ECT* European Centre for Theoretical Studies in Nuclear Physics and Related Areas

FONDAZIONE BRUNO KESSLER

Effects of divergent ghost loops on the QCD Green's functions

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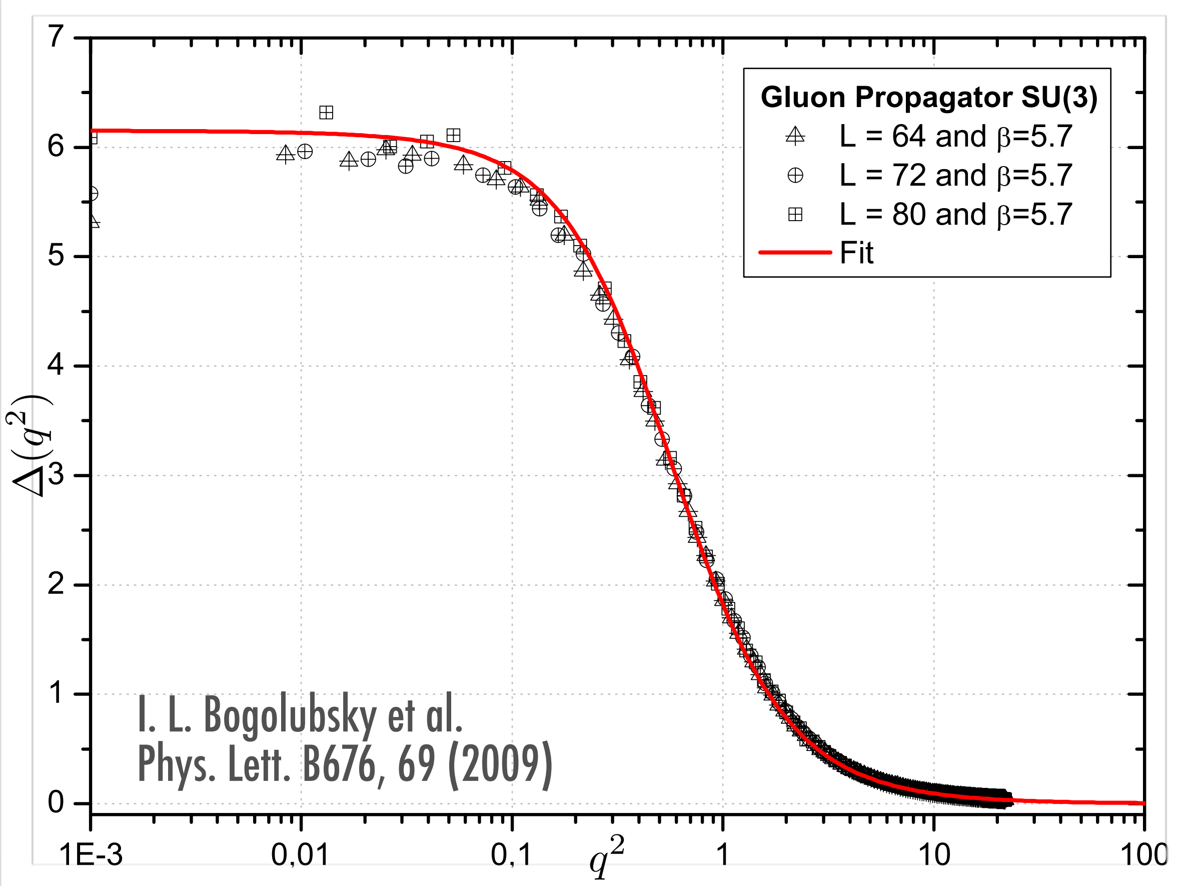
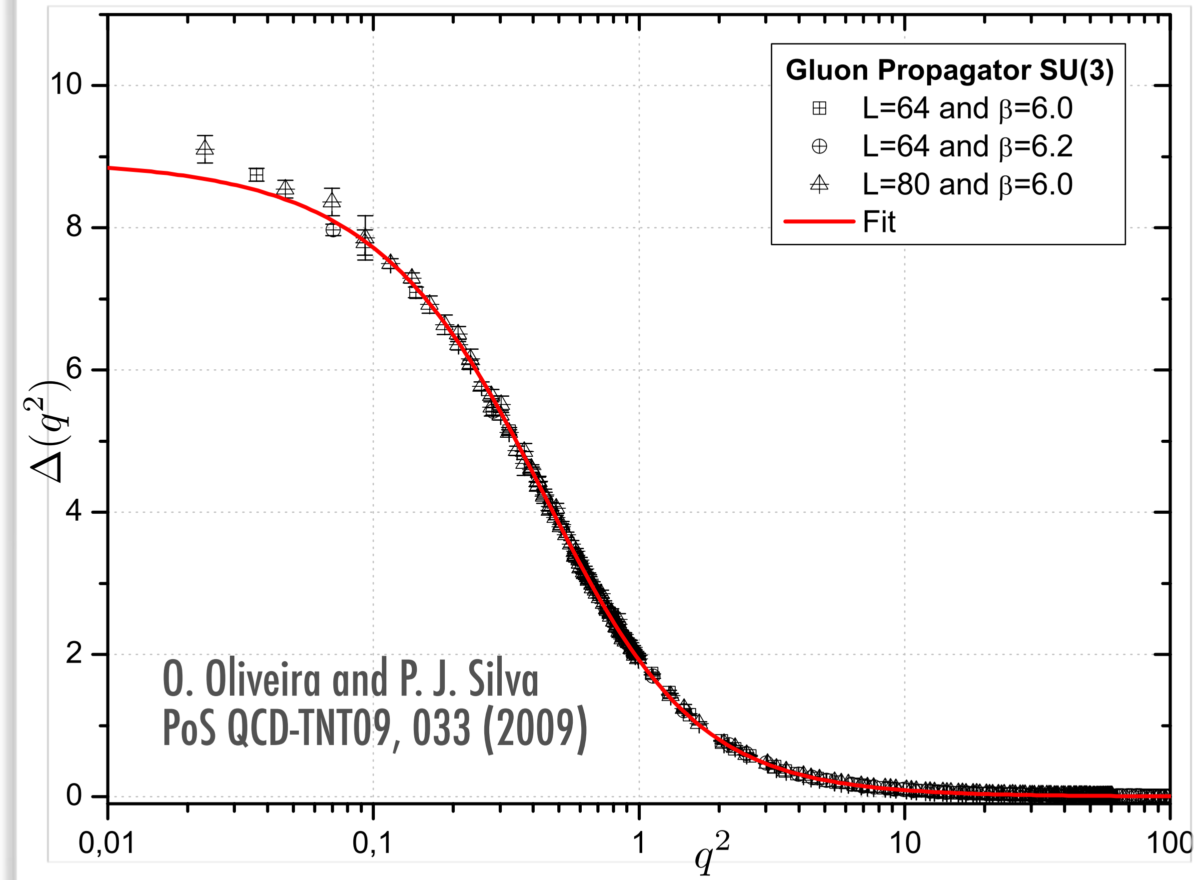
Prolegomena

