T Invariance and Neutrino Oscillations in a Symmetric Medium

J. C. D'Olivo
Univ. de Valencia

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✓Neutrino oscillations experiments can be interpreted in a 3ν framework, where the flavor eigenstates are linear combinations of the mass eigenstates.

✓The measurement of θ_{13} has open the door to CP violation searches in neutrino oscillations.

✓In most oscillation experiments neutrinos propagate considerable distances in matter. An accurate description of neutrino oscillations in a medium becomes an important ingredient in the analysis of the data.

✓For an arbitrary density profile the evolution equation for the flavor amplitudes admits no exact solution, even in the 2ν case.

✓Numerical integrations have been extensively used to examine the phenomenon.

✓Analytic solutions still useful: Significant insight into the physics of the problem and a better understanding of the dependence on the neutrino parameters and the properties of the medium.

Matter neutrino oscillations

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}\,|\nu_i\rangle \qquad (i = 1, 2, 3)$$

$$|\psi(t)\rangle = \sum_\beta \psi_\beta(t)\,|\nu_\beta\rangle \qquad (\beta = \mathrm{e}, \mu, \tau)$$

$$|\psi(t_0)\rangle = |\nu_\alpha\rangle, \qquad P(\nu_\alpha \to \nu_\beta) = |\psi_\beta(t)|^2$$

$L \simeq t - t_0$: distance from the production point

$\Psi(t) = U\,\Phi(t)$ $\quad U$: Mixing matrix

$$\Psi(t) = \begin{pmatrix} \psi_e(t) \\ \psi_\mu(t) \\ \psi_\tau(t) \end{pmatrix}, \qquad \Phi(t) = \begin{pmatrix} \phi_1(t) \\ \phi_2(t) \\ \phi_3(t) \end{pmatrix}$$

$$i\frac{d}{dt}\Psi(t) = H(t)\Psi(t) \qquad (\hbar = c = 1)$$

$$H(t) = UH_0U^\dagger + V(t)Y$$

$$H_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta_{21} & 0 \\ 0 & 0 & \Delta_{31} \end{pmatrix} \qquad Y = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Delta_{ij} = \frac{m_i^2 - m_j^2}{2E}, \qquad V(t) = V_e(t) - V_{\mu,\tau}(t) = \sqrt{2}\, G_F n_e(t)$$

Mixing matrix in vacuum

$$U = \mathcal{O}_{23}\, \Gamma \mathcal{O}_{13} \Gamma^* \mathcal{O}_{12}$$

$$\mathcal{O}_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \mathcal{O}_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}$$

$$\mathcal{O}_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}, \qquad \Gamma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix}$$

Mixing matrix in matter

$$U_m^\dagger(t) H(t) U_m(t) = H_D(t) = \begin{pmatrix} \mathcal{E}_1(t) & 0 & 0 \\ 0 & \mathcal{E}_2(t) & 0 \\ 0 & 0 & \mathcal{E}_3(t) \end{pmatrix}$$

$$[Y, \mathcal{O}_{23}] = [Y, \Gamma] = 0 \quad \Rightarrow \quad H(t) = \mathcal{O}_{23} \Gamma \widetilde{H}(t) \Gamma^* \mathcal{O}_{23}^{\mathrm{T}}$$

$$\widetilde{H}(t) = \mathcal{O} H_0 \mathcal{O}^{\mathrm{T}} + V(t) Y, \qquad \mathcal{O} = \mathcal{O}_{13} \mathcal{O}_{12}$$

T: transpose

- $\widetilde{H}(t)$is a real symmetric matrix and can be diagonalized by an orthogonal transformation $\mathcal{O}_m(t)$:

$$\mathcal{O}_m^{\mathrm{T}}(t) \widetilde{H}(t) \mathcal{O}_m(t) = H_D(t)$$

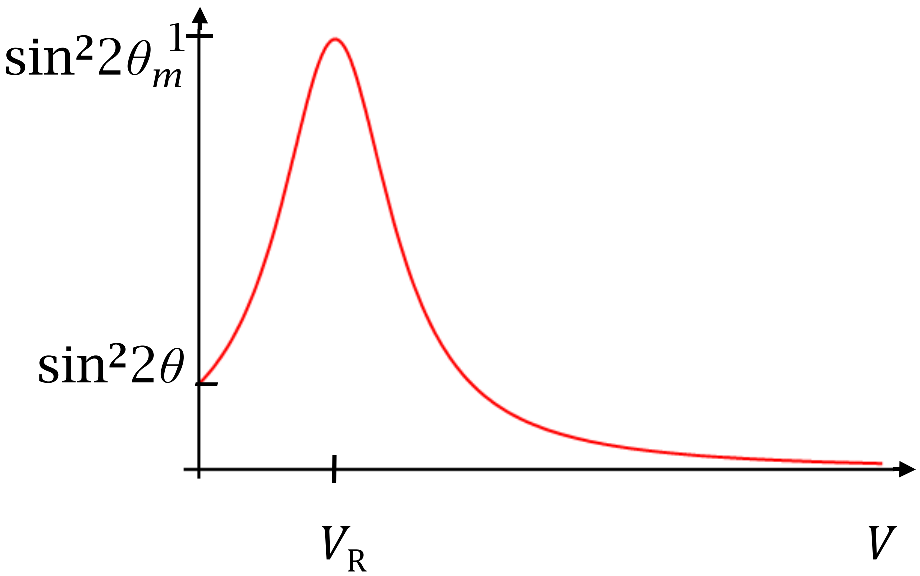
$$U_m(t) = \widetilde{U}_m(t) \Gamma^*, \qquad \widetilde{U}_m(t) = \mathcal{O}_{23} \Gamma \mathcal{O}_m(t)$$

