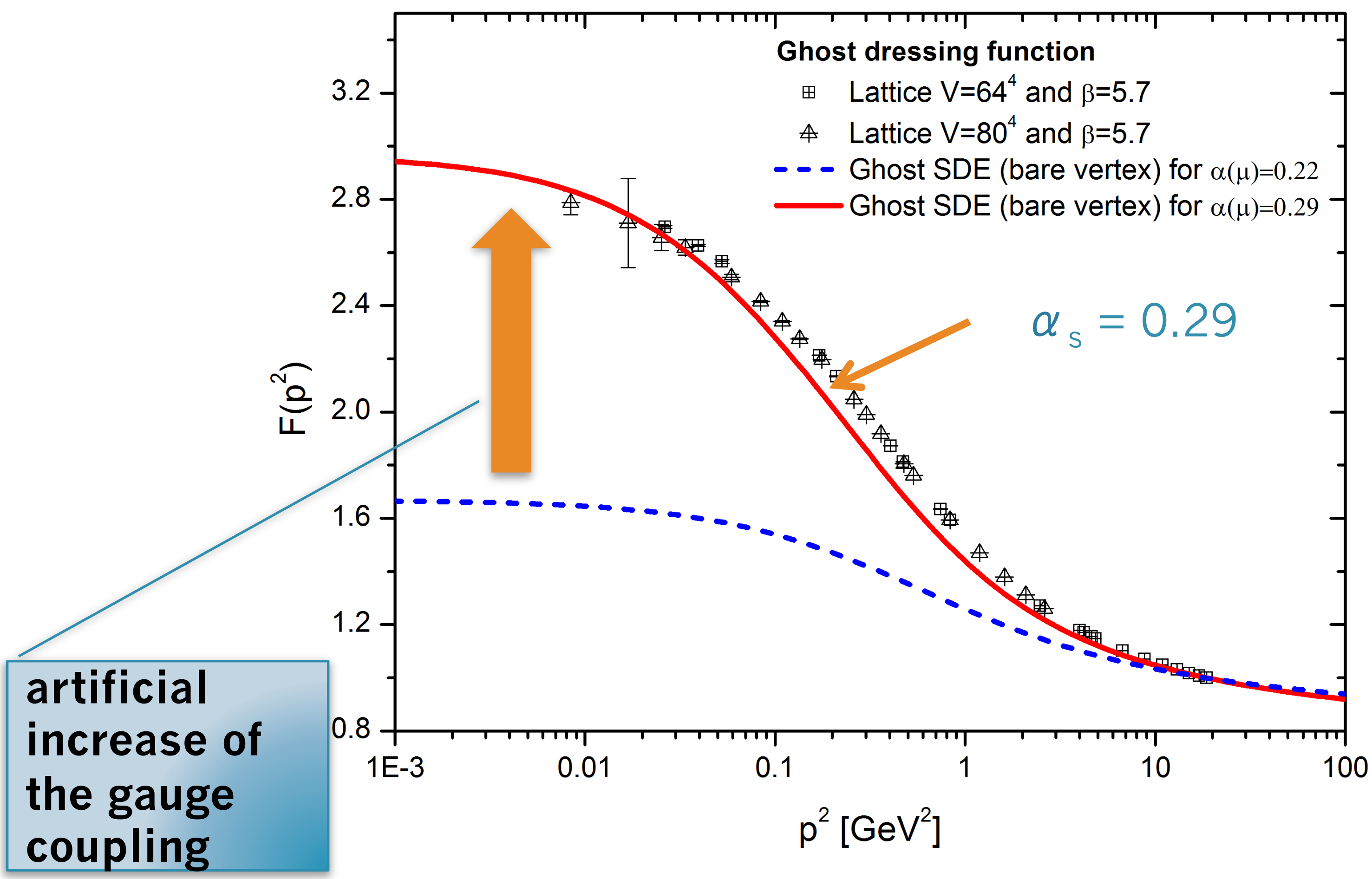
◎ When we use the bare gluon-ghost vertex the SDE result is



◎ However the lattice simulation tell us



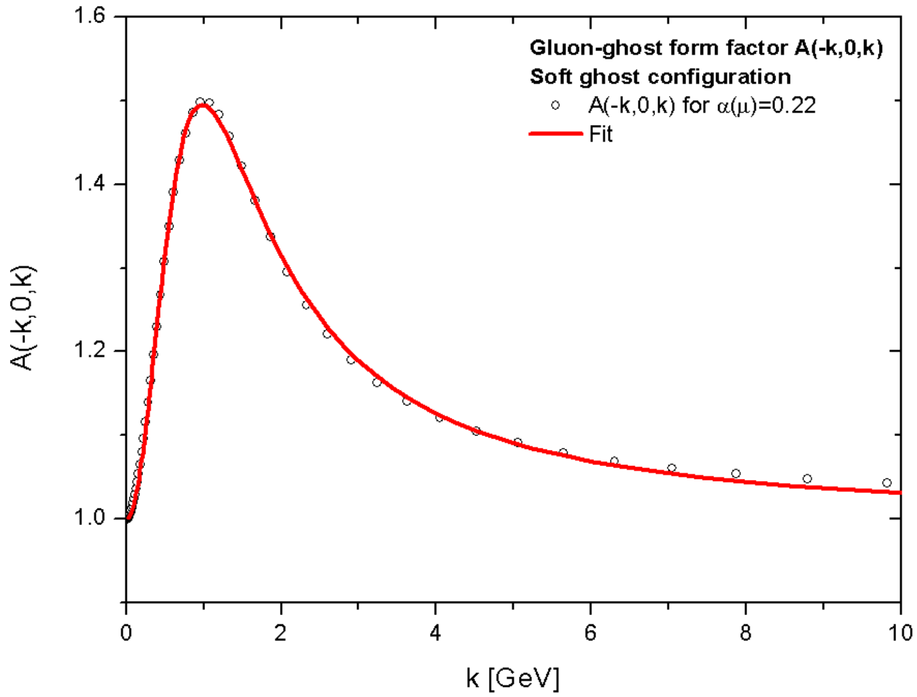
◎ One way of correcting → increase the value of the coupling