gluon and ghost propagators from lattice QCD

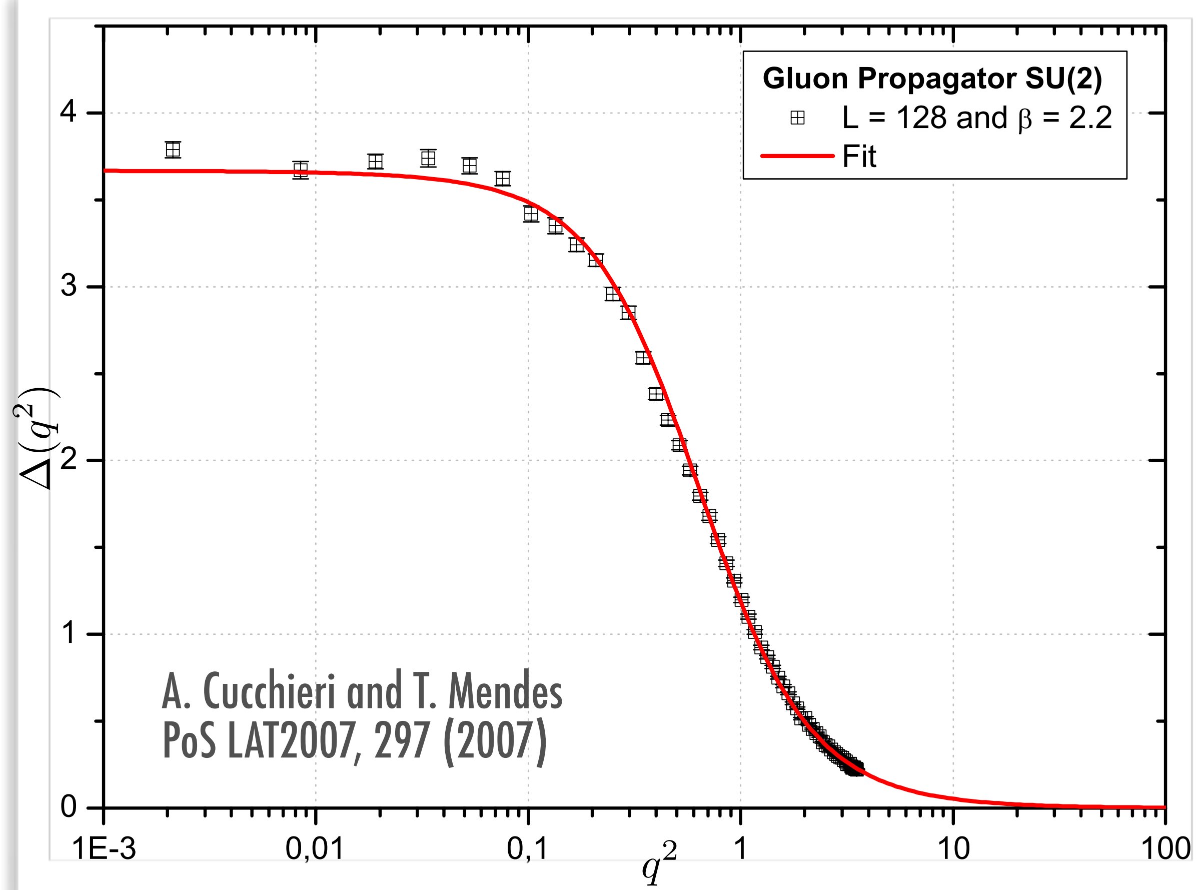
- **Massive gluon** propagator





- Fitted by the following function

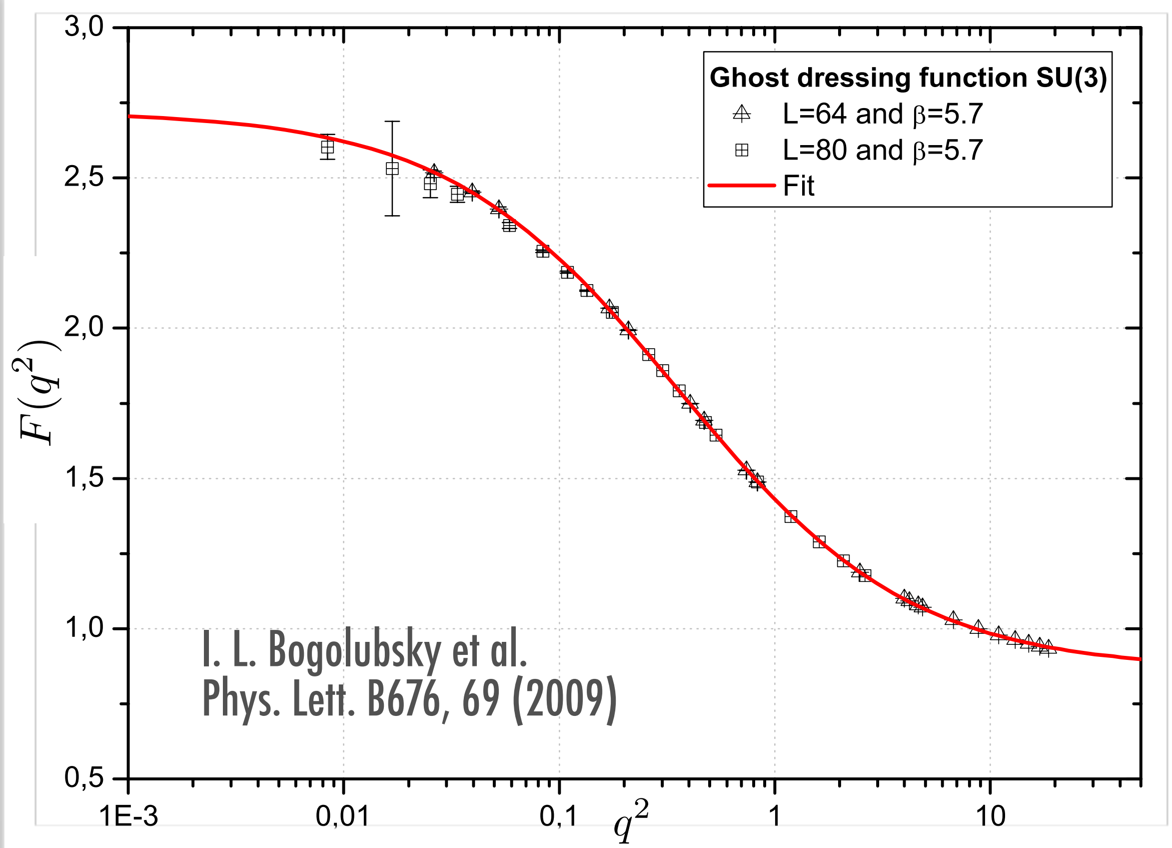
$$\Delta^{-1}(q^2) = m^2 + q^2 \left[1 + \frac{13 C_A g_f^2}{96\pi^2} \ln\left(\frac{q^2 + \rho m^2}{\mu^2} \right) \right]$$



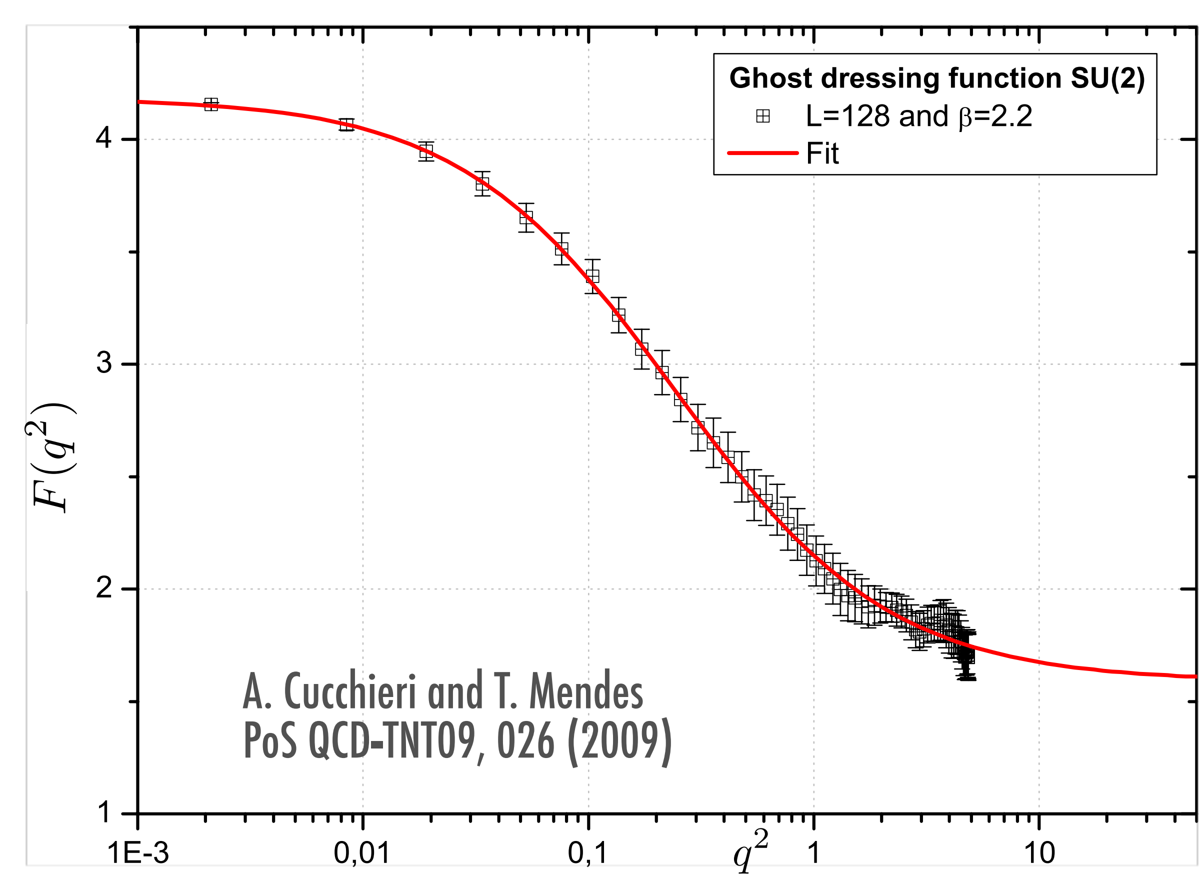
gluon and **ghost** propagators from **lattice QCD**

Ph. Boucaud et al., Phys. Rev. D74, 034505 (2006)

Ghost dressing function $F(q^2)$ **saturates**



- **Ghost** propagator **free** and IR divergent (no ghost mass)
- **Ghosts seem** to play a **marginal** role in the game (as opposed to ghost-dominance)
- **Kugo-Ojima** confinement criterion **does not work**