$$\mathcal{O}_m(t) \to \mathcal{O}_{13} \mathcal{O}_{12} \quad \text{for} \quad V(t) \to 0$$

$$\Delta m^2_{21} \ll |\Delta m^2_{31}| \quad \Rightarrow \quad \mathcal{O}_m(t) \simeq \mathcal{O}^m_{13}(t)\mathcal{O}^m_{12}(t)$$

$$\mathcal{O}^m_{12}(t) = \begin{pmatrix} c^m_{12}(t) & s^m_{12}(t) & 0 \\ -s^m_{12}(t) & c^m_{12}(t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathcal{O}^m_{13}(t) = \begin{pmatrix} c^m_{13}(t) & 0 & s^m_{13}(t) \\ 0 & 1 & 0 \\ -s^m_{13}(t) & 0 & c^m_{13}(t) \end{pmatrix}$$

$$s^m_{ij}(t) = \sin\theta^m_{ij}(t) \qquad c^m_{ij}(t) = \cos\theta^m_{ij}(t)$$



$$\sin 2\theta^m_{12}(t) = \frac{\Delta_{21}}{\Delta^m_{21}(t)} \sin 2\theta_{12}$$

$$\sin 2\theta^m_{13}(t) = \frac{\Delta_{31} - \Delta_{21} s^2_{12}}{\Delta^m_{31}(t)} \sin 2\theta_{13}$$

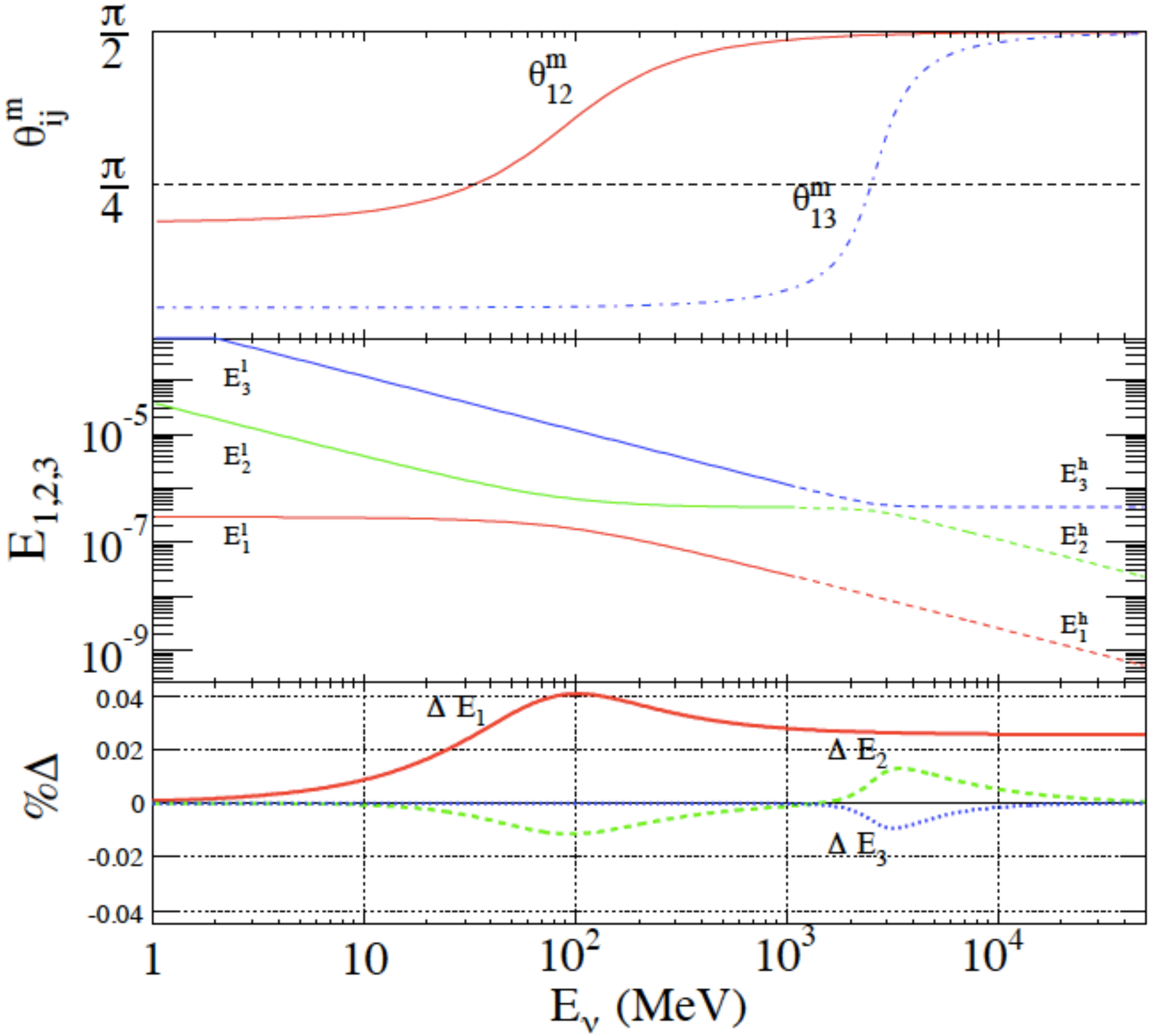
Low resonance $$V_\ell^{\mathrm{R}} = V(t_\ell) = \frac{\Delta_{21}}{c_{13}^2}\cos 2\theta_{12}$$

