DSE-inspired model for the pion GPD

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In collaboration with

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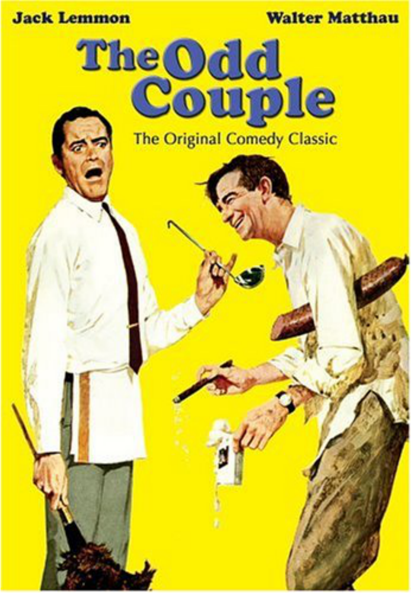
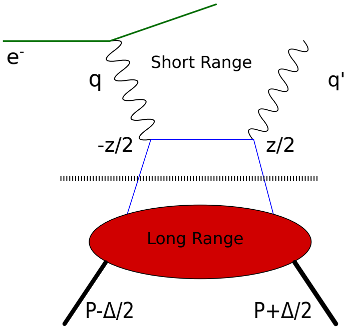
Describing a strongly coupled couple

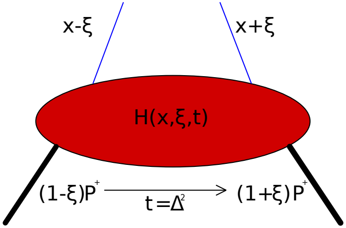


GPDs in a nutshell:

Deep-Virtual Compton Scattering (DVCS)



- Short range → perturbation theory.
- Long range → nonperturbative objects: GPDs,
- which encodes the hadrons 3D partonic and spin structure.
- *Universality*.



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- which encodes the hadrons 3D partonic and spin structure.
- *Universality*.
- H stands for the GPD,
- depending on the 3 variables : x, ξ, t.

Current GPDs models

- The most popular approach to model GPDs invokes the Double Distribution F et G

$$H(x,\xi,t) = \int_{|\alpha|+|\beta|\leq 1} \mathrm{d}\alpha\ \mathrm{d}\beta (F(\beta,\alpha,t) + \xi G(\beta,\alpha,t))\delta(x-\beta-\xi\alpha)$$

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How can we compute GPDs on the ground of QCD?

DSE and BSE approach:

quark propagator :

$$\begin{aligned} S(p;\mu) &= -i\gamma\cdot p\sigma_V(p;\mu)+\sigma_S(p;\mu), \\ &= \frac{1}{i\gamma\cdot pA(p;\mu)+B(p;\mu)}. \end{aligned}$$

$$S^{-1}(p)=Z_2(i\gamma\cdot p+m)+Z_1\int \mathrm{d}^4q\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\frac{\lambda^a}{2}\Gamma_\nu(q,p),$$

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Bethe-Salpeter amplitude:

$$\Gamma_\pi(q,P;\mu) = \gamma_5\left[iE_\pi(q,P;\mu) + \gamma\cdot PF_\pi(q,P;\mu) + q\cdot P\gamma\cdot qG_\pi(q,P;\mu) + \sigma_{\mu\nu}q^\mu P^\nu H_\pi(q,P;\mu)\right]$$

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Nakanishi representation in terms of complex conjugate poles
Lei Chang *et al.*, Phys.Rev.Lett. 110 (2013) 13, 132001

Formal definition:

$$H(x,\xi,t) = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \left\langle P+\frac{\Delta}{2}\right| \bar{q}\left(-\frac{z}{2}\right)\gamma^+\left[-\frac{z}{2};\frac{z}{2}\right] q\left(\frac{z}{2}\right)\left|P-\frac{\Delta}{2}\right\rangle_{z^+=0,z_\perp=0},$$

(*X. Ji, 1997; D. Müller, 1994; A. Radyushkin, 1997;*)

Scalar meson GPDs

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the Mellin moments of which can be formally expressed as (twist-2 operators)

$$\begin{aligned}\mathcal{M}_m(\xi,t) &= \int_{-1}^{1}\mathrm{d}x\; x^m H(x,\xi,t)\\ &= \frac{1}{2(P\cdot n)^{m+1}}\left\langle\pi,P+\frac{\Delta}{2}\right|\bar{\psi}(0)\gamma\cdot n(i\overleftrightarrow{D}\cdot n)^m\psi(0)\left|\pi,P-\frac{\Delta}{2}\right\rangle.\end{aligned}$$

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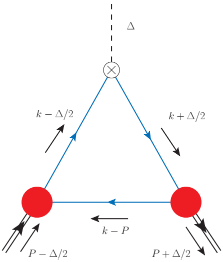
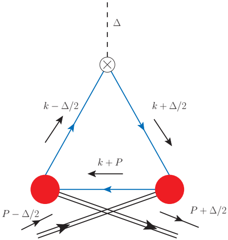
GPD can be reconstructed starting from its Mellin moments!!!

$$\langle x^m \rangle = \mathcal{M}_m(\xi, t) = \frac{1}{2(P\cdot n)^{m+1}} \left\langle \pi, P + \frac{\Delta}{2} \left| \bar{\psi}(0)\gamma\cdot n(i\overleftrightarrow{D}\cdot n)^m \psi(0) \right| \pi, P - \frac{\Delta}{2} \right\rangle .$$

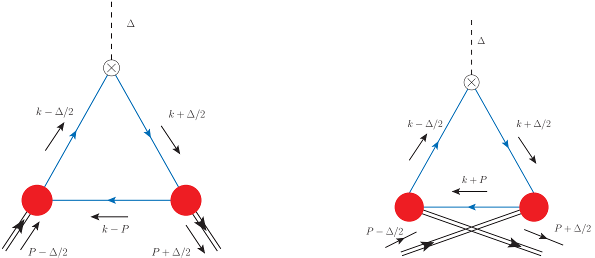
Direct and crossed triangle diagrams :





Simple analytical model for the Pion:

Direct and crossed triangle diagrams with fully dressed vertices and propagators:



$$
\begin{aligned}
2(P\cdot n)^{m+1}\langle x^m\rangle^u \quad = \quad & \mathrm{tr}_{CFD}\int \frac{\mathrm{d}^4k}{(2\pi)^4}(k\cdot n)^m\,\tau_+ i\Gamma_\pi\left(\eta(k-P)+(1-\eta)\left(k-\frac{\Delta}{2}\right),P-\frac{\Delta}{2}\right)\\
& S(k-\frac{\Delta}{2})\; i\Gamma^{\mathrm{e.m.}}\cdot n\; S(k+\frac{\Delta}{2})\\
& \tau_- i\bar{\Gamma}_\pi\left((1-\eta)\left(k+\frac{\Delta}{2}\right)+\eta(k-P),P+\frac{\Delta}{2}\right)S(k-P),
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DSE and BS inspired simple ansätze:

$$S(p) = \left[-i\gamma\cdot p+M\right]\Delta_M(p^2),$$
$$\Delta_M(s) = \frac{1}{s+M^2},$$
$$\Gamma_\pi(k,p) = i\gamma_5\frac{M}{f_\pi}M^{2\nu}\int_{-1}^{+1}\mathrm{d}z\,\rho_\nu(z)\;\left[\Delta_M(k^2_{+z})\right]^\nu;$$
$$\rho_\nu(z) = R_\nu(1-z^2)^\nu,$$

(*L. Chang et al.*, PRL110(2013)132001)

with $k_{\pm z}=k\mp(1-z)P/2$,
standing for the momentum fraction that the quark carries out.

Simple analytical model for the Pion:

Case $\xi = 0$ (dressing improved [H.L.L. Roberts et al. PRC83(2011)065206])

$$
\begin{aligned}
2(P\cdot n)^{m+1}\left\langle x^m\right\rangle^u &= \mathrm{tr}_{CFD}\int \frac{\mathrm{d}^4k}{(2\pi)^4}(k\cdot n)^m\,\tau_+ i\Gamma_\pi\left(\eta(k-P)+(1-\eta)\left(k-\frac{\Delta}{2}\right),P-\frac{\Delta}{2}\right) \\
& S(k-\frac{\Delta}{2})\; i\; P_T(-t=\Delta_\perp^2)\;\gamma\cdot n\; S(k+\frac{\Delta}{2}) \\
& \tau_- i\bar{\Gamma}_\pi\left((1-\eta)\left(k+\frac{\Delta}{2}\right)+\eta(k-P),P+\frac{\Delta}{2}\right)S(k-P),
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Results for the Mellin moments

$$
\begin{aligned}
\langle x^m \rangle^{I=0,1} \;=\;& \lambda \int \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}u\,\mathrm{d}v\,\mathrm{d}w\,\mathrm{d}z\,\mathrm{d}z' \,\left(\frac{M^2}{M'^2}\right)^{2\nu} \\
& \delta(1-x-y-u-v-w)x^{\nu-1}y^{\nu-1}\rho(z)\rho(z') \\
& \Big[(g-2\xi\ f)^m(g+1-2\xi f) \pm (-g-2\xi\ f)^m(-g-1-2\xi f) \\
& +\frac{1}{2}((-2\xi f+g-1)(g-2\xi\ f)^m \pm (-2\xi f-g+1)(-g-2\xi\ f)^m) \\
& +\frac{m}{2}\left((g-2\xi f)^{m-1}((g-2\xi f)^2-\xi^2) \pm (-g-2\xi f)^{m-1}((-g-2\xi f)^2-\xi^2)\right) \\
& +\frac{\Gamma(2\nu+1)}{2M'^2\Gamma(2\nu)}(g-2\xi f)^m\Big((g-2\xi f)(tf^2+P^2(g^2-2g)+\frac{t}{4}+M^2) \\
& +tf^2+P^2g^2-\frac{t}{4}+tf\xi+M^2\Big) \\
& \pm\frac{\Gamma(2\nu+1)}{2M'^2\Gamma 2\nu}(-g-2\xi f)^m\Big((-g-2\xi f)(tf^2+P^2(g^2-2g)+\frac{t}{4}+M^2) \\
& -tf^2-P^2g^2+\frac{t}{4}+tf\xi-M^2\Big)\Big]. \qquad (68)
\end{aligned}
$$

Results for the Mellin moments

$$f(x,y,v,w,z,z') = \frac{1}{2}\left(-\frac{1+z'}{2}y + \frac{1+z}{2}x + v - w\right), \qquad (55)$$

$$g(x,y,u,z,z') = \left(\frac{1-z'}{2}\right)y + x\frac{1-z}{2} + u, \qquad (56)$$

$$M'(t,P^2,x,y,u,v,w,z,z')^2 = M^2 + \frac{t}{4}\left(-4f^2 + y\left(\frac{1+z'}{2}\right)^2 + x\left(\frac{1+z}{2}\right)^2 + v + w\right)$$
$$+P^2\left(-g^2 + \left(\frac{1-z'}{2}\right)^2 y + \left(\frac{1-z}{2}\right)^2 x + u\right). \qquad (57)$$

$$+tf^2 + P^2g^2 - \frac{t}{4} + tf\xi + M^2\Big)$$

$$\pm\frac{\Gamma(2\nu+1)}{2M'^2\Gamma 2\nu}(-g-2\xi f)^m\Big((-g-2\xi f)(tf^2 + P^2(g^2-2g) + \frac{t}{4} + M^2)$$

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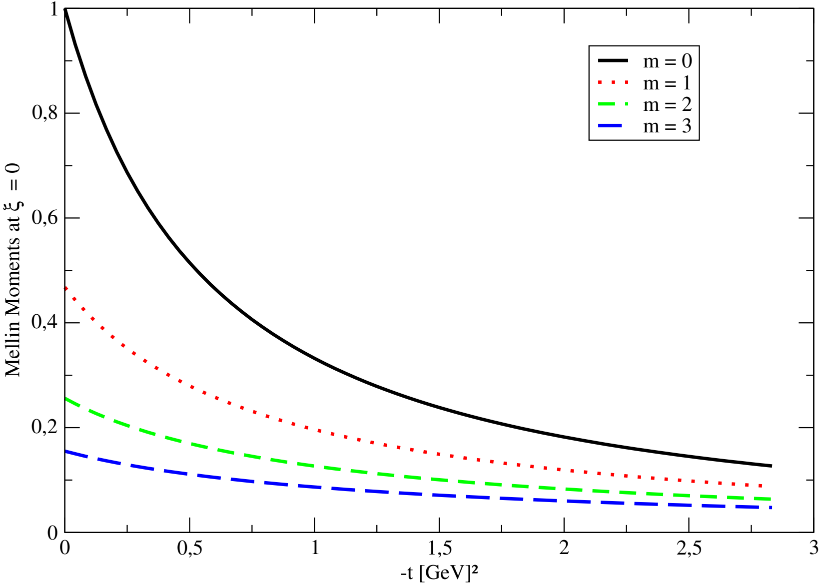
&\quad +P^2\left(-g^2+\left(\frac{1-z'}{2}\right)^2y+\left(\frac{1-z}{2}\right)^2x+u\right). && (57)
\end{aligned}
$$

