On the origin of neutrino oscillations through Lorentz violation

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DISCRETE 2014
London-UK
Higgs at the LHC → mass for quarks, charged leptons and vector bosons.

What about neutrino masses? Seesaw mechanism?

Another alternative: LV models that provide the possibility of generating neutrino masses and oscillations dynamically:
- by adding R-H fields, as in the seesaw mechanism;
- but also without the involvement of R-H neutrinos.

Lorentz violation is achieved by the addition of higher order space derivatives suppressed by a large mass scale $M$, e.g.

$$F_{\mu \nu} \left(1 - \frac{\Delta}{M^2}\right) F^{\mu \nu} \text{ or } \bar{\psi} \left[i \Phi \left(1 - \frac{\Delta}{M^2}\right)\right] \psi$$

Such operators have been shown to arise in the low-energy limit of some quantum gravity theories$^a$ and agree with those of the SM extension$^b$.

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$^a$N. E. Mavromatos, Phys. Rev. D 83, 025018 (2011)
Here, we present two LV models:

I - a massless bare neutrino doublet interacting with a LV $U(1)$ gauge field. The Schwinger-Dyson approach is used to calculate neutrino masses.

II - a LV massless bare neutrino doublet with four-fermion interactions. The effective potential method is applied.
Introduction

Here, we present two LV models:

I - a massless bare neutrino doublet interacting with a LV $U(1)$ gauge field.
The Schwinger-Dyson approach is used to calculate neutrino masses.

II - a LV massless bare neutrino doublet with four-fermion interactions.
The effective potential method is applied.

In both studies, after quantization we take the Lorentz symmetric limit:

$$M \to \infty \quad \text{and} \quad e \to 0,$$

so that classical effects are completely negligible, while quantum effects completely change the picture, leading to finite observables, i.e. neutrino masses and oscillations.
Lorentz-violating model I

- The Lagrangian of our first model is\(^{a}\)

\[
\mathcal{L}_1 = -\frac{1}{4} F_{\mu\nu} \left( 1 - \frac{\Delta}{M^2} \right) F^{\mu\nu} + \bar{\Psi} (i\partial - \tau \hat{A}) \Psi ,
\]

where \(\Delta = -\partial_i \partial^i\), \(\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}\), \(\tau = \text{diag}(e_1, e_2)\).

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- Is it possible to generate the following mass matrix \( M \) by quantum corrections?

\[ M = \begin{pmatrix} m_1 & \mu \\ \mu & m_2 \end{pmatrix}, \quad \lambda_{\pm} = \frac{m_1 + m_2}{2} \pm \frac{\sqrt{(m_1 - m_2)^2 + 4\mu^2}}{2}. \]

Lorentz-violating model I

- From now on, we make use of the non-perturbative Schwinger-Dyson (S-D) equations.
- An infinity hierarchy of integral equations for the Green’s functions of the theory.
- In particular, we consider the ladder approximation and focus only on the S-D equation for the fermion propagator:

\[
G^{-1} - G_B^{-1} = \int_p D^B_{\mu\nu} \tau \gamma^\mu G \tau \gamma^\nu.
\]

- For two flavours, the expression above becomes a set of 3 coupled equations.
- From this set of equations, the constraint below follows:

\[
\mu(m_1 + m_2)(e_2 m_1 + e_1 m_2)(e_1 - e_2) = 0.
\]
Solving the set of equations while taking into account this constraint, we find the following non-trivial solutions:

<table>
<thead>
<tr>
<th>Case</th>
<th>Implies</th>
<th>$\lambda_\pm$</th>
<th>$\theta$</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>I: $m_1 = m_2 \neq 0$</td>
<td>$e_1 = e_2; \mu^2 = m^2$</td>
<td>$2m; 0$</td>
<td>$\mp \pi/4 (\mu = \pm m)$</td>
<td>$m = \frac{M}{2} \exp(-X/e^2)$</td>
</tr>
<tr>
<td>II: $m_i = 0; \mu \neq 0$</td>
<td>—</td>
<td>$\pm \mu$</td>
<td>$-\pi/4$</td>
<td>$\mu = M \exp(-X/e_1e_2)$</td>
</tr>
<tr>
<td>III: $m_1 = -m_2 \neq 0$</td>
<td>$e_1 = e_2$</td>
<td>$\pm \sqrt{m^2 + \mu^2}$</td>
<td>$\arctan \frac{-\mu}{m^2 + \sqrt{m^2 + \mu^2}}$</td>
<td>$m^2 + \mu^2 = M^2 \exp(-\frac{2X}{e^2})$</td>
</tr>
<tr>
<td>IV: $\mu = 0$</td>
<td>—</td>
<td>$m_i$</td>
<td>0</td>
<td>$m_i = M \exp(-X/e_i^2)$</td>
</tr>
<tr>
<td>V: $e_2 m_1 + e_1 m_2 = 0; m_1^2 \neq m_2^2$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>$m \propto M$</td>
</tr>
</tbody>
</table>

**Oscillation probability:** $\mathcal{P}(\nu_{\beta_1} \rightarrow \nu_{\beta_2}) = \sin^2(2\theta) \sin^2 \left[ \frac{(\lambda^2_+ - \lambda^2_-)}{4E} L \right]$.

Thus, in case I, since $(\lambda^2_1 - \lambda^2_2) \neq 0$ and $\theta \neq 0$, oscillations are allowed.
Lorentz-violating model II

- LV kinematics with 4-fermion interactions:\(^a\):

\[
\mathcal{L}_2 = \bar{\Psi} \left[ i(\partial_0 \gamma^0 - \vec{\partial} \cdot \vec{\gamma}) \left(1 - \frac{\Delta}{M^2}\right) + \frac{\Delta}{M}\right] \Psi + \frac{1}{M^2} (\bar{\Psi} \tau_2 \Psi)^2,
\]

with \(\Delta = -\partial_i \partial^i\), \(\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}\), \(\tau_2 = \begin{pmatrix} g_1 & g_3 \\ g_3 & g_2 \end{pmatrix}\).

- We can rewrite \(\mathcal{L}_2\) by coupling \(\Psi\) to an auxiliary field \(\phi\) using a Yukawa coupling as:

\[
\mathcal{L}'_2 = \bar{\Psi} \left[ i(\partial_0 \gamma^0 - \vec{\partial} \cdot \vec{\gamma}) \left(1 - \frac{\Delta}{M^2}\right) + \frac{\Delta}{M}\right] \Psi - \frac{M^2}{4} \phi^2 - \phi \bar{\Psi} \tau_2 \Psi,
\]

- \(\mathcal{L}_2\) is obtained from \(\mathcal{L}'_2\) by integrating over the scalar field.

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\(^a\)J. Alexandre, J. Leite and N. E. Mavromatos, Phys. Rev. D 90, 045026 (2014)
Lorentz-violating models

**Lorentz-violating model II**

- Then, we integrate $\mathcal{L}_2'$ over fermions to find the effective potential below:

$$V = \frac{M^2}{4} \phi^2 + i \text{tr} \int \frac{d^4 p}{(2\pi)^4} \left( \ln o_+ - \ln o_- \right),$$

where $o_\pm$ are the eigenvalues in flavour space of the operator

$$\mathcal{O} = \begin{pmatrix}
(\omega \gamma^0 - \vec{p} \cdot \vec{\gamma})(1 + \frac{p^2}{M^2}) - \frac{p^2}{M} - g_1 \phi & -g_3 \phi \\
-g_3 \phi & (\omega \gamma^0 - \vec{p} \cdot \vec{\gamma})(1 + \frac{p^2}{M^2}) - \frac{p^2}{M} - g_2 \phi
\end{pmatrix}.$$  

- Finally, we look for a non-trivial minimum, $\phi_0$, of this effective potential by calculating $(dV/d\phi)_{\phi_0} = 0$ and we find that

$$\phi_0 = \alpha M (g_1 + g_2).$$

- Therefore, from the Yukawa coupling term, neutrino masses are generated:

$$\mathcal{M} = \alpha M (g_1 + g_2) \tau_2.$$
Lorentz-violating model II

- The Lorentz-symmetric limit, consists in taking the simultaneous limits

\[ M \to \infty \quad \text{and} \quad g \to 0, \]

in such a way that the mass matrix \( M \) remains finite.