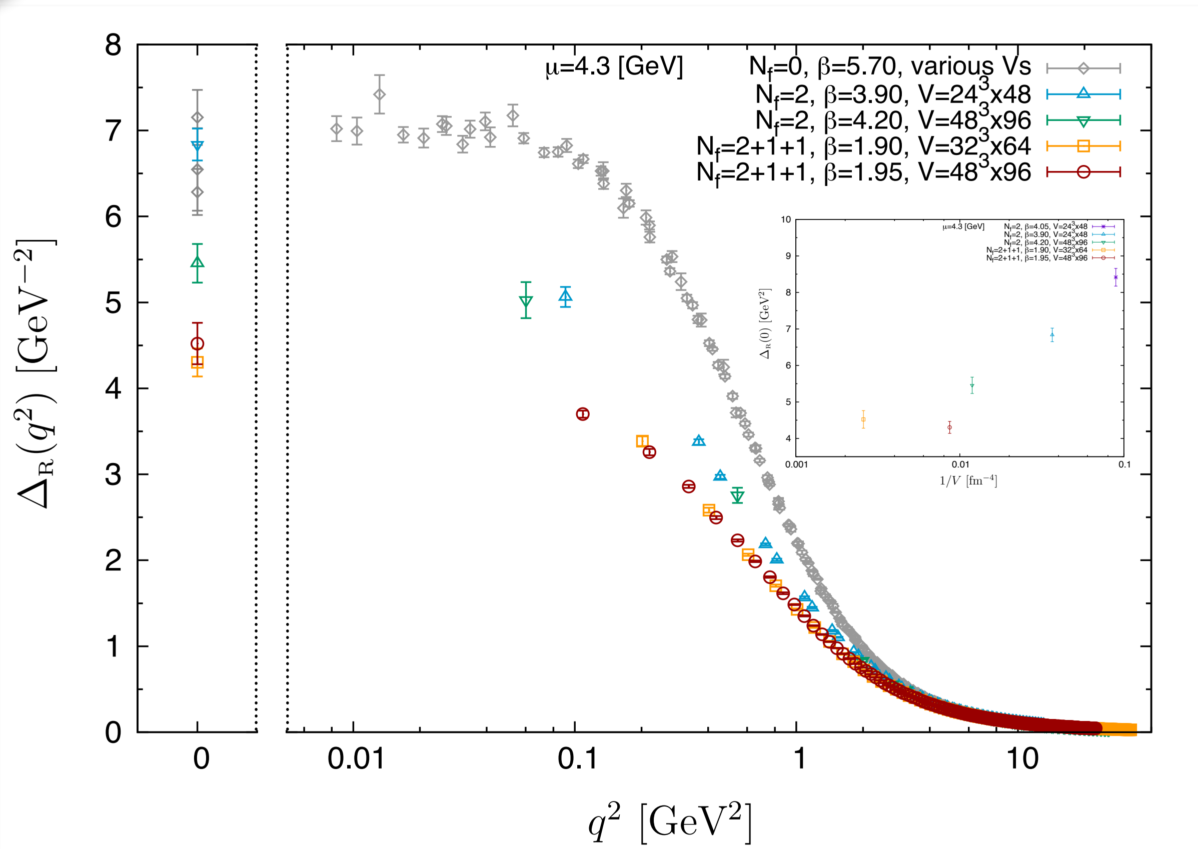


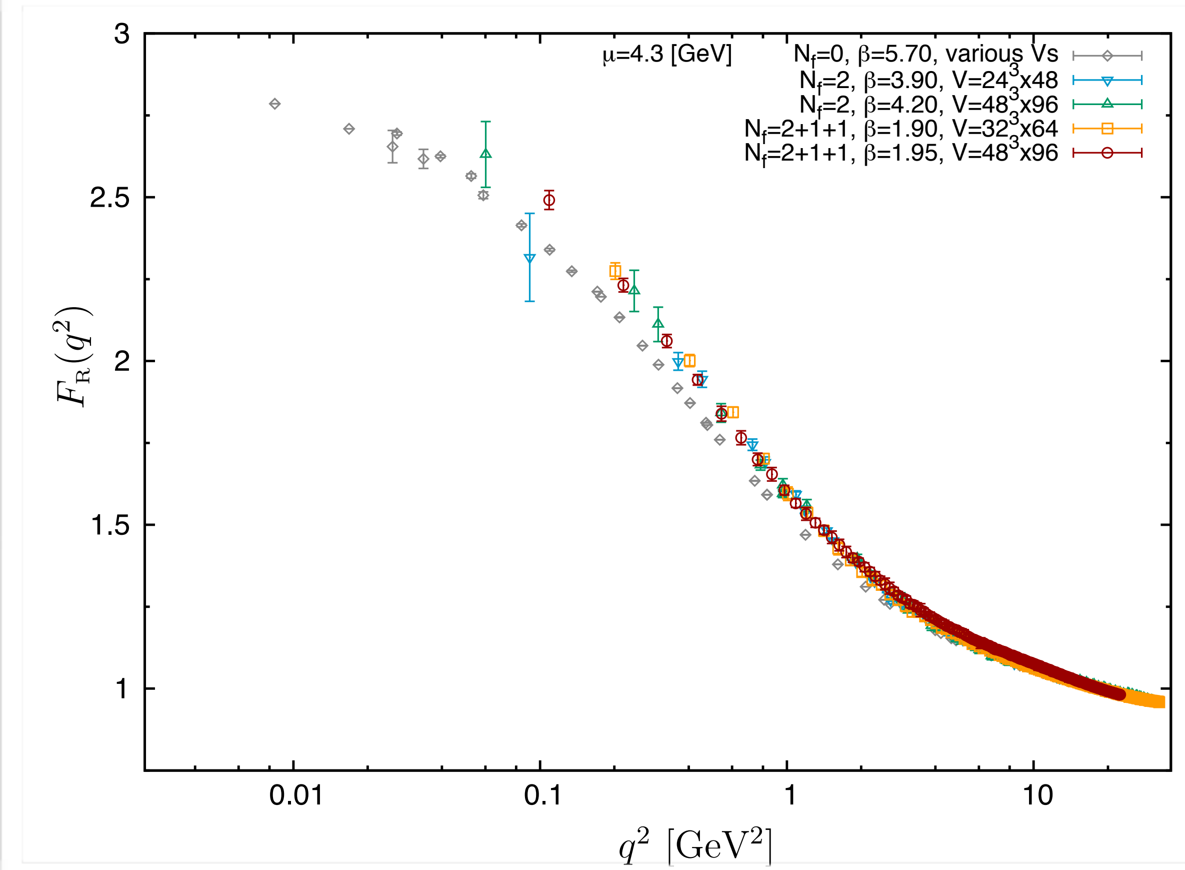
Fitted by the following function

$$F(p^2) = \frac{a_1 - a_2}{1 + (p^2/p_1^2)^\gamma} + a_2$$

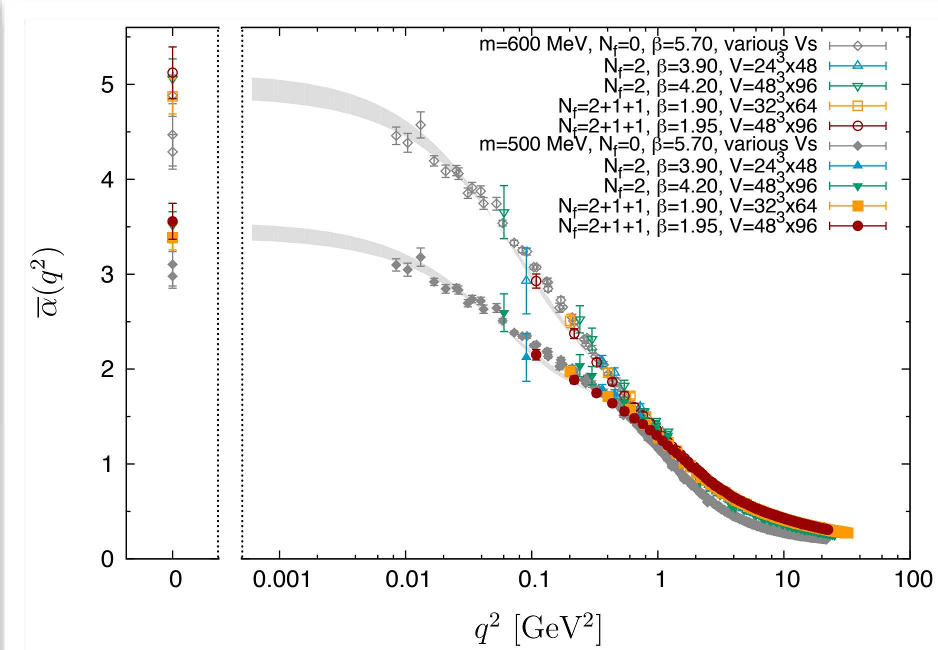
gluon and ghost propagators from lattice QCD

Results persist in full QCD





A. Ayala, A. Bashir, D. B., M. Cristoforetti and J. Rodriguez-Quintero, Phys. Rev. D86, 074512 (2012)



gluon and ghost propagators from
Schwinger-Dyson equations

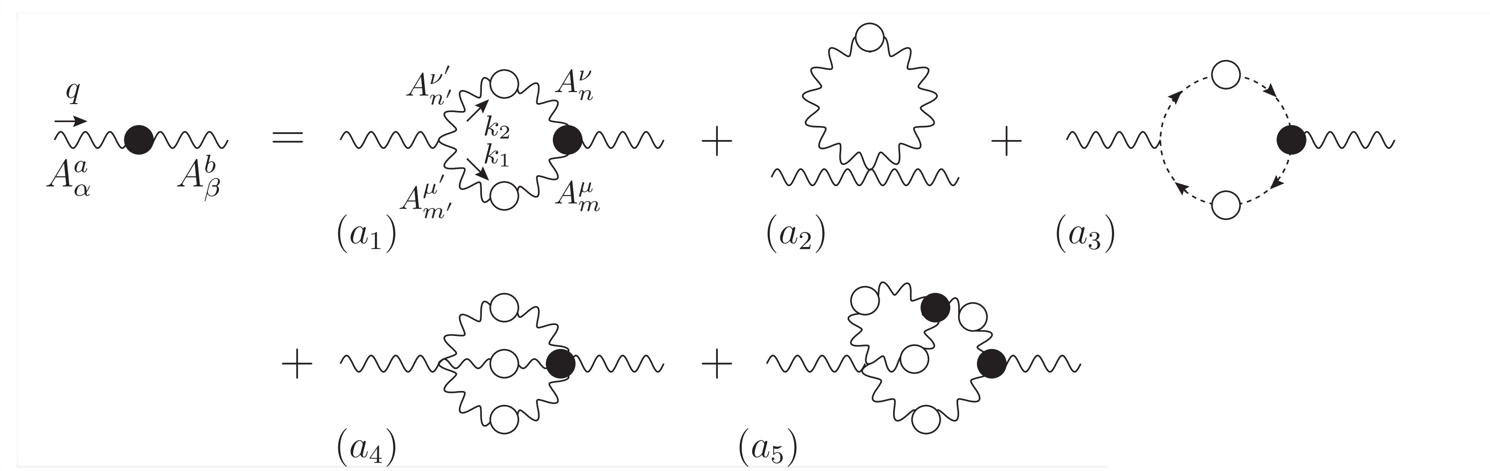
- Schwinger-Dyson eqs: way of **treating purely non-perturbative phenomena** (e.g., mass gap generation)

- Infinite system of **coupled non-linear integral equations**
  - captures the **full quantum e.o.m.**
  - expansion about the free-field vev, but finally no reference to it

- Require a **truncation scheme**
  - gauge and renormalization group invariance should be respected

Gluon propagator

- BRST demands $q^{\alpha}\sum_{i=1}^{5}(a_i)_{\alpha\beta} = 0$
  - very difficult diagrammatic verification
  - **cannot truncate in any obvious way**



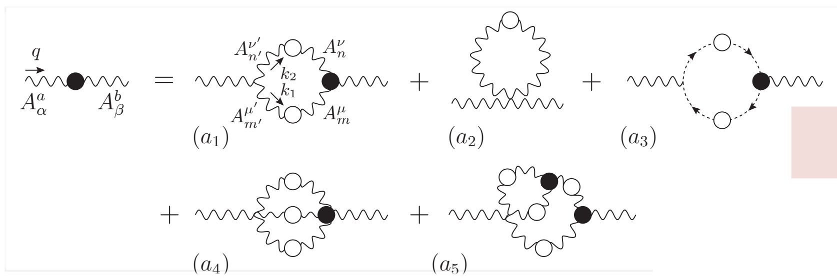
- Retaining (a_1) and (a_2) only is not correct even at one loop
  - $q^{\alpha}\Pi_{\alpha\beta}(q)|_{(a_1)+(a_2)} \neq 0$