High resonance $$V_h^{\mathrm{R}} = V(t_h) = \left(\Delta_{31} - \Delta_{21} s_{12}^2\right)\cos 2\theta_{13}$$

$$\Delta_{21}^m = c_{13}^2\sqrt{(V - V_\ell^{\mathrm{R}})^2 + (V_\ell^{\mathrm{R}}\tan 2\theta_{12})^2}$$
$$\Delta_{32}^m = \sqrt{(V - V_h^{\mathrm{R}})^2 + (V_h^{\mathrm{R}}\tan 2\theta_{13})^2}$$

$$\mathcal{E}_2 - \mathcal{E}_1 \simeq \Delta_{21}^m(t), \qquad t \sim t_\ell$$
$$\mathcal{E}_3 - \mathcal{E}_1 \simeq \Delta_{31}^m(t), \qquad t \sim t_h$$

$$\mathcal{E}_1(t) \simeq \tfrac{1}{2}\Big[\,(\Delta_{21} + V c_{13}^2) - \Delta_{21}^m(t)\Big]$$
$$\mathcal{E}_2(t) \simeq \tfrac{1}{2}\Big[\,(\Delta_{31} + \Delta_{21} c_{12}^2 + V s_{13}^2) + (\Delta_{21}^m - \Delta_{32}^m)(t)\Big]$$
$$\mathcal{E}_3(t) \simeq \tfrac{1}{2}\Big[\,(\Delta_{31} + \Delta_{21} s_{12}^2 + V) + \Delta_{32}^m(t)\Big]$$



$V = 4.54 \times 10^{-4}\ \mathrm{eV}^2/\mathrm{GeV}$ (Earth's core)

$\Delta m^2_{21} = 7.59 \times 10^{-5}\ \mathrm{eV}^2, \quad |\Delta m^2_{31}| = 2.35 \times 10^{-3}\ \mathrm{eV}^2$

$\theta_{12} = 34°, \quad \theta_{13} = 8.9°, \quad \delta = 0$

Adiabatic basis

$$H(t)|\nu_i^a(t)\rangle = \mathcal{E}_i(t)|\nu_i^a(t)\rangle$$

$$|\psi(t)\rangle = \sum_i \phi_i^a(t)|\nu_i^a(t)\rangle \qquad (i = 1, 2, 3)$$

$$\Psi(t) = \widetilde{U}_m(t)\tilde{\Phi}_a(t), \qquad \tilde{\Phi}_a(t) = \Gamma^* \begin{pmatrix} \phi_1^a(t) \\ \phi_2^a(t) \\ \phi_3^a(t) \end{pmatrix}$$

$$i\frac{d}{dt}\tilde{\Phi}_a(t) = \mathcal{H}(t)\tilde{\Phi}_a(t), \qquad \mathcal{H}(t) = H_D(t) - i\mathcal{O}_m^{\mathrm{T}}(t)\dot{\mathcal{O}}_m(t)$$

$$\mathcal{O}_m^{\mathrm{T}}\dot{\mathcal{O}}_m \simeq \dot{\theta}_{12}^m \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \dot{\theta}_{13}^m \begin{pmatrix} 0 & 0 & c_{12}^m \\ 0 & 0 & s_{12}^m \\ -c_{12}^m & -s_{12}^m & 0 \end{pmatrix}$$

Time reversal

$$\boxed{\begin{aligned} T\big(a|\phi\rangle + b|\psi\rangle\big) &= a^*|T\phi\rangle + b^*|T\psi\rangle \\ \langle T\phi|T\psi\rangle &= \langle\phi|\psi\rangle^* = \langle\psi|\phi\rangle \qquad \big(|T\psi\rangle = T|\psi\rangle\big) \end{aligned}}$$

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = \hat{H}(t)|\psi(t)\rangle \qquad t_0 \leqslant t \leqslant t_f$$

- If $T\hat{H}(2\bar{t}-t)T^\dagger = \hat{H}(t)$, $\quad \bar{t} = (t_f + t_0)/2 \quad (T^\dagger = T^1)$ the system is *time-reversal invariant* (more properly, *motion reversal invariant*).
- $\hat{H}$ time independent $\Longrightarrow$ $\big[\hat{H}, T\big] = 0$
- For any $|\psi(t)\rangle$ there exist another solution of the Sch. eq. $|\psi'(t)\rangle = T|\psi(2\bar{t}-t)\rangle$ which satisfies the initial condition $|\psi'(t_0)\rangle = T|\psi(t_f)\rangle$.