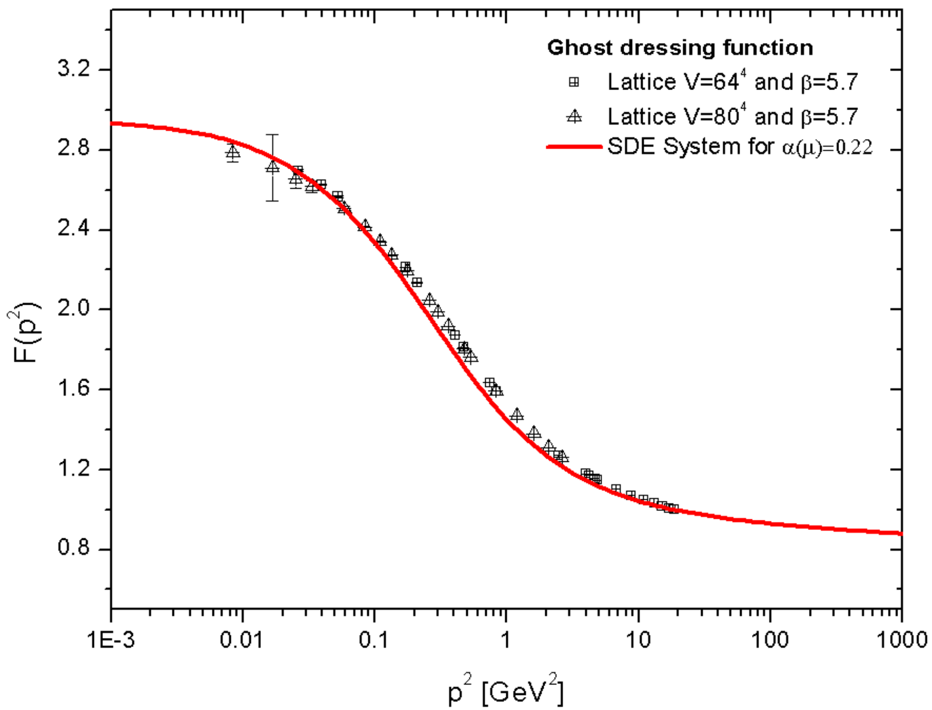


However the correct procedure is …

to add the contribution of the vertex, and solve the system for the

ghost gluon vertex + ghost SDEs



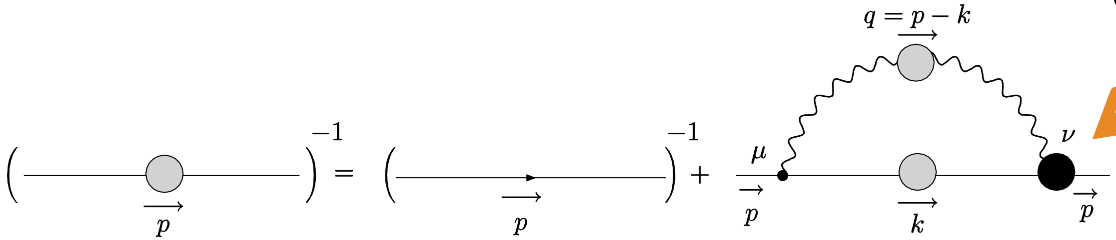


A.C. A., D. Ibáñez and J. Papavassiliou, Phys. Rev. D87, 114020 (2013)
D. Dudal, O. Oliveira and J. Rodriguez-Quintero, Phys. Rev. D86, 105005 (2012)
I. L. Bogolubsky, et al. PoS LATTICE, 290 (2007).

Quark Sector

Constituent quark masses

Crucial ingredient



$$S^{-1}(p) = \not{p} - m_0 - iC_{\rm r} g^2 \int \frac{d^4k}{(2\pi)^4} \gamma_\mu S(k) \Gamma_\nu(k,p) \Delta^{\mu\nu}(p-k)$$

Gap Equation

Full quark propagator

$$S(p) = \frac{1}{A(p^2)[\not{p} - \mathcal{M}(p^2)]}$$

Dynamical quark mass

$$\mathcal{M}(p) = \frac{B(p)}{A(p)}$$

Chiral Symmetry breaking occurs when $\mathcal{B} \neq 0$

- Considered as the most efficient mechanism for mass generation (*mass from nothing*!)
- **Responsible for** the generation of **98%** of the **visible mass of the Universe**!

Simple Ansatz for Γ_μ

- The quark dynamical mass equation is given by

$$\mathcal{M}(p^2) = 4\int_k \mathcal{K}(p,k)\frac{\mathcal{M}(k^2)}{k^2+\mathcal{M}^2(k^2)}$$

- The kernel $\mathcal{K}(p,k)$ depends on the approximation used for the quark gluon vertex

- A simple Ansatz is the Abelian approximation for Γ_μ (satisfies the Ward identity). In this case

$$\mathcal{K}(p,k) \propto g^2\Delta(p-k)$$

- However, the kernel does not have enough strength for generating the quark mass

Inflating the kernel

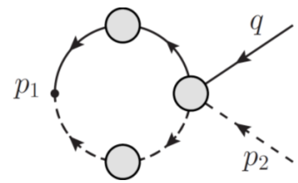
means better knowledge of the quark-gluon vertex

- Use an improved quark-gluon vertex (abelianization not good)
  - ✓ STI instead of WI

$$q^\mu \Gamma_\mu(k,p) = F(q^2)[S^{-1}(p)H(q,p) - \overline{H}(q,p)S^{-1}(k)]\,,$$

  - ✓ Include ghost sector (numerically crucial)

