Numerical Results for Gauge Theories near the Conformal Window

Biagio Lucini
Swansea University

Discrete 2014, King's College, December 2014

Credits

Collaborators: A. Athenodorou, E. Bennett, G. Bergner, F. Bursa, L. Del Debbio, D. Henty, E. Kerrane, A. Patella, T. Pickup, C. Pica, A. Rago, E. Rinaldi, R. Sabin

Standard Model and beyond

- The strong and electroweak interactions successfully described by the standard model (QCD for the strong sector, $SU(2)_L \otimes U(1)_Y$ with Higgs mechanism for the electroweak sector)
- The strong sector is believed to be valid at all energies, while the weak sector has a natural cut-off at the scale of the TeV
- Among the various models formulated to extend the electroweak sector of the SM above the TeV, strongly interacting BSM dynamics is based on the existence of a new strong interaction
- The lattice provide a natural framework to perform calculations in strongly coupled gauge theories from first principles

Standard Model and beyond

- The strong and electroweak interactions successfully described by the standard model (QCD for the strong sector, $SU(2)_L \otimes U(1)_Y$ with Higgs mechanism for the electroweak sector)
- The strong sector is believed to be valid at all energies, while the weak sector has a natural cut-off at the scale of the TeV
- Among the various models formulated to extend the electroweak sector of the SM above the TeV, strongly interacting BSM dynamics is based on the existence of a new strong interaction
- The lattice provide a natural framework to perform calculations in strongly coupled gauge theories from first principles

Standard Model and beyond

- The strong and electroweak interactions successfully described by the standard model (QCD for the strong sector, $SU(2)_L \otimes U(1)_Y$ with Higgs mechanism for the electroweak sector)
- The strong sector is believed to be valid at all energies, while the weak sector has a natural cut-off at the scale of the TeV
- Among the various models formulated to extend the electroweak sector of the SM above the TeV, strongly interacting BSM dynamics is based on the existence of a new strong interaction
- The lattice provide a natural framework to perform calculations in strongly coupled gauge theories from first principles

Standard Model and beyond

- The strong and electroweak interactions successfully described by the standard model (QCD for the strong sector, $SU(2)_L \otimes U(1)_Y$ with Higgs mechanism for the electroweak sector)
- The strong sector is believed to be valid at all energies, while the weak sector has a natural cut-off at the scale of the TeV
- Among the various models formulated to extend the electroweak sector of the SM above the TeV, strongly interacting BSM dynamics is based on the existence of a new strong interaction
- The lattice provide a natural framework to perform calculations in strongly coupled gauge theories from first principles

Outline

Outline

DEWSB in QCD

As a result of chiral symmetry breaking, in QCD there is a quark condensate

$$\langle \bar{u}u + \bar{d}d \rangle \approx (200\ \text{MeV})^3$$

that is not invariant under $SU(2)_L \otimes U(1)_Y$

Not enough for accounting for the symmetry breaking of the Standard Model:

$$\langle \phi \rangle = 246\ \text{GeV}$$

Can a scaled-up version of QCD work?

From Colour to Technicolour

- New strongly interacting gauge theory with N_{TC} colours and N_{TCf} fermions
- The (bilinear) fermionic condensate replaces the Higgs condensate
- Some of the Goldstone bosons of the techni-chiral symmetry are absorbed by three gauge bosons, which become the massive $W^{\pm}$, Z, while others acquire a mass of the order of the TeV
- The chiral condensate provide standard model quarks with a non-zero mass (Extended Technicolour)

From Colour to Technicolour

- New strongly interacting gauge theory with N_{TC} colours and N_{TCf} fermions
- The (bilinear) fermionic condensate replaces the Higgs condensate
- Some of the Goldstone bosons of the techni-chiral symmetry are absorbed by three gauge bosons, which become the massive $W^{\pm}$, Z, while others acquire a mass of the order of the TeV
- The chiral condensate provide standard model quarks with a non-zero mass (Extended Technicolour)