$$
\begin{aligned}
H^{I=0}(x,\xi,t) &= H^u_{\pi^\pm}(x,\xi,t)+H^d_{\pi^\pm}(x,\xi,t)\\
&= H^u_{\pi^0}(x,\xi,t)+H^d_{\pi^0}(x,\xi,t),\\
H^{I=1}(x,\xi,t) &= H^u_{\pi^+}(x,\xi,t)-H^d_{\pi^+}(x,\xi,t)\\
&= -(H^u_{\pi^-}(x,\xi,t)-H^d_{\pi^-}(x,\xi,t)),\\
0 &= H^u_{\pi^0}(x,\xi,t)-H^d_{\pi^0}(x,\xi,t).
\end{aligned}
$$

Results for the Mellin moments



Polynomiality:

$$\left\langle \pi, P+\frac{\Delta}{2} \left| \bar{\psi}(0)\gamma\cdot n(i\overleftrightarrow{D}\cdot n)^m\psi(0) \right| \pi, P-\frac{\Delta}{2} \right\rangle$$

$$= n_\mu n_{\mu_1}\ldots n_{\mu_m} P^{\{\mu} \sum_{j=0}^{m} \binom{m}{j} F_{m,j}(t) P^{\mu_1}\ldots P^{\mu_j} \left(-\frac{\Delta}{2}\right)^{\mu_{j+1}} \ldots \left(-\frac{\Delta}{2}\right)^{\mu_m\}}$$

$$-n_\mu n_{\mu_1}\ldots n_{\mu_m} \frac{\Delta}{2}^{\{\mu} \sum_{j=0}^{m} \binom{m}{j} G_{m,j}(t) P^{\mu_1}\ldots P^{\mu_j} \left(-\frac{\Delta}{2}\right)^{\mu_{j+1}} \ldots \left(-\frac{\Delta}{2}\right)^{\mu_m\}}$$

$\xi = -\frac{\Delta\cdot n}{2P\cdot n} \Rightarrow \mathcal{M}_m(\xi,t)$ is a polynomial in ξ of order $m+1$.

Properties of Mellin moments

Double distributions:

$$
\begin{aligned}
F_{m,j}(t) &= \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \; \beta^{m-j} \alpha^{j} \; F(\beta,\alpha,t) \\
G_{m,j}(t) &= \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \; \beta^{m-j} \alpha^{j} \; G(\beta,\alpha,t)
\end{aligned}
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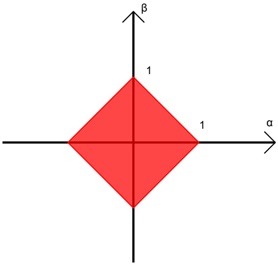

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\mathcal{M}_m(\xi,t) \;=\;& n_\mu n_{\mu_1} \ldots n_{\mu_m} \sum_{j=0}^{m} \binom{m}{j} \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \; \beta^{m-j} \alpha^{j} \\
& F(\beta,\alpha,t) P^{\{\mu} P^{\mu_1} \ldots P^{\mu_j} \left(-\frac{\Delta}{2}\right)^{\mu_{j+1}} \ldots \left(-\frac{\Delta}{2}\right)^{\mu_m\}} \\
& -G(\beta,\alpha,t) \frac{\Delta}{2}^{\{\mu} P^{\mu_1} \ldots P^{\mu_j} \left(-\frac{\Delta}{2}\right)^{\mu_{j+1}} \ldots \left(-\frac{\Delta}{2}\right)^{\mu_m\}}
\end{aligned}
$$

$$\mathcal{M}_m(\xi,t) = n_\mu n_{\mu_1}...n_{\mu_m}\sum_{j=0}^{m}\binom{m}{j}\int_{-1}^{1}\mathrm{d}\beta\int_{-1+|\beta|}^{1-|\beta|}\mathrm{d}\alpha\;\beta^{m-j}\alpha^j$$
$$F(\beta,\alpha,t)P^{\{\mu}P^{\mu_1}...P^{\mu_j}\left(-\frac{\Delta}{2}\right)^{\mu_{j+1}}...\left(-\frac{\Delta}{2}\right)^{\mu_m\}}$$
$$-G(\beta,\alpha,t)\frac{\Delta}{2}^{\{\mu}P^{\mu_1}...P^{\mu_j}\left(-\frac{\Delta}{2}\right)^{\mu_{j+1}}...\left(-\frac{\Delta}{2}\right)^{\mu_m\}}$$

Time Rerversal Invariance

$$\Delta \to -\Delta$$

$\mathcal{M}_m(\xi,t)$ is an *even* polynomial in ξ of order $m+1$.

$F(\beta,\alpha)$ is **even** in α.
$G(\beta,\alpha)$ is **odd** in α.

