The YM Vacuum Wave-Functional 35 Years Later

Štefan Olejník
Institute of Physics, Slovak Academy of Sciences, Bratislava, Slovakia

Thanks First, and References

- ***To organizers of DISCRETE 2014,***

  *especially **Joannis Papavassiliou**, the convener of this session, for inviting me to this city, this place, and this workshop.*

- ***To Jeff Greensite***

  for collaboration on this and many other topics.

- ***To other people***

  who collaborated with us on a part of the material that will be presented in my talk.

➡ J. Greensite, ŠO, **Dimensional reduction and the Yang–Mills vacuum state in 2+1 dimensions**, Phys. Rev. D **77** (2008) 065003, arXiv:0707.2860.

➡ J. Greensite, ŠO, **Coulomb confinement from the Yang–Mills vacuum state in 2+1 dimensions**, Phys. Rev. D **81** (2010) 074504, arXiv:1002.1189.

➡ J. Greensite, H. Matevosyan, ŠO, M. Quandt, H. Reinhardt, A. P. Szczepaniak, **Testing proposals for the Yang–Mills vacuum wavefunctional by measurement of the vacuum**, Phys. Rev. D **83** (2011) 114509, arXiv:1102.3941.

➡ J. Greensite, ŠO, **Numerical study of the Yang-Mills vacuum wavefunctional in D=3+1 dimensions**, Phys. Rev. D **89** (2014) 034504, arXiv:1310.6706.

Outline of the Talk

Introduction

Formulation of the Problem

- *In the Schrödinger representation, one would like to find the vacuum wave functional (VWF):*

$$\Psi_0\left[u^i_A(x), d^i_A(x), s^i_A(x), c^i_A(x), b^i_A(x), t^i_A(x); A^a_\mu(x)\right]$$
$$A = 1,2,3,4; \quad i = 1,2,3; \quad a = 1,2,\ldots,8; \quad \mu = 0,1,2,3$$

- *A damn' difficult, hopeless task:*
  - *with **six** flavours of quarks with **three** colours, each represented by a Dirac spinor of **four** components, and with **eight four**-vector gluons: **104 fields at each point in space** (not taking gauge invariance into account).*
- *Some simplifications:*
  - ***omit** quarks;*
  - *use **two** instead of **three** colours;*
  - ***discretize** space;*
  - *(maybe) go to **lower** dimensions.*

$$\hat{\mathcal{H}}\Psi_0[A] = E_0\Psi_0[A] \quad \ldots \quad \textit{Schrödinger equation}$$
$$\hat{\mathcal{H}} = \int d^dx \left\{-\frac{1}{2}\frac{\delta^2}{\delta A^a_k(x)^2} + \frac{1}{4}F^a_{ij}(x)^2\right\}$$
$$\left(\delta^{ac}\partial_k + g\epsilon^{abc}A^b_k\right)\frac{\delta}{\delta A^c_k}\Psi[A] = 0 \quad \ldots \quad \textit{Gauß' law}$$

A Few Well-Known Facts

Free-Field Limit

- *If we set* $g \to 0$*, SchE reduces to that of (3 copies of)* ***electrodynamics*** *and the solution is well-known:*

$$\Psi_0[A] \stackrel{g=0}{=} \mathcal{N} \exp\left[-\frac{1}{4}\int d^dx\, d^dy\, F^a_{ij}(x) \left(\frac{\delta^{ab}}{\sqrt{-\nabla^2}}\right)_{xy} F^b_{ij}(y)\right]$$

- *The ground state is (must be)* ***gauge-invariant****, e.g.*

$$\Psi_0[A] \;=\; \mathcal{N} \exp\left[-\frac{1}{4}\int d^dx\, d^dy\, F^a_{ij}(x)\, \mathcal{K}^{ab}(x,y)\, F^b_{ij}(y)\right]$$

$\mathcal{K}^{ab}(x,y)$ … *an adjoint-representation kernel.*

➡ *J. A. Wheeler,* ***Geometrodynamics****, Acad. Press, NY–London, 1962.*

Dimensional Reduction

- *At long distances –* ***"magnetic disorder"****:*

$$\Psi_0[A] = \mathcal{N} \exp\left\{-\tfrac{1}{2}\mu \int d^3x\, \mathrm{Tr}\left[F_{ij}(x)^2\right]\right\}$$

- ***Dimensional reduction:***

$$\begin{aligned} W(C) = \langle \mathrm{Tr}[U(C)]\rangle^{D=3+1} &= \left\langle \Psi_0^{(3)} \middle| \mathrm{Tr}[U(C)] \middle| \Psi_0^{(3)} \right\rangle \\ \sim \langle \mathrm{Tr}[U(C)]\rangle^{D=2+1} &= \left\langle \Psi_0^{(2)} \middle| \mathrm{Tr}[U(C)] \middle| \Psi_0^{(2)} \right\rangle \\ \sim \langle \mathrm{Tr}[U(C)]\rangle^{D=1+1} & \quad \ldots \quad \textit{area law} \Rightarrow \textit{confinement!} \end{aligned}$$

But this ***cannot*** *be the whole truth: some unwanted consequences (e.g.* ***Casimir scaling at all distance scales****).*

➡ *J. P. Greensite, Nucl. Phys. B* **158***, 469 (1979).*

➡ *M. B. Halpern, Phys. Rev. D* **19***, 517 (1979).*

➡ *M. Kawamura, K. Maeda, M. Sakamoto, Prog. Theor. Phys.* **97***, 939 (1997), arXiv:hep-th/9607176.*

Some Approaches

Different Strategies

- ***Strong-coupling expansion*** *of the vacuum wave-functional.*

  ➡ *J. Greensite (1979–1980), Shuo-Hong Guo, Qi-Zhou Chen, Lei Li (1994)*

- ***Weak-coupling expansion*** *of the vacuum wave-functional.*

  ➡ *B. F. Hatfield (1984), S. Krug, A. Pineda (2013–2014), see especially S. Krug's PhD thesis (2014)*

- *Formulate the theory in cleverly selected variables and express the vacuum WF in terms of these* ***new variables****.*

  ➡ *D. Karabali, C. Kim, V. P. Nair (1998), R. D. Leigh, D. Minic, A. Yelnikov (2005), L. Freidel (2006)*

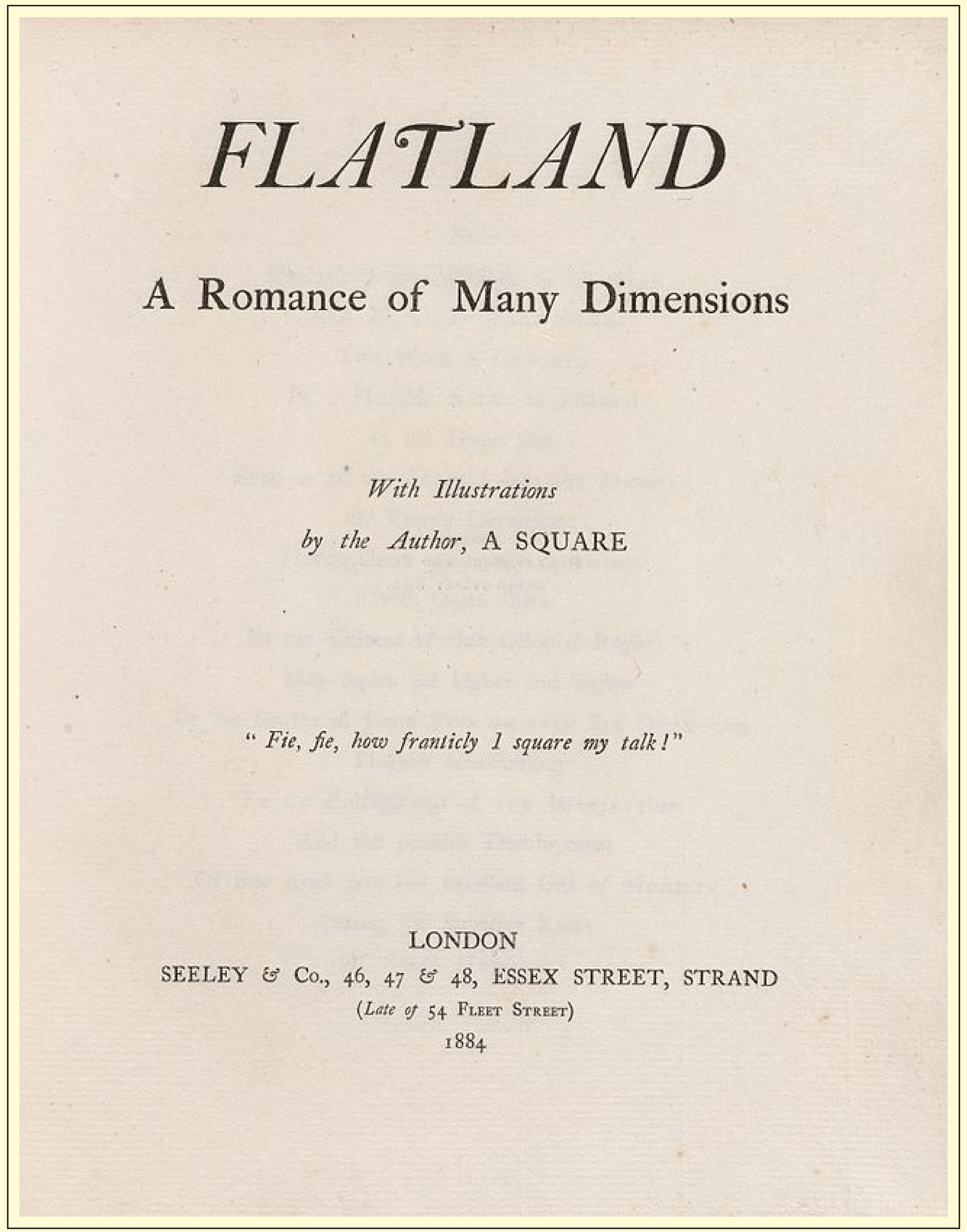
- ***Variational Ansatz*** *for the wave-functional in a certain gauge; its parameters then follow (e.g.) from minimizing the expectation value of the Yang–Mills hamiltonian.*

  ➡ *series of investigations in the Coulomb gauge by H. Reinhardt et al., A. Szczepaniak et al.*