From Colour to Technicolour

- New strongly interacting gauge theory with N_{TC} colours and N_{TCf} fermions
- The (bilinear) fermionic condensate replaces the Higgs condensate
- Some of the Goldstone bosons of the techni-chiral symmetry are absorbed by three gauge bosons, which become the massive $W^{\pm}$, Z, while others acquire a mass of the order of the TeV
- The chiral condensate provide standard model quarks with a non-zero mass (Extended Technicolour)

From Colour to Technicolour

- New strongly interacting gauge theory with N_{TC} colours and N_{TCf} fermions
- The (bilinear) fermionic condensate replaces the Higgs condensate
- Some of the Goldstone bosons of the techni-chiral symmetry are absorbed by three gauge bosons, which become the massive $W^{\pm}$, Z, while others acquire a mass of the order of the TeV
- The chiral condensate provide standard model quarks with a non-zero mass (Extended Technicolour)

From QCD-like to walking dynamics

Known problems: large flavour changing neutral currents (or quark mass always small), quark mass hierarchy, prediction for the S parameter, . . .

The problems of the technicolour models can be traced back to the logarithmic running of the coupling in QCD

Ultimately, QCD-like dynamics will dominate in the infrared (confinement) and in the ultraviolet (asymptotic freedom)

A very slowly running (*walking*) coupling in an intermediate energy domain could determine a natural mass hierarchy, suppress flavour changing neutral currents and give a small contribution to the S parameter, but needs a large (order 1) anomalous dimension of the chiral condensate

From QCD-like to walking dynamics

Known problems: large flavour changing neutral currents (or quark mass always small), quark mass hierarchy, prediction for the S parameter, ...

The problems of the technicolour models can be traced back to the logarithmic running of the coupling in QCD

Ultimately, QCD-like dynamics will dominate in the infrared (confinement) and in the ultraviolet (asymptotic freedom)

A very slowly running (*walking*) coupling in an intermediate energy domain could determine a natural mass hierarchy, suppress flavour changing neutral currents and give a small contribution to the S parameter, but needs a large (order 1) anomalous dimension of the chiral condensate

Perturbative IR fixed point

The running of the coupling in SU(N) gauge theories with N_f fermion flavours transforming in the representation R is determined by the β-function

$$\mu \frac{dg}{d\mu} = -b_0 g^3 - b_1 g^5 + \ldots \ ,$$

with

$$b_0 = \frac{1}{(4\pi^2)} \left(\frac{11}{3} N - \frac{4}{3} T_R N_f \right) \ , \qquad b_1 = \frac{1}{(4\pi)^4} \left[\frac{34}{3} N^2 - \frac{20}{3} N T_R N_f - 4 \frac{N^2 - 1}{d_R} N_f \right]$$

Banks-Zaks (perturbative) fixed point (two-loops):

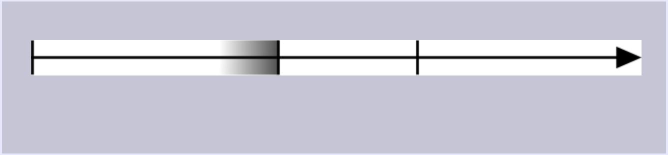
$$\left. \frac{dg}{d\mu} \right|_{2-L} = 0 \qquad \Rightarrow \qquad g^{\star} \simeq -\frac{b_0}{b_1} \ll 1$$

Starting from $g < g^{\star}$ in the ultraviolet, in the infrared $g \to g^{\star}$ (IR fixed point)

Phases of a gauge theory

At fixed N a critical number of flavours N_f^{cu} exists above which asymptotic freedom is lost

Banks and Zaks conjectured that an N_f^{lu} exists such that a non-trivial infrared fixed point appears for $N_f^{lu} \leq N_f \leq N_f^{cu}$ (conformal window)



