Anomaly-free chiral fermion sets and gauge coupling unification

C. Simões

IFPA, AGO, University of Liege (ULg)

Based on:

Minimal anomaly-free chiral fermion sets and gauge coupling unification

Luís M. Cebola, D. Emmanuel-Costa, R. González Felipe and C. S.

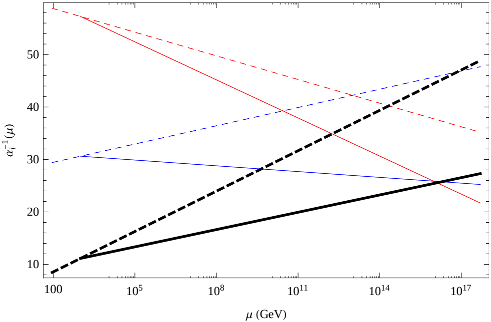
[arXiv:1409.0805]

Supported by: University of Liege and the EU in the context of the FP7-PEOPLE-COFUND-BeIPD project

Outline

- Motivation
- Minimal anomaly-free chiral fermion sets
  - Gauge coupling unification
- SU(5)-inspired anomaly-free chiral fermion sets
  - Gauge coupling unification
- Gauge coupling unification at string scale
- Conclusions

Motivation



- Each generation of Standard Model (SM) is anomaly-free
- No gauge coupling unification (GCU) in the SM

Goal:

- To find the minimal anomaly-free chiral sets of fermions **beyond the SM** that are simultaneously vector-like particles under $SU(3)_c$ and $U(1)_{em}$
- Does this new fermion content allow for GCU at a high energy scale?
- What is the minimal anomaly-free chiral set of fermions that belong to SU(5)-multiplets?
- Would we obtain GCU at a high scale?

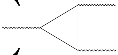
Anomalies (i)

Anomaly: is the breaking of the symmetry of the Lagrangian at the quantum level

- Triangular chiral gauge anomaly

[S.Adler, 1969]
[J.Bell & R.Jackiw, 1969]

$$\sum$$



$$= 0$$

"If a unified field theory is to be renormalizable, it must be free of triangular anomalies"

[D.Gross & R.Jackiw, 1972]

$$\mathrm{Tr}\left[\{T^a, T^b\}\, T^c\right] = 0$$

- Mixed gauge-gravitational anomaly

[R.Delbourgo & A.Salam, 1972]
[L.Alvarez-Gaumé & E.Witten, 1983]

- Witten's anomaly

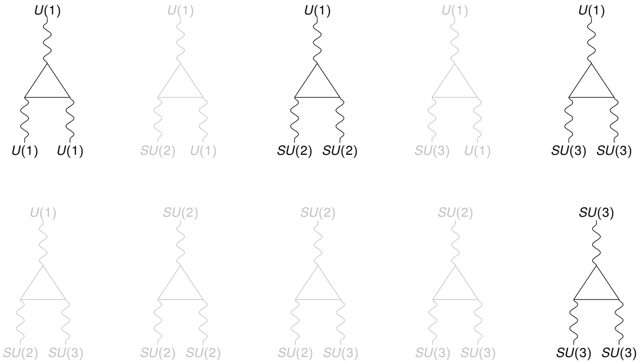
Any SU(2) gauge theory with an odd number of Weyl doublets is mathematically inconsistent!

$$\mathrm{Tr}\,[T\; T] \equiv \sum_R t_2(R) \;\text{ must be an integer number}$$

[E.Witten, 1982]

Anomalies (ii)

- Triangular anomaly: $\mathrm{Tr}\left[\{T^a, T^b\}\, T^c\right]$



- mixed gauge-gravitational anomaly

Anomalies (iii)

Anomaly conditions that must be verified:

anomaly index, $a(R)$

[SU(3) - SU(3) - SU(3)] : $$\sum_R a_3(R)\, d_2(R) = 0$$

[SU(3) - SU(3) - U(1)] : $$\sum_R y_R\, t_3(R)\, d_2(R) = 0$$

Dynkin index, $t(R)$

[SU(2) - SU(2) - U(1)] : $$\sum_R y_R\, t_2(R)\, d_3(R) = 0$$

[U(1) - U(1) - U(1)] : $$\sum_R y_R^3\, d_2(R)\, d_3(R) = 0$$

[gravity - gravity - U(1)] : $$\sum_R y_R\, d_2(R)\, d_3(R) = 0$$

Anomalies (iv)

Anomaly index

$$a(R)\ \mathrm{Tr}\left(\left\{t^i, t^j\right\} t^k\right) = \mathrm{Tr}\left(\left\{T_R^i, T_R^j\right\} T_R^k\right)$$ [J.Banks & H.Georgi, 1976]

T_R^i generator for the representation R

t^i generator for the fundamental representation

Dynkin index $t(R)\delta^{ab} \equiv \mathrm{Tr}\left[T^a T^b\right]$

$U(1)_Y$:

$$t_1(R) = y_R^2$$

SU(2):

R	2	3	4	5	6
Young diagram	□	□□	□□□	□□□□	□□□□□
Dynkin label	(1)	(2)	(3)	(4)	(5)
t_2	$\frac{1}{2}$	2	5	10	$\frac{23}{2}$

SU(3):

R	**3**	**6**	**8**	**10**	**15**	**15**$'$
Young diagram	□	□□	□□ □	□□□	□□□ □	□□□□
Dynkin label	(1,0)	(2,0)	(1,1)	(3,0)	(2,1)	(4,0)
t_3	$\frac{1}{2}$	$\frac{5}{2}$	3	$\frac{15}{2}$	10	$\frac{35}{2}$
a_3	1	7	0	27	14	77

Anomaly cancellation in the SM

"The imposition of all three types of anomaly conditions on the SM gauge group leads to correct minimal number of Weyl representations and their hypercharges."

[Geng & Marshak, 1989]

1 SM generation: $(3,2)_{y_Q}, (\overline{3},1)_{y_u}, (\overline{3},1)_{y_d}, (1,2)_{y_L}, (1,1)_{y_e}$

$$[SU(3)\text{ - }SU(3)\text{ - }SU(3)]: \quad 1\cdot 3\cdot 2 - 1\cdot 3\cdot 1 - 1\cdot 3\cdot 2 = 0$$

$$[SU(3)\text{ - }SU(3)\text{ - }U(1)]: \quad y_Q\cdot\frac{1}{2}\cdot 2 + y_u\cdot\frac{1}{2}\cdot 1 + y_d\cdot\frac{1}{2}\cdot 1 = 0$$

$$[SU(2)\text{ - }SU(2)\text{ - }U(1)]: \quad y_Q\cdot\frac{1}{2}\cdot 3 + y_L\cdot\frac{1}{2}\cdot 1 = 0$$

$$[U(1)\text{ - }U(1)\text{ - }U(1)]: \quad y_Q^3\cdot 3\cdot 2 + y_u^3\cdot 3\cdot 1 + y_d^3\cdot 3\cdot 1 + y_L^3\cdot 1\cdot 2 + y_e^3\cdot 1\cdot 1 = 0$$

Witten's anomaly : $3+1$ Weyl doublets

- This conditions are not enough to determine hypercharge relations!

$$[\text{gravity - gravity - }U(1)]: \quad y_Q\cdot 3\cdot 2 + y_u\cdot 3\cdot 1 + y_d\cdot 3\cdot 1 + y_L\cdot 1\cdot 2 + y_e\cdot 1\cdot 1 = 0$$

$$\boxed{y_Q = -\frac{1}{6}y_e} \qquad \boxed{y_u = \frac{2}{3}y_e} \qquad \boxed{y_d = -\frac{1}{3}y_e} \qquad \boxed{y_L = \frac{y_e}{2}}$$

$$y_e = -1$$

Anomaly-free chiral sets beyond SM (i)