For a spinless system $T^2 = I$ and a T-invariant vector basis $\{|\alpha_i\rangle, i = 1, 2, \cdots\}$ can be constructed: $T|\alpha_i\rangle = |\alpha_i\rangle$

$$i\hbar \frac{d}{dt}\Psi(t) = H(t)\Psi(t)$$

$$H(t) = \begin{pmatrix} h_{11}(t) & h_{12}(t) & \cdots \\ h_{21}(t) & h_{22}(t) & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}, \qquad \Psi(t) = \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \\ \vdots \end{pmatrix}$$

$$h_{ij}(t) = \langle \alpha_i | \hat{H}(t)\alpha_j \rangle, \qquad \psi_i(t) = \langle \alpha_i | \psi(t) \rangle$$

$$h^*_{ij}(2\bar{t} - t) = h_{ij}(t) \qquad H^*(2\bar{t} - t) = H(t)$$

$$\psi'_i(t) = \psi^*_i(2\bar{t} - t) \qquad \Psi'(t) = \Psi^*(2\bar{t} - t)$$

Evolution operator

$$\Psi(t) = \mathcal{U}(t,\tilde{t})\Psi(\tilde{t}), \qquad \Psi'(t) = \mathcal{U}(t,\tilde{t})\Psi'(\tilde{t})$$

$$i\hbar\frac{\partial}{\partial t}\mathcal{U}(t,\tilde{t}) = H(t)\mathcal{U}(t,\tilde{t}), \qquad \mathcal{U}(\tilde{t},\tilde{t}) = I$$

$$\mathcal{U}^{\dagger}(t,\tilde{t}) = \mathcal{U}^{-1}(t,\tilde{t}) = \mathcal{U}(\tilde{t},t)$$

$$\Psi'(t_f) = \Psi^*(t_0) \qquad \mathcal{U}^*(2\bar{t}-t,t_0) = \mathcal{U}(t,t_f)$$

$$\Psi'(\bar{t}) = \Psi^*(\bar{t}) \qquad \mathcal{U}^*(2\bar{t}-t,\bar{t}) = \mathcal{U}(t,\bar{t})$$

$$t = t_f$$

$$\boxed{\mathcal{U}(t_f,\bar{t}) = \mathcal{U}^*(t_0,\bar{t}) = \mathcal{U}^{\mathrm{T}}(\bar{t},t_0)}$$

T: transpose of the matrix

$$\mathcal{U}(t_f,t_0) = \mathcal{U}(t_f,\bar{t})\mathcal{U}(\bar{t},t_0) = \mathcal{U}^{\mathrm{T}}(\bar{t},t_0)\mathcal{U}(\bar{t},t_0)$$

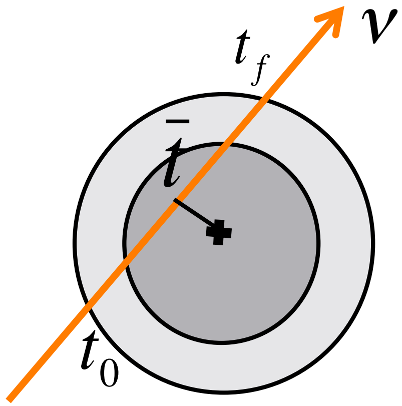
$$\mathcal{U}^{\mathrm{T}}(t_f,t_0) = \mathcal{U}(t_f,t_0)$$

$\mathcal{U}(t, t_0)$: Evolution op. in the flavor basis
$\mathcal{U}_a(t, t_0)$: Evolution op. in the adiabatic basis

$$\mathcal{U}(t, t_0) = \widetilde{U}_m(t)\,\mathcal{U}_a(t, t_0)\,\widetilde{U}_m^{\dagger}(t_0)$$

$$i\frac{\partial}{\partial t}\mathcal{U}_a(t, t_0) = \mathcal{H}(t)\,\mathcal{U}_a(t, t_0), \qquad \mathcal{U}_a(t_0, t_0) = I$$

Symmetric Potential



$$\mathcal{U}_a(t_f, t_0) = \mathcal{U}_a(t_f, \bar{t})\,\mathcal{U}_a(\bar{t}, t_0)$$

$$V(2\bar{t} - t) = V(t) \quad \Rightarrow \quad \mathcal{H}^*(2\bar{t} - t) = \mathcal{H}(t)$$

$$\mathcal{U}_a(\bar{t}, t_0) = \mathcal{U}_a^{\mathrm{T}}(t_f, \bar{t})$$

Adiabatic evolution

$$
\begin{aligned}
\mathcal{P}(t,\bar{t}) &= \exp\left(-i\int_{\bar{t}}^{t} dt' H_D(t')\right), \qquad \alpha_j(t,\bar{t}) = \int_{\bar{t}}^{t} dt' \mathcal{E}_j(t') \\
&= \begin{pmatrix} e^{-i\alpha_1(t,\bar{t})} & 0 & 0 \\ 0 & e^{-i\alpha_2(t,\bar{t})} & 0 \\ 0 & 0 & e^{-i\alpha_3(t,\bar{t})} \end{pmatrix}
\end{aligned}
$$

$$\mathcal{U}_a(t,\bar{t}) = \mathcal{P}(t,\bar{t})\check{\mathcal{U}}(t,\bar{t})$$

Non adiabatic corrections

$$i\frac{d}{dt}\check{\mathcal{U}}(t,\bar{t}) = \check{\mathcal{H}}(t)\check{\mathcal{U}}(t,\bar{t}), \qquad \check{\mathcal{U}}(t,\bar{t}) = I$$

$$\check{\mathcal{H}}(t) = -i\mathcal{P}^{\dagger}(t,\bar{t})\left[\mathcal{O}_m^T(t)\dot{\mathcal{O}}_m(t)\right]\mathcal{P}(t,\bar{t})$$

$$\check{\mathcal{H}}(t) = \mathcal{H}_\ell(t) + \mathcal{H}_h(t)$$

$$\mathcal{H}_\ell(t) = \begin{pmatrix} 0 & \varrho_l(t) & 0 \\ \varrho_l^*(t) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathcal{H}_h(t) \cong \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \varrho_h(t) \\ 0 & \varrho_h^*(t) & 0 \end{pmatrix}$$