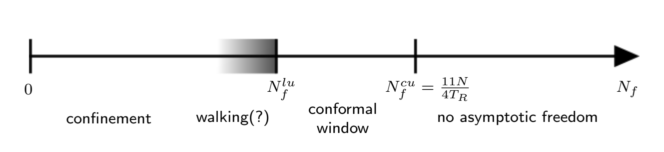
At fixed fermion representation N_f^{lu} depends on the number of flavours

Near the BZ point naive scaling arguments can not be applied and walking can arise

Phases of a gauge theory

At fixed N a critical number of flavours N_f^{cu} exists above which asymptotic freedom is lost

Banks and Zaks conjectured that an N_f^{lu} exists such that a non-trivial infrared fixed point appears for $N_f^{lu} \le N_f \le N_f^{cu}$ (conformal window)



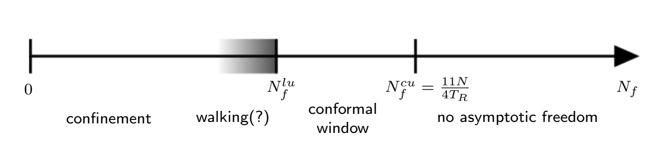
At fixed fermion representation N_f^{lu} depends on the number of flavours

Near the BZ point naive scaling arguments can not be applied and walking can arise

Phases of a gauge theory

At fixed N a critical number of flavours N_f^{cu} exists above which asymptotic freedom is lost

Banks and Zaks conjectured that an N_f^{lu} exists such that a non-trivial infrared fixed point appears for $N_f^{lu} \leq N_f \leq N_f^{cu}$ (conformal window)



At fixed fermion representation N_f^{lu} depends on the number of flavours

Near the BZ point naive scaling arguments can not be applied and walking can arise

A light scalar?

- Near the onset of the conformal window dilatation symmetry can be considered spontaneously broken
- The corresponding Goldstone boson, the technidilaton, has quantum numbers 0^{++}
- The proximity of the onset of the conformal window protects the technidilaton from acquiring a mass of the order of the typical energy scale of the interaction $\Rightarrow$ light Higgs?
- The SM couplings of the technidilaton are very similar to those of the Higgs
- The current LHC data are compatible with the Higgs being a technidilaton

(S. Matsuzaki and K. Yamawaki, arXiv:1206.6703)

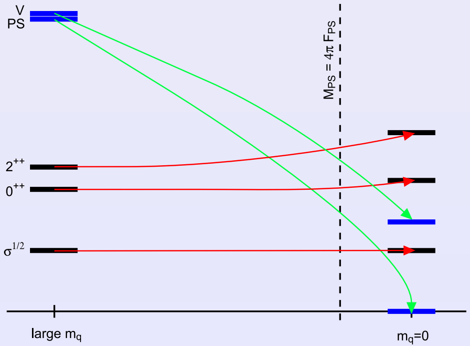
Towards a quantitative understanding

- Can we identify gauge theories within the conformal window?
- Can we determine the extent of the conformal window?
- Can we see a walking behaviour just below the conformal window?
- Do large anomalous dimensions arise near the edge of the conformal window?
- Do (near-)conformal gauge theories have parametrically light scalars?

The inherently non-perturbative nature of the problem requires an approach from first principles like lattice calculations

Outline

The spectrum for a QCD-like theory



- At high fermion masses the theory is nearly-quenched
- At low fermion masses the relevant degrees of freedoms are the pseudoscalar mesons

IR Conformality, Walking and Confinement

1. Confinement (and χSB) characterised by a string tension (and a chiral condensate) of the order of the dynamically generated scale
2. Conformality characterised by power-law behaviour of correlators (unparticles)
3. A small mass term m in a conformal theory generates dynamical scales, a meson spectrum scaling as m^{ρ} and a non-trivial running of the coupling
4. A walking theory is confining in the infrared, but presents features of a IR-conformal theory in an intermediate energy range