❖ SM + 1 extra chiral fermion

$R_1 \,:\, (d_3\,(R_1)\,,\,d_2\,(R_1))_{y_{R_1}}$

$$[\mathrm{SU(3)}\text{-}\mathrm{SU(3)}\text{-}\mathrm{SU(3)}]: \quad a_3(R_1)\,d_2(R_1) = 0$$

$$[\mathrm{SU(3)}\text{-}\mathrm{SU(3)}\text{-}\mathrm{U(1)}]: \quad y_{R_1}\,t_3(R_1)\,d_2(R_1) = 0$$

$$[\mathrm{SU(2)}\text{-}\mathrm{SU(2)}\text{-}\mathrm{U(1)}]: \quad y_{R_1}\,t_2(R_1)\,d_3(R_1) = 0$$

$$[\mathrm{U(1)}\text{-}\mathrm{U(1)}\text{-}\mathrm{U(1)}]: \quad y_{R_1}^3\,d_2(R_1)\,d_3(R_1) = 0$$

$$[\text{gravity}\text{-}\text{gravity}\text{-}\mathrm{U(1)}]: \quad y_{R_1}\,d_2(R_1)\,d_3(R_1) = 0$$

▷ R_1 must be in the **adjoint representation** with **zero hypercharge**

Anomaly-free chiral sets beyond SM (ii)

- SM + 2 extra multiplets chiral fermion: R_1 and R_2

$$[SU(3) - SU(3) - SU(3)]: \quad a_3(R_1)\, d_2(R_1) + a_3(R_2)\, d_2(R_2) = 0$$

$$[SU(3) - SU(3) - U(1)]: \quad y_{R_1} t_3(R_1)\, d_2(R_1) + y_{R_2} t_3(R_2)\, d_2(R_2) = 0$$

$$[SU(2) - SU(2) - U(1)]: \quad y_{R_1} t_2(R_1)\, d_3(R_1) + y_{R_2} t_2(R_2)\, d_3(R_2) = 0$$

$$[U(1) - U(1) - U(1)]: \quad y^3_{R_1} d_2(R_1)\, d_3(R_1) + y^3_{R_2} d_2(R_2)\, d_3(R_2) = 0$$

$$[\text{gravity} - \text{gravity} - U(1)]: \quad y_{R_1} d_2(R_1)\, d_3(R_1) + y_{R_2} d_2(R_2)\, d_3(R_2) = 0$$

$$\mathbf{y^2_{R_1} = y^2_{R_2}}$$

▷ $\mathbf{y_{R_1} = y_{R_2} = y_R}$ $\qquad y_R\,(d_2(R_1)\, d_3(R_1) + d_2(R_2)\, d_3(R_2)) = 0 \implies y_R = 0$

2 types of solutions:

- anomaly cancellation **between** multiplets, e.g. $(\mathbf{6}, \mathbf{1})_0 \oplus (\bar{\mathbf{3}}, \mathbf{7})_0$
- anomaly cancellation **for each** multiplet, i.e. $\mathbf{a_3(R_1) = 0 = a_3(R_2)}$

▷ $\mathbf{y_{R_1} = -y_{R_2} \neq 0}$

$$\frac{d_3(R_1)}{d_3(R_2)} = \frac{t_3(R_1)}{t_3(R_2)} = -\frac{a_3(R_1)}{a_3(R_2)} = 1$$

Vector-like particles after the electroweak symmetry breaking:

▷ vector-like set: $\quad (\mathbf{d}, \mathbf{d'})_y \oplus (\bar{\mathbf{d}}, \mathbf{d'})_{-y}$

▷ chiral fermions in **adjoint representations** with **zero hypercharge**

Anomaly-free chiral sets beyond SM (iii)

- SM + 3 extra chiral multiplets: R_1, R_2 and R_3

[SU(3) - SU(3) - SU(3)] : $a_3(R_1)\, d_2(R_1) + a_3(R_2)\, d_2(R_2) + a_3(R_3)\, d_2(R_3) = 0$

[SU(3) - SU(3) - U(1)] : $y_{R_1} t_3(R_1)\, d_2(R_1) + y_{R_2} t_3(R_2)\, d_2(R_2) + y_{R_3} t_3(R_3)\, d_2(R_3) = 0$

[SU(2) - SU(2) - U(1)] : $y_{R_1} t_2(R_1)\, d_3(R_1) + y_{R_2} t_2(R_2)\, d_3(R_2) + y_{R_3} t_2(R_3)\, d_3(R_3) = 0$

[U(1) - U(1) - U(1)] : $y^3_{R_1} d_2(R_1)\, d_3(R_1) + y^3_{R_2} d_2(R_2)\, d_3(R_2) + y^3_{R_3} d_2(R_3)\, d_3(R_3) = 0$

[gravity - gravity - U(1)] : $y_{R_1} d_2(R_1)\, d_3(R_1) + y_{R_2} d_2(R_2)\, d_3(R_2) + y_{R_3} d_2(R_3)\, d_3(R_3) = 0$

$y \to z\, y$

Anomaly-free chiral sets beyond SM (iv)

- $d_3(R) \leq 10,\ d_2(R) \leq 5,$ rational hypercharges

Set	Particle content				
P1	$(\mathbf{d}, \mathbf{1})_{5z/6}$	$\oplus$	$(\mathbf{d}, \mathbf{2})_{-2z/3}$	$\oplus$	$(\bar{\mathbf{d}}, \mathbf{3})_{z/6}$
P2	$(\mathbf{d}, \mathbf{1})_{7z/6}$	$\oplus$	$(\mathbf{d}, \mathbf{3})_{-5z/6}$	$\oplus$	$(\bar{\mathbf{d}}, \mathbf{4})_{z/3}$
P3	$(\mathbf{d}, \mathbf{1})_{3z/2}$	$\oplus$	$(\mathbf{d}, \mathbf{4})_{-z}$	$\oplus$	$(\bar{\mathbf{d}}, \mathbf{5})_{z/2}$
P4	$(\mathbf{d}, \mathbf{2})_{4z/3}$	$\oplus$	$(\mathbf{d}, \mathbf{3})_{-7z/6}$	$\oplus$	$(\bar{\mathbf{d}}, \mathbf{5})_{z/6}$

Solutions with different values of $d_3(R)$: $(\mathbf{15}, \mathbf{1})_{z/6} \oplus (\bar{\mathbf{6}}, \mathbf{2})_{-z/3} \oplus (\mathbf{1}, \mathbf{3})_{z/2}$

- vector-like particles after electroweak symmetry breaking:

$$(6,2)_x \to 6_{-\frac{1}{2}+x} \oplus 6_{\frac{1}{2}+x}$$

$$\sum_{p=1}^{3} \sum_{j_p} [j_p + y_p(z)]^m = 0$$

m is an odd positive integer

- $m = 1$ or $m = 3$: ✓ automatically for any z
- $m = 5$ determines z: $|z| = 0, 1$ or 3

Gauge coupling unification (i)

Renormalisation group equations (RGEs) for gauge couplings at 1-loop level :

$$\frac{d}{dt}\alpha_i^{-1} = -\frac{b_i}{2\pi\kappa_i} \longrightarrow \boxed{\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi\kappa_i}\ln\left(\frac{\mu}{M_Z}\right)}$$

$$b_i^{SM} = \left(\frac{41}{6}, -\frac{19}{6}, -7\right)$$

[A.Pérez-Lorenzana & W.Ponce, 99]

$$\boxed{\kappa_i \equiv \frac{Tr(T_i^2)}{Tr(T^2)}}$$

For N intermediate particles

$$\alpha_U^{-1} = \alpha_i^{-1}(M_Z) - \frac{1}{2\pi\kappa_i}\left(b_i^{SM} + \sum_{I=1}^{N} b_i^I r_I\right)\ln\left(\frac{\Lambda}{M_Z}\right)$$

with

$$r_I = \frac{\ln(\Lambda/M_I)}{\ln(\Lambda/M_Z)} \qquad M_Z \leq M_I \leq \Lambda \qquad 0 \leq r_I \leq 1$$

$$b_i(R) = \frac{1}{3}\sum_R s(R)t_i(R)\prod_{j\neq i} d_j(R) \qquad s(R) = \begin{cases} 1/2 & \text{real scalar} \\ 1 & \text{complex scalar} \\ 2 & \text{chiral fermions} \\ 4 & \text{vector-like fermions} \\ -11 & \text{gauge bosons} \end{cases}$$