From Mellin moments to Double Distributions (DD)

DD are directed linked to H:

$$H(x,\xi,t) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \; \left(F(\beta,\alpha,t) + \xi G(\beta,\alpha,t)\right) \delta(x-\beta-\alpha\xi)$$

Double Distributions are the Radon transform of the GPD

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Double Distributions are the Radon transform of the GPD

PDF case:

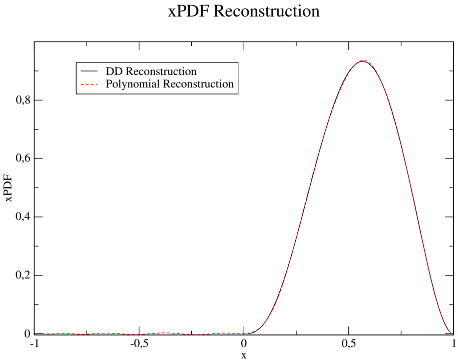
$$q(x) = H(x,0,0) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \; F(\beta,\alpha,t)\delta(x-\beta)$$

Form Factor case:

$$\mathcal{F}(t) = \int_{-1}^{1} \mathrm{d}x \; H(x,\xi,t) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \; F(\beta,\alpha,t)$$

Comparison with polynomial reconstruction

$$H(x,0,0) = \int_0^1 \mathrm{d}\beta \int_{-1+\beta}^{1-\beta} \mathrm{d}\alpha \delta(x-\beta) F(\beta,\alpha,t)$$



- Forward case : $\xi = 0$ and $t = 0$.
- Very good agreement.
- The polynomial reconstruction describes well the support properties of valence GPD: $x \in [-\xi, 1]$

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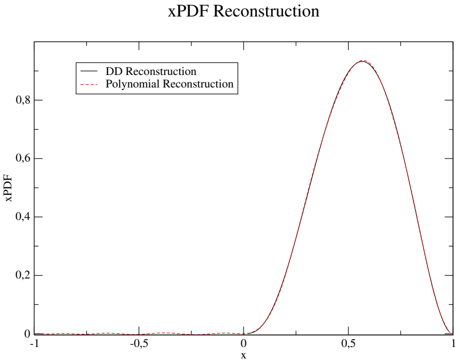

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If the polynomial reconstruction works, why should we care about DDs?

Advantages of DDs

- Support is stricly respected.

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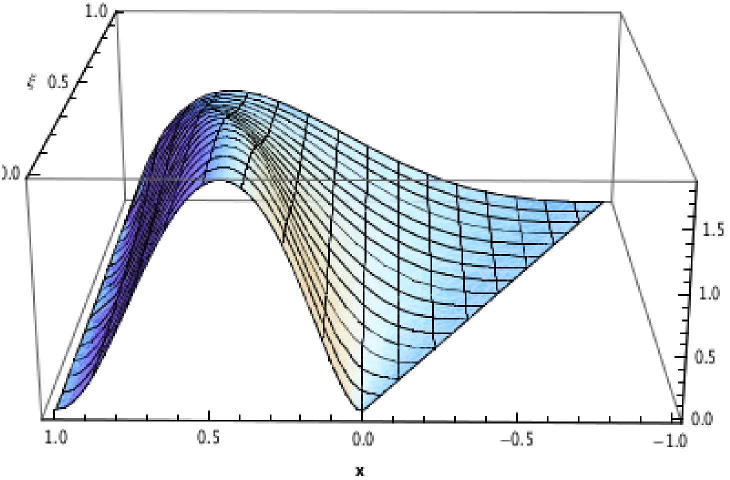
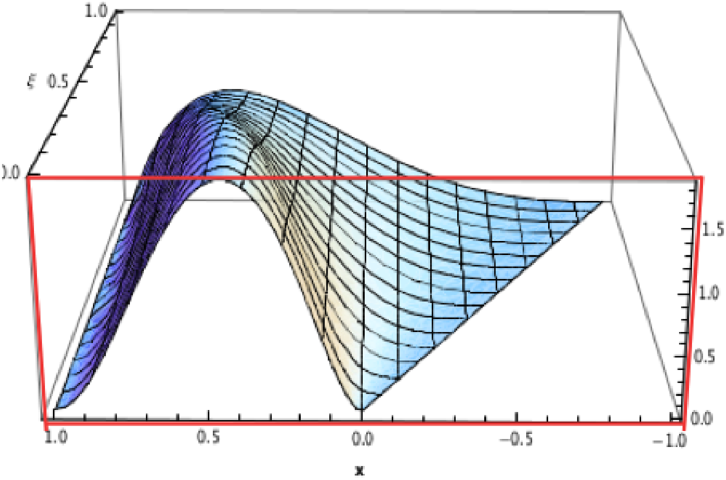
- Reconstruction is exact even at $\xi \neq 0$ (no numerical noise).
- We can get analytic expressions. For the PDF($\nu = 1$):

$$
\begin{aligned}
q(x) &= \frac{72}{25}\left(x^3(x(-2(x-4)x-15)+30)\log(x)\right. \\
&\quad + \left(2x^2+3\right)(x-1)^4\log(1-x) \\
&\quad \left. + x(x(x(2x-5)-15)-3)(x-1)\right)
\end{aligned}
$$

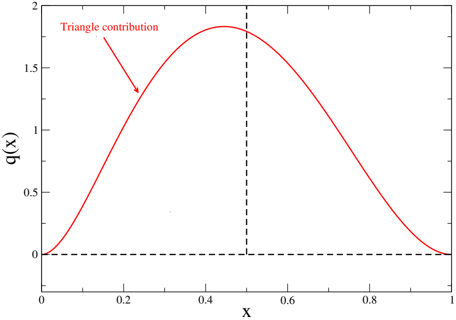
Analytical "resummation" ($t = 0$)



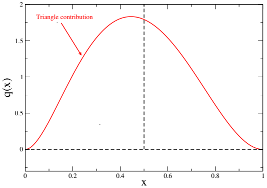
Analytical "resummation" ($t = 0, \xi = 0$)



Limits of the triangle diagrams



- The PDF appears not to be symmetric around $x = \frac{1}{2}$.



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Heuristic example: Light-cone wave function for a bound-state of two scalar particles [Bukardt, Int.J.Mod.Phys.A18(2003)173]

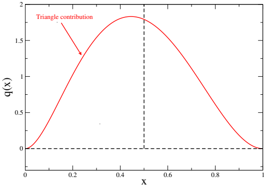
$$\psi(x, k_\perp^2) = \sqrt{\frac{15}{2\pi\,\sigma^2}}\,\frac{\sqrt{x(1-x)}}{1 + k_\perp^2/(4\,\sigma^2 x(1-x))}\theta(x)\theta(1-x)\,.$$

Non-skewed GPD overlap representation:

$$\begin{aligned} H_\sigma(x, 0, -\Delta_\perp^2) &= \int d^2k_\perp\,\psi(x, k_\perp + (1-x)\Delta_\perp)\,\psi(x, k_\perp) \\ &= 30(1-x)^2 x^2 \theta(x)\theta(1-x) C\left(\frac{\Delta_\perp^2}{4x^2\sigma^2}(1-x)\right)\;; \end{aligned}$$

$C(z)$ decreasing monotonically away from its maximum value $C(0) = 1$, and encoding the $(x \leftrightarrow 1-x)$-asymmetry.