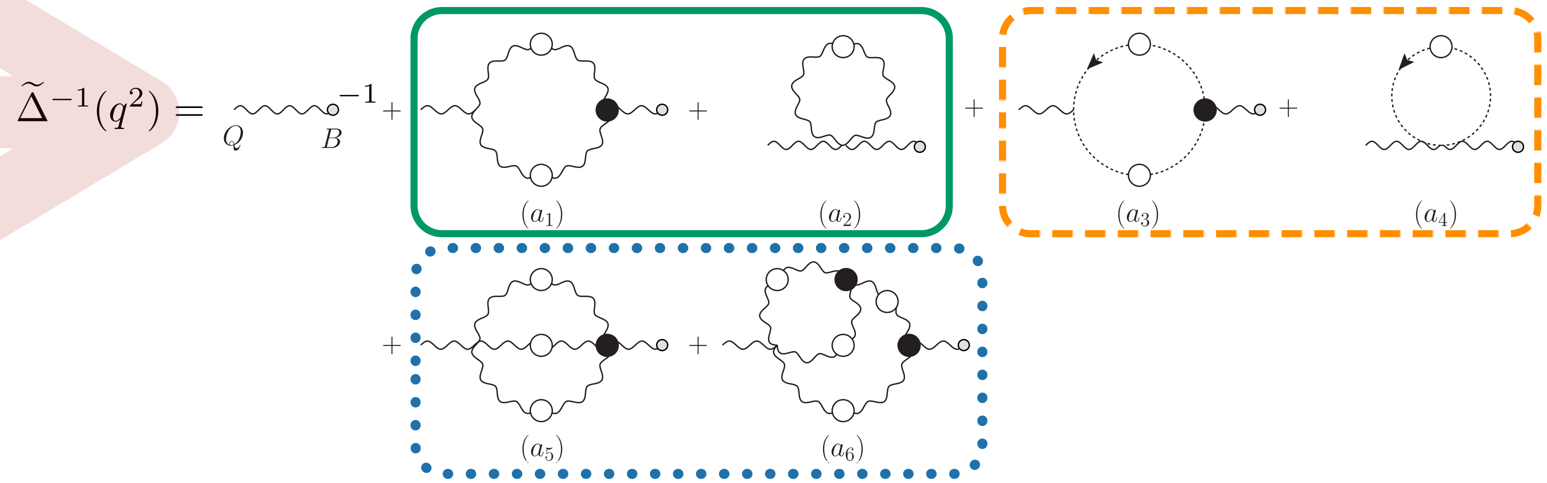
- Adding (a_3) is not sufficient for a full analysis; beyond one loop
  - $q^{\alpha}\Pi_{\alpha\beta}(q)|_{(a_1)+(a_2)+(a_3)} \neq 0$

D. B. & J. Papavassiliou, Phys. Rev. D77, 061702(R) (2008)
D. B. & J. Papavassiliou, JHEP 11, 063 (2008)

PT-BFM resummed Schwinger-Dyson series

- Apply the pinch technique to the Schwinger-Dyson equation of the gluon propagator





- graphs made out of **new vertices**, (inside **conventional props**)
- new vertices corresponds to **BFM vertices**
  - **external gluons** dynamically converted into **background gluons**

- New Schwinger-Dyson equation has a **special structure**
  - **Subgroups** (one-/two-loop dressed gluon/ghost) are **individually transverse**

Problem?

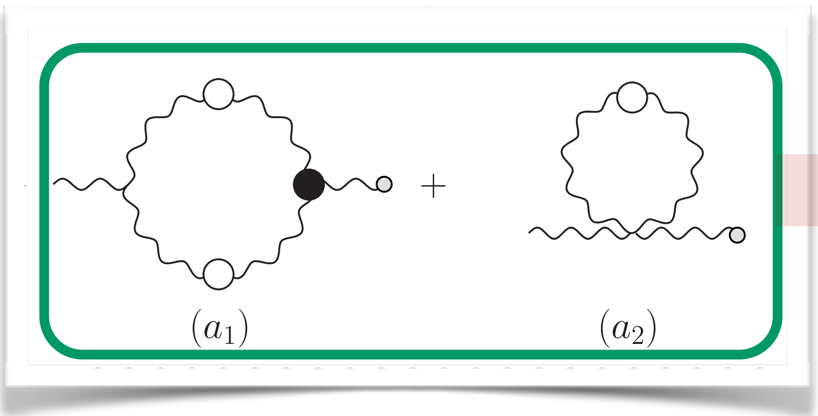
Not a genuine Schwinger-Dyson equation (**mixes BFM** and **conventional** propagators)

- Express the **Schwinger-Dyson eq** in terms of a **BQI**
  - $$\Delta^{-1}(q^2)[1+G(q^2)]P_{\alpha\beta}(q) = q^2 P_{\alpha\beta} + \sum_{i=1}^{6}(a_i)_{\alpha\beta}$$
    $$\widetilde{\Delta}^{-1}(q^2) = [1+G(q^2)]\Delta^{-1}(q^2)$$
  - Since in $4d$ L=0 the function G is directly related to the inverse of the ghost dressing function

$$F^{-1}(q^2) \approx 1 + G(q^2)$$

A.C. Aguilar and J. Papavassiliou, Phys. Rev. D 81, 034003 (2010)

no **mass** without
dynamical poles



$\lim_{q^2 \to 0}$

Seagull identity (sets to zero quadratic divergences)

$$\int_k k^2 \frac{\partial \Delta(k^2)}{\partial k^2} + \frac{d}{2} \int_k \Delta(k) = 0$$

Two (related) problems:

- How do we **evade** the **seagull cancellation** and generate a mass?
- How do we do that **preserving gauge invariance**?

dynamically generated

gluon mass

J. S. Schwinger, Phys. Rev. 125, 397 (1962)
J. S. Schwinger, Phys. Rev. 128, 2425 (1962)

Dyson resum

$$\Delta(q^2) = \frac{1}{q^2\,[1 + \Pi(q^2)]}$$

Idea

If $\Pi(q^2)$ has a pole at $q^2 = 0$ the gauge boson is massive even though it is massless in the absence of interactions

- Requires **massless, longitudinally coupled** Goldstone like **poles** (~ $1/q^2$)
- **Occur dynamically** (even in the **absence** of canonical **scalar fields**) as **composite excitations** in a **strongly coupled** gauge theory

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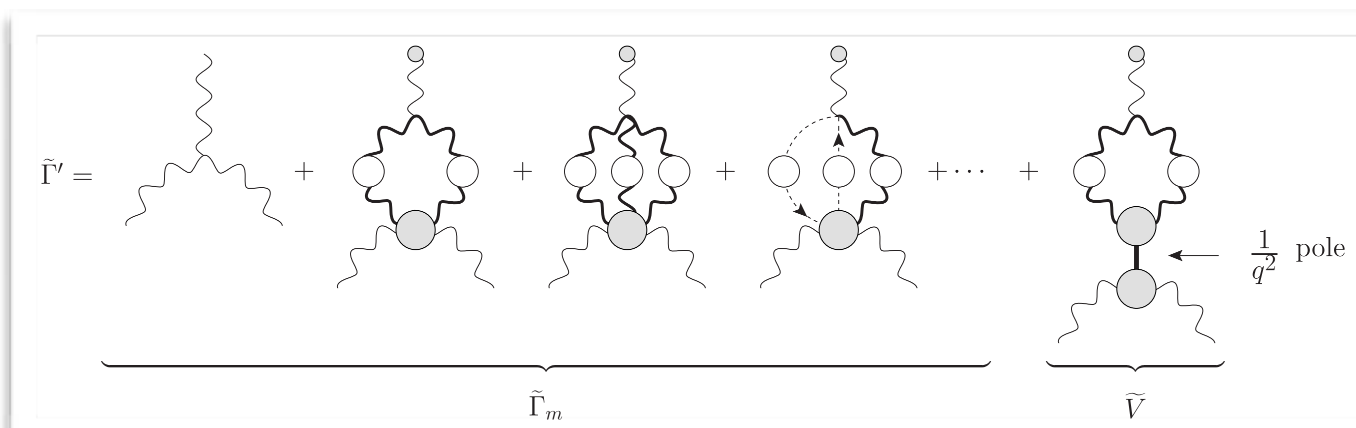
R. Jackiw and K. Johnson, Phys. Rev. D8, 2386 (1973)
J. M. Cornwall and R. E. Norton, Phys. Rev. D8, 3338 (1973)
E. Eichten and F. Feinberg, Phys. Rev. D10, 3254 (1974)

Dynamics enters through the **three-gluon vertex**

- **Longitudinally coupled** massless **poles**
  - **Not** a **kinematic singularity**, rather **bound states poles** non-perturbatively produced
  - **Do not appear** in the S matrix of the theory ("eaten-up" by the gluons to become massive)

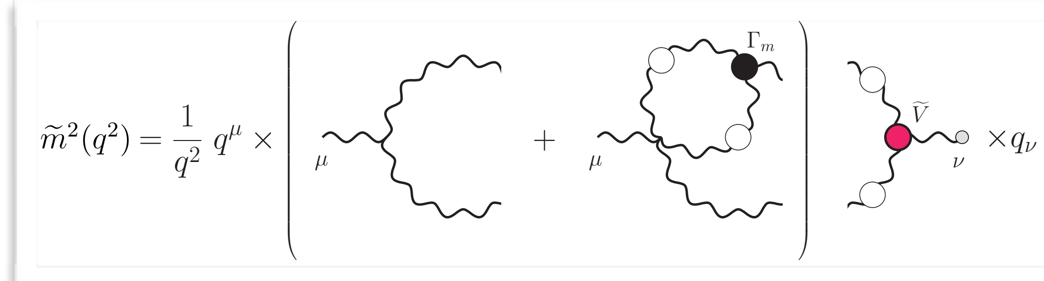


Instrumental for ensuring that

- $\Delta^{-1}(0) > 0$

dynamically generated
gluon mass

A. C. Aguilar, D. B. and J. Papavassiliou, Phys. Rev . D84, 085026 (2011)
D. B., D. Ibañez and J. Papavassiliou, Phys. Rev. D86, 085033 (2012)
A. C. Aguilar, D. B. and J. Papavassiliou, Phys. Rev . D89, 085032 (2014)