Gauge coupling unification (ii)

For each set with 3 intermediate multiplets

$$\alpha_i^{-1}(\Lambda) = \alpha_i^{-1}(M_Z) - \frac{1}{2\pi\kappa_i}\left(b_i^{SM} + b_i^1 r_1 + b_i^2 r_2 + b_i^3 r_3\right)\ln\left(\frac{\Lambda}{M_Z}\right)$$

At unification scale $\alpha_1^{-1} = \alpha_2^{-1}$ and $\alpha_2^{-1} = \alpha_3^{-1}$:

▷ $r_2,\ r_3 \in [0,1]$

$$r_1 = \frac{\mathbf{B}\,B'_{12} - B'_{23}}{\Delta^1_{23} - \mathbf{B}\,\Delta^1_{12}} \qquad \ln\left(\frac{\Lambda}{M_Z}\right) = \frac{\widetilde{\mathbf{B}}}{B_1 - B_2}$$

where

[A.Giveon, L.Hall & U.Sarid, 1991]

$$B'_{ij} = B'_i - B'_j\,, \qquad B'_i = \frac{1}{\kappa_i}\left(b_i^{SM} + b_i^2 r_2 + b_i^3 r_3\right), \qquad \Delta^1_{ij} = \frac{b_i^1}{\kappa_i} - \frac{b_j^1}{\kappa_j}\,, \qquad B_i = B'_i + \frac{1}{\kappa_i} b_i^1 r_1$$

$$\mathbf{B} \equiv \frac{\sin^2\theta_w - \dfrac{\kappa_2\,\alpha}{\kappa_3\,\alpha_s}}{\dfrac{\kappa_2}{\kappa_1} - \left(1 + \dfrac{\kappa_2}{\kappa_1}\right)\sin^2\theta_w}\,, \qquad \widetilde{\mathbf{B}} \equiv \frac{2\pi}{\alpha}\left[\frac{1}{\kappa_1} - \left(\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right)\sin^2\theta_w\right].$$

[Particle Data Group, 2012]

$\kappa_i = \left(\frac{5}{3}, 1, 1\right)$: $\mathbf{B} = 0.718 \pm 0.003$ $\widetilde{\mathbf{B}} = 185.0 \pm 0.2$

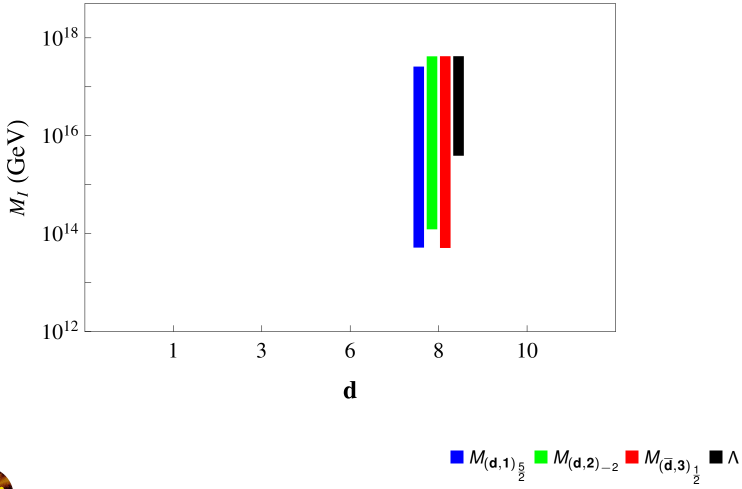
▷ $\kappa_i = (1, 1, 1)$: $\mathbf{B} = 0.308 \pm 0.001$ $\widetilde{\mathbf{B}} = 431.4 \pm 0.1$

$$\alpha^{-1} = 127.944 \pm 0.014$$
$$\alpha_s = 0.1185 \pm 0.0006$$
$$\sin^2\theta_w = 0.23126 \pm 0.00005$$

Gauge coupling unification: results (i)

P1: $(\mathbf{d},\mathbf{1})_{\frac{5}{6}z} \oplus (\mathbf{d},\mathbf{2})_{-\frac{2}{3}z} \oplus (\overline{\mathbf{d}},\mathbf{3})_{\frac{z}{6}}$

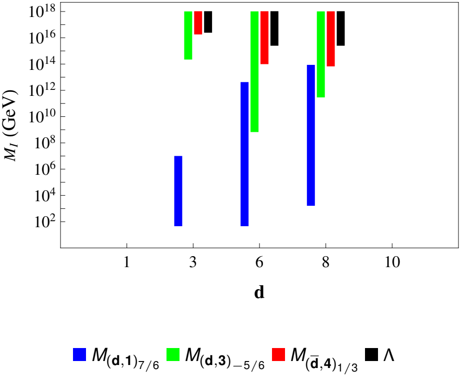
$z = 3$



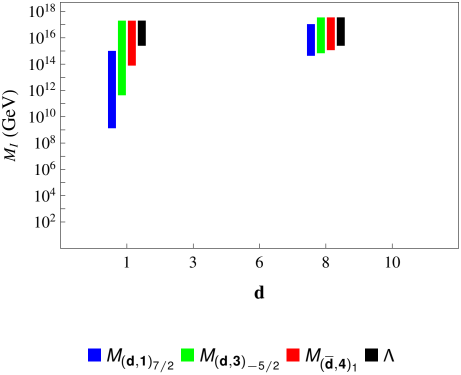
Gauge coupling unification: results (ii)

P2: $(\mathbf{d},\mathbf{1})_{\frac{7}{6}z} \oplus (\mathbf{d},\mathbf{3})_{-\frac{5}{6}z} \oplus (\overline{\mathbf{d}},\mathbf{4})_{\frac{z}{3}}$

$z = 1$



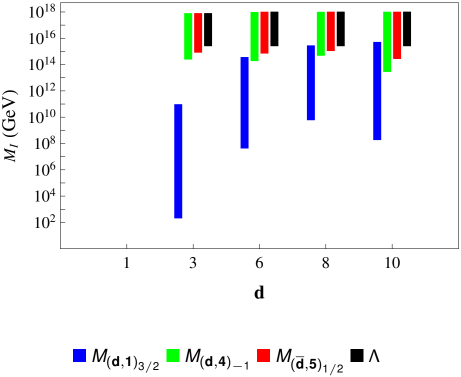
$z = 3$



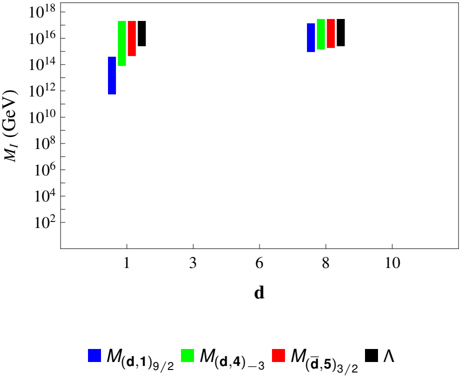
Gauge coupling unification: results (iii)

P3: $(\mathbf{d},\mathbf{1})_{3z/2} \oplus (\mathbf{d},\mathbf{4})_{-z} \oplus (\overline{\mathbf{d}},\mathbf{5})_{z/2}$

$z = 1$



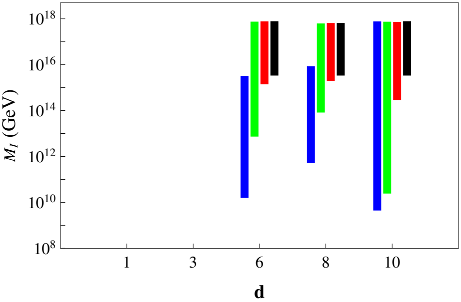
$z = 3$



Gauge coupling unification: results (iv)

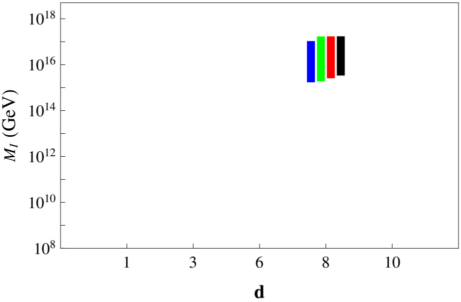
P4: $(\mathbf{d},\mathbf{2})_{4z/3} \oplus (\mathbf{d},\mathbf{3})_{-7z/6} \oplus (\bar{\mathbf{d}},\mathbf{5})_{z/6}$

$z = 1$



$M_{(\mathbf{d},\mathbf{2})_{4/3}}$ $M_{(\mathbf{d},\mathbf{3})_{-7/6}}$ $M_{(\bar{\mathbf{d}},\mathbf{5})_{1/6}}$ Λ

$z = 3$



$M_{(\mathbf{d},\mathbf{2})_{4}}$ $M_{(\mathbf{d},\mathbf{3})_{-7/2}}$ $M_{(\bar{\mathbf{d}},\mathbf{5})_{1/2}}$ Λ

Which is the minimal anomaly-free chiral fermion sets inspired in SU(5) multiplets?