- Thus, in this limit, the LV loop corrections to the kinetic term vanish.

- Moreover, the oscillation probability is given by

\[ P(\nu_{\beta_1} \to \nu_{\beta_2}) = \sin^2(2\theta) \sin^2 \left[ \frac{(\lambda^2_+ - \lambda^2_-)L}{4E} + (\lambda_+ - \lambda_-) \frac{EL}{2M} + \ldots \right], \]

where \( \lambda_{\pm} \) are the mass eigenvalues.

- The last term would be the correction due to the LV kinematics, however, it goes to zero when the Lorentz symmetric limit is taken.
Lorentz-violating model II

- **Majorana neutrinos**: \((\nu^M)^c \equiv C(\bar{\nu}^M)^T = \nu^M \to \nu_R = (\nu_L)^c\).
- **Extension I**: Left-handed neutrinos only \((\Psi \to \nu_L)\).
- Majorana mass terms with L-H neutrinos only, \(\propto \bar{\nu}_L M^M (\nu_L)^c + h.c..\)
- Rewriting our Lagrangian in the mass basis and choosing \(\tau_2\) to be diagonal:

\[
\mathcal{L}^M = \frac{1}{2} \sum_{j=1,2} \bar{\nu}_j \left[ i(\partial_0 \gamma^0 - \vec{\partial} \cdot \vec{\gamma}) \left( 1 - \frac{\Delta}{M^2} \right) + \frac{\Delta}{M} \right] \nu_j + \frac{1}{M^2} (g_j \bar{\nu}_j \nu_j)^2,
\]

where \(\nu_i = \nu_{iL} + (\nu_{iL})^c\) is a Majorana field.
- After calculating the minimum of the effective potential \(V(\phi)\), we get

\[
m_j = \alpha' M (g_1 + g_2) g_j.
\]
- Thus, once it is rewritten in terms of the flavour eigenstates, flavour mixing will be naturally generated and oscillations allowed.
Lorentz-violating models

Lorentz-violating model II

- **Extension II**: Seesaw-type.

Here, a fermion doublet with a L-H and a R-H neutrino is considered, so that

\[
\mathcal{L}_2^M = \bar{\Psi}_M \left[ i(\partial_0 \gamma^0 - \vec{\partial} \cdot \vec{\gamma}) \left( 1 - \frac{\Delta}{M^2} \right) + \frac{\Delta}{M} \right] \Psi_M + \frac{1}{M^2} (\bar{\Psi}_M \tau_2 \Psi_M)^2 ,
\]

with \( \Psi_M = \begin{pmatrix} \nu_L \\ N_R \end{pmatrix} \)

In this case, using the same mechanism as before, the following Dirac \( m_D \) and Majorana \( m_L \) and \( m_R \) mass matrix is generated

\[
\mathcal{M} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} = \alpha M g_2 \begin{pmatrix} g_1 & g_3 \\ g_3 & g_2 \end{pmatrix} ,
\]

So that, if \( g_1 = 0 \) and \( g_3 \ll g_2 \) (type-I seesaw), we find

\[
m_1 \approx m_D^2 / m_R \ll m_D \quad m_2 \approx m_R \gg m_D .
\]
Using LV extensions of the SM, we have presented alternative mechanisms to generate neutrino masses and oscillations.

In model I, only one solution allowed for neutrino oscillations. However, in such a case, one of the neutrinos is left massless and $\theta = \pi/4$.

On the other hand, the second model presented a wide range of interesting solutions, with arbitrary masses and mixing angles.

Then, two possible extensions to Majorana neutrinos were also considered:
- only L-H fields: masses and oscillations can be generated dynamically.
- Adding R-H neutrinos as in seesaw mechanisms: mass for heavy sterile neutrinos can be generated.

For both models, when the Lorentz-symmetric limit is taken, the only observable effects are the dynamical generation of mass and oscillations.
Conclusions and Outlook

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- An extension of these works consists in deriving LV operators from a LV alternative gravitational model, as Horava-Lifshitz gravity for example.