- ***Guess*** *an approximate form of the VWF; test consequences of the guess.*

  ➡ *S. Samuel (1997), J. Greensite, ŠO (2008–)*

Flatland: A Romance of Two Dimensions



FLATLAND

A Romance of Many Dimensions

With Illustrations

by the Author, A SQUARE

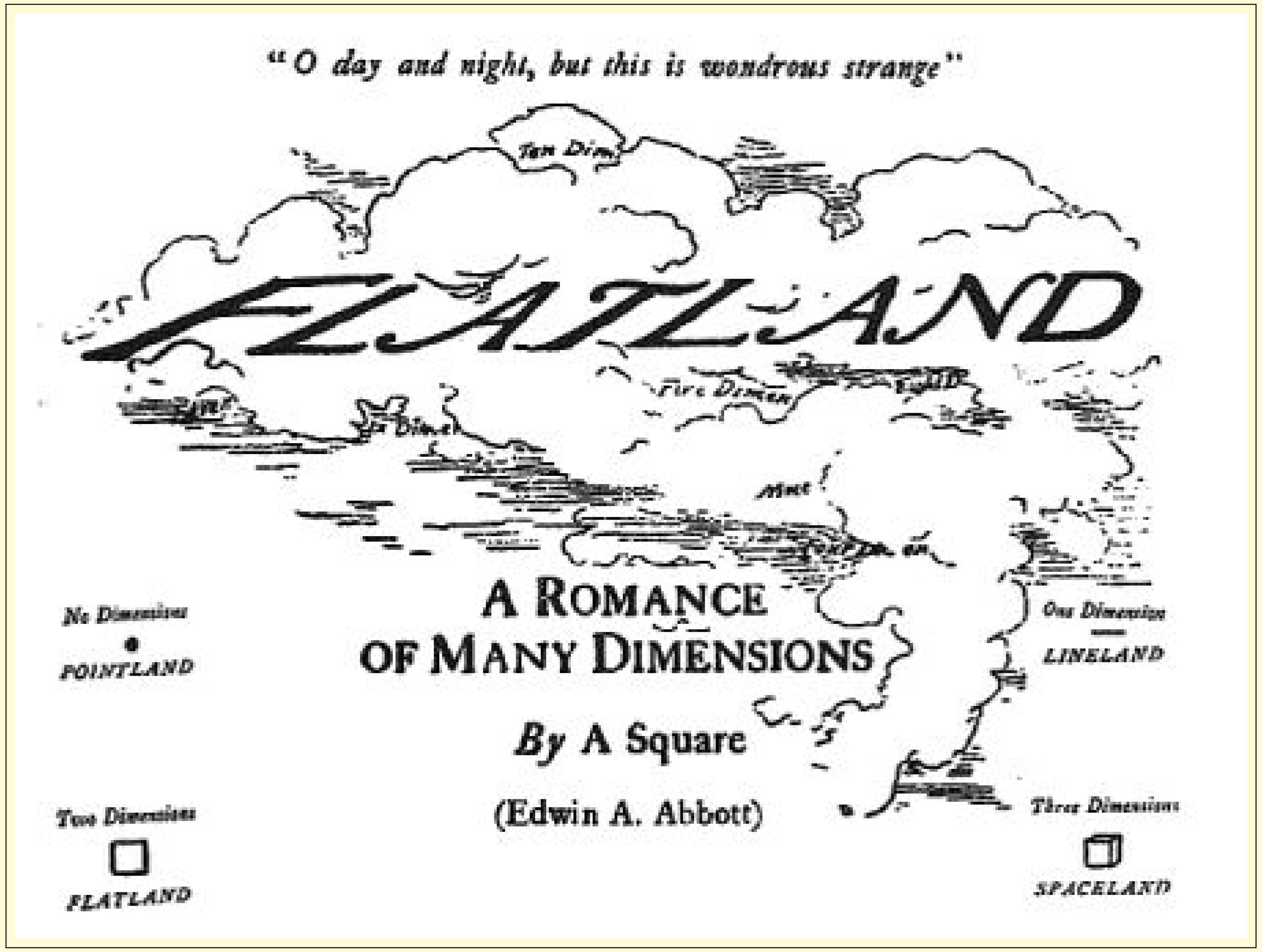
" *Fie, fie, how franticly I square my talk!* "

LONDON

SEELEY & Co., 46, 47 & 48, ESSEX STREET, STRAND

(*Late of* 54 FLEET STREET)

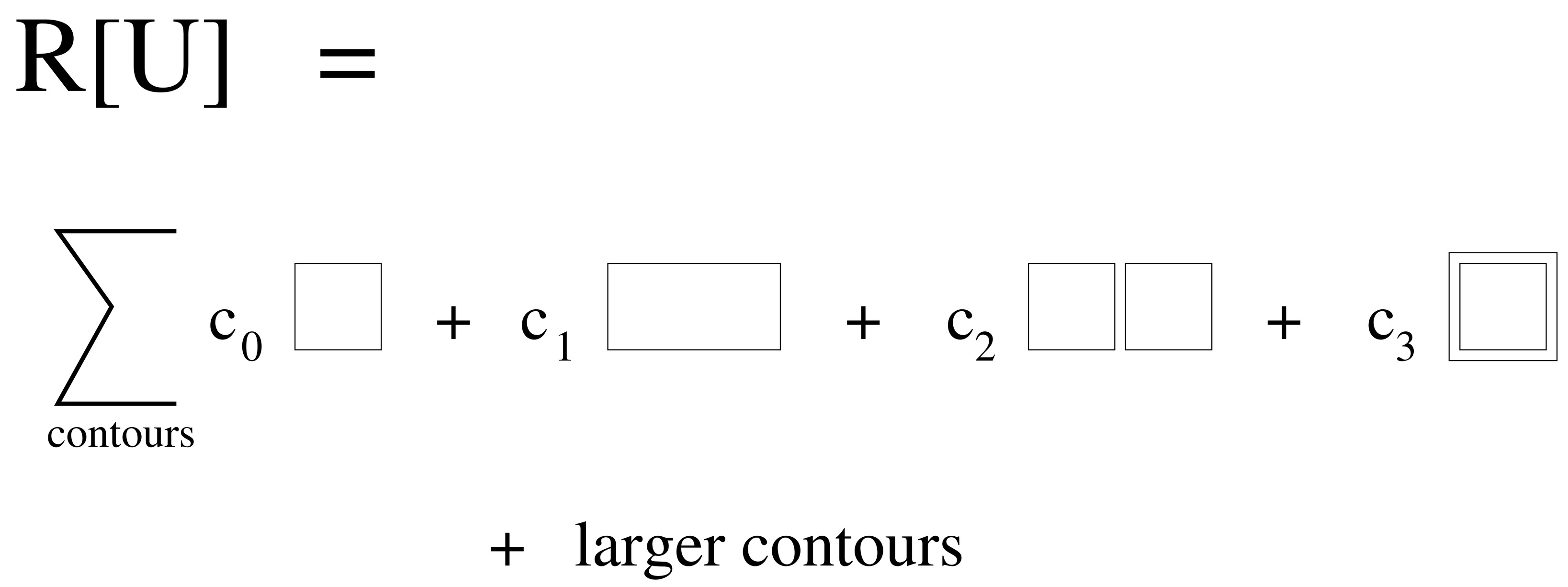
1884



➡ *A Square [Edwin Abbott Abbott],* ***Flatland: A Romance of Many Dimensions****, Seeley & Co, 46–48, Essex Street, Strand, London (1884)*

HINTS FROM STRONG-COUPLING EXPANSION

$$\Psi_0 = \mathcal{N} \exp(-R[U])$$



$$R[U] \propto \frac{1}{\beta}\left(a\kappa_0 Tr\,[B^2] - a^3\kappa_2 Tr\,[B(-\mathcal{D}^2)B] + \dots\right)$$

$$\kappa_0 = \tfrac{1}{2}c_0 + 2(c_1 + c_2 + c_3), \qquad \kappa_2 = \tfrac{1}{4}c_1$$

$$c_0 = \mathcal{O}(\beta^2), \qquad c_1, c_2, c_3 = \mathcal{O}(\beta^4)$$

➡ *Shuo-Hong Guo, Qi-Zhou Chen and Lei Li, Phys. Rev. D* **49**, *507 (1994).*

$$\Psi_0[A] \;=\; \mathcal{N} \exp\left[-\tfrac{1}{2}\int d^2x\, d^2y\, B^a(x)\, \mathcal{K}^{ab}_{xy}[-\mathcal{D}^2]\, B^b(y)\right]$$

The Proposal/Guess of Samuel

Interpolating Expression

- *Almost 20 years ago, **Samuel** proposed a simple form, interpolating between strong- and weak-coupling limits, and estimated with its use the 0^{++} glueball mass to be about 1.5 GeV:*

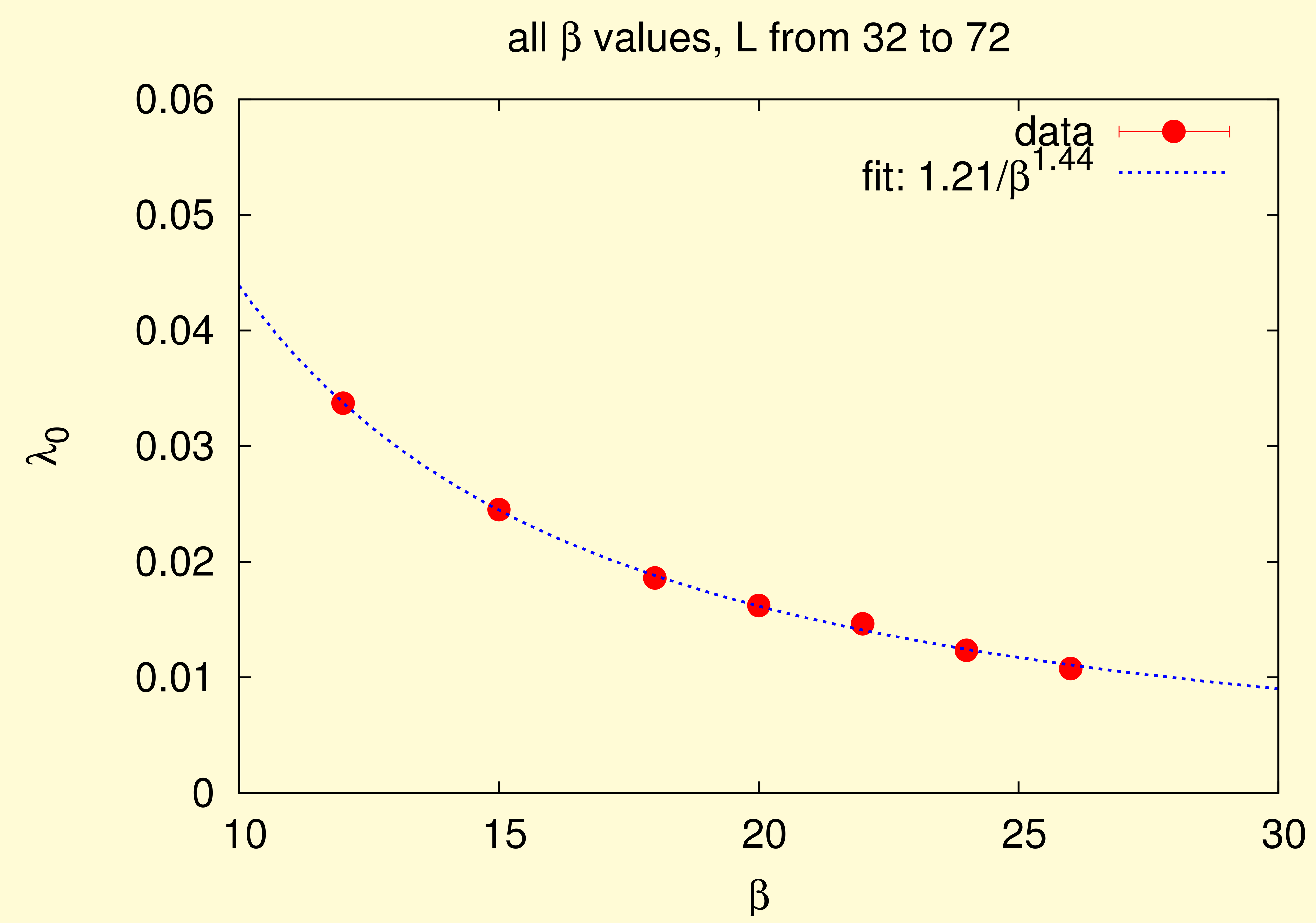
$$\Psi_0[A] = \mathcal{N} \exp\left[-\frac{1}{2} \int d^2x \, d^2y \, B^a(x) \left(\frac{1}{\sqrt{-\mathcal{D}^2 + m_0^2}} \right)^{ab}_{xy} B^b(y) \right]$$