SU(5)-*inspired anomaly-free chiral sets (i)*

Label	Multiplet	SU(5)-rep	Label	Multiplet	SU(5)-rep	Label	Multiplet	SU(5)-rep
1	$(\mathbf{1},\mathbf{2})_{1/2}$	5, 45	8	$(\mathbf{1},\mathbf{4})_{-3/2}$	35	15	$(\bar{\mathbf{3}},\mathbf{1})_{4/3}$	45
2	$(\mathbf{3},\mathbf{1})_{-1/3}$	5, 45, 50	9	$(\bar{\mathbf{3}},\mathbf{3})_{-2/3}$	35, 40	16	$(\bar{\mathbf{3}},\mathbf{2})_{-7/6}$	45, 50
3	$(\mathbf{1},\mathbf{1})_{1}$	10	10	$(\bar{\mathbf{6}},\mathbf{2})_{1/6}$	35, 40	17	$(\bar{\mathbf{6}},\mathbf{1})_{-1/3}$	45
4	$(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$	10, 40	11	$(\overline{\mathbf{10}},\mathbf{1})_{1}$	35	18	$(\mathbf{8},\mathbf{2})_{1/2}$	45, 50
5	$(\mathbf{3},\mathbf{2})_{1/6}$	10, 15, 40	12	$(\mathbf{1},\mathbf{2})_{-3/2}$	40	19	$(\mathbf{1},\mathbf{1})_{-2}$	50
6	$(\mathbf{1},\mathbf{3})_{1}$	15	13	$(\mathbf{8},\mathbf{1})_{1}$	40	20	$(\bar{\mathbf{6}},\mathbf{3})_{-1/3}$	50
7	$(\mathbf{6},\mathbf{1})_{-2/3}$	15	14	$(\mathbf{3},\mathbf{3})_{-1/3}$	45	21	$(\mathbf{6},\mathbf{1})_{4/3}$	50

SU(5)-*inspired anomaly-free chiral sets (ii)*

[SU(3) - SU(3) - SU(3)] : $n_2 - n_4 + 2n_5 + 7n_7 - 3n_9 - 14n_{10} - 27n_{11} + 3n_{14} - n_{15} - 2n_{16} - 7n_{17} - 21n_{20} + 7n_{21} = 0$

[SU(3) - SU(3) - U(1)] : $n_2 + 2n_4 - n_5 + 10n_7 + 6n_9 - 5n_{10} - 45n_{11} - 18n_{13} + 3n_{14} - 4n_{15} + 7n_{16} + 5n_{17} - 18n_{18} + 15n_{20} - 20n_{21} = 0$

[SU(2) - SU(2) - U(1)] : $n_1 + n_5 + 8n_6 - 30n_8 - 16n_9 + 2n_{10} - 3n_{12} - 8n_{14} - 7n_{16} + 8n_{18} - 16n_{20} = 0$

[U(1) - U(1) - U(1)] : $9n_1 - 4n_2 + 36n_3 - 32n_4 + n_5 + 108n_6 - 64n_7 - 486n_8 - 96n_9 + 2n_{10} + 360n_{11} - 243n_{12} + 288n_{13} - 12n_{14} + 256n_{15} - 343n_{16} - 8n_{17} + 72n_{18} - 288n_{19} - 24n_{20} + 512n_{21} = 0$

[gravity - gravity - U(1)] : $n_1 - n_2 + n_3 - 2n_4 + n_5 + 3n_6 - 4n_7 - 6n_8 - 6n_9 + 2n_{10} + 10n_{11} - 3n_{12} + 8n_{13} - 3n_{14} + 4n_{15} - 7n_{16} - 2n_{17} + 8n_{18} - 2n_{19} - 6n_{20} + 8n_{21} = 0$

SU(5)-inspired anomaly-free chiral sets (iii)

- ▷ Vector-like particles after electroweak symmetry breaking

$(\mathbf{3},\mathbf{2})_{1/3} \Longrightarrow \mathbf{3}_{2/3} \oplus \mathbf{3}_{-1/3}$

$$\mathbf{1}_1 : \; n_1 + n_3 + n_6 - n_8 - n_{12} = 0$$

$$\mathbf{3}_{-1/3} : \; n_2 + n_5 - n_9 + n_{14} = 0$$

$$\overline{\mathbf{3}}_{-2/3} : \; n_4 - n_5 + n_9 - n_{14} + n_{16} = 0$$

$$\mathbf{1}_2 : \; n_6 - n_8 - n_{12} - n_{19} = 0$$

$$\mathbf{6}_{-2/3} : \; n_7 - n_{10} - n_{20} = 0$$

$$\mathbf{1}_{-3} : \; n_8 = 0$$

$$\overline{\mathbf{3}}_{-5/3} : \; n_9 + n_{16} = 0$$

$$\overline{\mathbf{6}}_{-1/3} : \; n_{10} + n_{17} + n_{20} = 0$$

$$\overline{\mathbf{10}}_1 : \; n_{11} = 0$$

$$\mathbf{8}_1 : \; n_{13} + n_{18} = 0$$

$$\mathbf{3}_{-4/3} : \; n_{14} - n_{15} = 0$$

$$\overline{\mathbf{6}}_{-4/3} : \; n_{20} - n_{21} = 0$$

[SU(2) - SU(2) - U(1)] : $2n_1 + 9n_2 + 3n_3 + 17n_4 - 9n_5 - 5n_6 + 16n_7 - 18n_{10} + 8n_{13} = 0$

[U(1) - U(1) - U(1)] : $54n_1 + 243n_2 + 81n_3 + 459n_4 - 243n_5 - 135n_6 + 432n_7 - 486n_{10} + 216n_{13} = 0$

✓ [SU(3) - SU(3) - SU(3)], [SU(3) - SU(3) - U(1)] and [gravity - gravity - U(1)]