A study in the chiral limit is mandatory

IR Conformality, Walking and Confinement

1. Confinement (and χSB) characterised by a string tension (and a chiral condensate) of the order of the dynamically generated scale
2. Conformality characterised by power-law behaviour of correlators (unparticles)
3. A small mass term m in a conformal theory generates dynamical scales, a meson spectrum scaling as m^{ρ} and a non-trivial running of the coupling
4. A walking theory is confining in the infrared, but presents features of a IR-conformal theory in an intermediate energy range

A study in the chiral limit is mandatory

IR Conformality, Walking and Confinement

1. Confinement (and χSB) characterised by a string tension (and a chiral condensate) of the order of the dynamically generated scale
2. Conformality characterised by power-law behaviour of correlators (unparticles)
3. A small mass term m in a conformal theory generates dynamical scales, a meson spectrum scaling as m^{ρ} and a non-trivial running of the coupling
4. A walking theory is confining in the infrared, but presents features of a IR-conformal theory in an intermediate energy range

A study in the chiral limit is mandatory

IR Conformality, Walking and Confinement

1. Confinement (and χSB) characterised by a string tension (and a chiral condensate) of the order of the dynamically generated scale
2. Conformality characterised by power-law behaviour of correlators (unparticles)
3. A small mass term m in a conformal theory generates dynamical scales, a meson spectrum scaling as m^{ρ} and a non-trivial running of the coupling
4. A walking theory is confining in the infrared, but presents features of a IR-conformal theory in an intermediate energy range

A study in the chiral limit is mandatory

IR Conformality, Walking and Confinement

1. Confinement (and χSB) characterised by a string tension (and a chiral condensate) of the order of the dynamically generated scale
2. Conformality characterised by power-law behaviour of correlators (unparticles)
3. A small mass term m in a conformal theory generates dynamical scales, a meson spectrum scaling as m^{ρ} and a non-trivial running of the coupling
4. A walking theory is confining in the infrared, but presents features of a IR-conformal theory in an intermediate energy range

A study in the chiral limit is mandatory

Mass-deformation of the IR-conformal theory

A mass term drive the system outside the IR conformal point

Running of the mass

$$m(\mu) = m(\mu_0) \exp\left\{ - \int_{g(\mu_0)}^{g(\mu)} \frac{\gamma(z)}{\beta(z)} dz \right\} \equiv Z(\mu, \mu_0, \Lambda) m(\mu_0)$$

Close to the IR fixed point we assume a regular behaviour for the RG functions:

$$g \to g_* : \begin{cases} \beta(g) \simeq \beta_* (g - g_*) \\ \gamma(g) \simeq \gamma_* \end{cases}$$

Define a renormalised mass M from the condition $m(M) = M$

A large M destroys conformality and the theory looks like Yang-Mills with heavy sources

$$m_{mes} = 2M$$
$$m_{glue} = B_{glue} \Lambda$$

We are interested in the opposite regime $M \ll \Lambda$

Mass-deformation near the BZ fixed point

V. A. Miransky. Dynamics in the Conformal Window in QCD like theories. hep-ph/9812350.

$$\Lambda_{YM} = M\, e^{-\frac{1}{2b_0^{YM} g_*^2}} \ll M \ll \Lambda$$

- At energies much lower than M, the original theory is effectively described by a pure Yang-Mills theory with scale Λ_{YM}.
- Glueballs are lighter than mesons.
- A deconfinement transition occurs at a temperature $T_c \simeq \Lambda_{YM}$.
- Mesons are effectively quenched. The mesons are bound states of the quark-antiquark pair interacting via the YM static potential, the bound energy is small with respect to the mass of the fermions, and the correction to the potential due to quark-antiquark pair creation are negligible.
- As the mass M is reduced, the IR physics is always the same, provided that all the masses are rescaled with M.