$$\begin{aligned} \varrho_l(t) &= -i\,\dot{\theta}_{12}^m(t)\exp[-i\phi_{21}(t,\bar{t}\,)] \\ \varrho_h(t) &\cong -i\,\dot{\theta}_{13}^m(t)\exp[-i\phi_{32}(t,\bar{t}\,)] \end{aligned}$$

$$\dot{\theta}_{12}^m(t) = \frac{\sin^2 2\theta_{12}^m(t)}{2\sin 2\theta_{12}}\,\frac{c_{13}^2\,\dot{V}(t)}{\Delta_{21}}, \quad \dot{\theta}_{13}^m(t) = \frac{\sin^2 2\theta_{13}^m(t)}{2\sin 2\theta_{13}}\,\frac{\dot{V}(t)}{\Delta_{31} - \Delta_{21}s_{12}^2}$$

$$\phi_{ij}(t,\bar{t}) = \int_{\bar{t}}^{t} dt'\,\left[\mathcal{E}_i(t') - \mathcal{E}_j(t')\right]$$

$$H = H_1 + H_2$$

$$\mathcal{U}(t, t_0) = \mathcal{U}_1(t, t_0)\,\mathcal{U}'(t, t_0)$$

$$i\hbar \frac{\partial}{\partial t} \mathcal{U}_1 = H_1 \mathcal{U}_1 \qquad i\hbar \frac{\partial}{\partial t} \mathcal{U}' = H' \mathcal{U}'$$

$$H' = \mathcal{U}_1^\dagger H_2 \mathcal{U}_1$$

$$[\mathcal{U}_1, H_2] = 0 \quad \forall\, t \qquad \Rightarrow \qquad \mathcal{U}' = \mathcal{U}_2$$

$$\mathcal{U}(t, t_0) = \mathcal{U}_1(t, t_0)\,\mathcal{U}_2(t, t_0)$$

$\mathcal{U}_\ell(t,\bar{t})$: Evolution operator corresponding to $\mathcal{H}_\ell(t)$

$\mathcal{U}_h(t,\bar{t})$: Evolution operator corresponding to $\mathcal{H}_h(t)$

Decreasing potential in the second half part of the trajectory:

- High density region

$$\theta_{12}^m(t) \simeq \frac{\pi}{2}\,, \qquad \dot{\theta}_{12}^m(t) \simeq 0 \qquad \Longrightarrow \qquad \mathcal{H}_\ell(t) \simeq 0\,, \qquad \mathcal{U}_\ell(t,\bar{t}) \simeq I$$

$$\mathcal{H}_h(t) \neq 0\,, \qquad \mathcal{U}_h(t,\bar{t}) \neq I$$

- Low density region

$$\theta_{13}^m(t) \simeq \theta_{13}\,, \qquad \dot{\theta}_{13}^m(t) \simeq 0 \qquad \Longrightarrow \qquad \mathcal{H}_h(t) \simeq 0\,, \qquad \mathcal{U}_h(t,\bar{t}) \neq I$$

$$\mathcal{H}_\ell(t) \neq 0 \qquad \mathcal{U}_\ell(t,\bar{t}) \neq I$$

$$\Longrightarrow \qquad \left[\mathcal{U}_\ell(t,\bar{t}), \mathcal{H}_h(t)\right] \simeq 0\,, \qquad \bar{t} \leq t \leq t_f$$

$$\check{\mathcal{U}}(t,\bar{t}) \simeq \mathcal{U}_\ell(t,\bar{t})\,\mathcal{U}_h(t,\bar{t})$$

$$\mathcal{P}(t_f,\bar{t}) = e^{-i\alpha_2}\mathcal{P}_{12}\mathcal{P}_{32}$$

$$\mathcal{P}_{12} = \begin{pmatrix} e^{-i\bar{\phi}_{12}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \mathcal{P}_{32} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\bar{\phi}_{32}} \end{pmatrix}$$

$$\bar{\phi}_{ij} = \phi_{ij}(t_f,\bar{t}) = \frac{1}{2}\phi_{ij}(t_f,t_0)$$

To second order in the Magnus expansion:

$$\mathcal{U}_{\ell,h}(t,\bar{t}) = \exp\left[\Omega_{\ell,h}(t,\bar{t})\right]$$

$$\Omega_{\ell,h}(t,\bar{t}) \simeq \Omega^{(1)}_{\ell,h}(t,\bar{t}) + \Omega^{(2)}_{\ell,h}(t,\bar{t})$$

Magnus Expansion

$$\mathcal{U}(t,t_0) = \exp[\Omega(t,t_0)]$$

$$\Omega(t,t_0) = \sum_{n=1}^{\infty} \Omega^{(n)}(t,t_0)$$

$\Omega^{(n)}$: Sum of integrals of n-fold nested commutarors or H(t)

Unitary preserved order by order

$$\Omega^{(n)\dagger}(t,t_0) = -\Omega^{(n)}(t,t_0)$$

$$\boxed{\begin{aligned}\Omega^{(1)}(t,t_0) &= -\frac{i}{\hbar}\int_{t_0}^{t} dt_1 H(t_1)\\ \Omega^{(2)}(t,t_0) &= -\frac{1}{2\hbar^2}\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 [H(t_1),H(t_2)]\end{aligned}}$$