- *However, there may be a problem with this form: the adjoint covariant laplacian needs to be **regularized**.*

➡ *S. Samuel, Phys. Rev. D* **55**, *4189 (1997), arXiv:hep-ph/9604405.*

The Eigenvalue Spectrum of $-\mathcal{D}^2$

- *$-\mathcal{D}^2$ has a positive definite spectrum, finite with a lattice regularization, with the lowest eigenvalue tending to infinity for typical configurations in the continuum limit.*



$$\lim_{\beta\to\infty} \beta^2 \lambda_0(\beta) \to \infty$$

Our Guesswork

The GO Proposal

- *With somewhat different motivation, we proposed a simple form with a seemingly small, but crucial difference from that of Samuel:*

$$\Psi_0[A] = \mathcal{N} \exp\left[-\frac{1}{2}\int d^2x\, d^2y\, B^a(x) \left(\frac{1}{\sqrt{-\mathcal{D}_R^2 + m^2}}\right)^{ab}_{xy} B^b(y)\right]$$

where $\mathcal{D}_k[A]$ *and* $\mathcal{D}^2 = \mathcal{D}_k \cdot \mathcal{D}_k$ *are the covariant derivative and covariant laplacian in the adjoint representation:*

$$\left(-\mathcal{D}^2\right)^{ab}_{xy} = \sum_{k=1}^{2}\left[2\delta^{ab}\delta_{xy} - \mathcal{U}^{ab}_k(x)\delta_{y,x+\hat{k}} - \mathcal{U}^{\dagger ba}_k(x-\hat{k})\delta_{y,x-\hat{k}}\right]$$

$$\mathcal{U}^{ab}_k(x) = \frac{1}{2}\mathrm{Tr}\left[\sigma^a U_k(x)\sigma^b U^\dagger_k(x)\right]$$

$$\mathcal{D}^2_R = \mathcal{D}^2 - \lambda_0$$

λ_0 *is the lowest eigenvalue of* $(-\mathcal{D}^2)$, *and* m *a mass parameter that vanishes in the free-field limit* ($g \to 0$).

How to Test the Proposal

- *Do some* ***analytic calculations*** *with the proposed vacuum wave-functional.*
- ***Generate configurations*** *distributed according to (the square) of the proposed VWF, compute vacuum expectation values of physical quantities in this distribution, compare with results for these quantities in the full theory.*
- ***Measure (compute numerically) weights*** *of various ensembles of test configurations in the YM vacuum, compare with predictions based on the proposed VWF.*

Some Evidence in favour of the Proposed VWF

Analytic Arguments

- *(By construction)* Ψ_0 *becomes the well-known VWF of electrodynamics in the* ***free-field limit*** *(for* $g \to 0$*).*

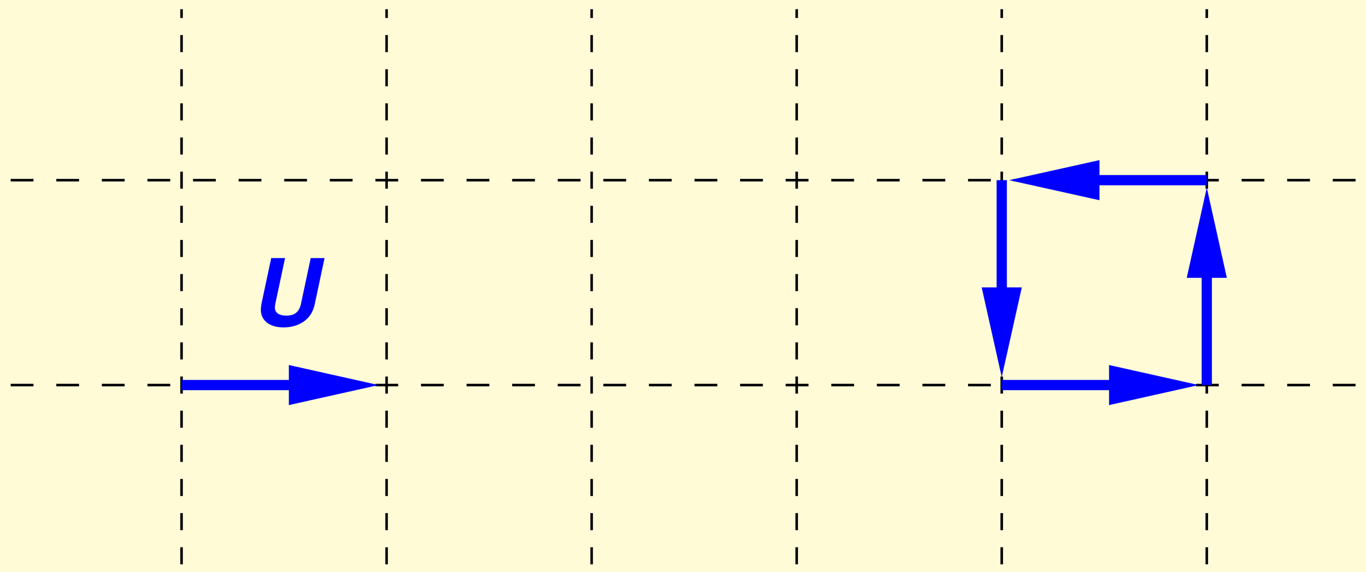
- *The proposed form is a good approximation to the true vacuum also for* ***strong fields constant in space*** *and* ***varying only in time****.*

- *If we divide the magnetic field strength* $B(x)$ *into "fast" and "slow" components, the part of the VWF that depends on* B_{slow} *takes on the* ***dimensional-reduction form****. The fundamental string tension is then easily computed as*

$$\sigma_F = 3mg^2/16 = 3m/4\beta.$$

- *If one takes the mass* m *in the wave-functional as a free variational parameter and computes (approximately) the expectation value of the YM hamiltonian, one finds that a* ***non-zero (finite) value of*** m ***is energetically preferred****.*

SPACE-TIME LATTICE



$$\mathbf{A}_\mu(x) = A^a_\mu(x)\mathbb{T}^a \quad \rightarrow \quad U_\mu(x) = \exp[iga\mathbf{A}_\mu(x)]$$

$$\mathbf{F}_{\mu\nu}(x) \quad \rightarrow \quad U_{\mu\nu}(x) = U_\mu(x)U_\nu(x+\hat{\mu})U^\dagger_\mu(x+\hat{\nu})U^\dagger_\nu(x)$$

$$\mathcal{S} = \frac{1}{2}\int d^4x \ \mathrm{Tr}\left[\mathbf{F}_{\mu\nu}(x)\mathbf{F}_{\mu\nu}(x)\right] \quad \rightarrow$$

$$\mathcal{S}_W = a^4\,\beta \sum_P \left[1 - \frac{1}{N}\mathrm{Re\ Tr}\ U_P\right], \quad \beta = 2N/g^2$$

VACUUM EXPECTATION VALUES

$$\langle 0|\widehat{Q}|0\rangle = \int [dU]\ \Psi^*_0[U]\ \widehat{Q}\ \Psi_0[U]$$

- *In the path-integral formulation:*

$$\langle\widehat{Q}\rangle = \frac{1}{Z}\int [dU_\mu(x)]\ Q[U]\ \exp(-\mathcal{S}_W[U])$$

- *In the numerical (Monte Carlo) simulation one computes in fact:*

$$\langle\widehat{Q}\rangle \approx \frac{1}{N_{\mathrm{conf}}}\sum_{i=1}^{N_{\mathrm{conf}}} Q[\{C_i\}],$$

an average over a (large) number N_{conf} *of gauge-field configurations* $\{C_i\}$ *distributed according to the probability distribution* $\sim \exp(-\mathcal{S}_W[U]) \sim |\Psi_0[U]|^2$.

Probability Distributions

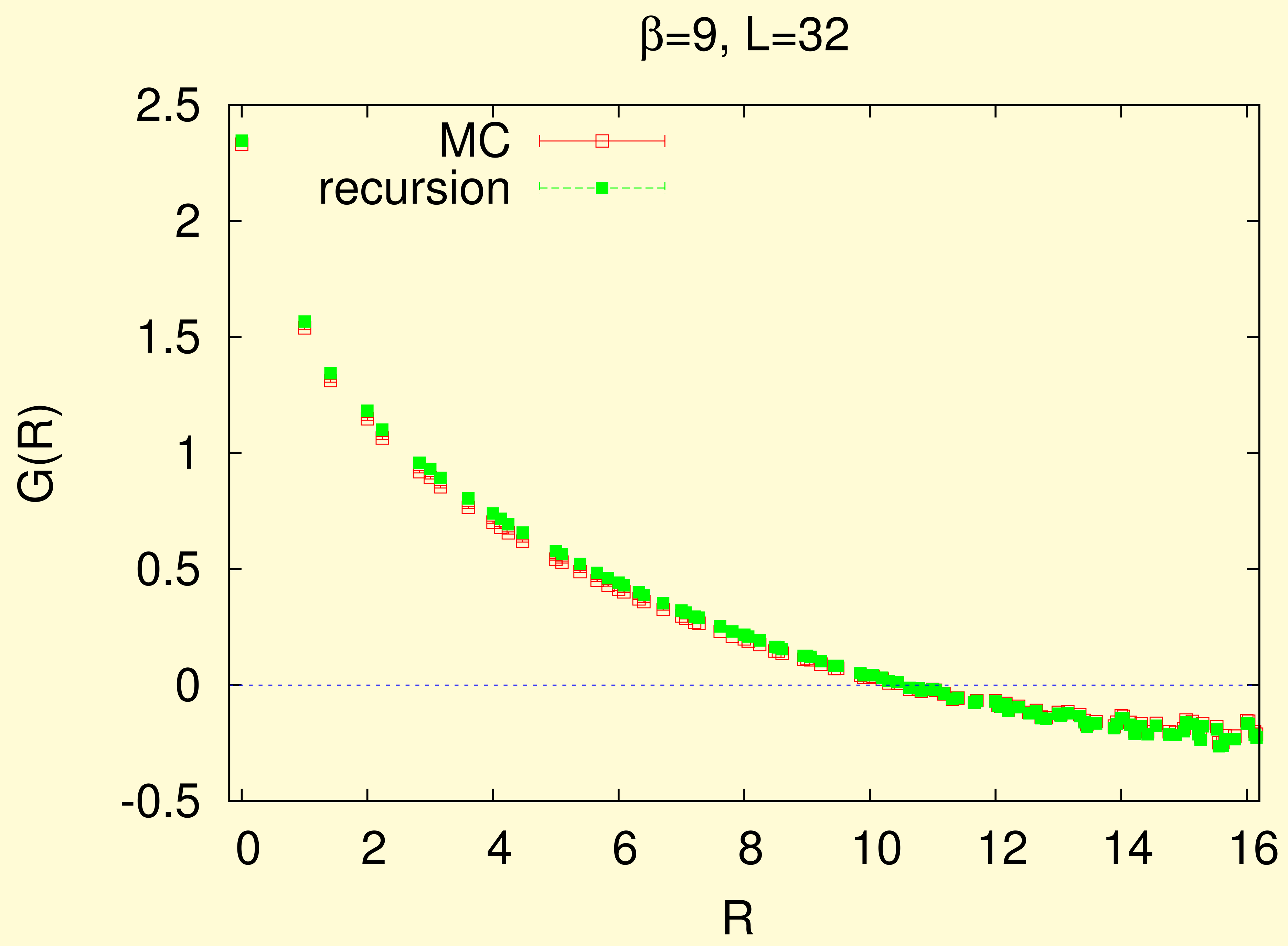
$$\langle \Psi_0^{\text{true}} | \widehat{Q}[A] | \Psi_0^{\text{true}} \rangle = \langle Q \rangle_{\text{MC}}$$

vs

$$\langle \Psi_0^{\text{guess}} | \widehat{Q}[A] | \Psi_0^{\text{guess}} \rangle = \langle Q \rangle_{\text{recursion}}$$