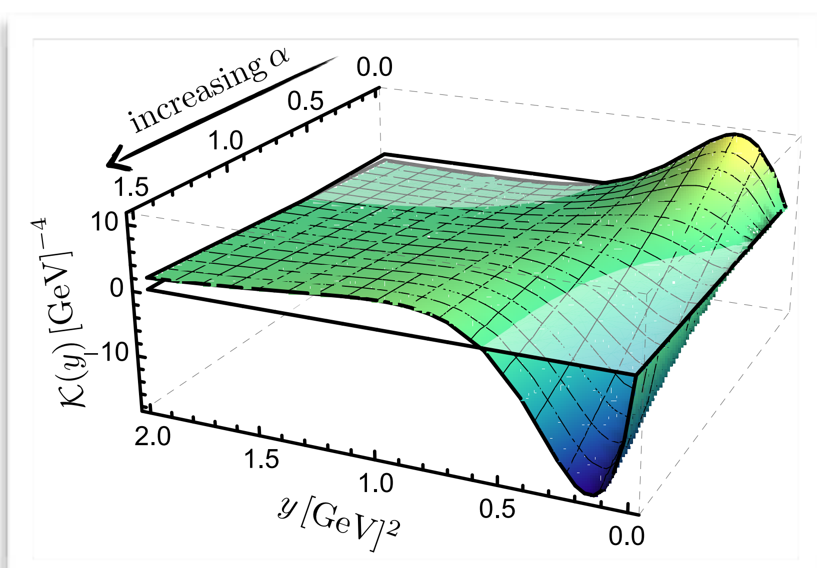


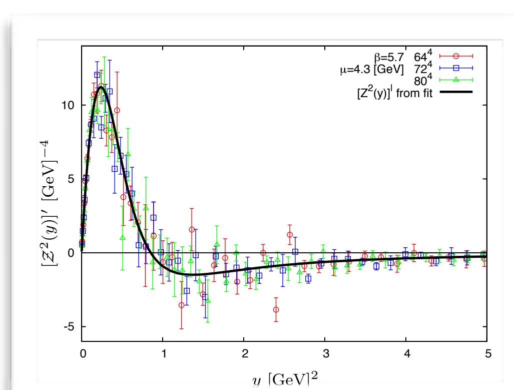
- Equation known to **all orders**

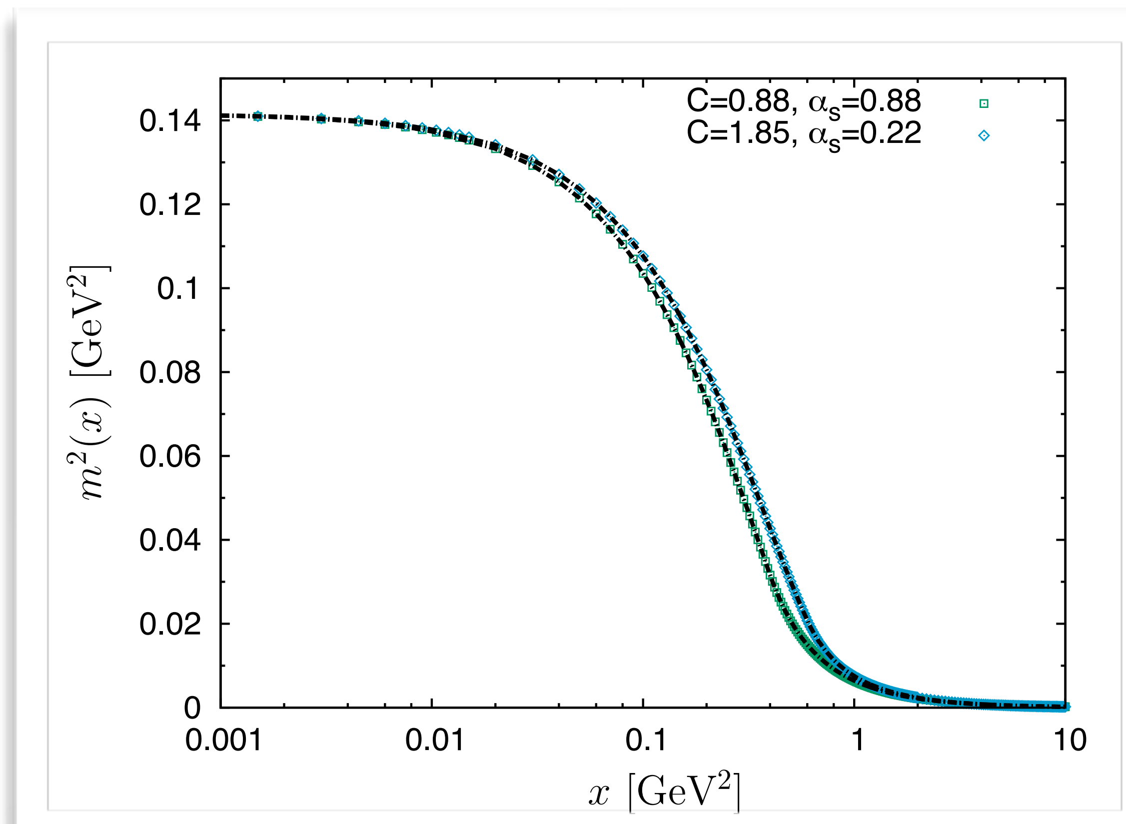
$$m^2(q^2) = \frac{ig^2 C_A}{1+G(q^2)}\frac{1}{q^2}\int_k m^2(k^2)[(k+q)^2-k^2]\Delta^{\alpha\rho}(k)\Delta_{\alpha\rho}(k+q)\left\{1+\frac{3}{4}ig^2C_A[Y(k+q)+Y(k)]\right\}$$
$$-\frac{3}{4}\frac{g^4C_A^2}{1+G(q^2)}\frac{1}{q^2}(q^2g_{\delta\gamma}-2q_\delta q_\gamma)\int_k m^2(k^2)[Y(k+q)-Y(k)]\Delta^{\delta}_{\epsilon}(k)\Delta^{\gamma\epsilon}(k+q)$$