W. Magnus, Commun. Pure Appl. Maths. 7, 649 (1954)
S. Blanes, F. Casas, J. A. Oteo & J. Ross, Phys. Rep. 470. 151 (2009).

$$\mathcal{U}_{\mathrm{a}}(t_f,t_0) \simeq \mathcal{U}_{\mathrm{a}}^{\ell\,\mathtt{T}}(\bar{t},t_0)\,\mathcal{U}_{\mathrm{a}}^{h}(t_f,t_0)\,\mathcal{U}_{a}^{\ell}(\bar{t},t_0)$$
$$= \begin{pmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} & \mathcal{A}_{13} \\ \mathcal{A}_{12} & \mathcal{A}_{22} & \mathcal{A}_{23} \\ \mathcal{A}_{13} & \mathcal{A}_{23} & \mathcal{A}_{33} \end{pmatrix} = \mathcal{U}_{\mathrm{a}}^{\mathtt{T}}(t_f,t_0)$$

$$\mathcal{A}_{11} = \left(u_{11}^2 + u_{12}^{*2}\,v_{11}\right)e^{i2\bar{\phi}_{21}}$$
$$\mathcal{A}_{22} = u_{12}^2 + u_{11}^{*2}\,v_{11}$$
$$\mathcal{A}_{33} = v_{11}^{*}\,e^{-i2\bar{\phi}_{32}}$$
$$\mathcal{A}_{12} = \left(u_{11}u_{12} - u_{11}^{*}u_{12}^{*}v_{11}\right)e^{i\bar{\phi}_{21}}$$
$$\mathcal{A}_{13} = -u_{12}^{*}\,v_{12}\,e^{i\bar{\phi}_{21}}\,e^{-i\bar{\phi}_{32}}$$
$$\mathcal{A}_{23} = u_{11}^{*}v_{12}\,e^{-i\bar{\phi}_{32}}$$

$$u_{11} = \cos\xi_\ell - i\,\frac{\sin\xi_\ell}{\xi_\ell}\,\xi_\ell^{(2)}$$
$$u_{12} = i\,\frac{\sin\xi_\ell}{\xi_\ell}\,\xi_\ell^{(1)}$$

$$v_{11} = \cos\xi_h - i\,\frac{\sin\xi_h}{\xi_h}\,\xi_h^{(2)}$$
$$v_{12} = i\,\frac{\sin\xi_h}{\xi_h}\,\xi_h^{(1)}$$

$$\xi_\ell^{(1)} = i\int_{t_0}^{\bar{t}} dt' \dot{\theta}_{12}^m(t') \exp[-i\phi_{21}(t',\bar{t})]$$

$$\xi_\ell^{(2)} = \int_{t_0}^{t_f} dt' \int_{t_0}^{t'} dt'' \dot{\theta}_{12}^m(t') \dot{\theta}_{12}^m(t'') \sin[\phi_{21}(t'',t')]$$

$$\xi_h^{(1)} = i\int_{t_0}^{t_f} dt' \dot{\theta}_{32}^m(t') \exp[-i\phi_{32}(t',\bar{t})]$$

$$\xi_h^{(2)} = \int_{t_0}^{t_f} dt' \int_{t_0}^{t'} dt'' \dot{\theta}_{13}^m(t') \dot{\theta}_{13}^m(t'') \sin[\phi_{32}(t'',t')]$$

For any potential V(t)