SU(5)-*inspired anomaly-free chiral sets (iv)*

Set	n_S	Particle content	GUT	String
S1	4	$3\,(\mathbf{1},\mathbf{2})_{1/2} \oplus 2\,(\mathbf{1},\mathbf{1})_{-1} \oplus (\mathbf{1},\mathbf{2})_{-3/2} \oplus (\mathbf{1},\mathbf{1})_{2}$	–	–
S2	5	$(\mathbf{1},\mathbf{2})_{1/2} \oplus (\mathbf{3},\mathbf{1})_{-1/3} \oplus (\mathbf{1},\mathbf{1})_{-1} \oplus (\mathbf{3},\mathbf{1})_{2/3} \oplus (\bar{\mathbf{3}},\mathbf{2})_{-1/6}$	–	–
S3	5	$(\mathbf{1},\mathbf{1})_{-1} \oplus (\mathbf{1},\mathbf{3})_{1} \oplus (\mathbf{8},\mathbf{1})_{1} \oplus (\mathbf{8},\mathbf{2})_{-1/2} \oplus (\mathbf{1},\mathbf{1})_{-2}$	–	–
S4	5	$2\,(\mathbf{1},\mathbf{2})_{1/2} \oplus 2\,(\mathbf{1},\mathbf{1})_{-1} \oplus (\bar{\mathbf{6}},\mathbf{1})_{2/3} \oplus (\mathbf{6},\mathbf{2})_{-1/6} \oplus (\bar{\mathbf{6}},\mathbf{1})_{-1/3}$	–	–
S5	5	$(\mathbf{1},\mathbf{2})_{1/2} \oplus (\mathbf{1},\mathbf{1})_{1} \oplus (\mathbf{1},\mathbf{3})_{1} \oplus 3\,(\mathbf{1},\mathbf{2})_{-3/2} \oplus 2\,(\mathbf{1},\mathbf{1})_{2}$	–	–
S6	5	$2\,(\mathbf{1},\mathbf{2})_{1/2} \oplus 3\,(\mathbf{1},\mathbf{1})_{-1} \oplus (\mathbf{1},\mathbf{3})_{-1} \oplus 2\,(\mathbf{1},\mathbf{2})_{3/2} \oplus (\mathbf{1},\mathbf{1})_{-2}$	–	–
S7	5	$2\,(\mathbf{1},\mathbf{2})_{1/2} \oplus 2\,(\mathbf{3},\mathbf{1})_{-1/3} \oplus 2\,(\mathbf{1},\mathbf{1})_{-1} \oplus 2\,(\mathbf{3},\mathbf{1})_{2/3} \oplus 2\,(\bar{\mathbf{3}},\mathbf{2})_{-1/6}$	✓	–
S8	5	$2\,(\mathbf{1},\mathbf{1})_{-1} \oplus 2\,(\mathbf{1},\mathbf{3})_{1} \oplus 2\,(\mathbf{8},\mathbf{1})_{1} \oplus 2\,(\mathbf{8},\mathbf{2})_{-1/2} \oplus 2\,(\mathbf{1},\mathbf{1})_{-2}$	✓	–
S9	6	$(\bar{\mathbf{3}},\mathbf{1})_{1/3} \oplus (\bar{\mathbf{3}},\mathbf{2})_{-1/6} \oplus (\mathbf{3},\mathbf{3})_{2/3} \oplus (\mathbf{3},\mathbf{3})_{-1/3} \oplus (\bar{\mathbf{3}},\mathbf{1})_{4/3} \oplus (\bar{\mathbf{3}},\mathbf{2})_{-7/6}$	*	–
S10	6	$(\bar{\mathbf{3}},\mathbf{1})_{1/3} \oplus (\bar{\mathbf{3}},\mathbf{1})_{-2/3} \oplus (\mathbf{8},\mathbf{1})_{-1} \oplus (\mathbf{3},\mathbf{3})_{-1/3} \oplus (\bar{\mathbf{3}},\mathbf{1})_{4/3} \oplus (\mathbf{8},\mathbf{2})_{1/2}$	✓	✓
S11	6	$(\mathbf{3},\mathbf{1})_{2/3} \oplus (\bar{\mathbf{3}},\mathbf{2})_{-1/6} \oplus (\mathbf{3},\mathbf{3})_{2/3} \oplus (\mathbf{8},\mathbf{1})_{1} \oplus (\bar{\mathbf{3}},\mathbf{2})_{-7/6} \oplus (\mathbf{8},\mathbf{2})_{-1/2}$	✓	✓
S12	6	$(\mathbf{1},\mathbf{2})_{1/2} \oplus (\mathbf{6},\mathbf{1})_{-2/3} \oplus (\bar{\mathbf{6}},\mathbf{2})_{1/6} \oplus (\mathbf{1},\mathbf{2})_{-3/2} \oplus (\mathbf{6},\mathbf{1})_{1/3} \oplus (\mathbf{1},\mathbf{1})_{2}$	–	–
S13	6	$(\bar{\mathbf{6}},\mathbf{1})_{2/3} \oplus 2\,(\mathbf{8},\mathbf{1})_{1} \oplus (\bar{\mathbf{6}},\mathbf{1})_{-1/3} \oplus 2\,(\mathbf{8},\mathbf{2})_{-1/2} \oplus (\mathbf{6},\mathbf{3})_{1/3} \oplus (\bar{\mathbf{6}},\mathbf{1})_{-4/3}$	✓	✓
S14	6	$2\,(\bar{\mathbf{3}},\mathbf{1})_{1/3} \oplus 2\,(\mathbf{3},\mathbf{3})_{2/3} \oplus (\mathbf{6},\mathbf{2})_{-1/6} \oplus 2\,(\bar{\mathbf{3}},\mathbf{2})_{-7/6} \oplus (\bar{\mathbf{6}},\mathbf{3})_{-1/3} \oplus (\mathbf{6},\mathbf{1})_{4/3}$	✓	✓
S15	6	$2\,(\bar{\mathbf{3}},\mathbf{1})_{1/3} \oplus 2\,(\bar{\mathbf{3}},\mathbf{1})_{-2/3} \oplus 2\,(\mathbf{3},\mathbf{2})_{1/6} \oplus (\bar{\mathbf{6}},\mathbf{1})_{2/3} \oplus (\mathbf{6},\mathbf{2})_{-1/6} \oplus (\bar{\mathbf{6}},\mathbf{1})_{-1/3}$	✓	✓
S16	6	$2\,(\bar{\mathbf{3}},\mathbf{2})_{-1/6} \oplus (\bar{\mathbf{6}},\mathbf{2})_{1/6} \oplus 2\,(\mathbf{3},\mathbf{3})_{-1/3} \oplus 2\,(\bar{\mathbf{3}},\mathbf{1})_{4/3} \oplus (\mathbf{6},\mathbf{3})_{1/3} \oplus (\bar{\mathbf{6}},\mathbf{1})_{-4/3}$	✓	✓
S17	6	$(\mathbf{1},\mathbf{2})_{1/2} \oplus 2\,(\bar{\mathbf{3}},\mathbf{1})_{1/3} \oplus 2\,(\bar{\mathbf{3}},\mathbf{1})_{-2/3} \oplus 2\,(\mathbf{3},\mathbf{2})_{1/6} \oplus (\mathbf{1},\mathbf{2})_{-3/2} \oplus (\mathbf{1},\mathbf{1})_{2}$	✓	–
S18	6	$(\mathbf{1},\mathbf{2})_{1/2} \oplus 2\,(\mathbf{1},\mathbf{3})_{1} \oplus 3\,(\mathbf{1},\mathbf{2})_{-3/2} \oplus (\mathbf{8},\mathbf{1})_{1} \oplus (\bar{\mathbf{8}},\mathbf{2})_{-1/2} \oplus (\mathbf{1},\mathbf{1})_{2}$	–	–
S19	6	$3\,(\mathbf{1},\mathbf{2})_{1/2} \oplus 3\,(\mathbf{1},\mathbf{1})_{-1} \oplus (\mathbf{1},\mathbf{3})_{1} \oplus (\mathbf{1},\mathbf{2})_{-3/2} \oplus (\mathbf{8},\mathbf{1})_{1} \oplus (\bar{\mathbf{8}},\mathbf{2})_{-1/2}$	*	–
S20	6	$2\,(\mathbf{1},\mathbf{2})_{1/2} \oplus 2\,(\mathbf{1},\mathbf{1})_{-1} \oplus 2\,(\mathbf{1},\mathbf{3})_{-1} \oplus 2\,(\mathbf{1},\mathbf{2})_{3/2} \oplus (\bar{\mathbf{8}},\mathbf{1})_{-1} \oplus (\mathbf{8},\mathbf{2})_{1/2}$	*	–

$\Lambda_{\rm min} = 5.0 \times 10^{15}$ GeV $\Lambda^{\rm string}_{\rm min} = 3.0 \times 10^{17}$ GeV

gauge coupling unification: results (i)