$H(p_1, p_2, q) \;=$

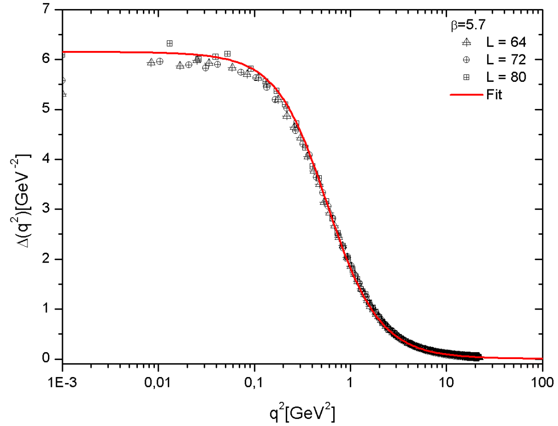


A. C. Aguilar and J. P. , Phys. Rev. D83, 014013 (2011)

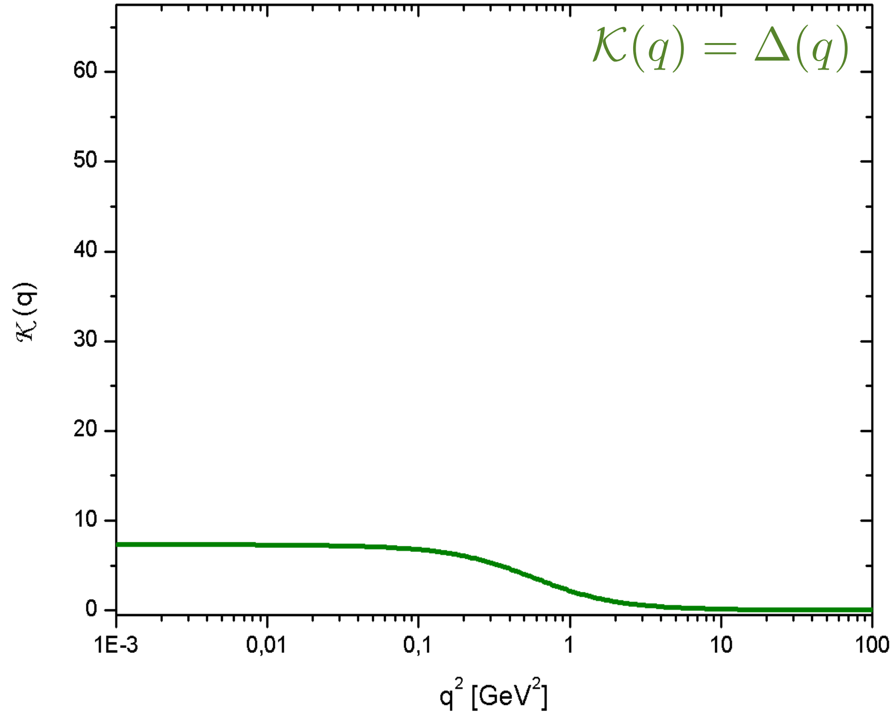
Generating the quark mass

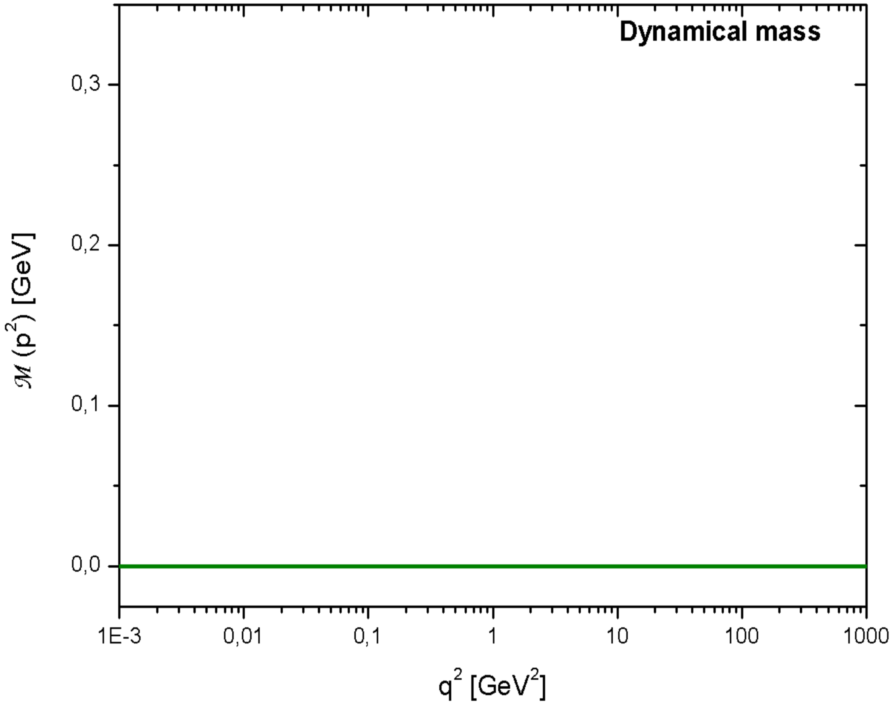
◎The dependence of the kernel on the non-perturbative ingredients:

✓ Linear in Δ



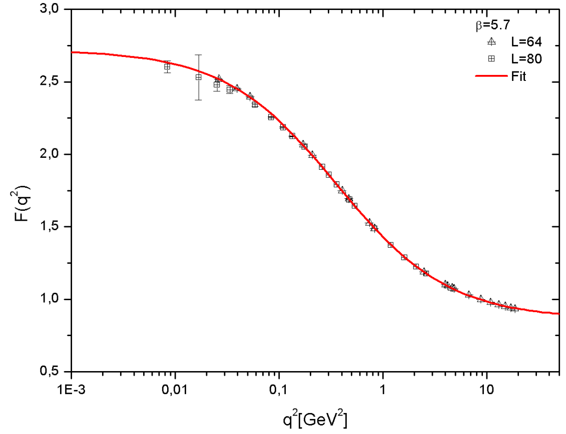
$$\mathcal{M}(p^2) = 4\int_k \underline{\mathcal{K}(p,k)} \frac{\mathcal{M}(k^2)}{k^2+\mathcal{M}^2(k^2)}$$



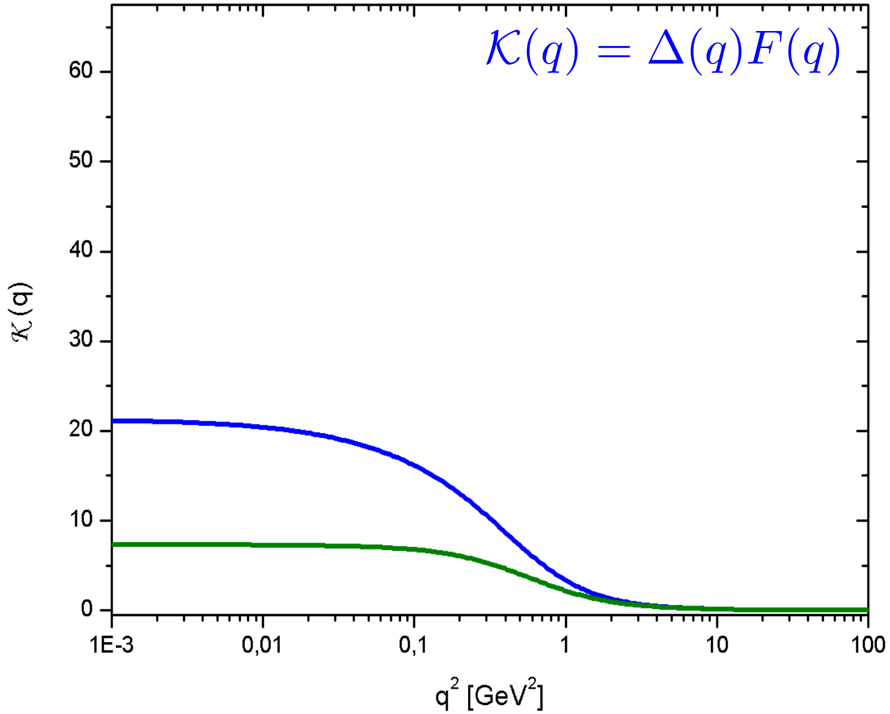


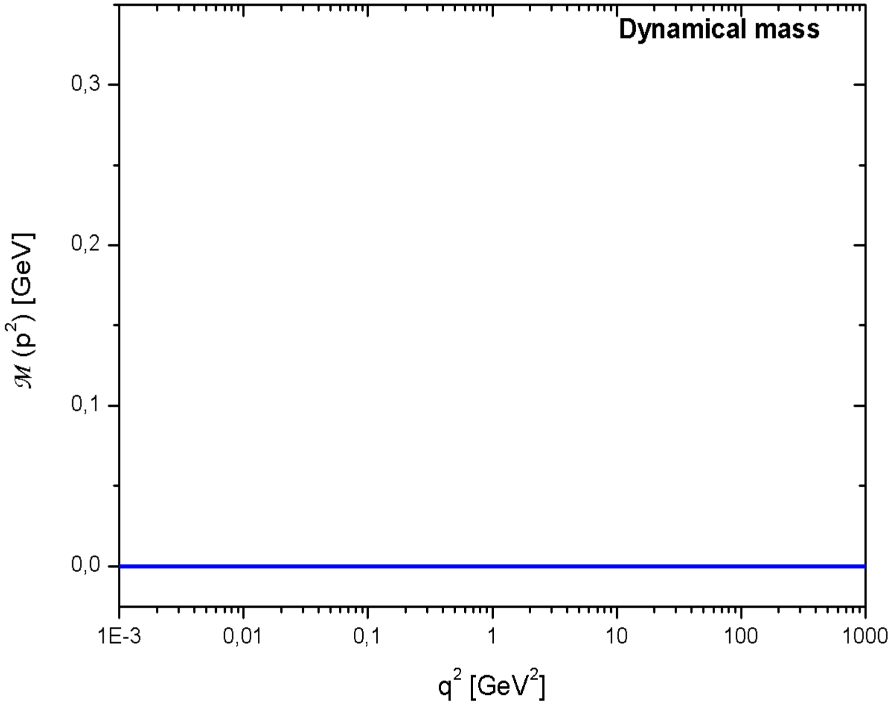
Generating the quark mass

- The dependence of the kernel on the non-perturbative ingredients:
  - ✓ Linear in Δ
  - ✓ Quadratic in F
    - ➢ one F from vertex



$$\mathcal{M}(p^2) = 4\int_k \mathcal{K}(p,k)\frac{\mathcal{M}(k^2)}{k^2+\mathcal{M}^2(k^2)}$$