$$\xi_{\ell,h} = \sqrt{|\xi_{\ell,h}^{(1)}|^2 + (\xi_{\ell,h}^{(2)})^2}$$

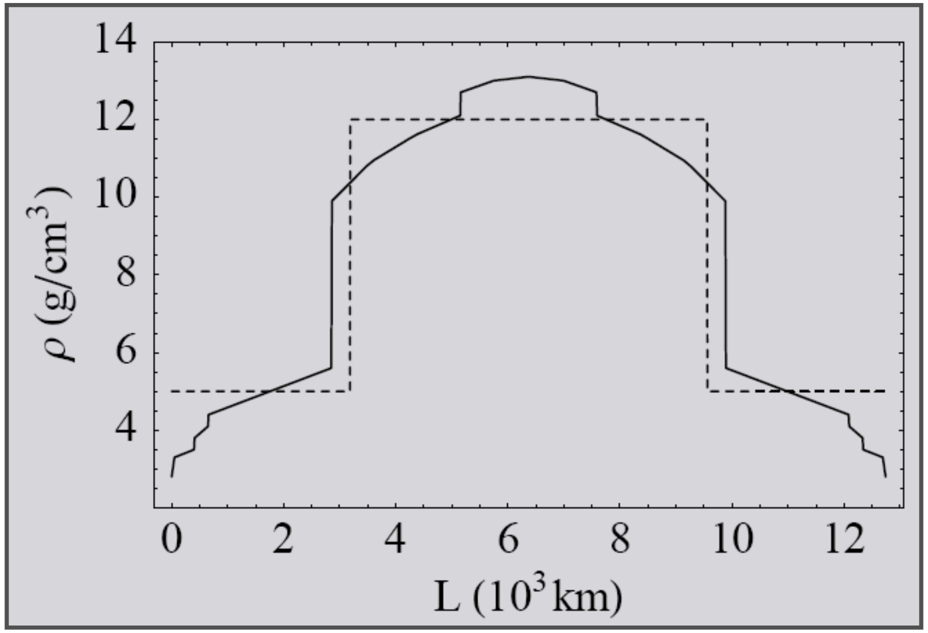
Atmospheric neutrinos

$$|\psi(t_0)\rangle = |\nu_\mu\rangle$$

$$\psi_\mu(t_0) = 1, \qquad \psi_e(t_0) = \psi_\tau(t_0) = 0$$

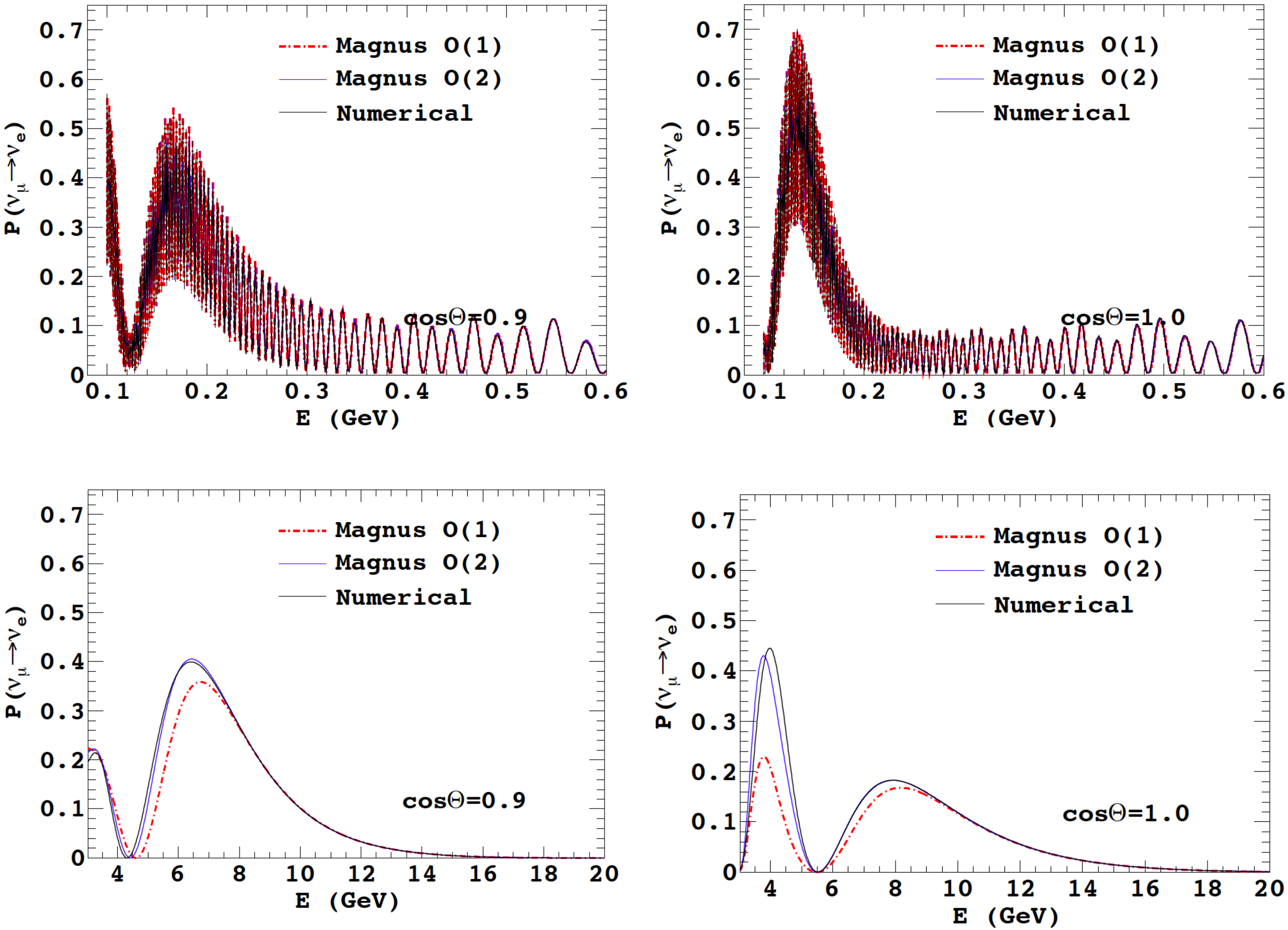
$$P(\nu_\mu \to \nu_\alpha) = |\psi_\alpha(t_f)|^2 = |\mathcal{U}_{\alpha\mu}(t_f, t_0)|^2$$

$$\mathcal{U}_{\alpha\mu}(t_f, t_0) = \sum_{i,j} U^m_{\alpha i}(t_0) U^{m*}_{\mu j}(t_0)) \mathcal{A}_{ij} \qquad \left(U^m(t_f) = U^m(t_0)\right)$$



$$n_e(r) = N_A \begin{cases} 5.95 \text{ cm}^{-3}, & r \le R_\oplus/2 \\ 2.48 \text{ cm}^{-3}, & R_\oplus/2 < r \le R_\oplus \end{cases}$$

Mantle-Core-Mantle Model



Magnus O(1)
Magnus O(2)
Numerical
cosΘ=0.9
cosΘ=1.0
P(ν_μ→ν_e)
E (GeV)