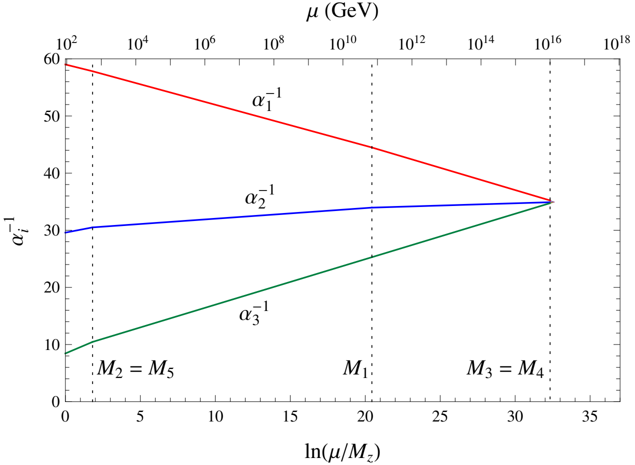
Set	α_U^{-1}	$\Lambda_{\max}$[GeV]	rep	min [GeV]	max [GeV]	rep	min [GeV]	max [GeV]
S7	$[30.5, 37.6]$	1.0×10^{17}	$(\mathbf{1},\mathbf{2})_{1/2}$	M_Z	7.6×10^{16}	$(\mathbf{3},\mathbf{1})_{-1/3}$	M_Z	1.3×10^{16}
			$(\mathbf{1},\mathbf{1})_{-1}$	8.0×10^{3}	7.8×10^{16}	$(\mathbf{3},\mathbf{1})_{2/3}$	2.6×10^{4}	9.4×10^{16}
			$(\bar{\mathbf{3}},\mathbf{2})_{-1/6}$	M_Z	5.8×10^{7}			
S8	$[1.4, 18.8]$	5.0×10^{17}	$(\mathbf{1},\mathbf{1})_{-1}$	2.2×10^{3}	2.8×10^{17}	$(\mathbf{1},\mathbf{3})_{1}$	M_Z	4.0×10^{5}
			$(\mathbf{8},\mathbf{1})_{1}$	8.9×10^{14}	4.7×10^{17}	$(\mathbf{8},\mathbf{2})_{-1/2}$	M_Z	6.2×10^{7}
			$(\mathbf{1},\mathbf{1})_{-2}$	9.4×10^{12}	3.4×10^{17}			
S10	$[8.0, 35.0]$	5.3×10^{17}	$(\bar{\mathbf{3}},\mathbf{1})_{1/3}$	M_Z	4.2×10^{17}	$(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$	M_Z	4.9×10^{17}
			$(\mathbf{8},\mathbf{1})_{-1}$	6.7×10^{3}	5.0×10^{17}	$(\mathbf{3},\mathbf{3})_{-1/3}$	M_Z	1.1×10^{12}
			$(\bar{\mathbf{3}},\mathbf{1})_{4/3}$	M_Z	5.0×10^{17}	$(\mathbf{8},\mathbf{2})_{1/2}$	M_Z	4.7×10^{13}
S11	$[4.0, 34.5]$	5.3×10^{17}	$(\mathbf{3},\mathbf{1})_{2/3}$	M_Z	4.9×10^{17}	$(\bar{\mathbf{3}},\mathbf{2})_{-1/6}$	M_Z	2.7×10^{17}
			$(\mathbf{3},\mathbf{3})_{2/3}$	M_Z	9.7×10^{14}	$(\mathbf{8},\mathbf{1})_{1}$	8.4×10^{3}	4.6×10^{17}
			$(\bar{\mathbf{3}},\mathbf{2})_{-7/6}$	4.0×10^{5}	5.1×10^{17}	$(\mathbf{8},\mathbf{2})_{-1/2}$	M_Z	4.3×10^{13}
S13	$[1.0, 35.5]$	5.3×10^{17}	$(\bar{\mathbf{6}},\mathbf{1})_{2/3}$	7.4×10^{4}	5.1×10^{17}	$(\mathbf{8},\mathbf{1})_{1}$	1.9×10^{7}	5.0×10^{17}
			$(\bar{\mathbf{6}},\mathbf{1})_{-1/3}$	5.5×10^{7}	5.0×10^{17}	$(\mathbf{8},\mathbf{2})_{-1/2}$	1.4×10^{13}	5.2×10^{17}
			$(\mathbf{6},\mathbf{3})_{1/3}$	M_Z	7.1×10^{13}	$(\bar{\mathbf{6}},\mathbf{1})_{-4/3}$	M_Z	5.0×10^{17}
S14	$[1.0, 35.9]$	5.3×10^{17}	$(\bar{\mathbf{3}},\mathbf{1})_{1/3}$	M_Z	4.7×10^{17}	$(\mathbf{3},\mathbf{3})_{2/3}$	6.4×10^{7}	4.9×10^{17}
			$(\mathbf{6},\mathbf{2})_{-1/6}$	M_Z	2.8×10^{16}	$(\bar{\mathbf{3}},\mathbf{2})_{-7/6}$	2.9×10^{3}	5.0×10^{17}
			$(\bar{\mathbf{6}},\mathbf{3})_{-1/3}$	2.5×10^{3}	4.4×10^{17}	$(\mathbf{6},\mathbf{1})_{4/3}$	M_Z	4.9×10^{17}
S15	$[32.3, 37.3]$	5.3×10^{17}	$(\bar{\mathbf{3}},\mathbf{1})_{1/3}$	9.0×10^{6}	4.3×10^{17}	$(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$	1.5×10^{11}	4.3×10^{17}
			$(\mathbf{3},\mathbf{2})_{1/6}$	M_Z	9.8×10^{5}	$(\bar{\mathbf{6}},\mathbf{1})_{2/3}$	2.1×10^{13}	5.0×10^{17}
			$(\mathbf{6},\mathbf{2})_{-1/6}$	8.1×10^{10}	8.5×10^{16}	$(\bar{\mathbf{6}},\mathbf{1})_{-1/3}$	5.4×10^{12}	4.8×10^{17}
S16	$[1.0, 37.1]$	5.3×10^{17}	$(\bar{\mathbf{3}},\mathbf{2})_{-1/6}$	M_Z	5.1×10^{17}	$(\bar{\mathbf{6}},\mathbf{2})_{1/6}$	M_Z	6.0×10^{15}
			$(\mathbf{3},\mathbf{3})_{-1/3}$	9.5×10^{7}	5.3×10^{17}	$(\bar{\mathbf{3}},\mathbf{1})_{4/3}$	M_Z	5.0×10^{17}
			$(\mathbf{6},\mathbf{3})_{1/3}$	880	4.9×10^{17}	$(\bar{\mathbf{6}},\mathbf{1})_{-4/3}$	M_Z	5.0×10^{17}
S17	$[31.9, 37.1]$	6.0×10^{16}	$(\mathbf{1},\mathbf{2})_{1/2}$	M_Z	2.0×10^{16}	$(\bar{\mathbf{3}},\mathbf{1})_{1/3}$	M_Z	8.3×10^{15}
			$(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$	2.9×10^{6}	3.3×10^{16}	$(\mathbf{3},\mathbf{2})_{1/6}$	M_Z	2.0×10^{7}
			$(\mathbf{1},\mathbf{2})_{-3/2}$	6.5×10^{10}	5.0×10^{16}	$(\mathbf{1},\mathbf{1})_{2}$	6.7×10^{10}	5.9×10^{16}

$24_H : \Sigma_3 \sim (1,3)_0,\ \Sigma_8 \sim (8,1)_0,\ X \sim (3,2)_{-\frac{5}{6}}\,,\quad 5_H : T \sim (3,1)_{-\frac{1}{3}}\,,\ H \sim (1,2)_{\frac{1}{2}}$

gauge coupling unification: results (ii)

S7: $2\,(\mathbf{1},\mathbf{2})_{1/2} \oplus 2\,(\mathbf{3},\mathbf{1})_{-1/3} \oplus 2\,(\mathbf{1},\mathbf{1})_{-1} \oplus 2\,(\mathbf{3},\mathbf{1})_{2/3} \oplus 2\,(\bar{\mathbf{3}},\mathbf{2})_{-1/6}$
$M_1 \quad M_2 \quad M_3 \quad M_4 \quad M_5$