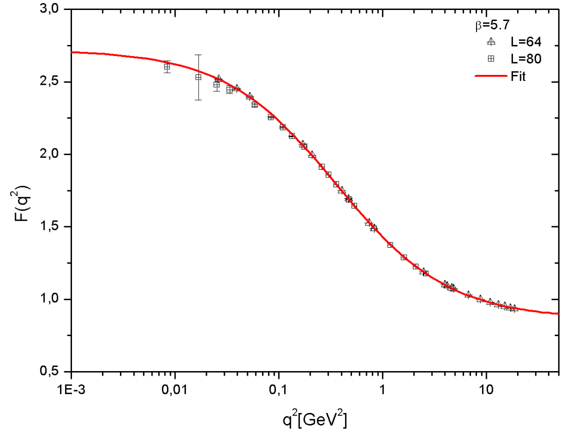




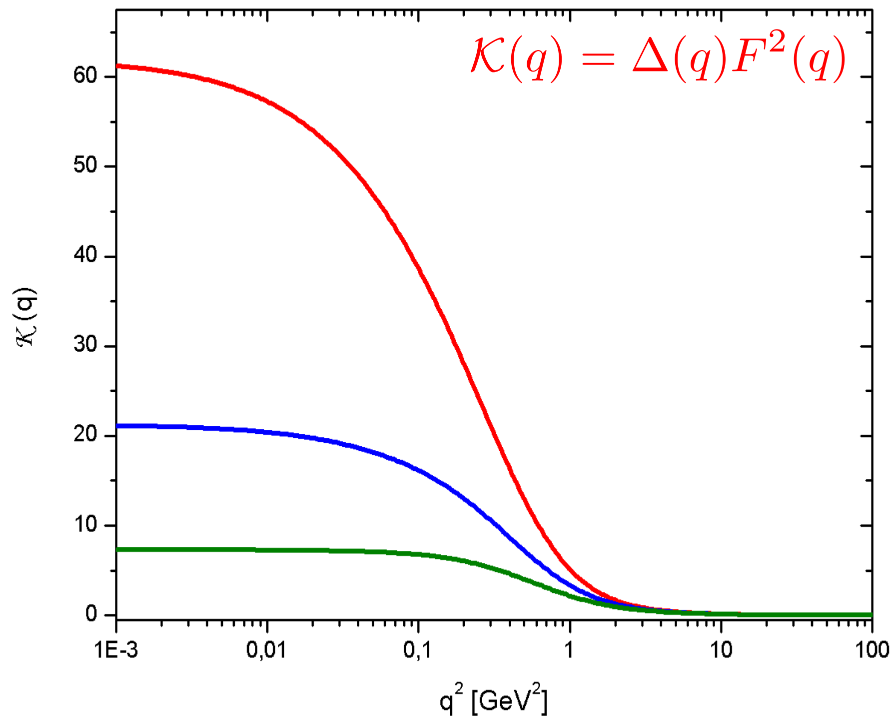
Generating the quark mass

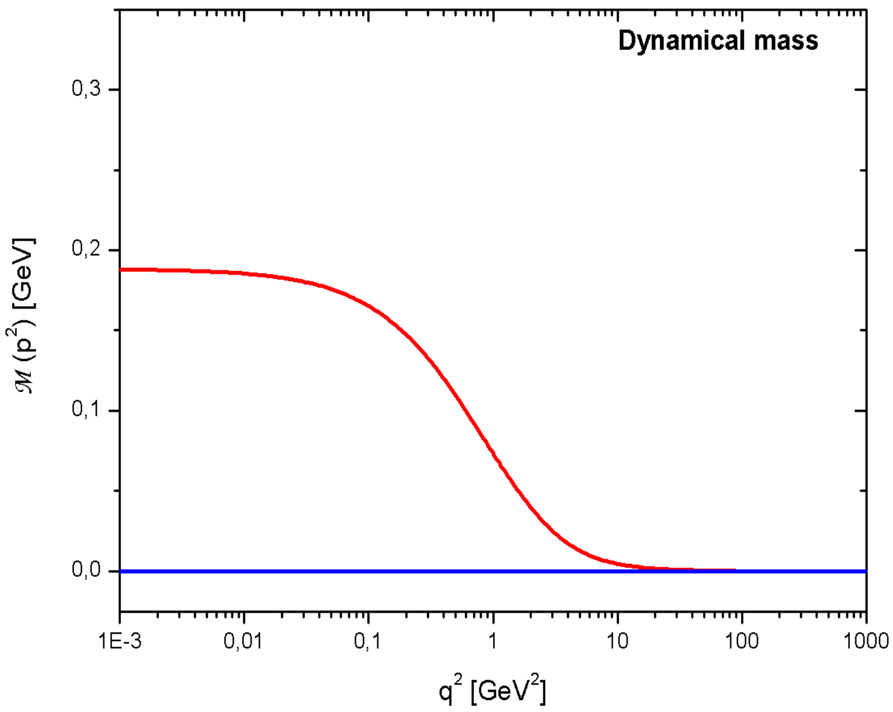
◎The dependence of the kernel on the non-perturbative ingredients:

- ✓ Linear in Δ
- ✓ Quadratic in F
  - ➢ one F from vertex
  - ➢ another for compatibility with RG



$$\mathcal{M}(p^2) = 4\int_k \underline{\mathcal{K}(p,k)}\frac{\mathcal{M}(k^2)}{k^2+\mathcal{M}^2(k^2)}$$