Limits of the triangle diagrams



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Non-skewed GPD overlap representation:

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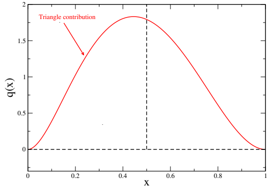
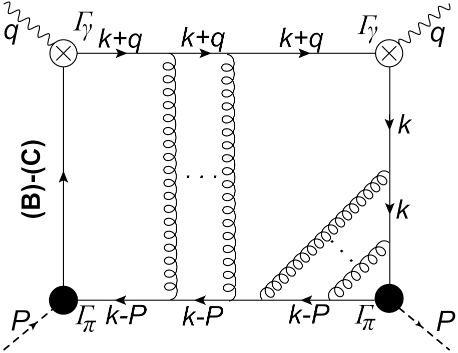
The PDF-like function is symmetric under $x \leftrightarrow 1 - x$.

Limits of the triangle diagrams





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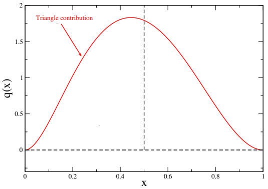
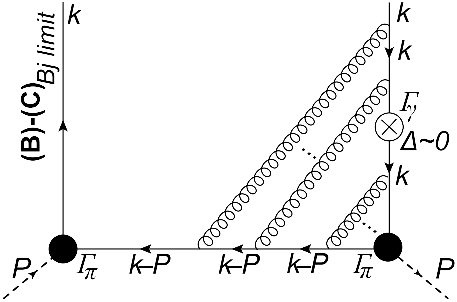
Limits of the triangle diagrams





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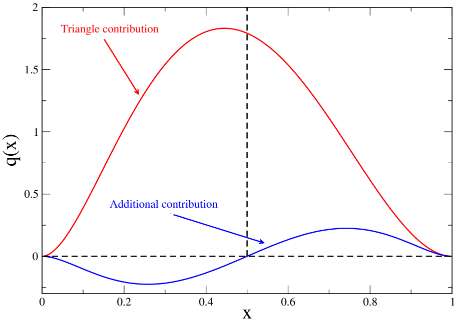

$$q^{\pi}_{BC}(x) = n_q \Big[x^3(2x([x-3]x+5)-15)\ln(x) - (2x^3+4x+9) \times (x-1)^3 \ln(1-x) - x(2x-1)([x-1]x-9)(x-1) \Big] . \quad (13)$$

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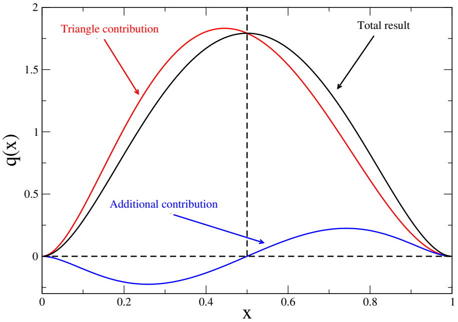

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- Adding this contribution allows us to recover a symmetric PDF [L. Chang *et al.*, Phys.Lett.B737(2014)2329].

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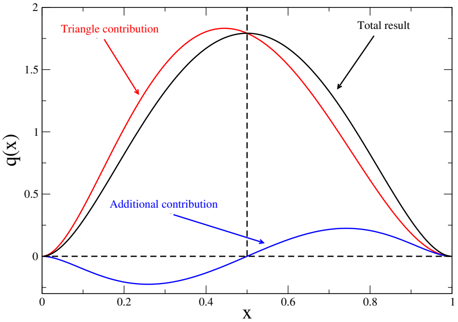

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Non forward case?

Pion form factor

QCD sum rule:

$$F_\pi^q(t) = \mathcal{M}_0(t) = \int_{-1}^{1} \mathrm{d}x \; H^q(x,\xi,t)$$

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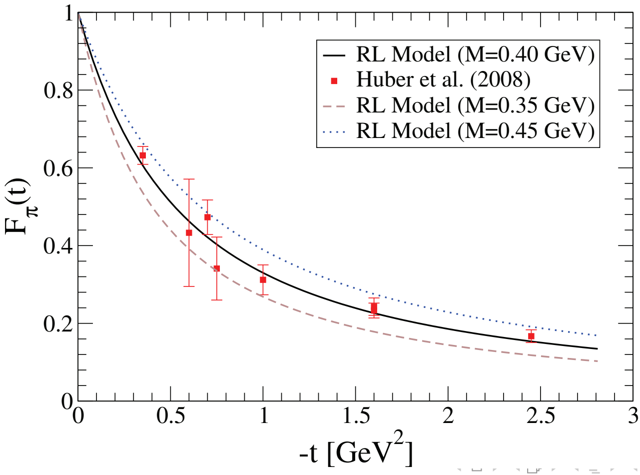
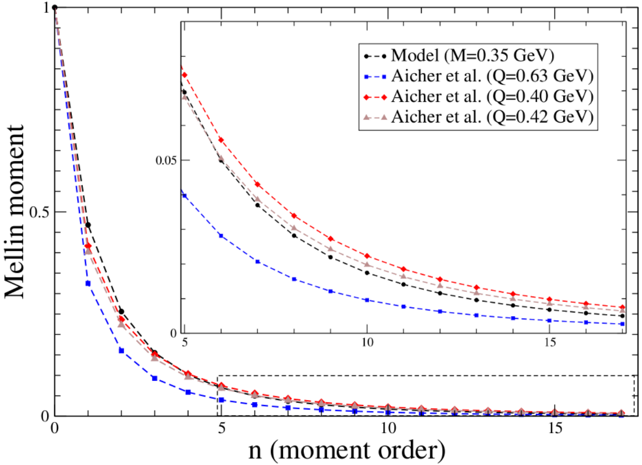