- Solutions known when approximating Y at lowest order in pt

$$Y_{\rm R}(k^2) = -\frac{1}{(4\pi)^2}\frac{15}{4}k^2\log\frac{k^2}{\mu^2}$$



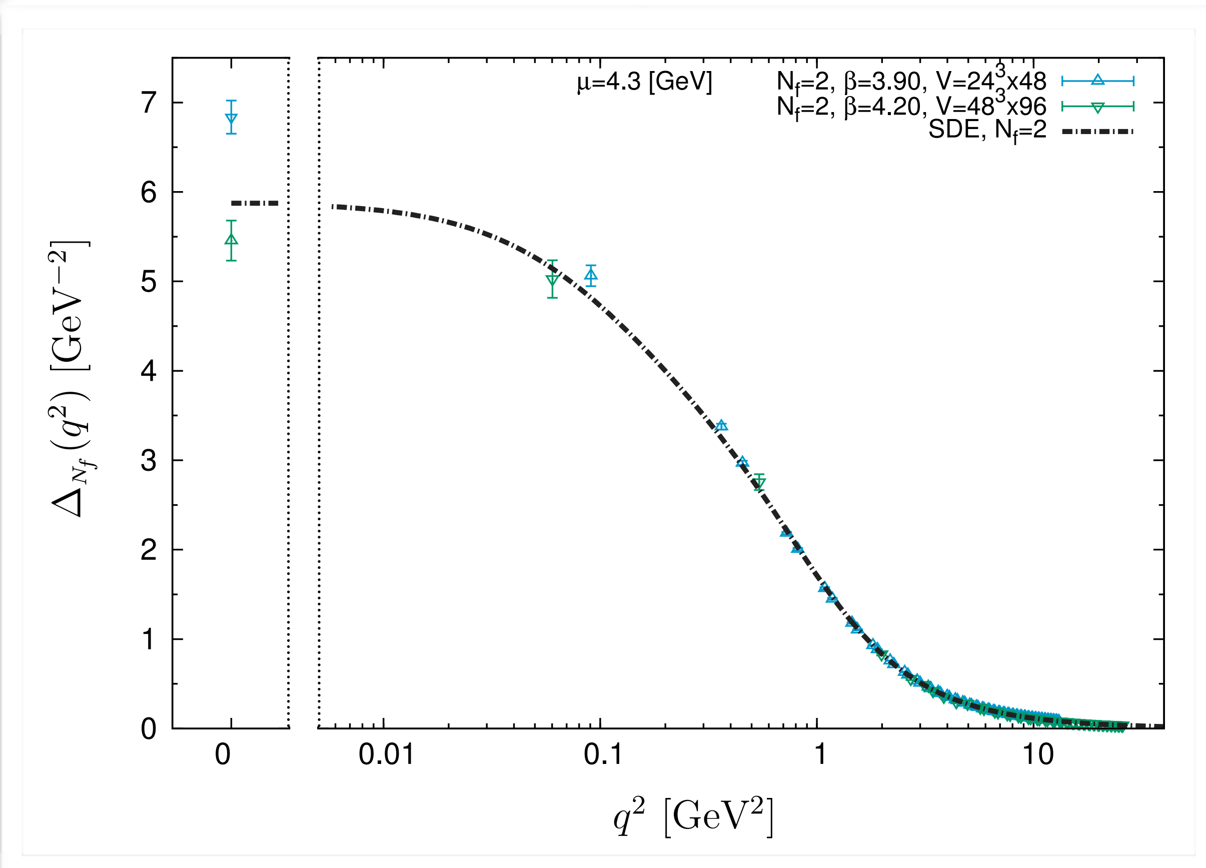


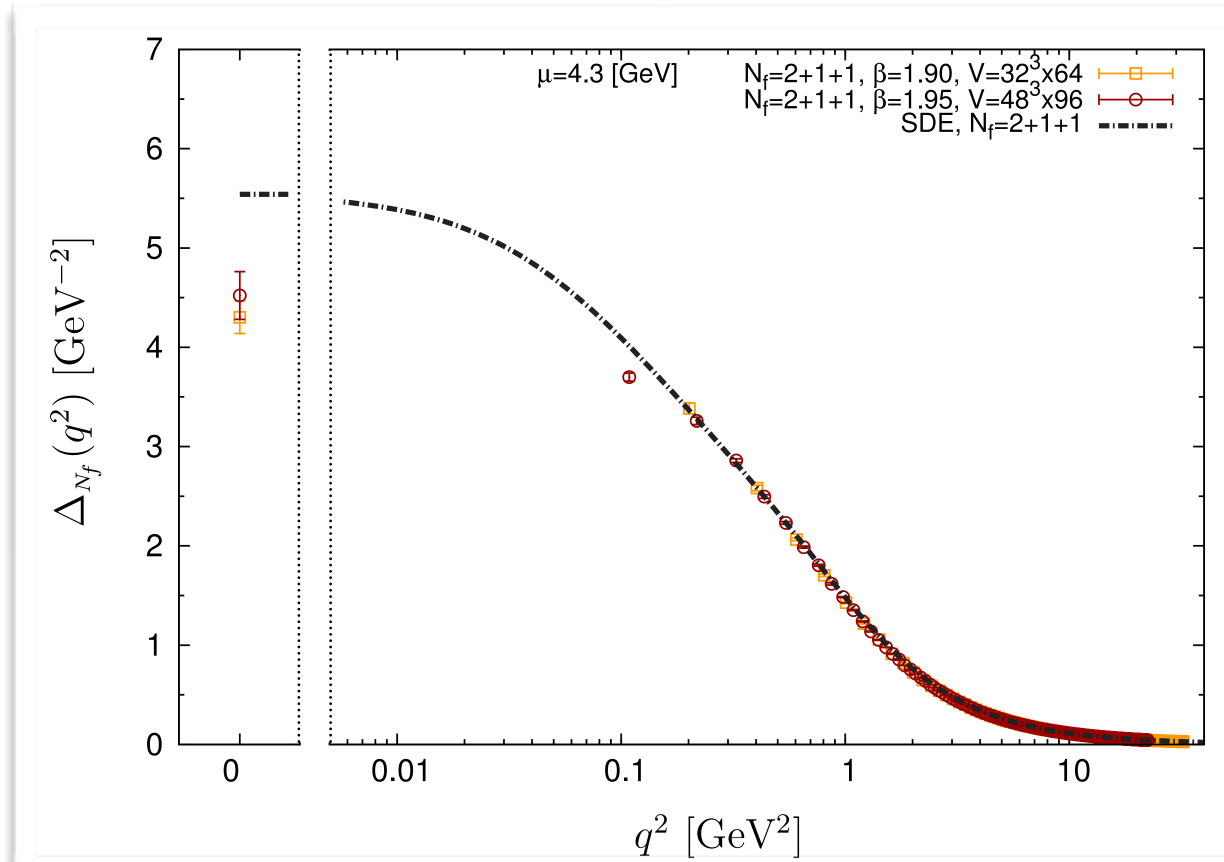


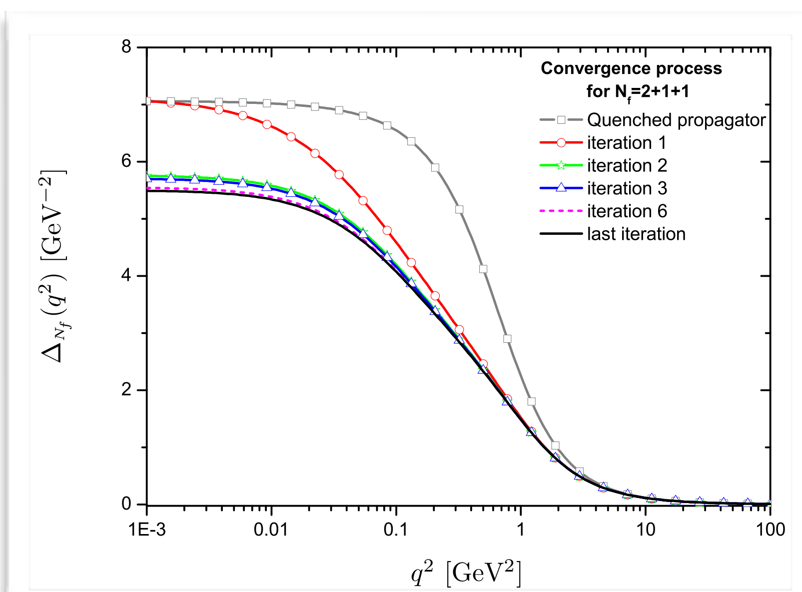
dynamically generated
gluon mass

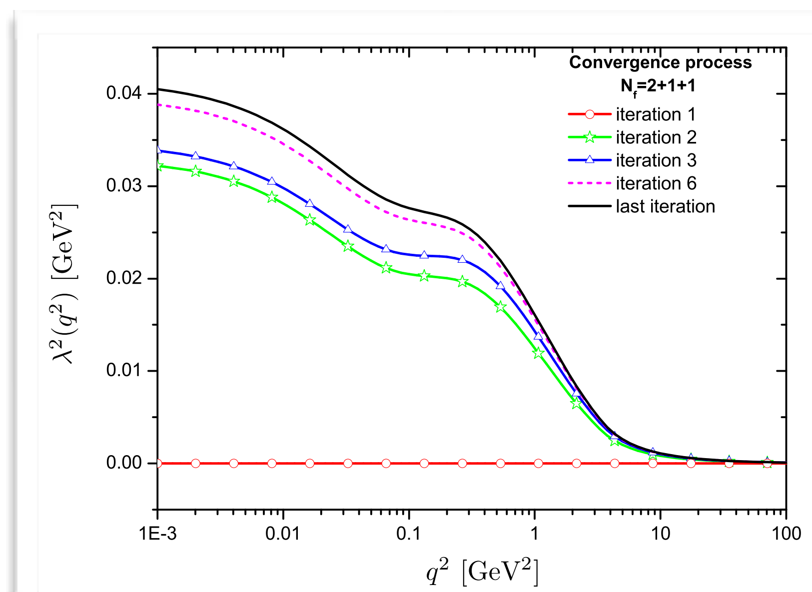
A. C. Aguilar, D. B. and J. Papavassiliou, Phys.Rev. D86, 014032 (2012)
A. C. Aguilar, D. B. and J. Papavassiliou, Phys. Rev . D88, 074010 (2013)

- Good description of the gluon propagator (even unquenched)



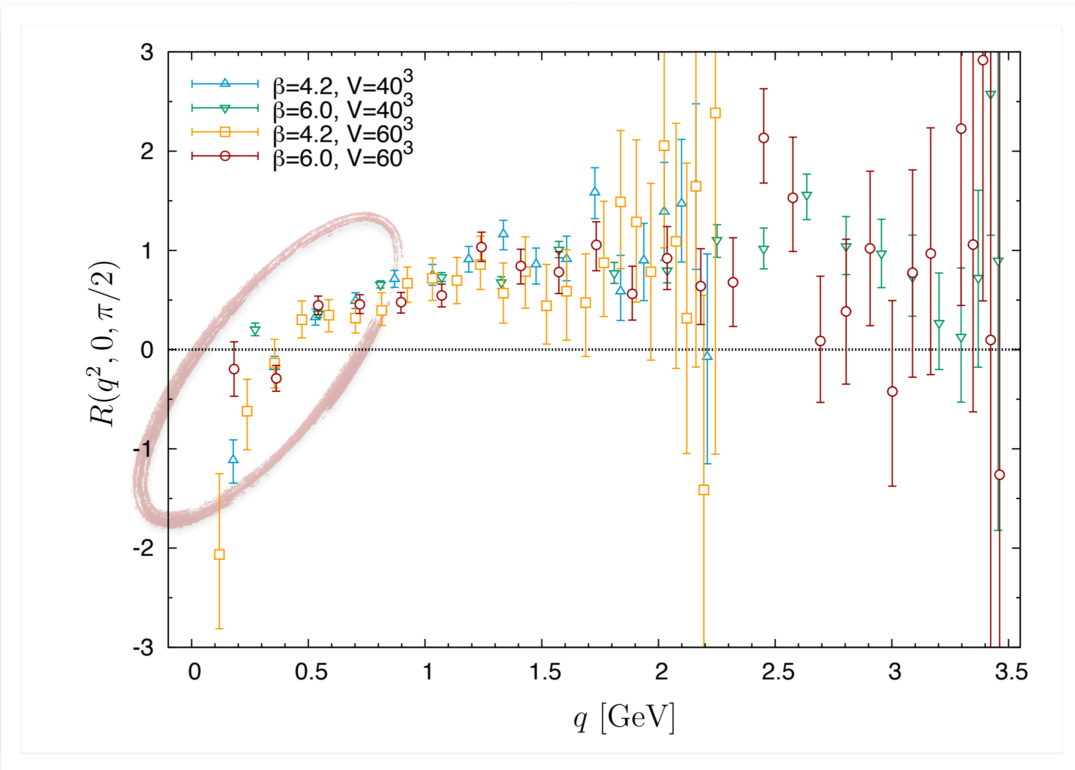


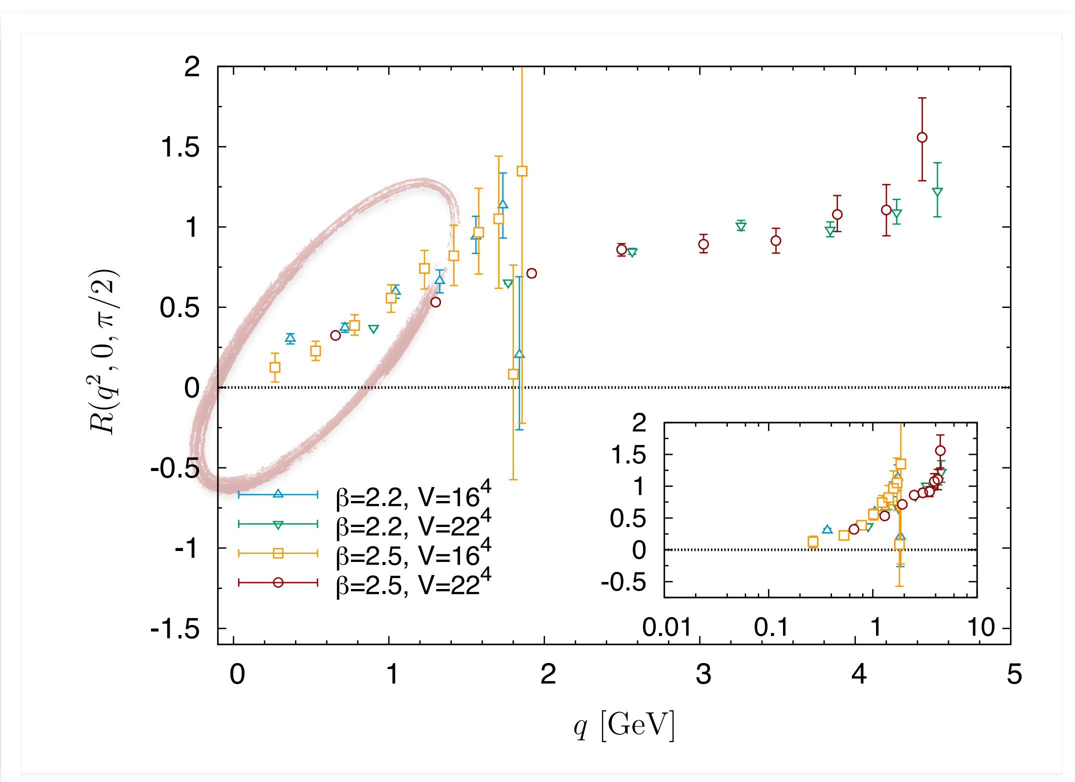




three gluon vertex in lattice QCD

SU(2) lattice data: A. Cucchieri, A. Maas and T. Mendes, Phys.Rev. D77 (2008) 094510