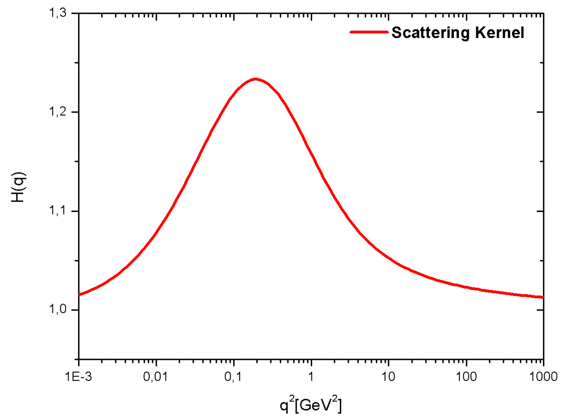




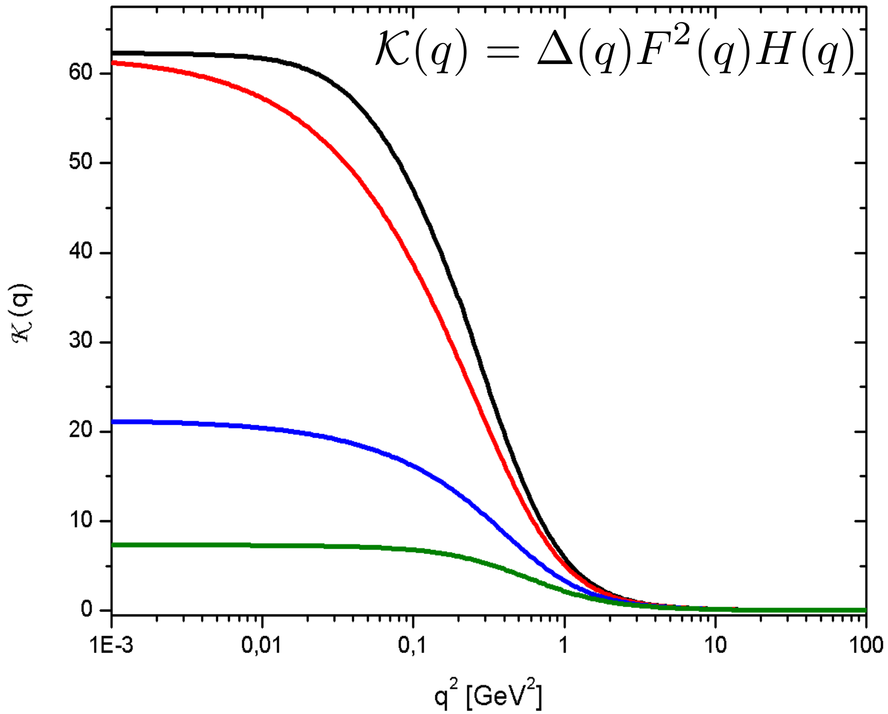
Generating the quark mass

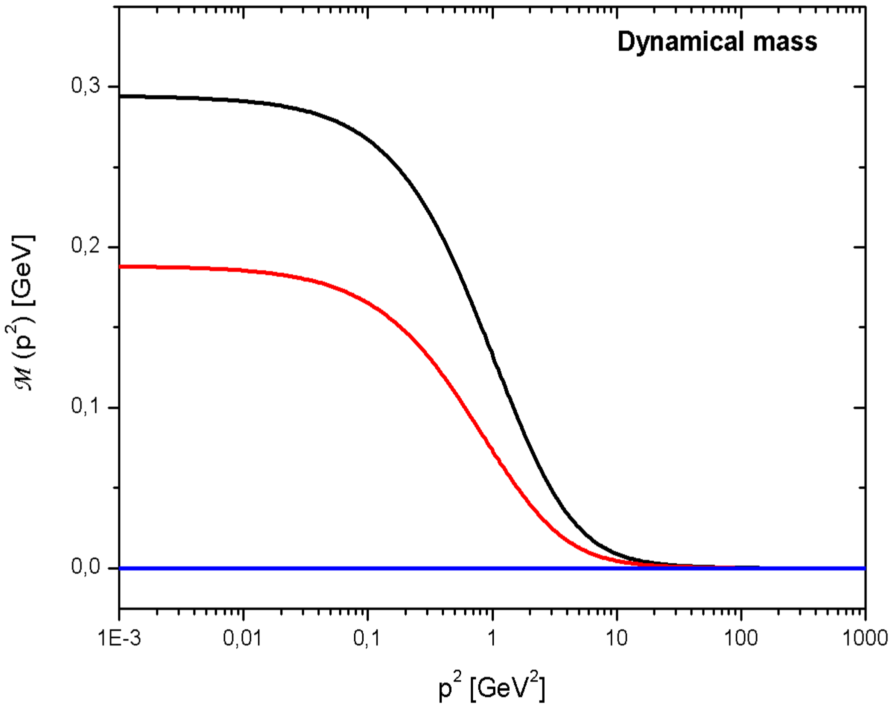
◎The dependence of the kernel on the non-perturbative ingredients:

- ✓ Linear in Δ
- ✓ Quadratic in F
  - ➢ one F from vertex
  - ➢ another for compatibility with RG
- ✓ Linear in H (from vertex)

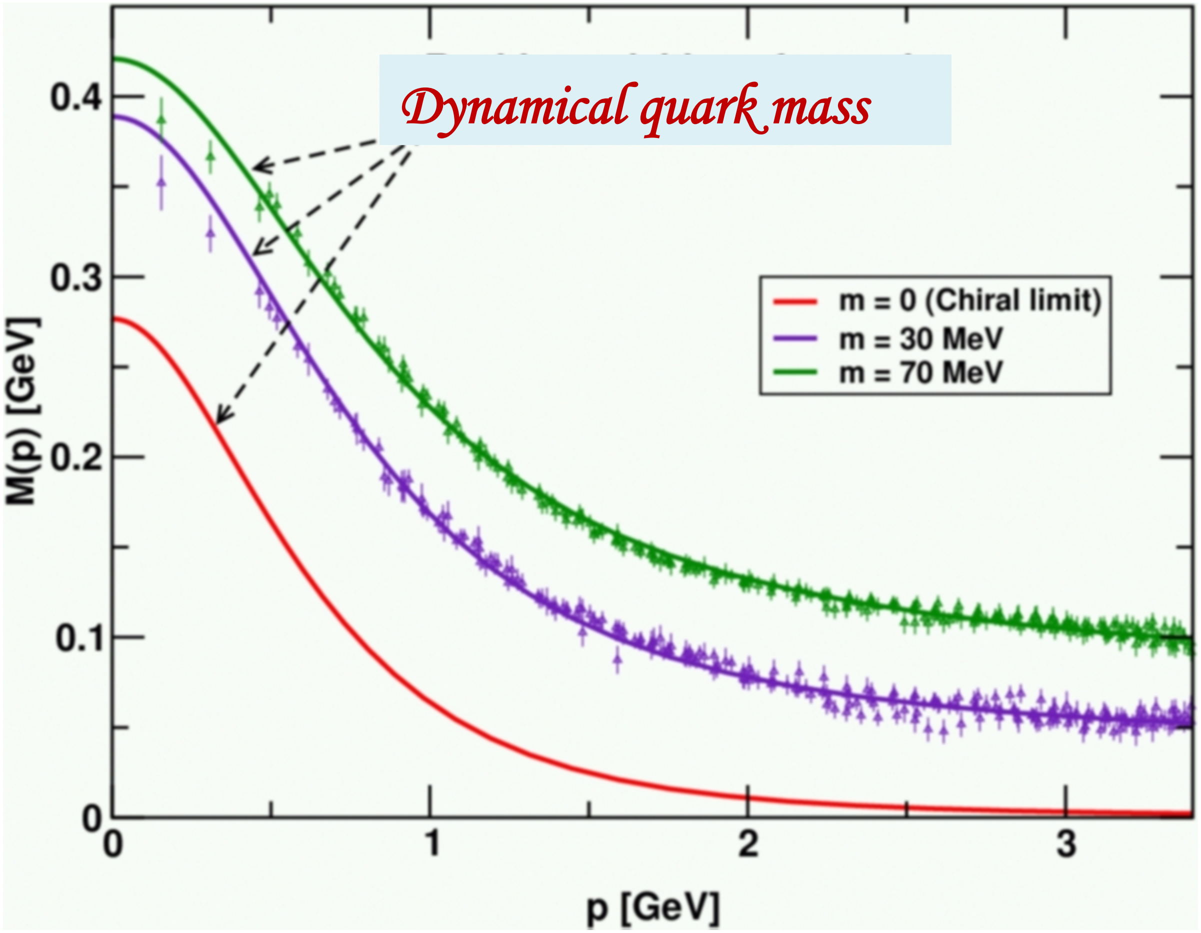


$$\mathcal{M}(p^2) = 4\int_k \underline{\mathcal{K}(p,k)}\frac{\mathcal{M}(k^2)}{k^2+\mathcal{M}^2(k^2)}$$