PDF Mellin moments

GPD forward limit:

$$PDF(x) = H^q(x,0,0)$$



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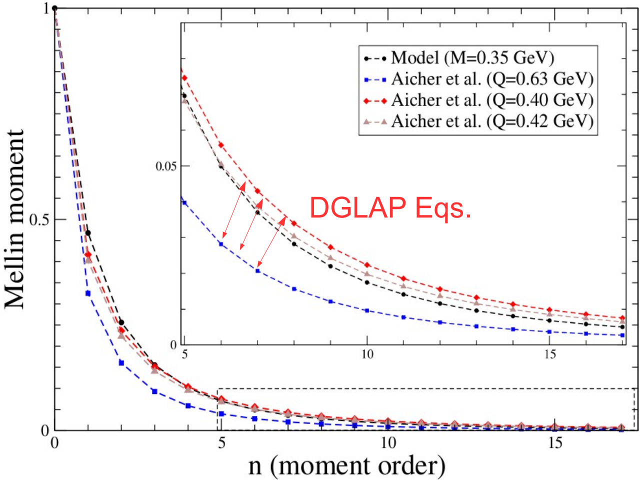
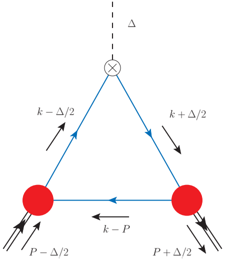


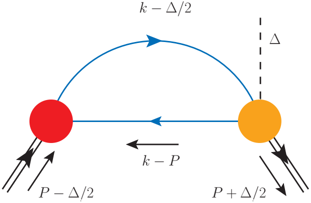
The standard triangle diagram does not fully keep all the contributions from the gluons which bind the dressed-quark to the pion



$$2(P\cdot n)^{m+1}\langle x^m\rangle^u = \mathrm{tr}_{CFD}\int\frac{d^4k}{(2\pi)^4}(k\cdot n)^m\,\tau_+ i\Gamma_\pi\left(\eta(k-P)+(1-\eta)\left(k-\frac{\Delta}{2}\right),P-\frac{\Delta}{2}\right)$$
$$S(k-\frac{\Delta}{2})\; i\gamma\cdot n\; S(k+\frac{\Delta}{2})$$
$$\tau_- i\bar{\Gamma}_\pi\left((1-\eta)\left(k+\frac{\Delta}{2}\right)+\eta(k-P),P+\frac{\Delta}{2}\right)S(k-P),$$

Non-forward pion GPD

The standard triangle diagram does not fully keep all the contributions from the gluons which bind the dressed-quark to the pion and a new contribution is needed.



$$2(P \cdot n)^{m+1} \langle x^m \rangle^u = \mathrm{tr}_{CFD} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} (k \cdot n)^m \, \tau_+ i\Gamma_\pi \left(\eta(k-P) + (1-\eta)\left(k - \frac{\Delta}{2}\right), P - \frac{\Delta}{2} \right)$$
$$S(k - \frac{\Delta}{2}) \tau_- \, \frac{\partial}{\partial k} \bar{\Gamma}_\pi \left((1-\eta)\left(k + \frac{\Delta}{2}\right) + \eta(k-P), P + \frac{\Delta}{2} \right) S(k-P)$$

Non-forward pion GPD

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$$F^{BC}(\beta,\alpha,t),\,G^{BC}(\beta,\alpha,t)$$

$$H^{BC}(x,\xi,t)=\int_{-1}^{1}d\beta\int_{-1+|\beta|}^{1-|\beta|}d\alpha\,\left(F^{BC}(\beta,\alpha,t)+\xi\,G^{BC}(\beta,\alpha,t)\right)\delta(x-\beta-\alpha\xi)$$

$$H^{BC}(x,0,0)=\int_{-1+|x|}^{1-|x|}d\alpha\;F^{BC}(x,\alpha,0)$$

Non-forward pion GPD

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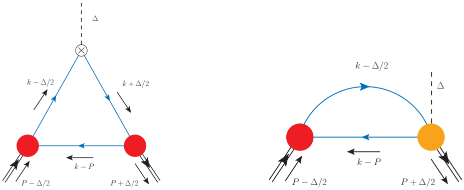
$$H^{BC}(x,0,0) = \int_{-1+|x|}^{1-|x|}\mathrm{d}\alpha\ F^{BC}(x,\alpha,0) \equiv q^{\pi}_{BC}(x)$$

The full model:



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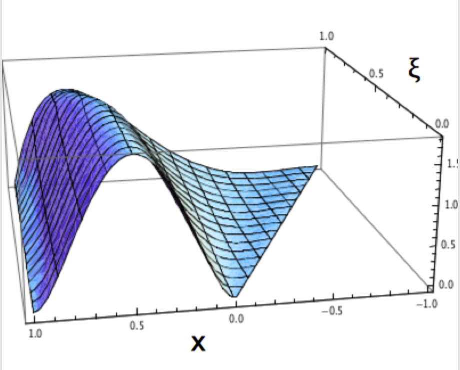
Non-forward pion GPD

$$H(x,\xi,0) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \;\left(F(\beta,\alpha,0) + \xi G(\beta,\alpha,0)\right)\delta(x-\beta-\alpha\xi)$$

GPD 3D-plot (t=0)



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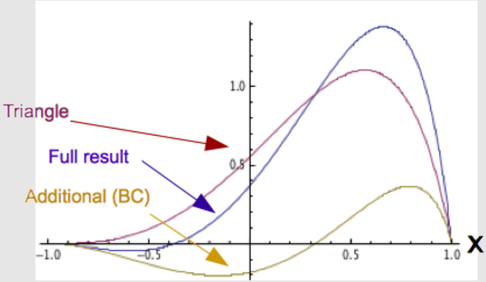
GPD (t=0, $\xi = 1$)



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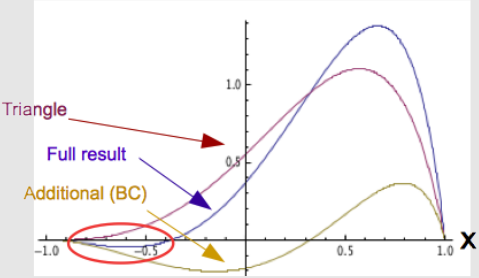


Problems at large ξ!!!