- ***Monte Carlo lattices**: ensemble of 2-d slices of configurations generated by MC simulations of 3-d euclidean SU(2) LGT with standard Wilson action at a coupling β; from each configuration, only one (random) slice at fixed euclidean time is taken.*
- ***"Recursion" lattices**: ensemble of independent 2-d lattice configurations generated with the probability distribution given by the proposed VWF, with m and g^2 fixed to get the correct value of the fundamental string tension.*
- *In this way we compared in both ensembles:*
  - *the mass gap,*
  - *the Coulomb-gauge ghost propagator,*
  - *the colour Coulomb potential.*
- ***Good agreement was found.***

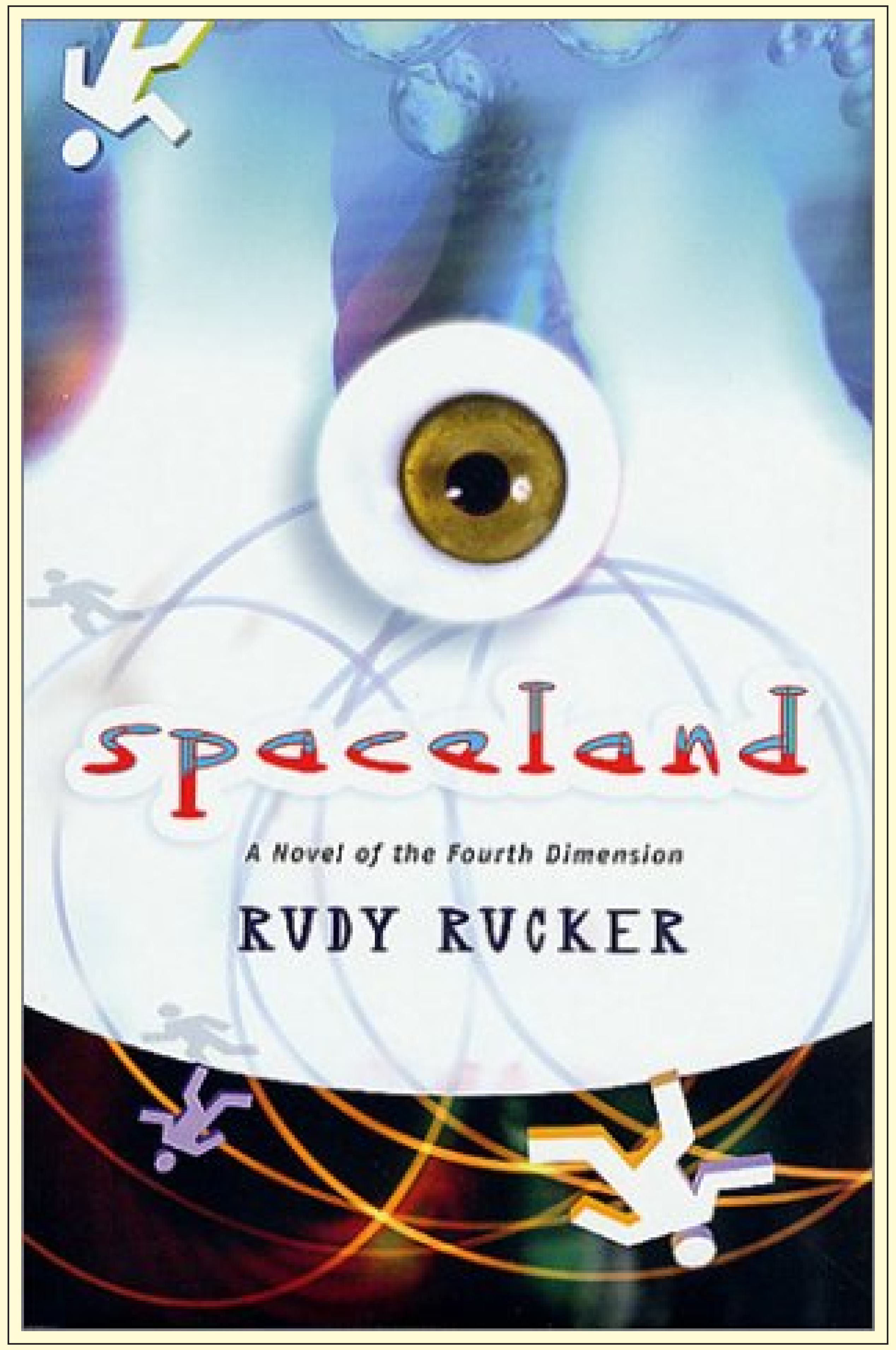
Example



- *In D=2+1 our Ansatz for the VWF seems **a fairly good approximation** to the true ground state of the theory.*
- *We have provided **analytic arguments** in favour of this choice, argued that its free parameter, m, is preferably non-zero, which implies non-zero string tension, i.e. confinement.*
- *We have demonstrated **numerically** on the lattice that a number of quantities computed in ensembles of gauge-field configurations distributed according to (the square of) the proposed VWF, agree reasonably with those computed in true YM vacuum configurations.*
- *For **test configurations**, numerical data are consistent with expectations based on the proposed VWF. **(Not covered in this talk.)***
- *Some other Ansätze interpolating between the free-field limit and the dimensional-reduction form also **work well**.*

— BUT WE DO NOT LIVE IN FLATLAND, OUR WORLD IS SPACELAND! —

Version in 3D or Spaceland



➡ *Rudy Rucker,* ***Spaceland: A Novel of the Fourth Dimension****, Tor Books, NYC (2002)*

- *We have proposed an analogous form:*

$$\Psi_0[A] = \mathcal{N} \exp\left[-\frac{1}{4} \int d^3x \, d^3y \, F^a_{ij}(x) \left(\frac{1}{\sqrt{-\mathcal{D}_R^2 + m^2}} \right)^{ab}_{xy} F^b_{ij}(y) \right]$$

where $\mathcal{D}_k[A]$ *and* $\mathcal{D}^2 = \mathcal{D}_k \cdot \mathcal{D}_k$ *are the covariant derivative and covariant laplacian in the adjoint representation:*

$$\left(-\mathcal{D}^2\right)^{ab}_{xy} = \sum_{k=1}^{3} \left[2\delta^{ab}\delta_{xy} - \mathcal{U}^{ab}_k(x)\delta_{y,x+\hat{k}} - \mathcal{U}^{\dagger ba}_k(x-\hat{k})\delta_{y,x-\hat{k}} \right]$$

$$\mathcal{U}^{ab}_k(x) = \frac{1}{2}\mathrm{Tr}\left[\sigma^a U_k(x) \sigma^b U^\dagger_k(x)\right]$$

$$\mathcal{D}_R^2 = \mathcal{D}^2 - \lambda_0$$

λ_0 *is the lowest eigenvalue of* $(-\mathcal{D}^2)$*, and* m *is a parameter with the dimension of mass (assumed to vanish for* $g \to 0$*).*

Positive News

Zero-Mode, Strong-Field Limit

- *Keep only the zero-mode of the A-field, i.e. fields* ***constant in space, varying in time****, and assume* ***A large*** $(|gA|^2 \gg m^2)$*. Denote:*

$$\ell^2 = \vec{A}_1^2 + \vec{A}_2^2 + \vec{A}_3^2$$

$$\mathcal{S}^2 = (\vec{A}_1 \times \vec{A}_2)^2 + (\vec{A}_2 \times \vec{A}_3)^2 + (\vec{A}_3 \times \vec{A}_1)^2$$

$$\mathcal{V}^2 = \left[\vec{A}_1 \cdot (\vec{A}_2 \times \vec{A}_3)\right]^2$$

The lagrangian and hamiltonian, in a finite volume V*, are:*

$$\mathcal{L} = \frac{1}{2}V\left[\sum_k \partial_t \vec{A}_k \cdot \partial_t \vec{A}_k - g^2 \mathcal{S}^2\right]$$

$$\hat{\mathcal{H}} = -\frac{1}{2V}\sum_k \frac{\partial^2}{\partial \vec{A}_k \cdot \partial \vec{A}_k} + \frac{1}{2}g^2 V \mathcal{S}^2$$

Natural choice – $1/V$ expansion:

$$\Psi_0 = \exp[-VR_0 + R_1 + V^{-1}R_2 + \ldots]$$

$$\mathcal{T}_0 \equiv V\left[-\sum_k \frac{\partial R_0}{\partial \vec{A}_k} \cdot \frac{\partial R_0}{\partial \vec{A}_k} + g^2 \mathcal{S}^2\right] = 0$$

- *The solution is:*

$$R_0 = \tfrac{1}{2} g \frac{\mathcal{S}^2}{\ell}$$

since

$$\mathcal{T}_0 = 0 + g^2 V\left(\frac{7\mathcal{S}^4}{4\ell^4} - \frac{3\mathcal{V}^2}{\ell^2}\right) = 0 + \mathcal{O}\left(\frac{1}{V}\right)$$

$$\vec{A}_1 = (a_1^1, a_1^2, \mathcal{A}_1^3) \qquad \vec{A}_2 = (a_2^1, a_2^2, \mathcal{A}_2^3) \qquad \vec{A}_3 = (a_3^1, a_3^2, \mathcal{A}_3^3)$$

$$VR_0 \sim gV\frac{a^2\mathcal{A}^2}{\mathcal{A}} \sim \mathcal{O}(1) \quad \Rightarrow \quad a \sim \frac{1}{\sqrt{g\mathcal{A}V}} \qquad \ldots \text{“abelian valley”}$$

- *In the proposed VWF* $\Psi_0 = \exp(-Q)$ *we have, in this limit:*

$$Q \approx \tfrac{1}{4} gV(\vec{A}_i \times \vec{A}_j)^a \left(\frac{\delta^{ab}}{\ell} - \frac{\delta^{a3}\delta^{b3}}{\ell} + \frac{\delta^{a3}\delta^{b3}}{m}\right)(\vec{A}_i \times \vec{A}_j)^b$$

$$Q_1 \sim gV\frac{(\vec{A}_i \times \vec{A}_j)\cdot(\vec{A}_i \times \vec{A}_j)}{\ell} \sim gV\mathcal{A}a^2 \sim \mathcal{O}(1)$$

$$Q_2 \text{ or } Q_3 \sim \frac{gV(\vec{A}_i \times \vec{A}_j)^3(\vec{A}_i \times \vec{A}_j)^3}{\ell \text{ or } m} \sim \frac{gVa^4}{\mathcal{A} \text{ or } m} \sim \mathcal{O}\left(V^{-1}\right)$$

and the same result as above is obtained in the leading order.