Quark masses



C. D. Roberts, Prog. Part. Nucl. Phys.61, 50 (2008).
P. O. Bowman et al., Phys. Rev. D71, 054507 (2005);

Confinement

Sign of the spectral density

- The gluon propagator can be written in terms of the spectral density $\rho(\sigma)$

$$\Delta(q^2) = \int_0^\infty d\sigma \frac{\rho(\sigma)}{q^2+\sigma}$$

- If $\Delta(q^2)$ has an inflection point at q_0^2

$$\Delta'(q_0^2) = -\int_0^\infty d\sigma \frac{\rho(\sigma)}{(q_0^2+\sigma)^2}$$

$$\Delta''(q_0^2) = 2\int_0^\infty d\sigma \frac{\rho(\sigma)}{(q_0^2+\sigma)^3} = 0$$

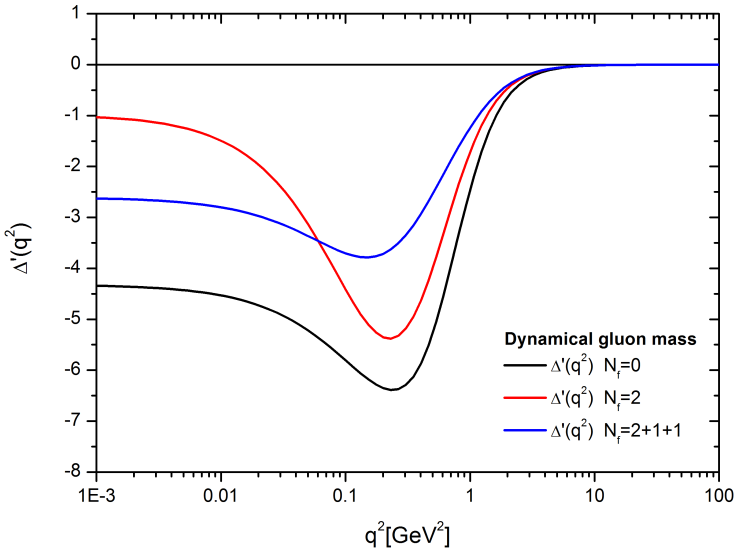
- Since $q_0^2 > 0$, then **$\rho(\sigma)$ is not positive definite**

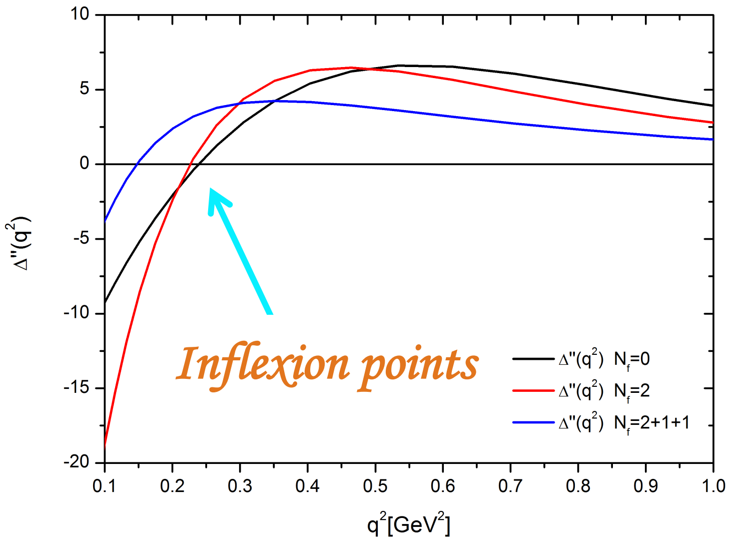
- The gluon cannot appear in the Hilbert space of observable states

Confinement

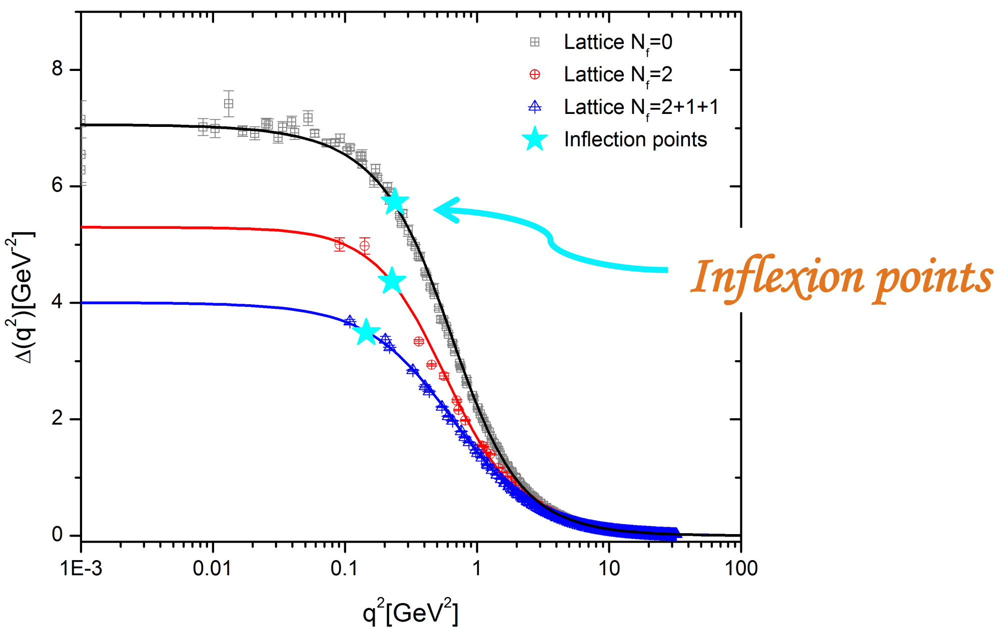
I. C. Cloet and C. D. Roberts, Prog. Part. Nucl. Phys.77, 1 (2014)