AVWT Identity $\Rightarrow$ Soft pion theorem [C. Mezrag *et al.*, arXiv:1411:6634]

Non-forward (non-skewed) pion GPD ($\xi = 0,\ t \neq 0$)

The pion GPD

$$H^q(x,0,t) = \int_{-1+|x|}^{1-|x|} \mathrm{d}\alpha \ \left(F^0(x,\alpha,t) + F^{BC}(x,\alpha,t) \right)$$

Non-forward (non-skewed) pion GPD ($\xi = 0,\ t \neq 0$)

The pion GPD

$$H^q(x,0,t) = \int_{-1+|x|}^{1-|x|} \mathrm{d}\alpha \left(F^0(x,\alpha,t) + F^{BC}(x,\alpha,t) \right)$$

$$\begin{aligned} H(x,0,t) &= H(x,0,0)\mathcal{N}(t) C_\pi(x,t) F_\pi(t)\,, \\ 1 &= \mathcal{N}(t) \int_{-1}^{1} dx\; H(x,0,0) C_\pi(x,t)\,. \end{aligned}$$

$$F(\beta,\alpha,t) = \frac{1}{\left(1+\frac{t}{4M^2}(1-\beta+\alpha)(1-\beta+\alpha)\right)^2} \times \left(F_S(\beta,\alpha) + t\,[\cdots]\right)$$

The pion GPD

$$H^q(x,0,t) = \int_{-1+|x|}^{1-|x|} \mathrm{d}\alpha \left(F^0(x,\alpha,t) + F^{BC}(x,\alpha,t) \right)$$

$$H(x,0,t) = H(x,0,0)\mathcal{N}(t)C_\pi(x,t)F_\pi(t)\,,$$

$$1 = \mathcal{N}(t)\int_{-1}^{1} dx\ H(x,0,0)\ C_\pi(x,t)\,.$$

$$F(\beta,\alpha,t) = \frac{1}{\left(1+\frac{t}{4M^2}(1-\beta)(1-\beta)\right)^2} \times F_S(\beta,\alpha)$$

Simplified analytical model:

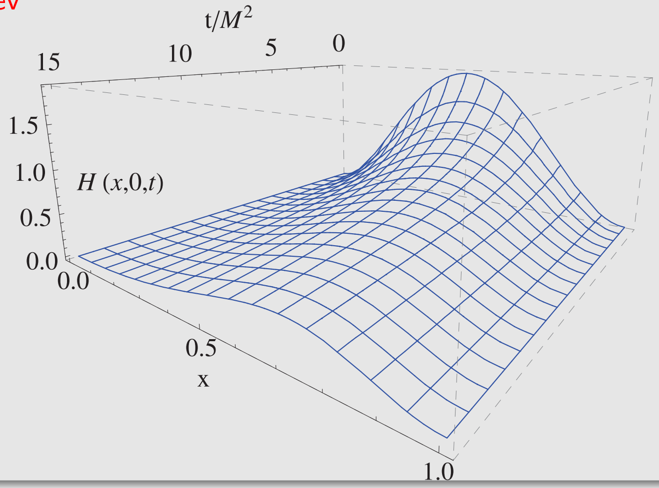
$$C(x,t) = \frac{1}{\left(1+\frac{t}{4M^2}(1-x)^2\right)^2}$$

Valuable to sketch the pion's valence-quark GPD [C. Mezrag et al., arXiv:1411.6634]

Non-forward (non-skewed) pion GPD ($\xi = 0$, $t \neq 0$)

3D plot of GPD at $\zeta = 0.4$ GeV

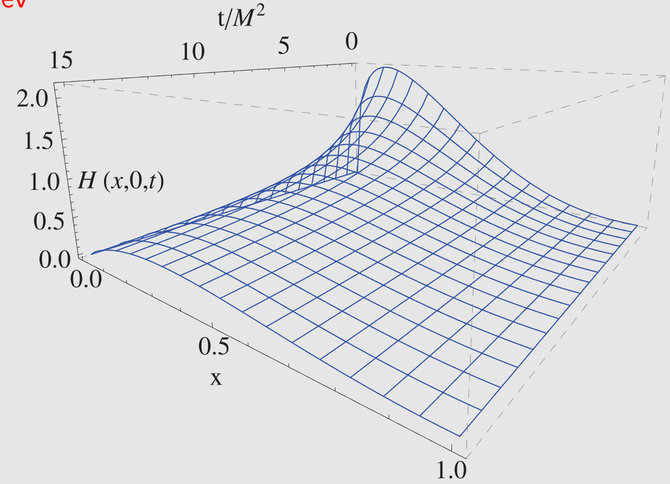
$M = 0.4$ GeV



Non-forward (non-skewed) pion GPD ($\xi = 0,\ t \neq 0$)

3D plot of GPD at $\zeta = 2$ GeV (DGLAP running; $x > \xi$)

$M = 0.4$ GeV



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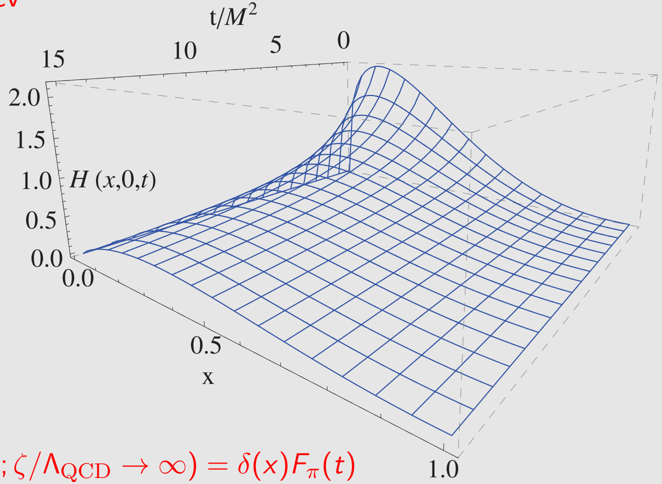


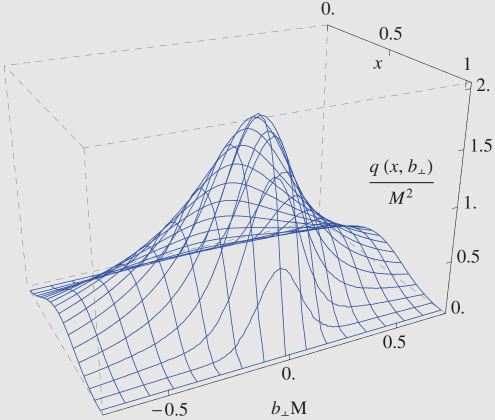
$H(x, 0, t; \zeta/\Lambda_{\rm QCD} \to \infty) = \delta(x) F_\pi(t)$