Locking

For a physical quantity

$$m_X = A_X \mu^{\frac{\gamma_*}{1+\gamma_*}} m(\mu)^{\frac{1}{1+\gamma_*}} \; .$$

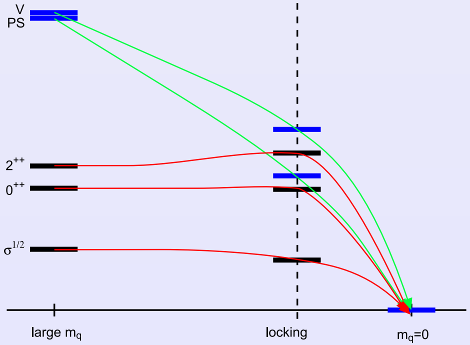
On the lattice, choosing $\mu = a^{-1}$ gives

$$am_X = A_X (am_0)^{\frac{1}{1+\gamma_*}}$$

Consequences

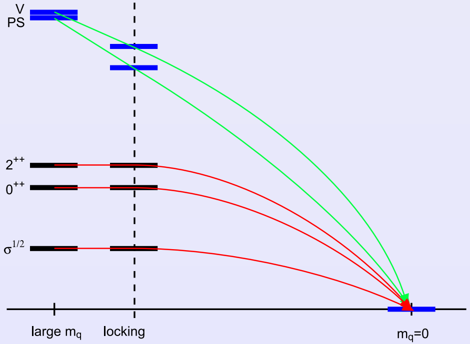
- Ratios of physical quantities with the same mass dimension are independent of the fermion mass if the latter is sufficiently small
- γ_* can be determined by looking at the small-mass scaling of a physical observable

Locking at intermediate M_{lock}



All spectral mass ratios depend very mildly on m below the locking scale

Locking at large M_{lock}



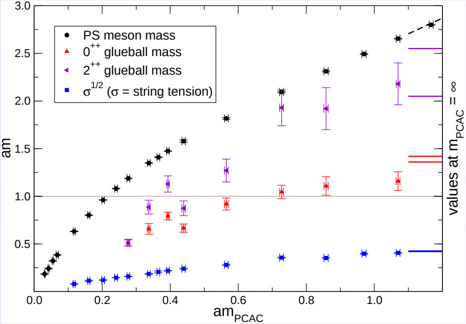
If this scenario is valid beyond BZ, at all scales $\ll \Lambda$

$$m_V/m_{PS} \simeq 1 + \epsilon$$
$$m_{PS} \gg \sigma^{1/2}$$
$$m_G/\sigma^{1/2} \simeq \left[m_G/\sigma^{1/2}\right]^{(YM)}$$

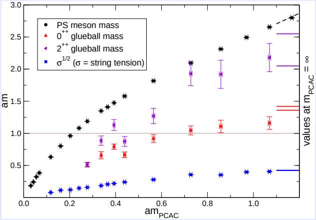
Outline

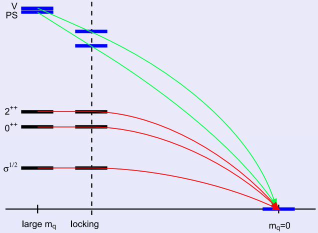
Spectrum hierarchy

(L. Del Debbio *et al.*, arXiv:0907.3896)



Spectrum hierarchy





The pseudoscalar is always higher in mass than the 0^{++} glueball

$\Downarrow$

The locking scenario at high fermion mass looks plausible

Condensate anomalous dimension – Results

Method	γ
FSS (Lucini:2009)	$0.05 < \gamma < 0.25$
SF (Bursa:2009)	$0.05 < \gamma < 0.56$
FSS (DelDebbio:2010)	$0.05 < \gamma < 0.20$
FSS (DelDebbio:2010)	0.22 ± 0.06
MCRG (Catterall:2011)	$-0.6 < \gamma < 0.6$
SF (DeGrand:2011)	0.31 ± 0.06
FSS (Giedt:2012)	0.51 ± 0.16
MNS (Patella:2012)	0.371 ± 0.020
Perturbative 4-loop (Pica:2010)	0.500
Schwinger-Dyson (Ryttov:2010)	0.653
All-orders hypothesis (Pica:2010)	0.46