Direct Measurement of the VWF

The Relative Weight Method

- *The squared VWF is given by the path integral:*

$$\Psi_0^2[U'] = \frac{1}{Z}\int [DU]\,\delta_{gf}(U_0)\prod_{\mathbf{x},i}\delta[U_i(\mathbf{x},0)-U'(\mathbf{x})]e^{-S[U]}$$

- *The* ***relative weight method*** *enables one to compute ratios* $\Psi_0^2[U^{(n)}]/\Psi_0^2[U^{(m)}]$ *for configurations belonging to a finite set* $\mathcal{U} = \left\{U_i^{(j)}(\mathbf{x}), j = 1,2,\ldots,M\right\}$ *(close in config'n space).*
- *Use MC simulations with the usual update algorithm (e.g. heat-bath) for all spacelike links at* $t \neq 0$ *and for timelike links.*
- *Update the spacelike links at* $t = 0$ *all at once selecting one configuration from the set* $\mathcal{U}$ *at random and accept/reject it via Metropolis. Then:*

$$\frac{\Psi_0^2[U^{(n)}]}{\Psi_0^2[U^{(m)}]} = \lim_{N_{\rm tot}\to\infty}\frac{N_n}{N_m}$$

N_n (N_m) is the number of times the n-th (m-th) configuration is accepted, and $N_{\rm tot}$ is the total number of updates.

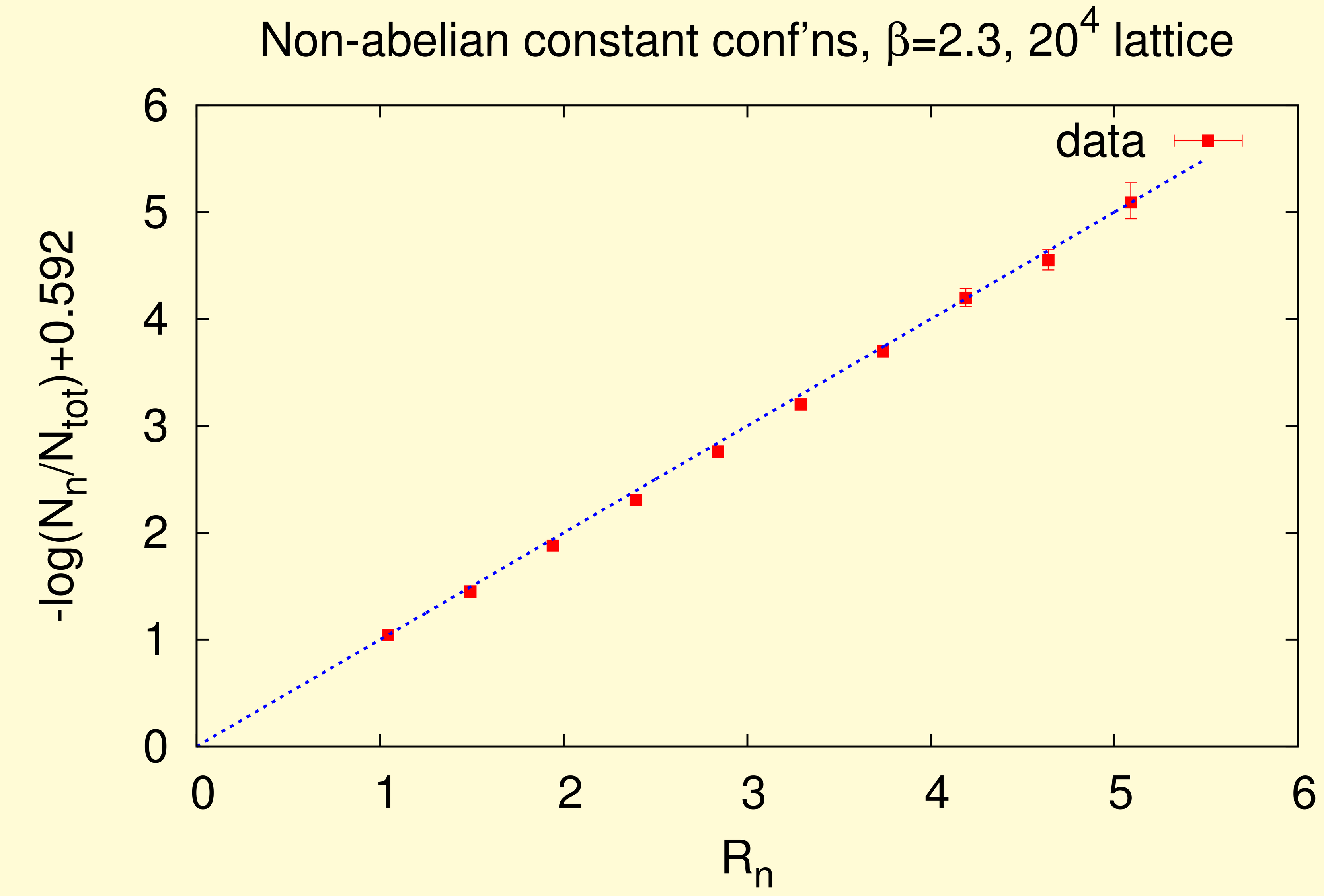
➡ *J. Greensite, J. Iwasaki, Phys. Lett. B* **223***, 207 (1989).*

An Example

- *If the VWF is assumed to be of the form*

$$\Psi_0^2[U] = \mathcal{N}e^{-R[U]}$$

then the measured values of $-\log(N_n/N_{\rm tot})$ should fall on a straight line with unit slope as function of $R_n \equiv R[U^{(n)}]$.



$-\log(N_n/N_{\rm tot})$ vs. $R_n = \mu\kappa n$ for $\mathcal{U}_{\rm NAC}$ with $\kappa = 0.14$; $\mu = 3.21(5)$.

Direct Measurement of the VWF

Non-Abelian Constant Configurations

$$\mathcal{U}_{\mathrm{NAC}} = \left\{ U_k^{(n)}(x) = \sqrt{1-(a^{(n)})^2}\,\mathbf{1} + i a^{(n)} \boldsymbol{\sigma}_k \right\}$$

$$a^{(n)} = \left(\frac{\kappa}{6L^3} n \right)^{1/4}, \qquad n = 1, 2, \ldots, 10$$

Fits for Non-Abelian Constant Configurations

$$-\log(N^{(n)}/N_{\mathrm{tot}}) = R_{\boldsymbol{n}}^{(n)} = \kappa n \times \mu$$

Abelian Plane-Wave Configurations

$$\mathcal{U}_{\mathrm{APW}} = \left\{ U_1^{(j)}(x) = \sqrt{1-\left(a_{\boldsymbol{n}}^{(j)}(x)\right)^2}\,\mathbf{1} + i a_{\boldsymbol{n}}^{(j)}(x) \boldsymbol{\sigma}_3, \right.$$

$$\left. U_2^{(j)}(x) = U_3^{(j)}(x) = \mathbf{1} \right\}, \qquad \boldsymbol{n} = (n_1, n_2, n_3)$$

$$a_{\boldsymbol{n}}^{(j)} = \sqrt{\frac{\alpha_{\boldsymbol{n}} + \gamma_{\boldsymbol{n}} j}{L^3}} \cos\left(\frac{2\pi}{L} \boldsymbol{n} \cdot \boldsymbol{x} \right), \qquad j = 1, 2, \ldots, 10$$

Fits for Abelian Plane-Wave Configurations

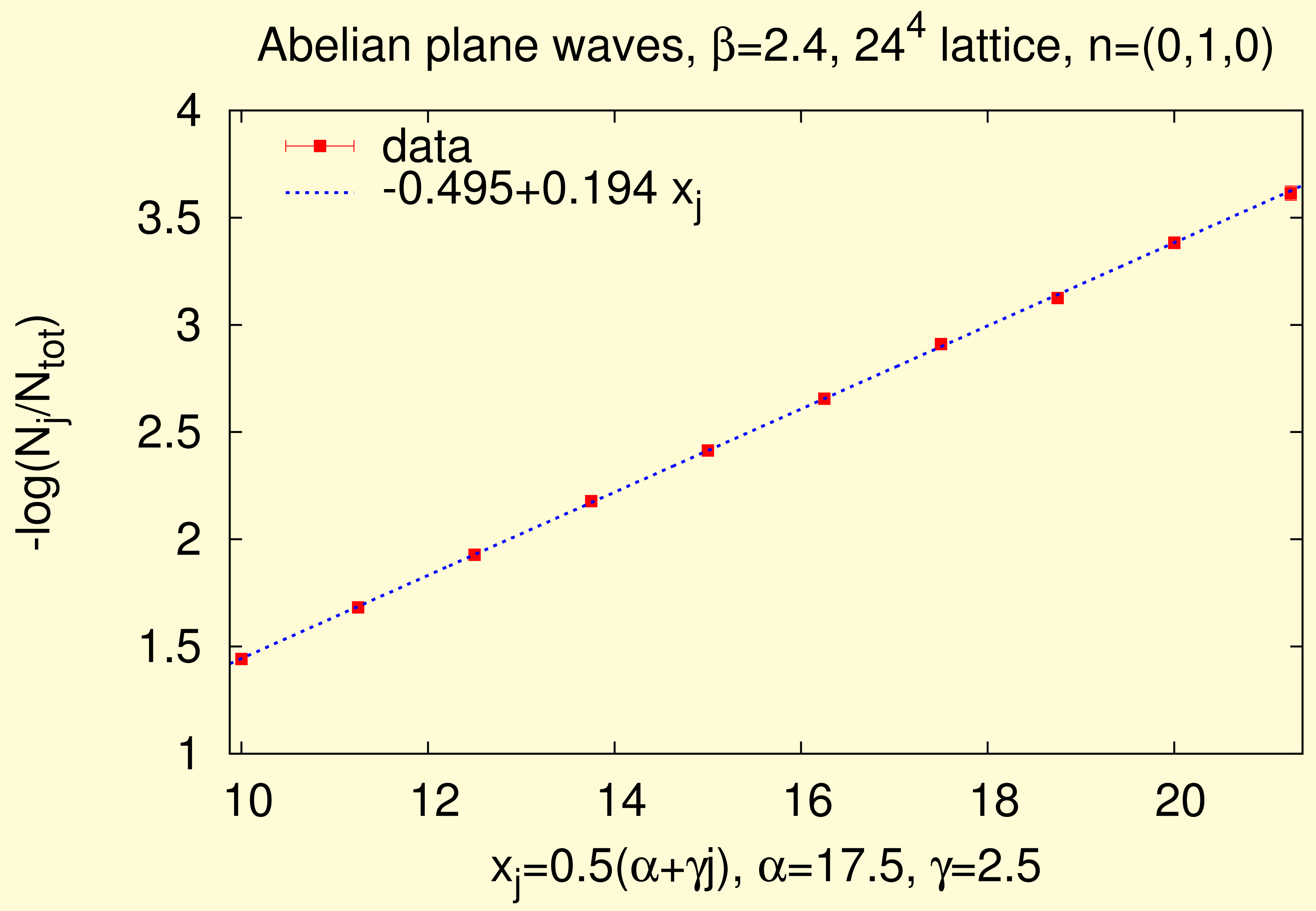
$$-\log(N_{\boldsymbol{n}}^{(j)}/N_{\mathrm{tot}}) = R_{\boldsymbol{n}}^{(j)} = \tfrac{1}{2}(\alpha_{\boldsymbol{n}} + \gamma_{\boldsymbol{n}} j) \times \omega(\boldsymbol{n})$$