Non-forward (non-skewed) pion GPD ($\xi = 0,\ t \neq 0$)

$$q(x, |\vec{b}|) = \int \frac{d|\vec{\Delta}_\perp|}{2\pi} |\vec{\Delta}_\perp| J_0(|\vec{b}_\perp||\vec{\Delta}_\perp|) H(x, 0, -\Delta_\perp^2)$$

Impact parameter space GPD at $\zeta = 0.4$ GeV

$M = 0.4$ GeV



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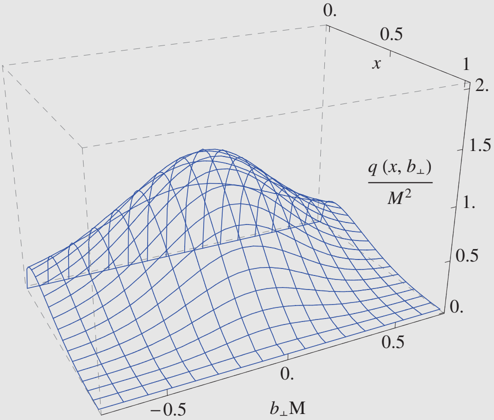

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Impact parameter space GPD at $\zeta = 2$ GeV

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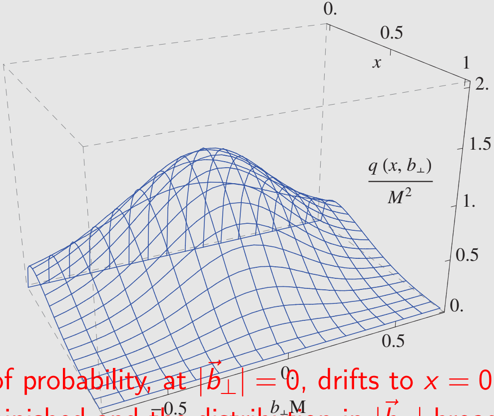


The peak of probability, at $|\vec{b}_\perp| = 0$, drifts to $x = 0$, its height is diminished and the distribution in $|\vec{b}_\perp|$ broadens.

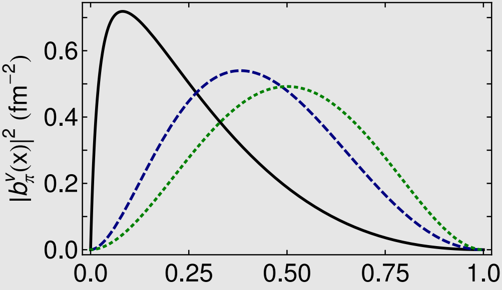
Non-forward (non-skewed) pion GPD ($\xi = 0,\ t \neq 0$)

$$q(x, |\vec{b}|) = \int \frac{d|\vec{\Delta}_\perp|}{2\pi} |\vec{\Delta}_\perp| J_0(|\vec{b}_\perp||\vec{\Delta}_\perp|) H(x, 0, -\Delta_\perp^2)$$

$$\langle |\vec{b}_\perp|^2 \rangle = \int_{-1}^{1} dx \, \langle |\vec{b}_\perp(x;\zeta)|^2 \rangle = \int_{-1}^{1} dx \int_0^\infty d|\vec{b}_\perp| |\vec{b}_\perp|^3 \int_0^\infty d\Delta \Delta J_0(\vec{b}_\perp |\Delta) F_\pi(\Delta^2)$$

Impact parameter space GPD

$\langle |\vec{b}_\perp|^2 \rangle = (0.52 \text{ fm})^2$



$\zeta = 2$ GeV; $\zeta = 0.4$ GeV; $\zeta = 0.4$GeV [c(x,t)=1].

Non-forward (non-skewed) pion GPD ($\xi = 0,\ t \neq 0$)

$$q(x, |\vec{b}|) = \int \frac{d|\vec{\Delta}_\perp|}{2\pi} |\vec{\Delta}_\perp| J_0(|\vec{b}_\perp||\vec{\Delta}_\perp|) H(x, 0, -\Delta_\perp^2)$$

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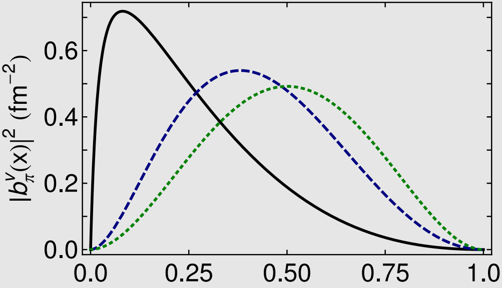
Impact parameter space GPD

$r_\pi = \sqrt{3/2\langle|\vec{b}_\perp|^2\rangle} = 0.674$ fm $\Longleftrightarrow$ $r_\pi = 0.672(8)$ fm [PRD86(2012)010001]



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Epilogue: conclusion and perspectives

- We described a calculation of the pion's valence dressed-quark GPD within the context of a RL-truncation of QCD Dyson-Schwinger equations and, in the appropriate limits, coupled the results with the pion's PDF and form factor.

Epilogue: conclusion and perspectives

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- Drawing analogy with the pion's valence dressed-quark PDF, we argued that the *impulse-approximation* invoked to derive the triangle-diagram GPD computation from the well-known *handbag diagram* contribution to DVCS gives incomplete results.

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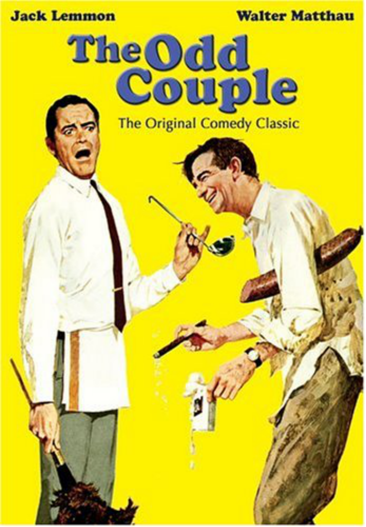
- In the future, more realistic forms for the dressed propagators and vertices and better extensions to the entire kinematic domain of ξ and t may be potentially helpful to relate the phenomenology of hadron GPDs to the properties of QCD.

Thank you


Jack Lemmon
Walter Matthau
The Odd Couple
The Original Comedy Classic