$\Lambda = 1.3 \times 10^{16}$ GeV, $\quad \alpha_U^{-1} \simeq 35$



$M_{\Sigma_3} = M_{\Sigma_8} = \Lambda \qquad M_2 = M_5 = 560$ GeV $\qquad M_1 = 6.9 \times 10^{10}$ GeV $\qquad M_3 = M_4 = 1.0 \times 10^{16}$ GeV

Anomaly-free sets within the SU(5) (i)

Which are the minimal anomaly-free sets of complete representations of SU(5)?

$$[SU(5) - SU(5) - SU(5)] : \quad \sum_R a_5(R) = 0$$

SU(5)-irrep	**5**	**10**	**15**	**24**	**35**	**40**	**45**	**50**
a_5	1	1	9	0	-44	-16	-6	-15

$$n_5 + n_{10} + 9n_{15} - 44n_{35} - 16n_{40} - 6n_{45} - 15n_{50} = 0$$

▷ low energy vector-like particles

Label	Multiplet	SU(5)-rep	Label	Multiplet	SU(5)-rep	Label	Multiplet	SU(5)-rep
1	$(\mathbf{1},\mathbf{2})_{1/2}$	5, 45	8	$(\mathbf{1},\mathbf{4})_{-3/2}$	35	15	$(\bar{\mathbf{3}},\mathbf{1})_{4/3}$	45
2	$(\mathbf{3},\mathbf{1})_{-1/3}$	5, 45, 50	9	$(\bar{\mathbf{3}},\mathbf{3})_{-2/3}$	35, 40	16	$(\bar{\mathbf{3}},\mathbf{2})_{-7/6}$	45, 50
3	$(\mathbf{1},\mathbf{1})_{1}$	10	10	$(\bar{\mathbf{6}},\mathbf{2})_{1/6}$	35, 40	17	$(\bar{\mathbf{6}},\mathbf{1})_{-1/3}$	45
4	$(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$	10, 40	11	$(\overline{\mathbf{10}},\mathbf{1})_{1}$	35	18	$(\mathbf{8},\mathbf{2})_{1/2}$	45, 50
5	$(\mathbf{3},\mathbf{2})_{1/6}$	10, 15, 40	12	$(\mathbf{1},\mathbf{2})_{-3/2}$	40	19	$(\mathbf{1},\mathbf{1})_{-2}$	50
6	$(\mathbf{1},\mathbf{3})_{1}$	15	13	$(\mathbf{8},\mathbf{1})_{1}$	40	20	$(\bar{\mathbf{6}},\mathbf{3})_{-1/3}$	50
7	$(\mathbf{6},\mathbf{1})_{-2/3}$	15	14	$(\mathbf{3},\mathbf{3})_{-1/3}$	45	21	$(\mathbf{6},\mathbf{1})_{4/3}$	50

Anomaly-free sets within the SU(5) (i)

Which are the minimal anomaly-free sets of complete representations of SU(5)?

$$[SU(5) - SU(5) - SU(5)] : \quad \sum_R a_5(R) = 0$$

SU(5)-irrep	**5**	**10**	**15**	**24**	**35**	**40**	**45**	**50**
a_5	1	1	9	0	-44	-16	-6	-15

$$n_5 + n_{10} + 9n_{15} - 44n_{35} - 16n_{40} - 6n_{45} - 15n_{50} = 0$$

▷ low energy vector-like particles

$$\begin{aligned} n_5 + n_{10} - n_{40} &= 0 \\ n_{15} - n_{40} - n_{50} &= 0 \\ n_{15} + n_{45} &= 0 \\ n_{35} &= 0 \end{aligned}$$

Anomaly-free sets within the SU(5) *(ii)*

Minimal anomaly-free sets of complete SU(5) representations:

$$\bar{5} \oplus 10 \qquad \text{1 SM generation}$$

$$\bar{5} \oplus \overline{40} \oplus 50$$

$$10 \oplus 40 \oplus \overline{50}$$

$$15 \oplus \overline{45} \oplus 50$$

$$\bar{5} \oplus \overline{15} \oplus \overline{40} \oplus 45$$

$$10 \oplus 15 \oplus 40 \oplus \overline{45} \qquad \text{1 SM generation}$$

SM in a non-standard way

$$3 \times 10 \oplus 4 \times 15 \oplus 3 \times 40 \oplus 4 \times \overline{45} \oplus 50$$

Q: Is it possible to obtain gauge coupling unification in this setup?

A: No, unless one allows for mass-splittings inside each SU(5)-multiplet!

Gauge coupling unification at string scale (i)

$$\alpha_{\text{string}} = \frac{2G_N}{\alpha'} = \kappa_i \alpha_i \longrightarrow \Lambda = \sqrt{4\pi\alpha_{\text{string}}}\,\Lambda_S$$

$$\Lambda_S = \frac{e^{\frac{1-\gamma}{2}}\, 3^{-\frac{3}{4}}}{4\pi} M_P \approx 5.27 \times 10^{17}\ \text{GeV}$$

[V.Kaplunovsky, 1987]

For **each** set of solutions

α' Regge slope, $\gamma = 0.577$, $M_P = 1.22 \times 10^{19}$GeV

$$\frac{1}{4\pi}\left(\frac{\Lambda}{\Lambda_S}\right)^2 = \alpha_i^{-1}(M_Z) - \frac{1}{2\pi\kappa_i}\left(b_i^{\text{SM}} + b_i^1 r_1 + b_i^2 r_2 + b_i^3 r_3\right) \ln\left(\frac{\Lambda}{M_Z}\right)$$

▷ $r_3 \in [0, 1]$

$$r_1 = \beta_2 - \frac{\widetilde{\beta}_2}{\ln(\Lambda/M_Z)} \qquad r_2 = -\beta_1 + \frac{\widetilde{\beta}_1}{\ln(\Lambda/M_Z)}$$

where

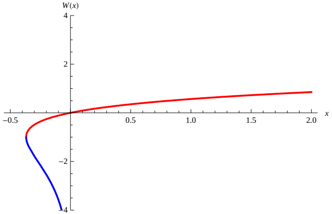
$$\beta_a = \frac{B''_{12}\,\Delta^a_{23} - B''_{23}\Delta^a_{12}}{\Delta^2_{12}\,\Delta^1_{23} - \Delta^2_{23}\,\Delta^1_{12}}\,, \qquad \widetilde{\beta}_a = \widetilde{\mathbf{B}}\,\frac{\Delta^a_{23} - \mathbf{B}\,\Delta^a_{12}}{\Delta^2_{12}\,\Delta^1_{23} - \Delta^2_{23}\,\Delta^1_{12}}$$

$$B''_{ij} = B''_i - B''_j\,, \qquad B''_i = \frac{1}{\kappa_i}\left(b_i^{\text{SM}} + b_i^3 r_3\right)\,, \qquad \Delta^a_{ij} = \frac{b^a_i}{\kappa_i} - \frac{b^a_j}{\kappa_j}$$

Gauge coupling unification at string scale (ii)

The unification scale is given by

$$\left(\frac{\Lambda_S}{\Lambda}\right)^2 = -f\, W_k\left(-\frac{\Lambda_S^2}{f\, M_Z^2}\, e^{-\frac{g}{4\pi f}}\right)$$



Λ is uniquely determined for each set!