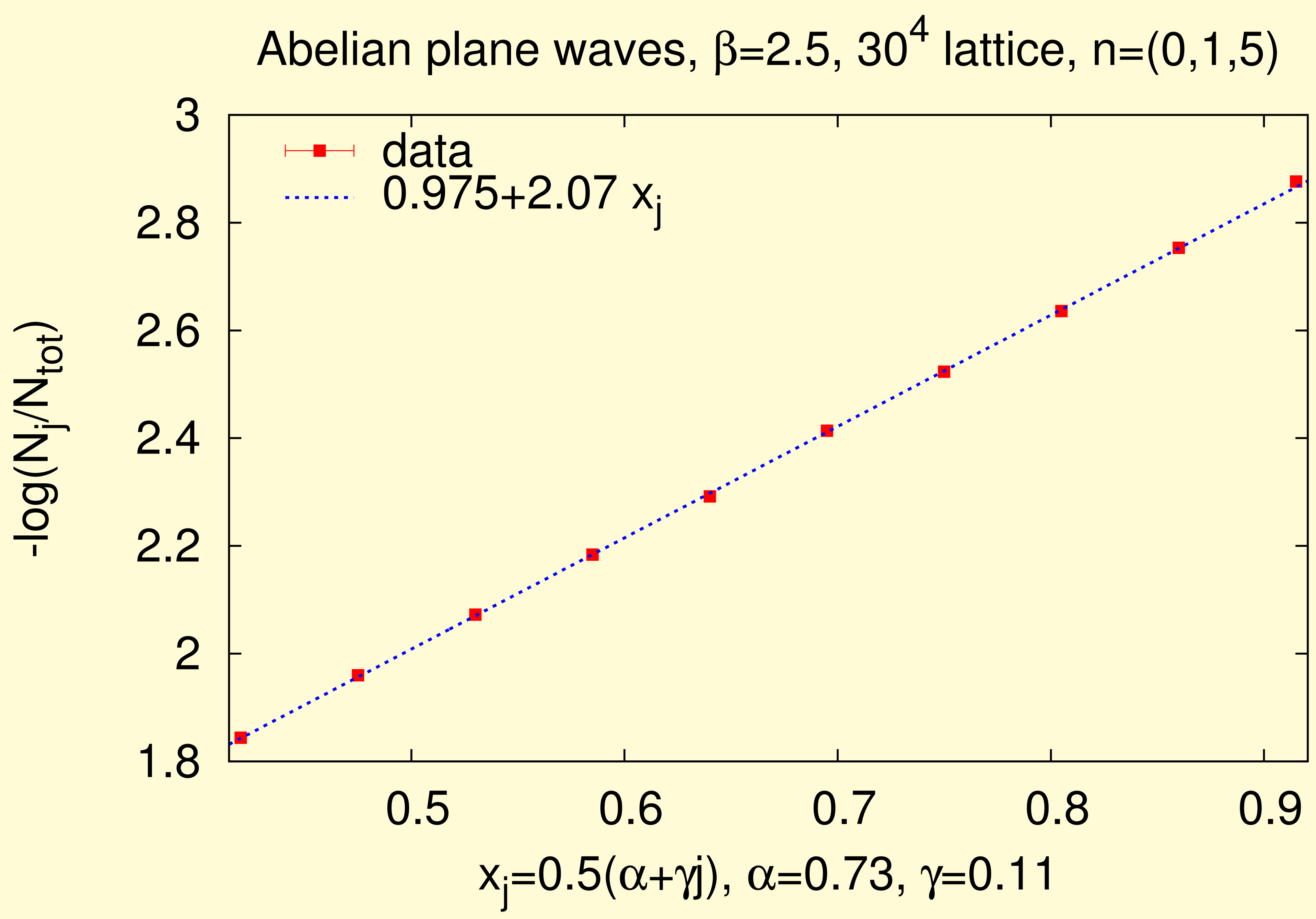
$$\omega(\boldsymbol{n}) = \begin{cases} b k^2(\boldsymbol{n}) & \ldots \textit{ dim. reduction} \\ \dfrac{c k^2(\boldsymbol{n})}{\sqrt{k^2(\boldsymbol{n}) + m^2}} & \ldots \textit{ GO proposal} \end{cases}$$

$$\textit{with} \qquad k^2(\boldsymbol{n}) = 2 \sum_i \left(1 - \cos \frac{2\pi n_i}{L} \right).$$

Abelian Plane Waves: How $\omega(n)$ is Extracted

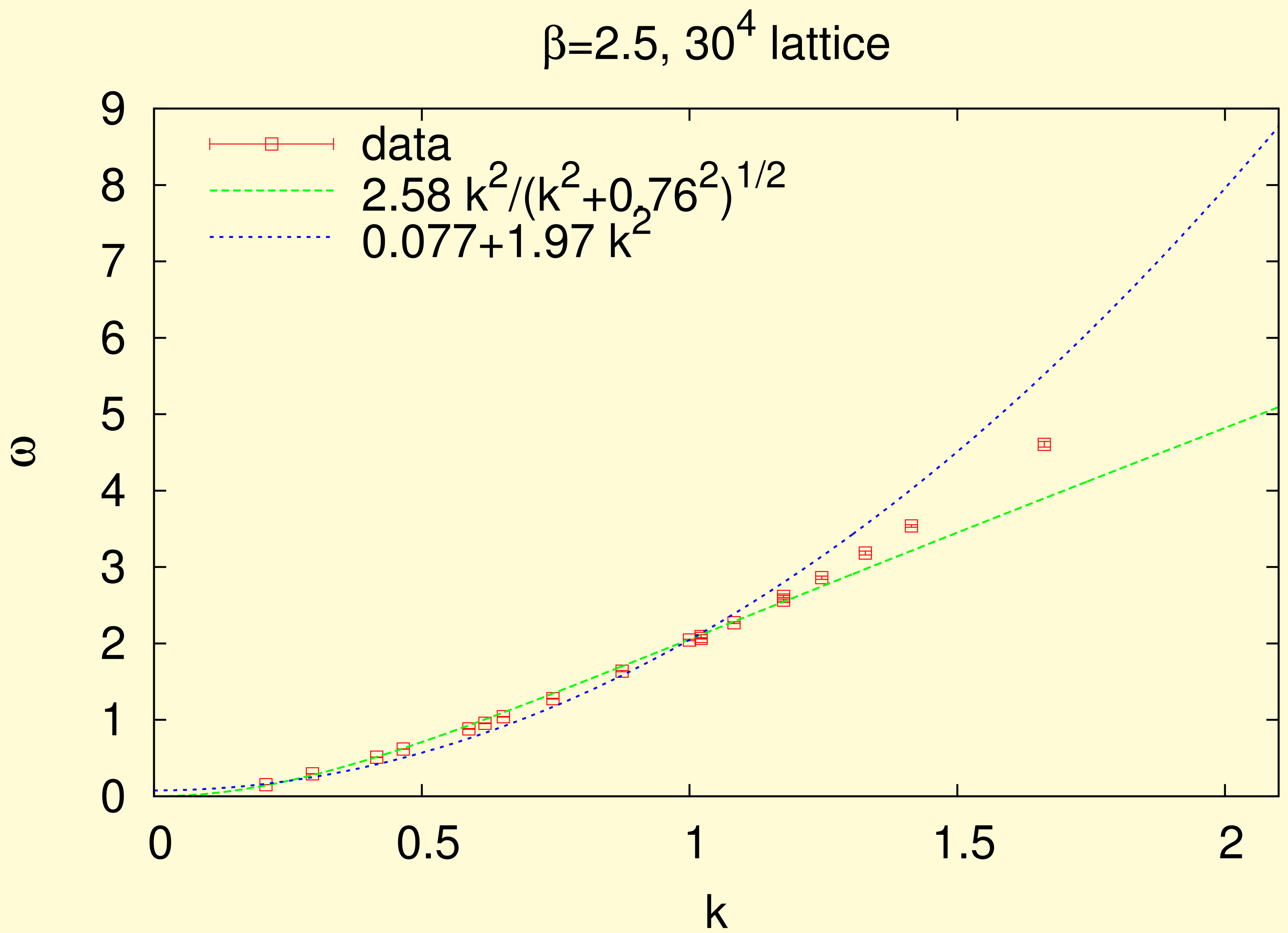
- *For each* n *we choose a suitable pair of parameters* α_n *and* γ_n; $\omega(n)$ *is the* ***slope*** *of* $\left[-\log(N^{(j)}/N_{\rm tot})\right]$ *vs.* $\frac{1}{2}(\alpha_n + \gamma_n j)$.
- *Two examples of results:*





ABELIAN PLANE WAVES: DR AND GO FITS

$$\omega(\boldsymbol{n}) = \begin{cases} a + bk^2(\boldsymbol{n}) & \ldots \quad \textit{dim. reduction} \\ \dfrac{ck^2(\boldsymbol{n})}{\sqrt{k^2(\boldsymbol{n}) + m^2}} & \ldots \quad \textit{GO proposal} \end{cases}$$



ω vs. k, DR and GO fits.

ABELIAN PLANE WAVES: BEST FIT

- *Improvement with **a single additional fit parameter**:*

$$\omega(\boldsymbol{n}) = \frac{ck^2(\boldsymbol{n})}{\sqrt{k^2(\boldsymbol{n}) + m^2}}\left(1 + \boxed{dk(\boldsymbol{n})}\right) \ldots \quad \textit{a guess}$$

- *This would correspond in the continuum limit to:*

$$\Psi_0[A] = \mathcal{N} \exp\left[-\tfrac{1}{4}\int d^dx\, d^dy\, F^a_{ij}(x)\, \mathcal{K}^{ab}_{xy}[-\mathcal{D}^2]\, F^b_{ij}(y)\right]$$

with

$$\mathcal{K}^{ab}_{xy}[-\mathcal{D}^2] \propto \left(\frac{1}{\sqrt{-\mathcal{D}^2_R + m^2_{\rm ph}}} + \boxed{d_{\rm ph}\sqrt{\frac{-\mathcal{D}^2_R}{-\mathcal{D}^2_R + m^2_{\rm ph}}}}\right)^{ab}_{xy}$$

$$-\mathcal{D}^2_R = -\mathcal{D}^2 - \lambda_{0,\rm ph} \qquad \lambda_{0,\rm ph} = \lambda_0/a^2$$

$$d_{\rm ph} = da \qquad m_{\rm ph} = m/a$$

Abelian Plane Waves: Best Fit

- *Improvement with* ***a single additional fit parameter****:*

$$\omega(\mathbf{n}) = \frac{c k^2(\mathbf{n})}{\sqrt{k^2(\mathbf{n}) + m^2}} \left(1 + \boxed{d k(\mathbf{n})}\right) \ldots \quad \textit{a guess}$$