All estimates are well below one

Discrepancies accounted for by finite size effects (study in progress)

The Emerging Picture

1. The spectrum of SU(2) gauge theory shows the following features:
   - Well-defined mass hierarchy $m_{PS,V} > m_G > \sqrt{\sigma}$
   - Dynamical quenching
   - Dynamically-generated scale sliding with m_{PCAC}
   - Anomalous dimension $\gamma \simeq 0.37(2)$ preliminary result!
2. These results are compatible with the idea that SU(2) with $N_f = 2$ flavours of Dirac fermions is in a (near-)conformal phase
3. However
   - The anomalous dimension is too small for fitting current models of DEWSB
   - A walking scenario can not be completely excluded at this stage

The Emerging Picture

1. The spectrum of SU(2) gauge theory shows the following features:
   - Well-defined mass hierarchy $m_{PS,V} > m_G > \sqrt{\sigma}$
   - Dynamical quenching
   - Dynamically-generated scale sliding with m_{PCAC}
   - Anomalous dimension $\gamma \simeq 0.37(2)$ preliminary result!
2. These results are compatible with the idea that SU(2) with $N_f = 2$ flavours of Dirac fermions is in a (near-)conformal phase
3. However
   - The anomalous dimension is too small for fitting current models of DEWSB
   - A walking scenario can not be completely excluded at this stage

The Emerging Picture

1. The spectrum of SU(2) gauge theory shows the following features:
   - Well-defined mass hierarchy $m_{PS,V} > m_G > \sqrt{\sigma}$
   - Dynamical quenching
   - Dynamically-generated scale sliding with m_{PCAC}
   - Anomalous dimension $\gamma \simeq 0.37(2)$ preliminary result!
2. These results are compatible with the idea that SU(2) with $N_f = 2$ flavours of Dirac fermions is in a (near-)conformal phase
3. However
   - The anomalous dimension is too small for fitting current models of DEWSB
   - A walking scenario can not be completely excluded at this stage

Outline

Hunting for large anomalous dimensions

- Large (i.e. O(1)) anomalous dimensions are crucial for the framework to be compatible with phenomenology
- Large anomalous dimensions might be realised near the onset of the conformal window
- With adjoint Dirac flavour and gauge group SU(2), the only remaining possibility is to look at the single flavour case
- The common lore is that this theory be confining and chiral symmetry breaking, similarly to QCD, but lattice calculations on reduced models can not confirm this scenario
- A lattice calculation from first principles can pin down the phase of the theory and perhaps (if the theory is conformal or near-conformal) unveil a large anomalous dimension

(A. Athenodorou, E. Bennett, G. Bergner, B. Lucini and A. Patella, arXiv:1311.4155 and in preparation)

The model

The one Dirac flavour theory can be equivalently seen as a theory with two Majorana flavours

$$\psi = \psi_{M1} + i\psi_{M2} \ , \qquad \psi_{M1} = \frac{\psi + C(\psi)}{2} \ , \psi_{M2} = \frac{\psi - C(\psi)}{2i}$$

in terms of which the action separates into two independent, identical contributions

$$S[\psi] = S_M[\psi_{M1}] + S_M[\psi_{M2}]$$

Alternatively, the action can be rewritten in terms of the Weyl spinor $\chi = (\psi_L, -\sigma_2\psi_R^\star)$, which unveils an $SU(2) \to SO(2)$ chiral symmetry breaking pattern and the structure of the spectrum

Note: this chiral symmetry breaking pattern can not account for $W^\pm$ and Z masses

