On the origin of neutrino oscillations through Lorentz violation

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Introduction

- Higgs at the LHC $\rightarrow$ mass for quarks, charged leptons and vector bosons.
- What about neutrino masses? Seesaw mechanism?

- Another alternative: LV models that provide the possibility of generating neutrino masses and oscillations dynamically:
  - by adding R-H fields, as in the seesaw mechanism;
  - but also without the involvement of R-H neutrinos.
- Lorentz violation is achieved by the addition of higher order space derivatives suppressed by a large mass scale M, *e.g.*

$$F_{\mu\nu}\left(1-\frac{\Delta}{M^2}\right)F^{\mu\nu} \quad \text{or} \quad \bar{\psi}\left[i\partial\!\!\!/\left(1-\frac{\Delta}{M^2}\right)\right]\psi$$

- Such operators have been shown to arise in the low-energy limit of some quantum gravity theories[a] and agree with those of the SM extension[b].

[a] N. E. Mavromatos, Phys. Rev. D **83**, 025018 (2011)
[b] A. Kostelecky and M. Mewes, Phys. Rev. D **85**, 096005 (2012).

Introduction

- Here, we present two LV models:
- **I** - a massless bare neutrino doublet interacting with a LV $U(1)$ gauge field.
- The Schwinger-Dyson approach is used to calculate neutrino masses.
- **II** - a LV massless bare neutrino doublet with four-fermion interactions.
- The effective potential method is applied.

Introduction

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- **II** - a LV massless bare neutrino doublet with four-fermion interactions.
- The effective potential method is applied.
- In both studies, after quantization we take the Lorentz symmetric limit:

$$M \to \infty \quad \text{and} \quad e \to 0 \ ,$$

so that classical effects are completely negligible, while quantum effects completely change the picture, leading to finite observables, *i.e.* neutrino masses and oscillations.

Lorentz-violating model I

- The Lagrangian of our first model is[a]

$$\mathcal{L}_1 = -\frac{1}{4}F_{\mu\nu}\left(1 - \frac{\Delta}{M^2}\right)F^{\mu\nu} + \bar{\Psi}(i\partial\!\!\!/ - \tau A\!\!\!/)\Psi \ ,$$

where $\Delta = -\partial_i \partial^i \ , \quad \Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \ , \quad \tau = diag(e_1, e_2)$.

- The LV operator (Δ/M^2) improves the convergence and also introduces a mass scale.

[a] J. Alexandre, J. Leite and N. E. Mavromatos, Phys. Rev. D **87**, 125029 (2013)

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- Is it possible to generate the following mass matrix **M** by quantum corrections?

$$\mathbf{M} = \begin{pmatrix} m_1 & \mu \\ \mu & m_2 \end{pmatrix} , \quad \lambda_{\pm} = \frac{m_1 + m_2}{2} \pm \frac{\sqrt{(m_1 - m_2)^2 + 4\mu^2}}{2} .$$

[a] J. Alexandre, J. Leite and N. E. Mavromatos, Phys. Rev. D **87**, 125029 (2013)

Lorentz-violating model I

- From now on, we make use of the non-perturbative Schwinger-Dyson (S-D) equations.
- An infinity hierarchy of integral equations for the Green's functions of the theory.
- In particular, we consider the ladder approximation and focus only on the S-D equation for the fermion propagator:

$$G^{-1} - G_B^{-1} = \int_p D^B_{\mu\nu} \; \tau\gamma^\mu \; G \; \tau\gamma^\nu \; .$$

- For two flavours, the expression above becomes a set of 3 coupled equations.
- From this set of equations, the constraint below follows:

$$\mu(m_1 + m_2)(e_2 m_1 + e_1 m_2)(e_1 - e_2) = 0.$$

Lorentz-violating model I

- Solving the set of equations while taking into account this constraint, we find the following non-trivial solutions:

Case	Implies	$\lambda_\pm$	θ	Solution
I: $m_1 = m_2 \neq 0$	$e_1 = e_2$; $\mu^2 = m^2$	$2m$; 0	$\mp\pi/4(\mu = \pm m)$	$m = \frac{M}{2}\exp(-X/e^2)$
II: $m_i = 0$; $\mu \neq 0$	—	$\pm\mu$	$-\pi/4$	$\mu = M\exp(-X/e_1 e_2)$
III: $m_1 = -m_2 \neq 0$	$e_1 = e_2$	$\pm\sqrt{m^2+\mu^2}$	$\arctan\frac{-\mu}{m^2+\sqrt{m^2+\mu^2}}$	$m^2+\mu^2 = M^2\exp(\frac{-2X}{e^2})$
IV: $\mu = 0$	—	m_i	0	$m_i = M\exp(-X/e_i^2)$
V: $e_2 m_1 + e_1 m_2 = 0$; $m_1^2 \neq m_2^2$	—	—	—	$m \propto M$

- Oscillation probability: $\mathcal{P}(\nu_{\beta_1} \to \nu_{\beta_2}) = \sin^2(2\theta)\sin^2\left[\frac{(\lambda_+^2 - \lambda_-^2)L}{4E}\right]$.
- Thus, in case I, since $(\lambda_1^2 - \lambda_2^2) \neq 0$ and $\theta \neq 0$, oscillations are allowed.

Lorentz-violating model II

- LV kinematics with 4-fermion interactions[a]:

$$\mathcal{L}_2 = \bar{\Psi}\left[i(\partial_0\gamma^0 - \vec{\partial}\cdot\vec{\gamma})\left(1-\frac{\Delta}{M^2}\right)+\frac{\Delta}{M}\right]\Psi + \frac{1}{M^2}(\overline{\Psi}\tau_2\Psi)^2,$$

$$\text{with} \quad \Delta = -\partial_i\partial^i \ , \quad \Psi = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} \ , \quad \tau_2 = \begin{pmatrix} g_1 & g_3 \\ g_3 & g_2\end{pmatrix} \ .$$

- We can rewrite $\mathcal{L}_2$ by coupling Ψ to an auxiliary field ϕ using a Yukawa coupling as:

$$\mathcal{L}_2' = \bar{\Psi}\left[i(\partial_0\gamma^0 - \vec{\partial}\cdot\vec{\gamma})\left(1-\frac{\Delta}{M^2}\right)+\frac{\Delta}{M}\right]\Psi - \frac{M^2}{4}\phi^2 - \phi\bar{\Psi}\tau_2\Psi \ ,$$

- $\mathcal{L}_2$ is obtained from $\mathcal{L}_2'$ by integrating over the scalar field.

[a] J. Alexandre, J. Leite and N. E. Mavromatos, Phys. Rev. D **90**, 045026 (2014)

Lorentz-violating model II

- Then, we integrate $\mathcal{L}_2'$ over fermions to find the effective potential below:

$$V = \frac{M^2}{4}\phi^2 + i\mathrm{tr}\int \frac{d^4 p}{(2\pi)^4}(\ln o_+ - \ln o_-) \ ,$$

where $o_\pm$ are the eigenvalues in flavour space of the operator

$$\mathcal{O} = \begin{pmatrix} (\omega\gamma^0 - \vec{p}\cdot\vec{\gamma})(1+\frac{p^2}{M^2}) - \frac{p^2}{M} - g_1\phi & -g_3\phi \\ -g_3\phi & (\omega\gamma^0 - \vec{p}\cdot\vec{\gamma})(1+\frac{p^2}{M^2}) - \frac{p^2}{M} - g_2\phi \end{pmatrix} \ .$$

- Finally, we look for a non-trivial minimum, ϕ_0, of this effective potential by calculating $(dV/d\phi)_{\phi_0} = 0$ and we find that

$$\phi_0 = \alpha M(g_1 + g_2) \ .$$

- Therefore, from the Yukawa coupling term, neutrino masses are generated:

$$\mathcal{M} = \alpha M(g_1 + g_2)\ \tau_2 \ .$$

Lorentz-violating model II

- The Lorentz-symmetric limit, consists in taking the simultaneous limits

$$M \to \infty \quad \text{and} \quad g \to 0,$$

in such a way that the mass matrix $\mathcal{M}$ remains finite.