- *This would correspond in the continuum limit to:*

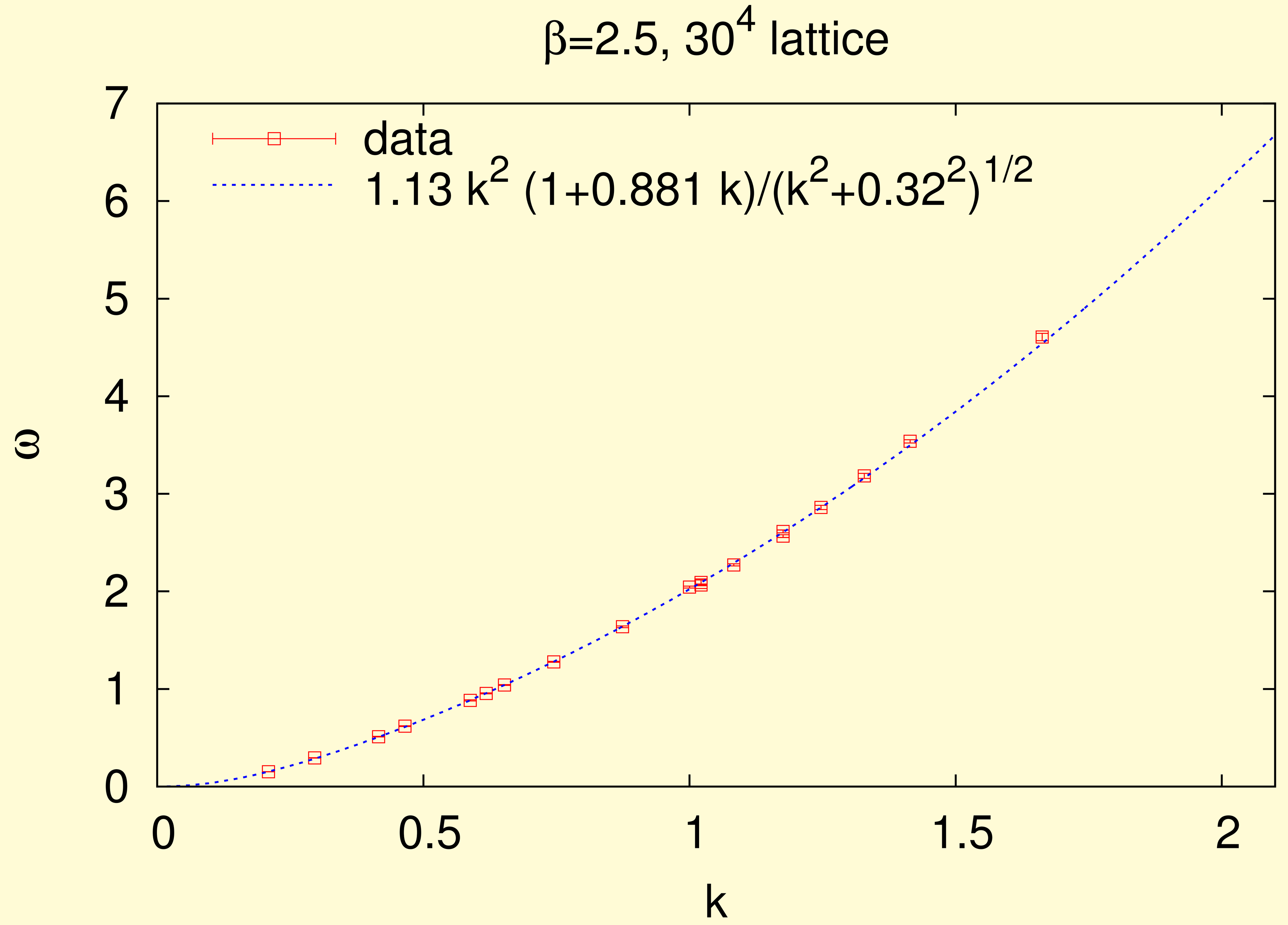
$$\Psi_0[A] \;=\; \mathcal{N} \exp\left[-\tfrac{1}{4}\int d^d x\, d^d y\, F^a_{ij}(x)\, \mathcal{K}^{ab}_{xy}[-\mathcal{D}^2]\, F^b_{ij}(y)\right]$$

with

$$\mathcal{K}^{ab}_{xy}[-\mathcal{D}^2] \propto \left(\frac{1}{\sqrt{-\mathcal{D}^2_R + m^2_{\mathrm{ph}}}} + \boxed{d_{\mathrm{ph}} \sqrt{\frac{-\mathcal{D}^2_R}{-\mathcal{D}^2_R + m^2_{\mathrm{ph}}}}}\right)^{ab}_{xy}$$

$$-\mathcal{D}^2_R = -\mathcal{D}^2 - \lambda_{0,\mathrm{ph}} \qquad \lambda_{0,\mathrm{ph}} = \lambda_0/a^2$$

$$d_{\mathrm{ph}} = da \qquad m_{\mathrm{ph}} = m/a$$



ω vs. k, best (= guessed) fit.

Consistency of NAC and APW Data

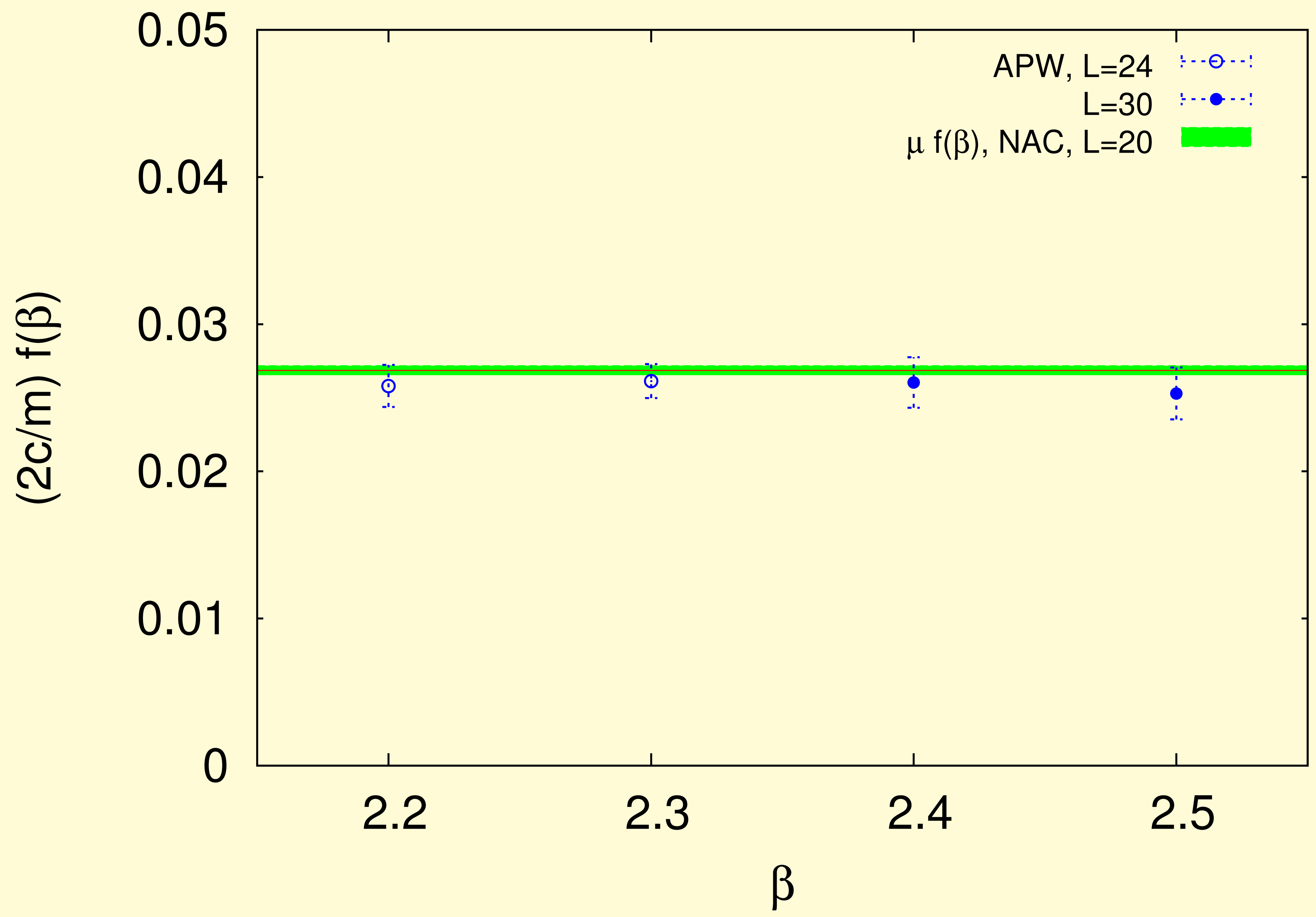
- *For **small-amplitude** constant configurations one can derive:*

$$\mu_{\mathrm{NAC}} \approx \frac{2c}{m}$$

- *Are values of μ_{NAC} extracted from data for sets of non-abelian constant configurations consistent with $2c_{\mathrm{APW}}/m_{\mathrm{APW}}$ obtained from data for abelian plane waves?*

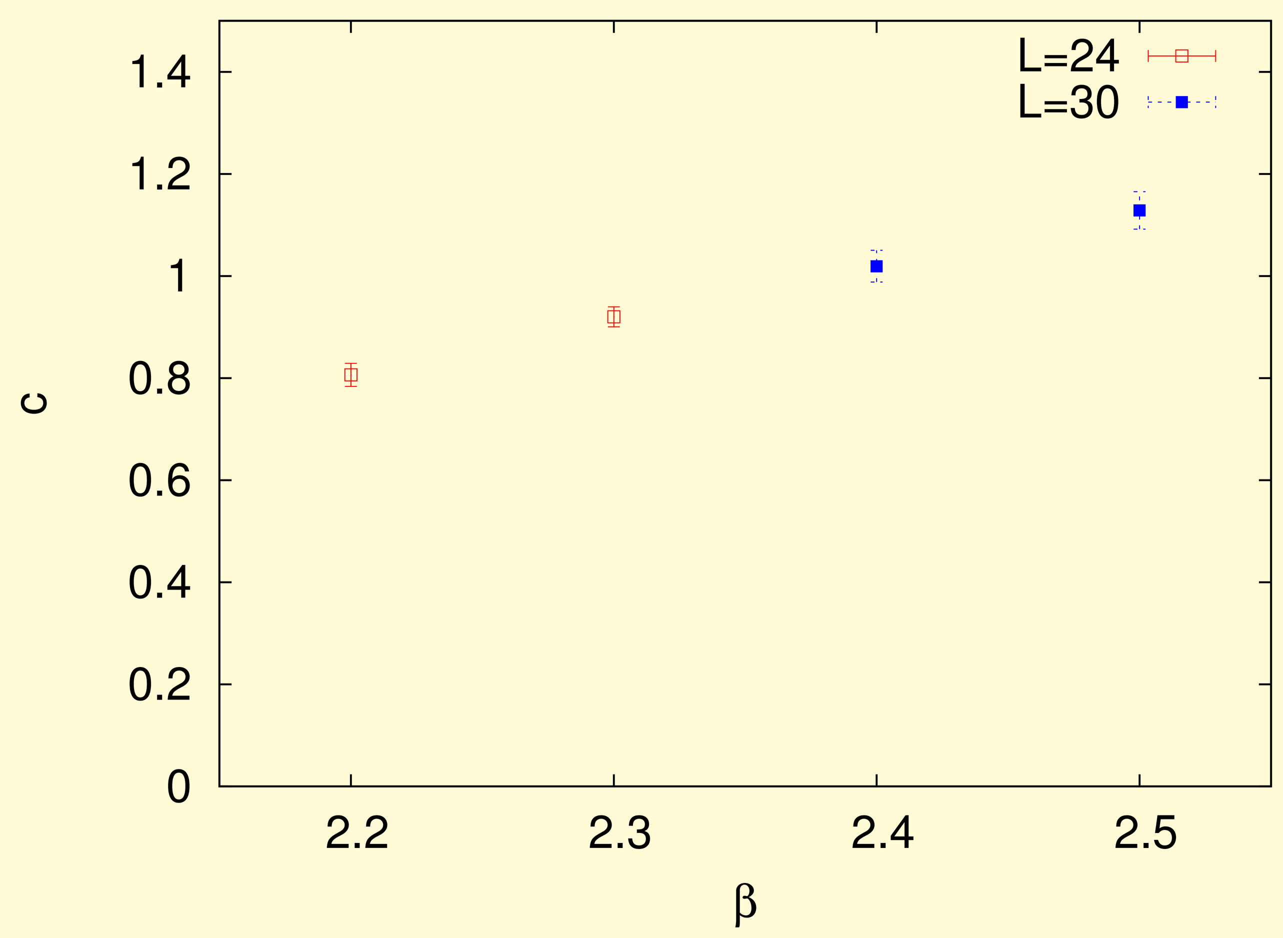
- *We compare $\mu_{\mathrm{NAC}}(\beta) f(\beta)$ to $(2c_{\mathrm{APW}}/m_{\mathrm{APW}}) f(\beta)$, where*