- Lattice results for the R projector

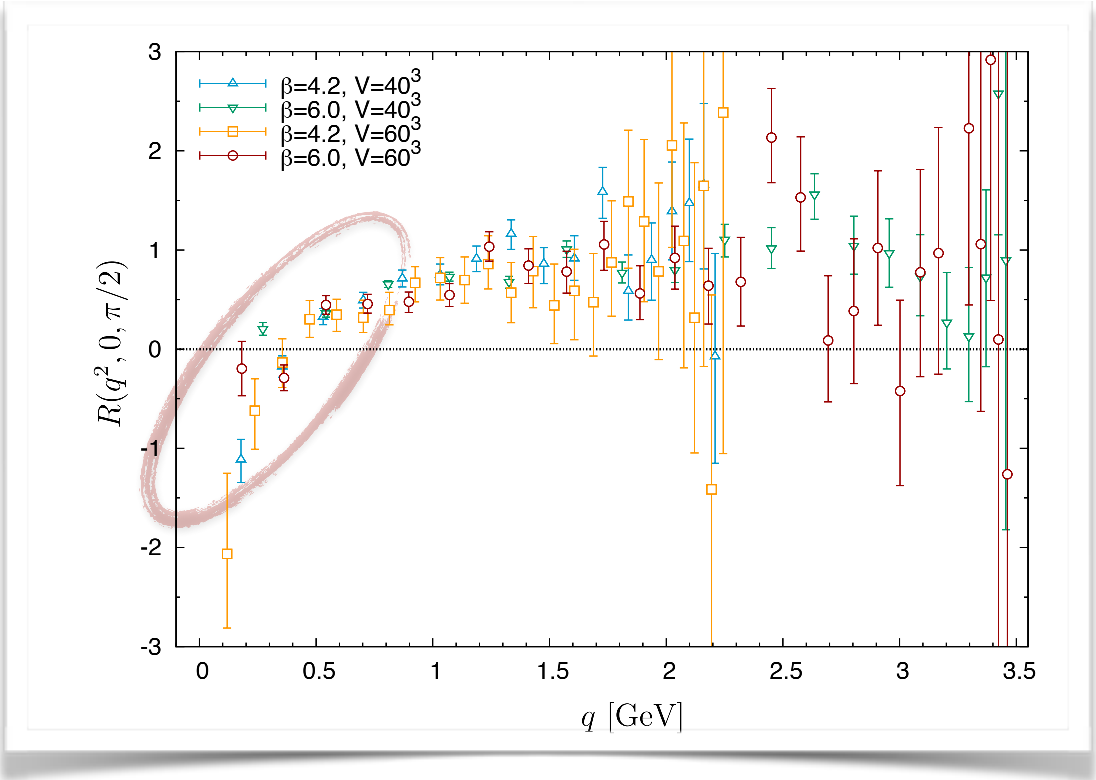
$$R(q,r,p) = \frac{\Gamma^{(0)}_{\alpha\mu\nu}(q,r,p)P^{\alpha\rho}(q)P^{\mu\sigma}(r)P^{\nu\tau}(p)\Gamma_{\rho\sigma\tau}(q,r,p)}{\Gamma^{(0)}_{\alpha\mu\nu}(q,r,p)P^{\alpha\rho}(q)P^{\mu\sigma}(r)P^{\nu\tau}(p)\Gamma^{(0)}_{\rho\sigma\tau}(q,r,p)}$$

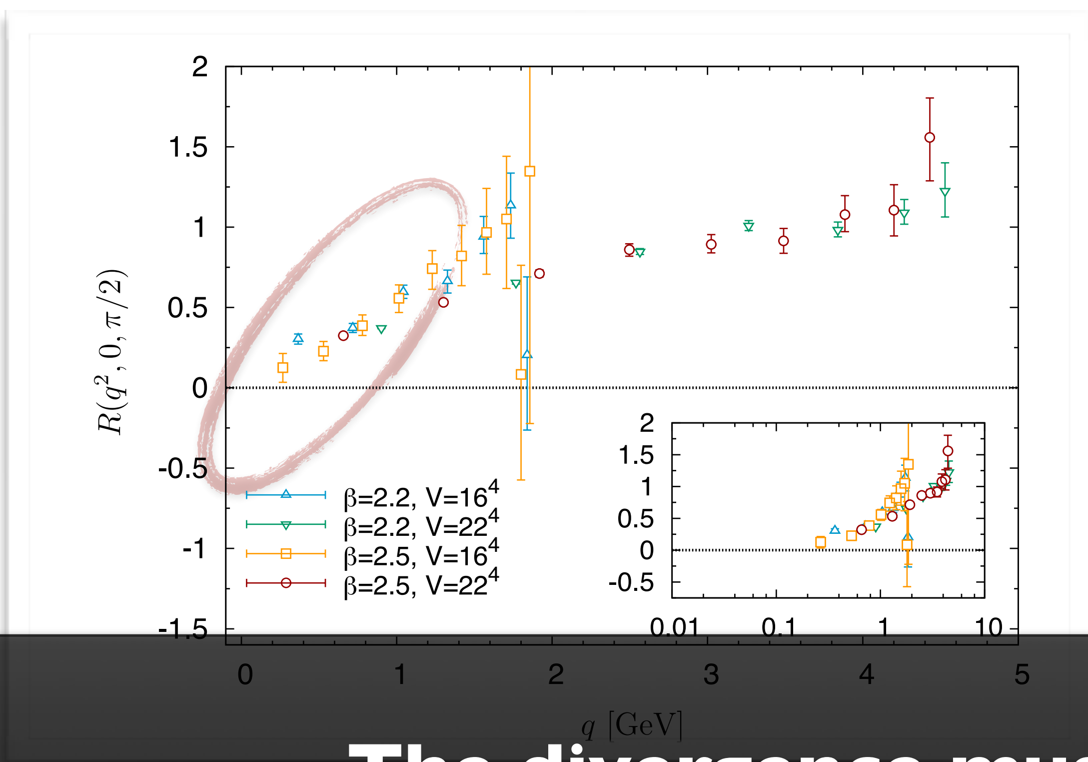
- Somewhat surprising results
  - Negative IR divergence in the deep IR (evidence in d=3, indication in d=4)
- How can we understand this in terms of IR finite propagators/dressing functions?
- In the "orthogonal" configuration one has
  - $R(q^2) = F(0)[q^2J(q^2)]' + R^{s\ell}(q^2)$

$$\Delta^{-1}(q^2) = q^2J(q^2) + m^2(q^2)$$

three gluon vertex in lattice QCD

SU(2) lattice data: A. Cucchieri, A. Maas and T. Mendes, Phys.Rev. D77 (2008) 094510





- Lattice results for the R projector

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- Somewhat surprising results
  - Negative IR divergence in the deep IR (evidence in d=3, indication in d=4)
- How can we understand this in terms of IR finite propagators/dressing functions?
- In the "orthogonal" configuration one has
  - $R(q^2)=F(0)[q^2J(q^2)]'+R^{s\ell}(q^2)$

$$\Delta^{-1}(q^2)=q^2J(q^2)+m^2(q^2)$$

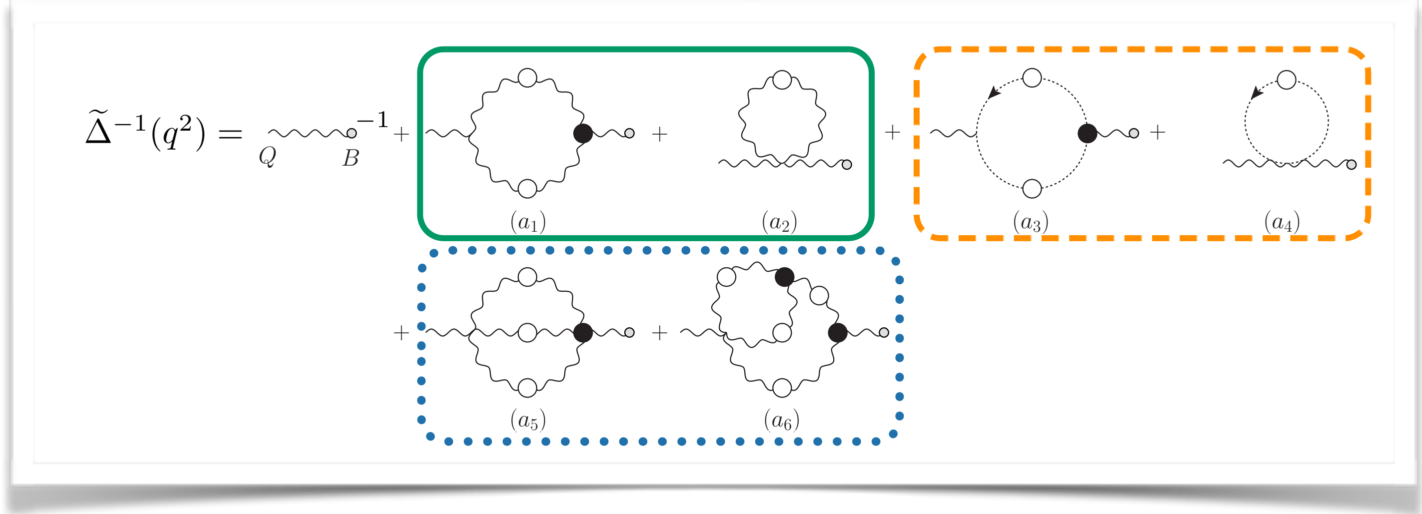
- The divergence must be coming from $J(q^2)$

The divergence must be coming from $J(q^2)$!

A toy model

a one-loop
toy model

A. C. Aguilar, D. B., D. Ibañez and J. Papavassiliou, Phys.Rev. D89, 085008 (2014)



- Perturbative one-loop setting with a **massive gluon** and a **massless ghost**
  - Gluon contribution to the inverse dressing

$$J_{a_1}(q^2) \sim \begin{cases} \ln\left[(q^2+m^2)/\mu^2\right], & d=4; \\ (1/q)\arctan(q/2m), & d=3, \end{cases}$$

  - Ghost contribution to the inverse dressing

$$J_{a_3}(q^2) \sim \begin{cases} \ln\left(q^2/\mu^2\right), & d=4; \\ 1/q, & d=3. \end{cases}$$

- Then, the gluon propagator becomes

$$\begin{aligned}\Delta^{-1}(q^2) &= q^2 J(q^2) + m^2 \\ &= q^2[1 + c_1 J_{a_1}(q^2) + c_3 J_{a_3}(q^2)] + m^2\end{aligned}$$

The coefficients are determined at the one-loop level

- **Four** dimensional case

$$c_1 = 2\left(\frac{\alpha C_A}{4\pi}\right); \quad c_3 = \frac{1}{6}\left(\frac{\alpha C_A}{4\pi}\right)$$

$c_i > 0$ and $c_1 \gg c_3$

- **Three** dimensional case

$$c_1 = -\left(\frac{25 g^2 C_A}{32\pi}\right); \quad c_3 = -\left(\frac{g^2 C_A}{32}\right)$$

$c_i < 0$ and $c_1 \gg c_3$

toy model
features

- The gluon propagator displays a maximum for any d

$$[\Delta^{-1}(q^2)]' = c_3 \ln\left(q^2/\mu^2\right) + \left\{1 + c_1 \ln\left[(q^2+m^2)/\mu^2\right] + \frac{c_1 q^2}{q^2+m^2} + c_3\right\}$$

$$[\Delta^{-1}(q^2)]'' = \frac{c_1}{q^2+m^2} + \frac{c_1 m^2}{(q^2+m^2)^2} + \frac{c_3}{q^2} > 0$$