- Thus, in this limit, the LV loop corrections to the kinetic term vanish.
- Moreover, the oscillation probability is given by

$$\mathcal{P}(\nu_{\beta_1} \to \nu_{\beta_2}) = \sin^2(2\theta)\sin^2\left[\frac{(\lambda_+^2 - \lambda_-^2)L}{4E} + (\lambda_+ - \lambda_-)\frac{EL}{2M} + ...\right] ,$$

where $\lambda_\pm$ are the mass eigenvalues.

- The last term would be the correction due to the LV kinematics, however, it goes to zero when the Lorentz symmetric limit is taken.

Lorentz-violating model II

- **Majorana neutrinos**: $(\nu^M)^c \equiv C(\bar{\nu}^M)^T = \nu^M \;\rightarrow\; \nu_R = (\nu_L)^c$.
- **Extension I**: Left-handed neutrinos only ($\Psi \rightarrow \nu_L$).
- Majorana mass terms with L-H neutrinos only, $\propto \bar{\nu}_L \; M^M \; (\nu_L)^c + h.c.$.
- Rewriting our Lagrangian in the mass basis and choosing τ_2 to be diagonal:

$$\mathcal{L}^M = \frac{1}{2} \sum_{j=1,2} \bar{\nu}_j \left[i(\partial_0 \gamma^0 - \vec{\partial} \cdot \vec{\gamma}) \left(1 - \frac{\Delta}{M^2}\right) + \frac{\Delta}{M} \right] \nu_j + \frac{1}{M^2} (g_j \bar{\nu}_j \nu_j)^2,$$

  where $\nu_i = \nu_{iL} + (\nu_{iL})^c$ is a Majorana field.
- After calculating the minimum of the effective potential $V(\phi)$, we get

$$m_j = \alpha' M (g_1 + g_2) g_j.$$

- Thus, once it is rewritten in terms of the flavour eigenstates, flavour mixing will be naturally generated and oscillations allowed.

Lorentz-violating model II

- **Entension II**: Seesaw-type.
- Here, a fermion doublet with a L-H and a R-H neutrino is considered, so that

$$\mathcal{L}_2^M = \bar{\Psi}_M \left[i(\partial_0\gamma^0 - \vec{\partial}\cdot\vec{\gamma})\left(1 - \frac{\Delta}{M^2}\right) + \frac{\Delta}{M}\right]\Psi_M + \frac{1}{M^2}(\bar{\Psi}_M\tau_2\Psi_M)^2 \ ,$$

$$\text{with} \quad \Psi_M = \begin{pmatrix} \nu_L \\ N_R \end{pmatrix}$$

- In this case, using the same mechanism as before, the following Dirac (m_D) and Majorana (m_L and m_R) mass matrix is generated

$$\mathcal{M} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} = \alpha M g_2 \begin{pmatrix} g_1 & g_3 \\ g_3 & g_2 \end{pmatrix} \ ,$$

- So that, if $g_1 = 0$ and $g_3 \ll g_2$ (type-I seesaw), we find

$$m_1 \approx m_D^2/m_R \ll m_D \qquad m_2 \approx m_R \gg m_D \ .$$

Conclusions and Outlook

- Using LV extensions of the SM, we have presented alternative mechanisms to generate neutrino masses and oscillations.
- In model I, only one solution allowed for neutrino oscillations. However, in such a case, one of the neutrinos is left massless and $\theta = \pi/4$.
- On the other hand, the second model presented a wide range of interesting solutions, with arbitrary masses and mixing angles.
- Then, two possible extensions to Majorana neutrinos were also considered:
  - only L-H fields: masses and oscillations can be generated dynamically.
  - Adding R-H neutrinos as in seesaw mechanisms: mass for heavy sterile neutrinos can be generated.
- For both models, when the Lorentz-symmetric limit is taken, the only observable effects are the dynamical generation of mass and oscillations.

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- For both models, when the Lorentz-symmetric limit is taken, the only observable effects are the dynamical generation of mass and oscillations.
- An extension of these works consists in deriving LV operators from a LV alternative gravitational model, as Horava-Lifshitz gravity for example.