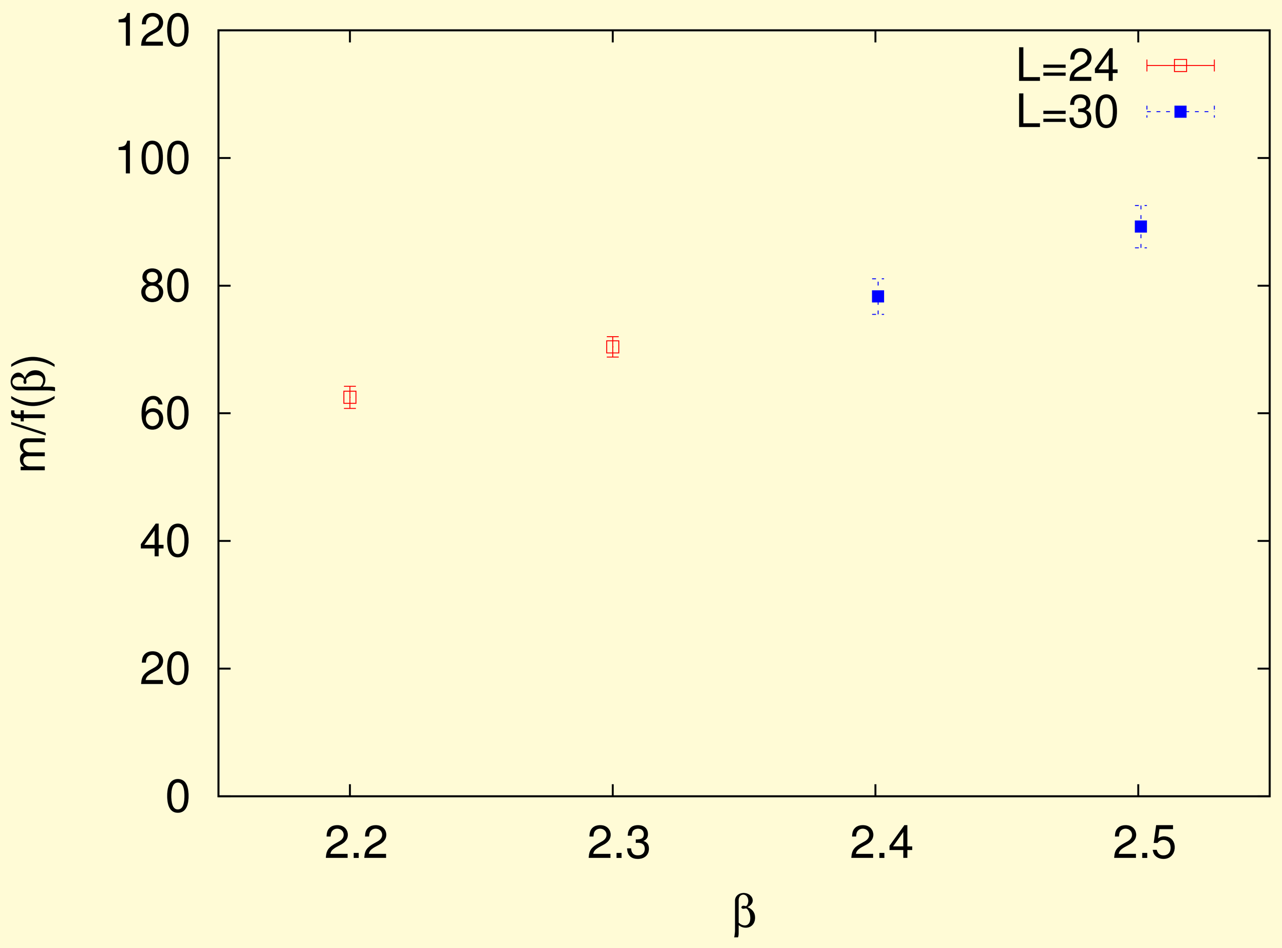
$$f(\beta) = \left(\frac{6\pi^2\beta}{11}\right)^{\frac{51}{121}} \exp\left(-\frac{3\pi^2\beta}{11}\right)$$

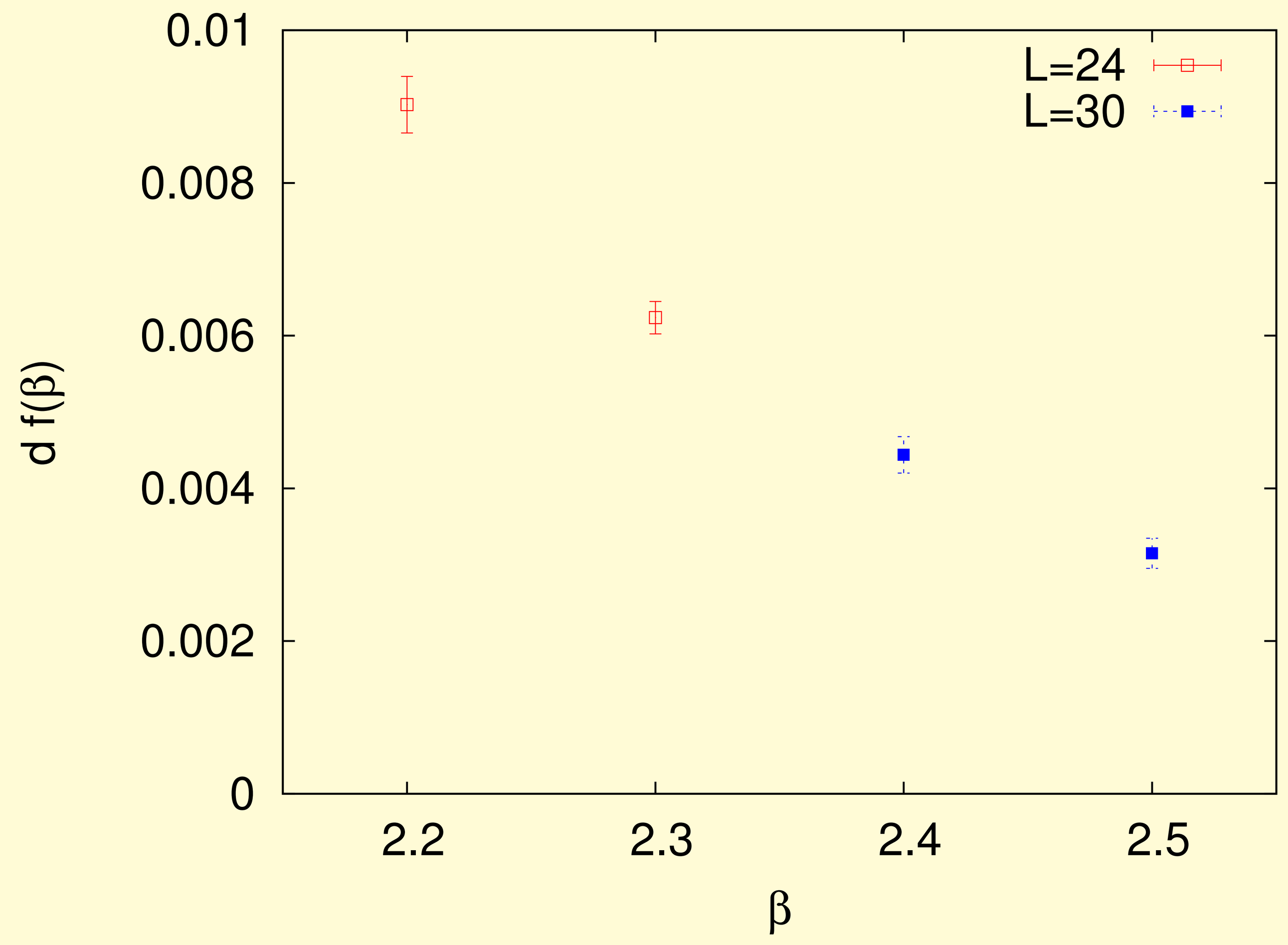


Scaling behaviour of $(2c_{\mathrm{APW}}/m_{\mathrm{APW}}) f(\beta)$; the band shows $\mu_{\mathrm{NAC}}(\beta) f(\beta) = 0.0269(3)$.

Scaling Behaviour of the Best-Fit Parameters







$$c(\beta) \stackrel{(?)}{\longrightarrow} const. \neq 0$$

$$m_{\text{ph}} \propto m(\beta)/f(\beta) \stackrel{(?)}{\longrightarrow} const. \neq 0$$

$$d_{\text{ph}} \propto d(\beta) f(\beta) \stackrel{(?)}{\longrightarrow} 0$$

Open End of the Romance

Thumbs Up

- *We have proposed an approximate form of the SU(2) YM vacuum wave-functional – looks good in D=2+1, somewhat worse in 3+1.*

- *There is a method to measure (on a lattice) relative probabilities of various gauge-field configurations in the YM vacuum.*

- *Both for NAC and for long-wavelength APW the measured probabilities are consistent with the dimensional reduction form, and the coefficients μ for these sets agree.*

- *The data are nicely described by a modification of our proposal, and the correction term may vanish in the continuum limit.*

Thumbs Down

- *In D=3+1, we have not found a method of generating configurations distributed according to the proposed vacuum wave functional.*

- *The method works reasonably well only for configurations rather close in configuration space.*

- *Neither the DR form, nor our proposal for the vacuum wave-functional describe data satisfactorily for larger plane-wave momenta.*

- *The configurations tested so far, NAC and APW, are rather atypical, not representatives of true vacuum fields.*

What Next?

Roads to Take

Compute, by the relative-weight method, weights of more realistic, "typical" configurations. E.g. one can generate ensembles of configurations, make their Fourier decomposition, switch on/off individual momentum modes, and compare the obtained momentum dependence with that following from the proposed VWF.

Find a way of generating field configurations distributed according (the square of) the proposed wave-functional. It was possible in D=2+1, but complicated (prohibitively?) by Bianchi constraints in 3+1.

Color screening?
N-ality dependence?
Dominant configurations?

???

Acknowledgements

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