High Energy (E >1GeV)

$$V \gg \Delta_{21}$$

$$\theta_{12}^m \simeq \frac{\pi}{2}, \qquad \dot{\theta}_{12}^m \simeq 0$$

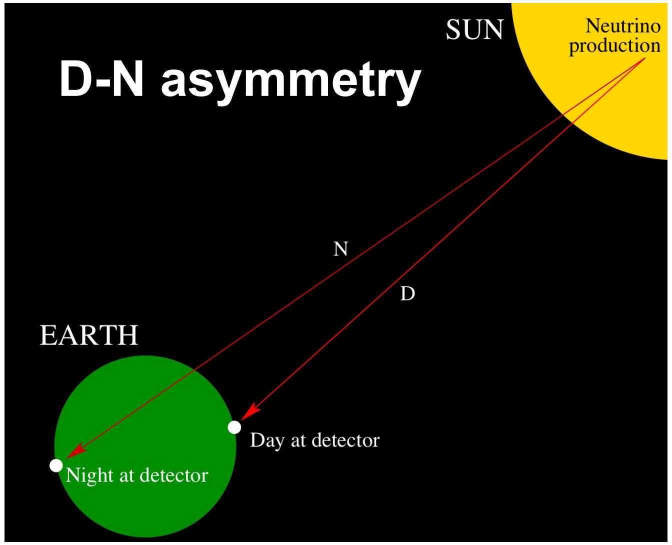
$$\xi_\ell^{(1,2)} = 0 \qquad \Longrightarrow \qquad u_{11} = u_{12} = 0$$

$$\mathcal{A}_{11} = e^{i\phi_{21}}, \qquad \mathcal{A}_{12} = \mathcal{A}_{13} = 0$$

$$P(\nu_\mu \to \nu_e) \;\simeq\; s_{23}^2\Big(\cos 2\theta_{13}^m(t_0)\,\mathcal{I}m(v_{12}) - \sin 2\theta_{13}^m(t_0)\,\mathcal{I}m(v_{11}e^{i\bar{\phi}_{32}})\Big)^2$$

2ν result

D. Supanitsky, J. C. D & G. Medina Tanco, Phys. Rev. D78 045024 (2008); A. N. Ioannisian& A. Yu Smirnov. Nucl. Phys. B816, 94 (2009).



At night, neutrinos coming from the Sun reach the detector after they propagate through the Earth.

Adiabatic transformation inside the Sun.

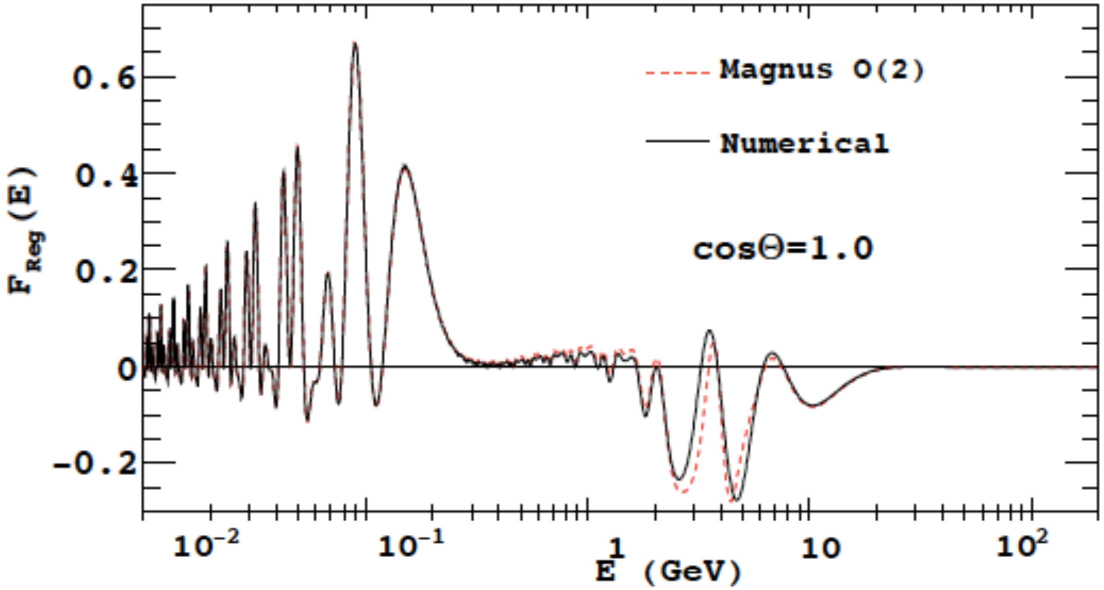
$$\overline{P}(\nu_e \to \nu_e) = \sin^2\theta + \cos 2\theta \cos^2\theta^0_\odot - \cos 2\theta^0_\odot \, f_{reg} \qquad (\theta = \theta_{12})$$

$\theta^0_\odot$: Matter mixing angle at the production point in the solar core

$$f_{reg} = P^{Earth}_{2e} - P^{vac}_{2e}$$

$$P^{Earth}_{2e} \to |\langle \nu_e | \mathcal{U}(t_f, t_0) | \nu_2 \rangle|^2$$

$$P^{vac}_{2e} \to |\langle \nu_e | \nu_2 \rangle|^2$$



Conclusions

- The Magnus expansion for the evolution operator (implemented in the adiabatic basis) provides an efficient formalism to describe three neutrino oscillations in a medium with an arbitrary density profile.

- In the case of neutrinos propagating through a symmetric medium, when the condition impose by time reversal is taken into account, the method renders simple semianalytical expressions for the transition probabilities.

- The results are valid in a wide interval of neutrino energies, making possible a simple (and accurate) description of Earth matter effects on the oscillations of solar and atmospheric neutrinos.

- The same formalism can be applied to the study of other situations of physical interest (for example, long baseline experiments).