Inflexion points





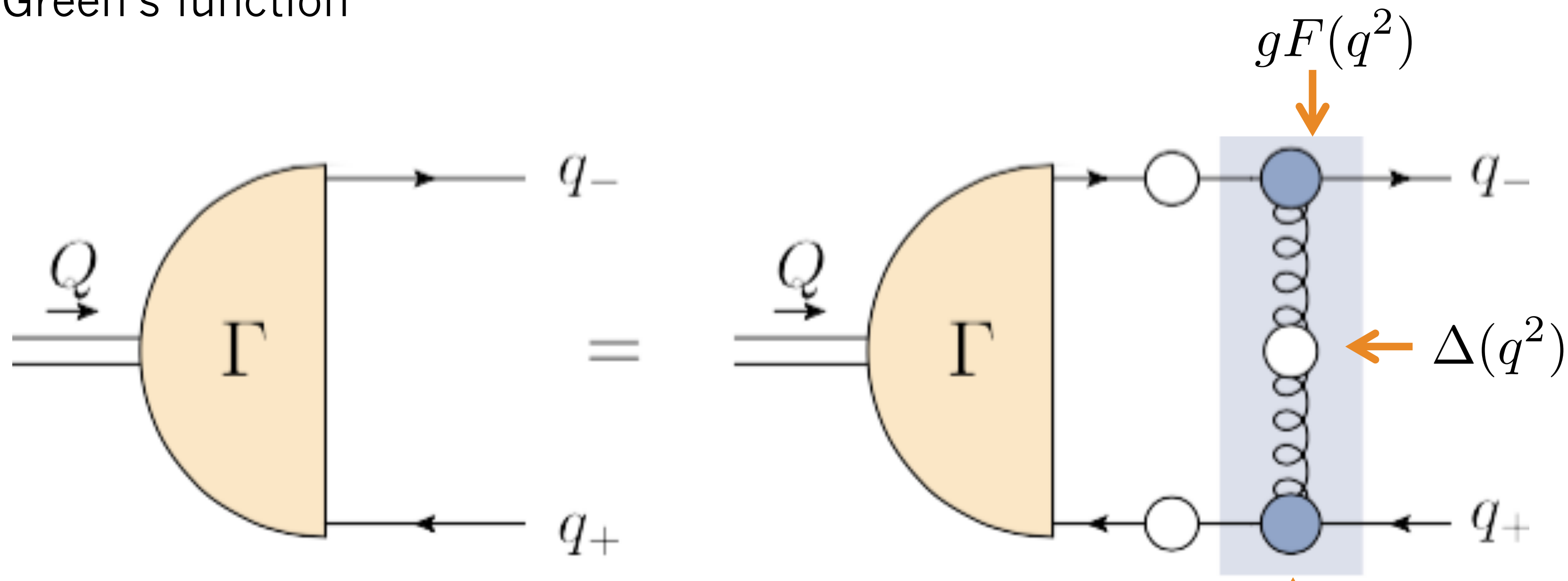
N_f	Inflection point (MeV)
0	488
2	476
2+1+1	380



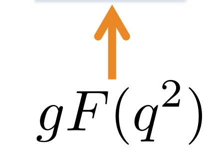
Bound States

Bethe-Salpeter equation

◎ Describes the formation of bound states (mesons) using the fundamental Green's function



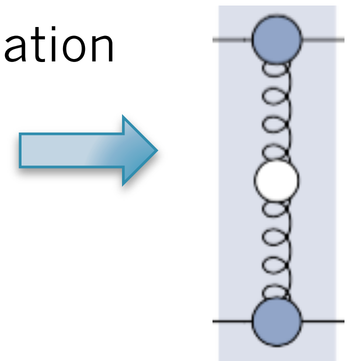
P. Maris and C. D. Roberts, Int. J. Mod. Phys. E 12, 297 (2003)
P. Maris and P. C. Tandy, Phys. Rev. C60, 055214 (1999)



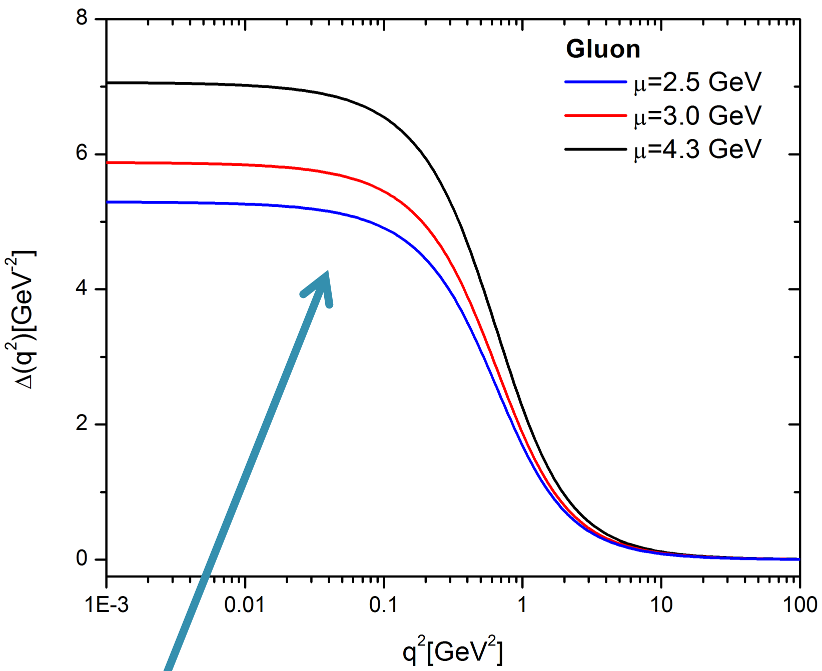
✓ Fundamental ingredient of the BS equation