Fermion bilinears

spin	$SU(2)^P$	Weyl bilinears	Dirac bilinears	Majorana bilinears	$U(1)^P$	correlators
(pseudo)scalars	1^-	$\chi^\dagger\chi$	$\bar\psi\gamma_0\gamma_5\psi$	$O_{++}(\gamma_0\gamma_5)+O_{--}(\gamma_0\gamma_5)$	0^-	singlet γ_5, $\gamma_0\gamma_5$
	3^-	$\chi^T\sigma_2\epsilon_2T_2^A\chi - h.c.$	$\bar\psi\gamma_5\psi$	$O_{++}(\gamma_5)+O_{--}(\gamma_5)$		
			$\psi^TC\psi$	$O_{++}(1)-O_{--}(1)+2iO_{+-}(1)$	2^-	triplet 1
			$\psi^\dagger C\psi^*$	$O_{++}(1)-O_{--}(1)-2iO_{+-}(1)$	-2^-	
	3^+	$\chi^T\sigma_2\epsilon_2T_2^A\chi + h.c.$	$\bar\psi\psi$	$O_{++}(1)+O_{--}(1)$	0^+	singlet 1, γ_0
			$\bar\psi\gamma_0\psi$	$O_{+-}(\gamma_0)$		
			$\psi^TC\gamma_5\psi$	$O_{++}(\gamma_5)-O_{--}(\gamma_5)+2iO_{+-}(\gamma_5)$	2^+	triplet γ_5, $\gamma_0\gamma_5$
			$\psi^TC\gamma_5\gamma_0\psi$	$O_{++}(\gamma_5\gamma_0)-O_{--}(\gamma_5\gamma_0)+2iO_{+-}(\gamma_5\gamma_0)$		
		$\chi^\dagger T_2^A\chi$	$\psi^\dagger C\gamma_5\psi^*$	$O_{++}(\gamma_5)-O_{--}(\gamma_5)-2iO_{+-}(\gamma_5)$	-2^+	
			$\psi^\dagger C\gamma_5\gamma_0\psi^*$	$O_{++}(\gamma_5\gamma_0)-O_{--}(\gamma_5\gamma_0)-2iO_{+-}(\gamma_5\gamma_0)$		
(pseudo)vectors	1^+	$\chi^\dagger\boldsymbol{\sigma}\chi$	$\bar\psi\gamma_5\boldsymbol{\gamma}\psi$	$O_{++}(\gamma_5\boldsymbol{\gamma})+O_{--}(\gamma_5\boldsymbol{\gamma})$	0^+	singlet $\gamma_5\boldsymbol{\gamma}$, $\gamma_0\gamma_5\boldsymbol{\gamma}$
		$\chi^T\sigma_2\boldsymbol{\sigma}\epsilon_2\chi + h.c.$	$\bar\psi\gamma_0\gamma_5\boldsymbol{\gamma}\psi$	$O_{+-}(\gamma_0\gamma_5\boldsymbol{\gamma})$		
	1^-	$\chi^T\sigma_2\boldsymbol{\sigma}\epsilon_2\chi - h.c.$	$\bar\psi\gamma_0\boldsymbol{\gamma}\psi$	$O_{+-}(\gamma_0\boldsymbol{\gamma})$	0^-	singlet $\boldsymbol{\gamma}$, $\gamma_0\boldsymbol{\gamma}$
	3^-	$\chi^\dagger\boldsymbol{\sigma}T_2^A\chi$	$\bar\psi\boldsymbol{\gamma}\psi$	$O_{+-}(\boldsymbol{\gamma})$		
			$\psi^TC\gamma_5\boldsymbol{\gamma}\psi$	$O_{++}(\gamma_5\boldsymbol{\gamma})-O_{--}(\gamma_5\boldsymbol{\gamma})+2iO_{+-}(\gamma_5\boldsymbol{\gamma})$	2^-	triplet $\gamma_5\boldsymbol{\gamma}$
			$\psi^\dagger C\gamma_5\boldsymbol{\gamma}\psi^*$	$O_{++}(\gamma_5\boldsymbol{\gamma})-O_{--}(\gamma_5\boldsymbol{\gamma})-2iO_{+-}(\gamma_5\boldsymbol{\gamma})$	-2^-	