$h(x) = xe^x \Rightarrow x = W_k\,(h(x))$
W_k: Lambert function

[R.M.Corless et al, 1996]

where

$$f = \frac{1}{16\pi^2}\left(\beta_2\frac{b_i^1}{\kappa_i} - \beta_1\frac{b_i^2}{\kappa_i} + B_i''\right) \qquad g = \alpha_i^{-1}(M_Z) + \frac{1}{2\pi}\left(\widetilde{\beta}_2\frac{b_i^1}{\kappa_i} - \widetilde{\beta}_1\frac{b_i^2}{\kappa_i}\right)$$

▷ $k = 0$ for $f < 0$ ▷ $k = -1$ for $f > 0$

GCU at string scale: results (i)

Set	z	$\mathbf{d}$	Λ [10^{17} GeV]	M_1 [GeV]	M_2 [GeV]	M_3 [GeV]
				$(\mathbf{d},\mathbf{1})_{5/2}$	$(\mathbf{d},\mathbf{2})_{-2}$	$(\bar{\mathbf{d}},\mathbf{3})_{1/2}$
P1	3	8	2.7	$[2.7, 7.3] \times 10^{16}$	$[1.2, 2.7] \times 10^{17}$	$[2.2, 2.7] \times 10^{17}$
				$(\mathbf{d},\mathbf{1})_{7/6}$	$(\mathbf{d},\mathbf{3})_{-5/6}$	$(\bar{\mathbf{d}},\mathbf{4})_{1/3}$
P2	1	3	2.7	$[2.1, 26] \times 10^{4}$	$[4.4, 26] \times 10^{16}$	$[1.3, 2.7] \times 10^{17}$
	1	6	3.0	$[3.7, 45] \times 10^{10}$	$[6.7, 40] \times 10^{14}$	$[1.4, 3.0] \times 10^{17}$
	1	8	2.9	$[9.4, 110] \times 10^{11}$	$[8.7, 52] \times 10^{15}$	$[1.4, 2.9] \times 10^{17}$
				$(\mathbf{d},\mathbf{1})_{3/2}$	$(\mathbf{d},\mathbf{4})_{-1}$	$(\bar{\mathbf{d}},\mathbf{5})_{1/2}$
P3	1	3	2.7	$[8.6, 12] \times 10^{9}$	$[2.2, 2.7] \times 10^{17}$	$[2.5, 2.7] \times 10^{17}$
	1	6	2.8	$[1.3, 4.4] \times 10^{13}$	$[1.2, 2.8] \times 10^{17}$	$[1.9, 2.8] \times 10^{17}$
	1	8	2.8	$[1.9, 4.1] \times 10^{14}$	$[1.7, 2.8] \times 10^{17}$	$[2.2, 2.8] \times 10^{17}$
	1	10	3.0	$[1.1, 9.2] \times 10^{14}$	$[4.7, 20] \times 10^{16}$	$[1.5, 3.0] \times 10^{17}$
				$(\mathbf{d},\mathbf{2})_{4/3}$	$(\mathbf{d},\mathbf{3})_{-7/6}$	$(\bar{\mathbf{d}},\mathbf{5})_{1/6}$
P4	1	6	2.8	$[3.5, 13] \times 10^{14}$	$[9.1, 28] \times 10^{16}$	$[2.4, 2.8] \times 10^{17}$
	1	8	2.8	$[2.2, 3.6] \times 10^{15}$	$[1.8, 2.8] \times 10^{17}$	$[2.6, 2.8] \times 10^{17}$
	1	10	3.2	$[3.1, 320] \times 10^{15}$	$[8.3, 490] \times 10^{14}$	$[1.6, 2.8] \times 10^{17}$

GCU at string scale: results (ii)

Set	$\Lambda\,[10^{17}$ GeV]	Intermediate mass scales [GeV] rep	min	max	rep	min	max
S10	[3.5, 5.3]	$(\bar{\mathbf{3}},\mathbf{1})_{1/3}$	M_Z	5.0×10^{17}	$(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$	M_Z	4.6×10^{17}
		$(\mathbf{8},\mathbf{1})_{-1}$	2.9×10^{7}	4.5×10^{17}	$(\mathbf{3},\mathbf{3})_{-1/3}$	3.5×10^{3}	2.4×10^{12}
		$(\bar{\mathbf{3}},\mathbf{1})_{4/3}$	9.8×10^{7}	5.2×10^{17}	$(\mathbf{8},\mathbf{2})_{1/2}$	M_Z	1.1×10^{8}
S11	[3.6, 5.3]	$(\mathbf{3},\mathbf{1})_{2/3}$	M_Z	4.3×10^{17}	$(\bar{\mathbf{3}},\mathbf{2})_{-1/6}$	M_Z	8.8×10^{16}
		$(\mathbf{3},\mathbf{3})_{2/3}$	9.6×10^{7}	3.0×10^{14}	$(\mathbf{8},\mathbf{1})_{1}$	1.3×10^{11}	4.4×10^{17}
		$(\bar{\mathbf{3}},\mathbf{2})_{-7/6}$	5.7×10^{12}	5.0×10^{17}	$(\mathbf{8},\mathbf{2})_{-1/2}$	M_Z	1.1×10^{8}
S13	[3.3, 5.3]	$(\bar{\mathbf{6}},\mathbf{1})_{2/3}$	7.9×10^{3}	5.2×10^{17}	$(\mathbf{8},\mathbf{1})_{1}$	7.7×10^{9}	5.2×10^{17}
		$(\bar{\mathbf{6}},\mathbf{1})_{-1/3}$	3.1×10^{6}	5.2×10^{17}	$(\mathbf{8},\mathbf{2})_{-1/2}$	8.3×10^{12}	5.2×10^{17}
		$(\mathbf{6},\mathbf{3})_{1/3}$	4.8×10^{5}	1.3×10^{14}	$(\bar{\mathbf{6}},\mathbf{1})_{-4/3}$	3.3×10^{6}	5.0×10^{17}
S14	[3.2, 5.3]	$(\bar{\mathbf{3}},\mathbf{1})_{1/3}$	M_Z	5.0×10^{17}	$(\mathbf{3},\mathbf{3})_{2/3}$	7.6×10^{10}	5.2×10^{17}
		$(\mathbf{6},\mathbf{2})_{-1/6}$	M_Z	3.3×10^{15}	$(\bar{\mathbf{3}},\mathbf{2})_{-7/6}$	5.2×10^{9}	5.1×10^{17}
		$(\bar{\mathbf{6}},\mathbf{3})_{-1/3}$	2.3×10^{7}	4.3×10^{17}	$(\mathbf{6},\mathbf{1})_{4/3}$	5.8×10^{5}	5.2×10^{17}
S15	[3.2, 3.3]	$(\bar{\mathbf{3}},\mathbf{1})_{1/3}$	1.3×10^{7}	3.3×10^{17}	$(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$	1.7×10^{11}	3.2×10^{17}
		$(\mathbf{3},\mathbf{2})_{1/6}$	M_Z	8.4×10^{5}	$(\bar{\mathbf{6}},\mathbf{1})_{2/3}$	1.0×10^{14}	3.2×10^{17}
		$(\mathbf{6},\mathbf{2})_{-1/6}$	1.8×10^{11}	5.2×10^{16}	$(\bar{\mathbf{6}},\mathbf{1})_{-1/3}$	7.8×10^{12}	3.2×10^{17}
S16	[3.2, 5.3]	$(\bar{\mathbf{3}},\mathbf{2})_{-1/6}$	M_Z	5.2×10^{17}	$(\bar{\mathbf{6}},\mathbf{2})_{1/6}$	M_Z	3.6×10^{12}
		$(\mathbf{3},\mathbf{3})_{-1/3}$	1.7×10^{11}	5.3×10^{17}	$(\bar{\mathbf{3}},\mathbf{1})_{4/3}$	8.5×10^{4}	5.2×10^{17}
		$(\mathbf{6},\mathbf{3})_{1/3}$	1.8×10^{8}	5.2×10^{17}	$(\bar{\mathbf{6}},\mathbf{1})_{-4/3}$	1.1×10^{5}	5.2×10^{17}

Conclusions

- We searched for minimal chiral sets of fermions beyond the SM that are anomaly-free and, simultaneously, vector-like particles with respect to $SU(3)_c$ and $U(1)_{em}$
- It was studied whether the addition of such particles to the SM particle content allows for the unification of gauge couplings at a high energy scale
- We looked for minimal anomaly-free chiral fermion sets that belong to SU(5) representations with dim ≤ 50
- It was showed that some of the sets lead to GCU at a scale above 5.0×10^{15} GeV

Thank you!