✓ RGI quantity → μ –independent

$$R(q^2) = \alpha_s(\mu)\Delta(q^2,\mu^2)F^2(q^2,\mu^2)$$

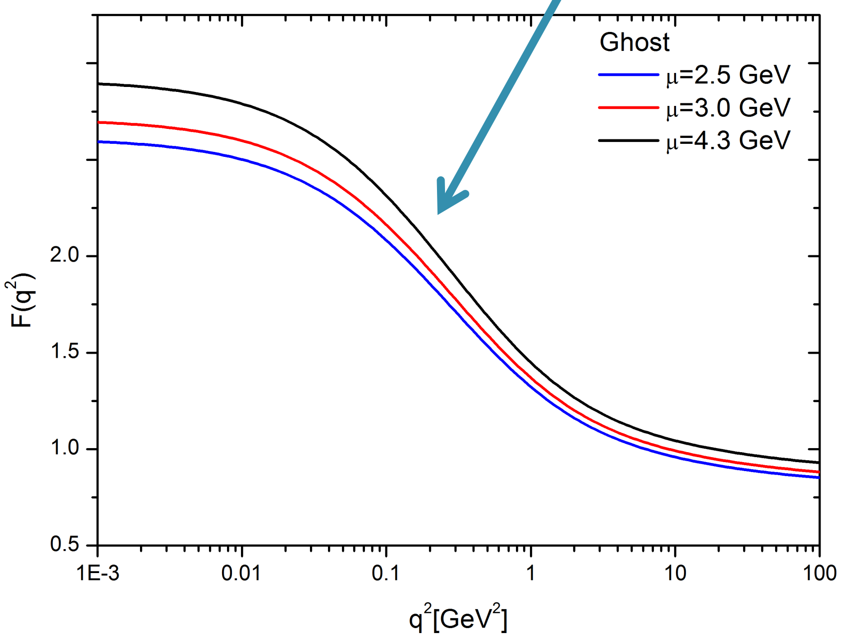


The μ-dependent ingredients



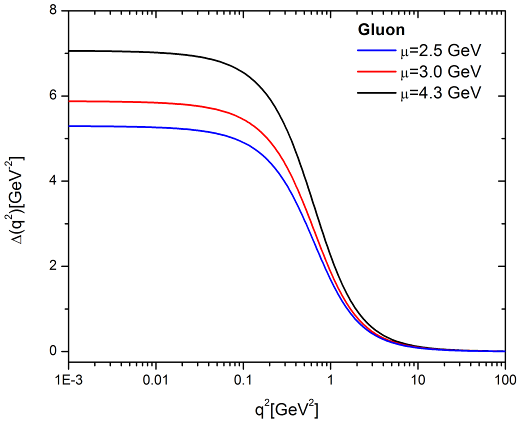
μ-dependence

μ-dependence

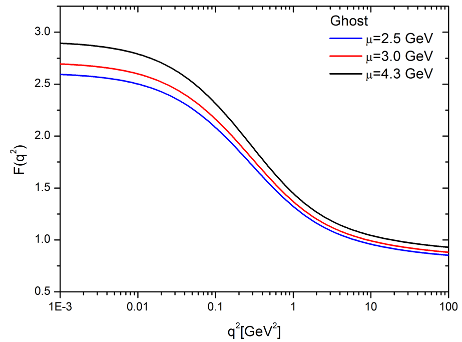


Forming the μ-independent product

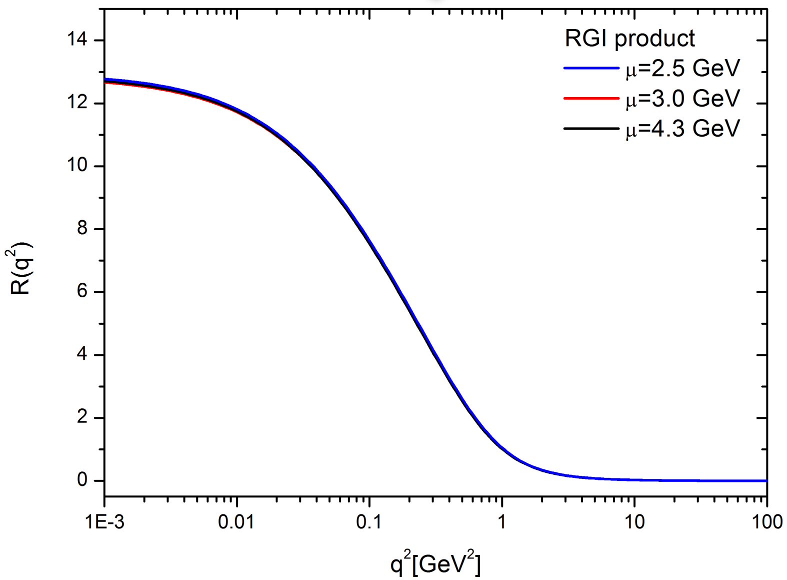
$\alpha_s(\mu)\times$



$\times$



2



$\alpha_s(\mu)$ fixed from ghost SDE → reproduce the lattice data

$\alpha_s(\mu) = 0.22$

$\alpha_s(\mu) = 0.30$

$\alpha_s(\mu) = 0.36$

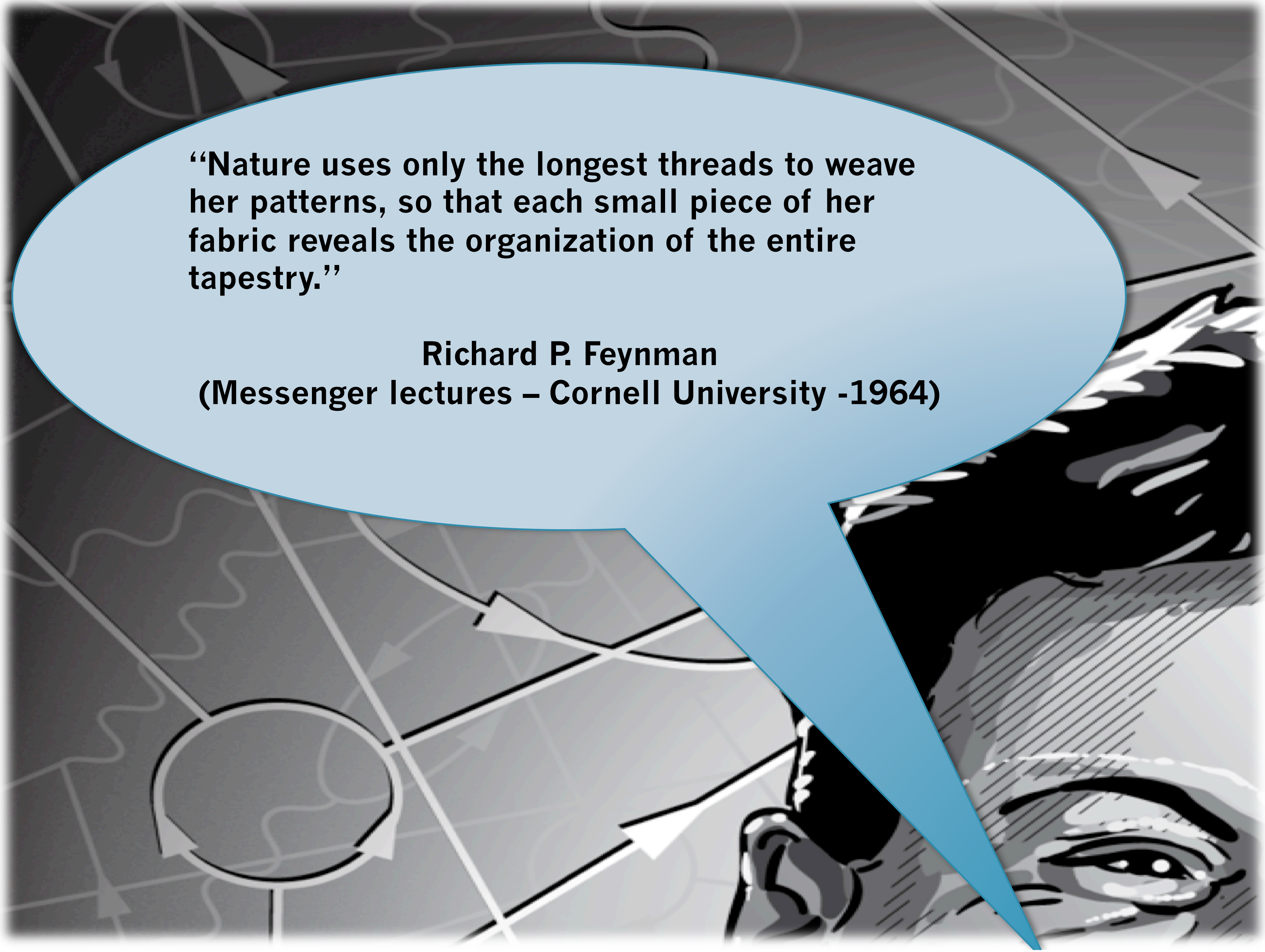
A. C. Aguilar, D. Binosi, J. P. and J. Rodriguez-Quintero, Phys. Rev. D80, 085018 (2009)

Conclusions

- The level of reliability of the SDEs has been improving steadily over the years.

- The synergy with the lattice opens new unexplored avenues. It must be nurtured and strengthened.

- The fundamental issue of mass generation can be addressed in a self-consistent framework.


"Nature uses only the longest threads to weave her patterns, so that each small piece of her fabric reveals the organization of the entire tapestry."
Richard P. Feynman
(Messenger lectures – Cornell University -1964)