The spectrum

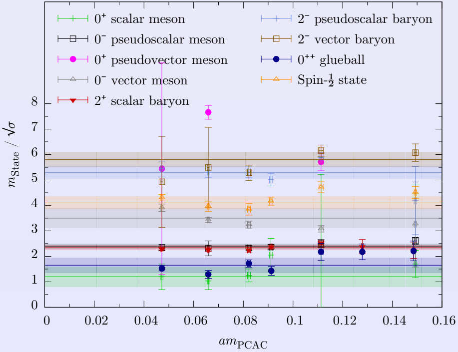
Conserved quantum numbers: $B = 0, \pm 2$ (baryon number), J (spin), P (parity)

Spectral states

- Mesons: two-fermion states with $B = 0$
- Baryons: two-fermion states with $B = \pm 2$
- Glueballs: pure gluonic states
- “Glueballino”: fermion-glueball bound state

Spectrum and Dirac MNS studied at a single $\beta = 2.05$ and for various lattices, paying attention that finite volume artefacts are negligible

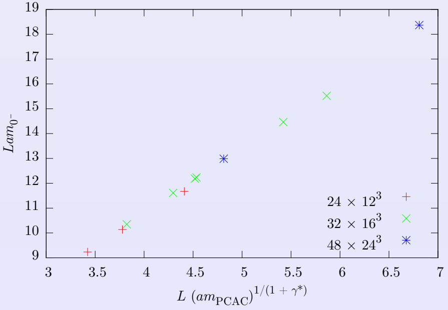
Mass ratios



Mass ratios are constant as am is taken to zero $\Rightarrow$ (near-)conformality?

Finite size scaling

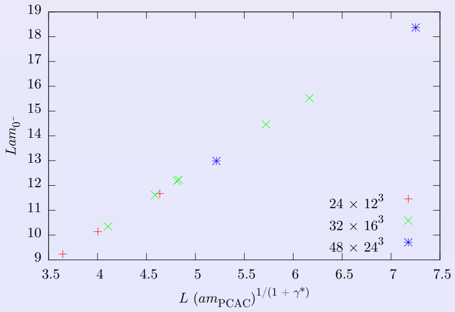
$\gamma = 0.9$



Finite size scaling consistent with near-conformality and large anomalous dimensions

Finite size scaling

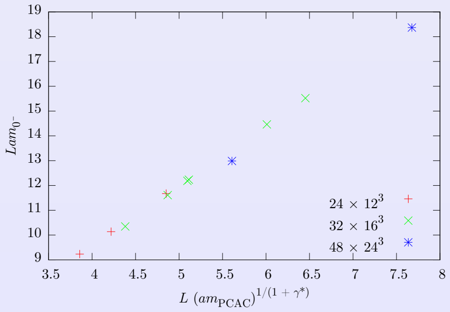
$\gamma = 1.0$



Finite size scaling consistent with near-conformality and large anomalous dimensions

Finite size scaling

$\gamma = 1.1$



Finite size scaling consistent with near-conformality and large anomalous dimensions

Outline

Conclusions

- Non-supersymmetric conformal gauge theories can be studied quantitatively with lattice techniques (e.g. SU(2) gauge theory with two adjoint fermions)
- We have (preliminary!) evidence that large anomalous dimensions can arise near the onset of the conformal window
- light scalars seem to arise naturally when the system has a diverging correlation length

The quest for a dynamical explanation of electroweak symmetry breaking is now open more than ever, and strong BSM dynamics is still on the table