The massless log is a sufficient condition for a maximum

$$[\Delta^{-1}(q^2)]' = 1 + \frac{c_1}{2q}\arctan(q/2m) + \frac{c_3}{2q} + \frac{c_1 m}{q^2+4m^2}$$

$$q_\triangle/m = -\frac{c_3/m}{2+c_1/m} \qquad \frac{m}{2g^2} \gtrsim 0.14$$

- The combination $q^2J(q^2)$ displays a minimum (located at the same position)

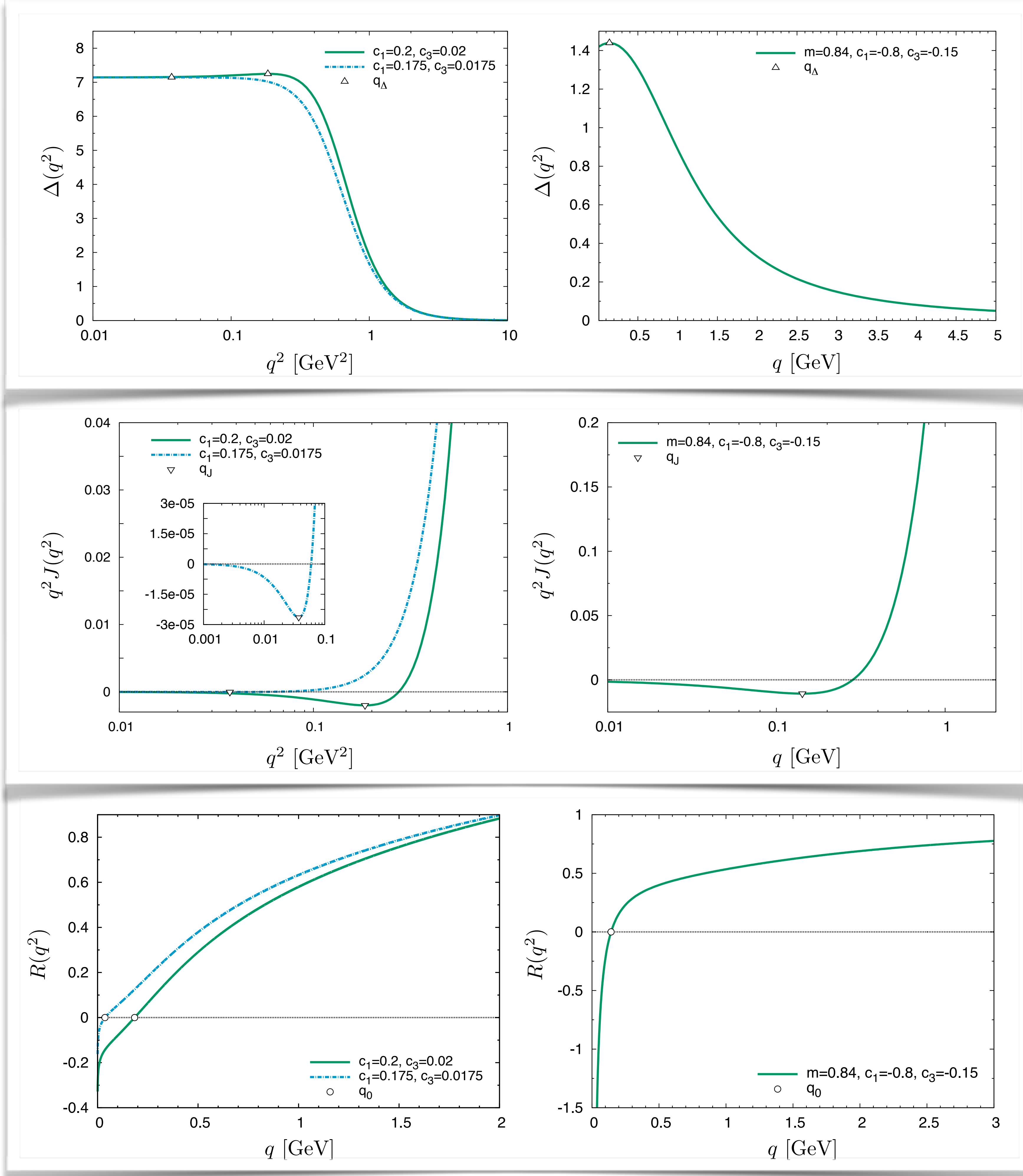
- The R projector displays a (negative) divergence

  - $R(q^2) \sim [q^2J(q^2)]'$

$$R(q^2) \underset{q^2\to 0}{\sim} c_3 J_{a_3}(q^2) \sim \begin{cases} \ln\left(q^2/\mu^2\right), & d=4; \\ -1/q, & d=3. \end{cases}$$

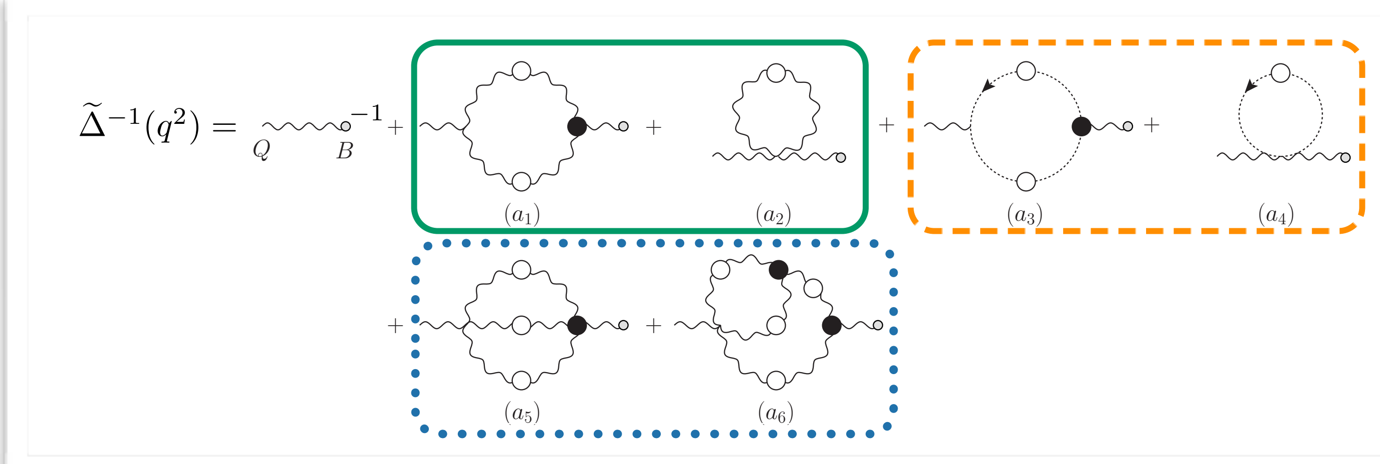
  - Zero crossing happens at the same position of the minimum of $q^2J(q^2)$

toy model
numerics



Full np analysis

non perturbative
calculation-I



- Need to evaluate the full ghost contributions to $q^2J(q^2)$

$$q^2 J_c(q^2) = C_d\, F(q^2)\left[4T(q^2) + q^2 S(q^2)\right]$$

$$T(q^2) = \int_k \frac{F(k+q) - F(k)}{(k+q)^2 - k^2} + \left(\frac{d}{2} - 1\right)\int_k \frac{F(k)}{k^2}$$

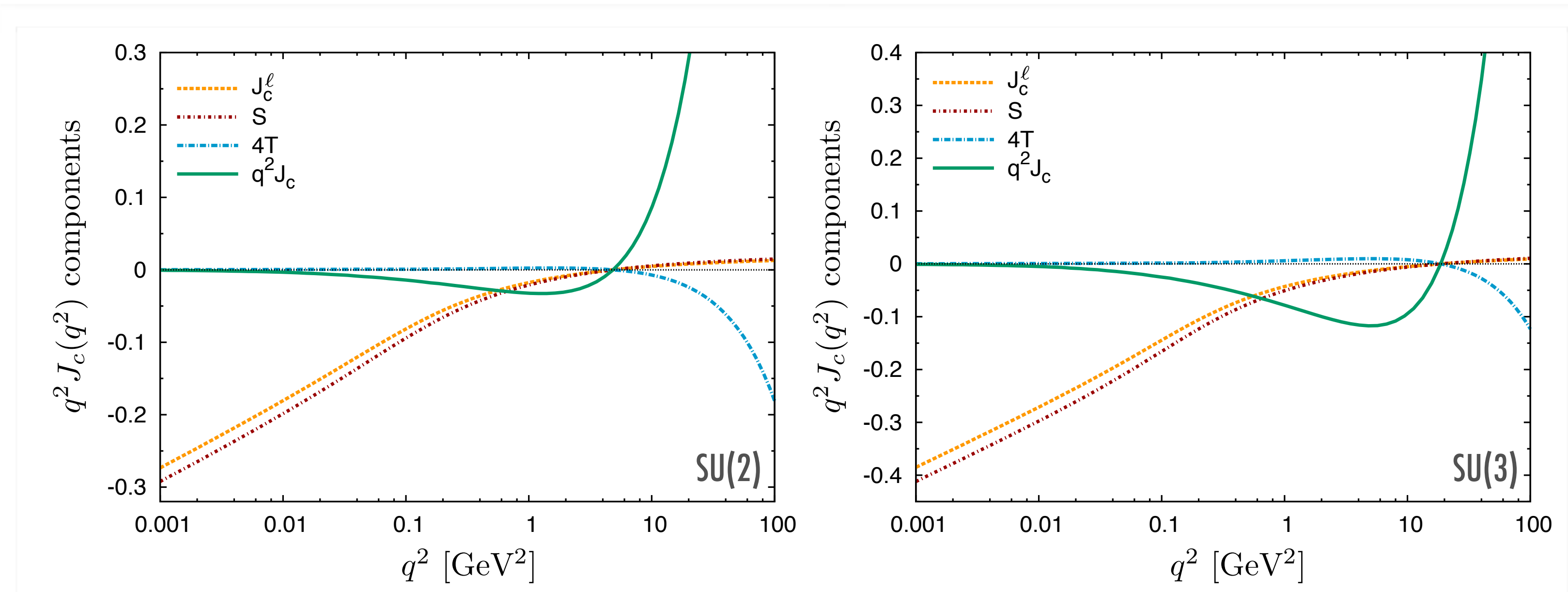
$$S(q^2) = \int_k \frac{F(k)}{k^2(k+q)^2} - \int_k \frac{F(k+q) - F(k)}{k^2[(k+q)^2 - k^2]}$$

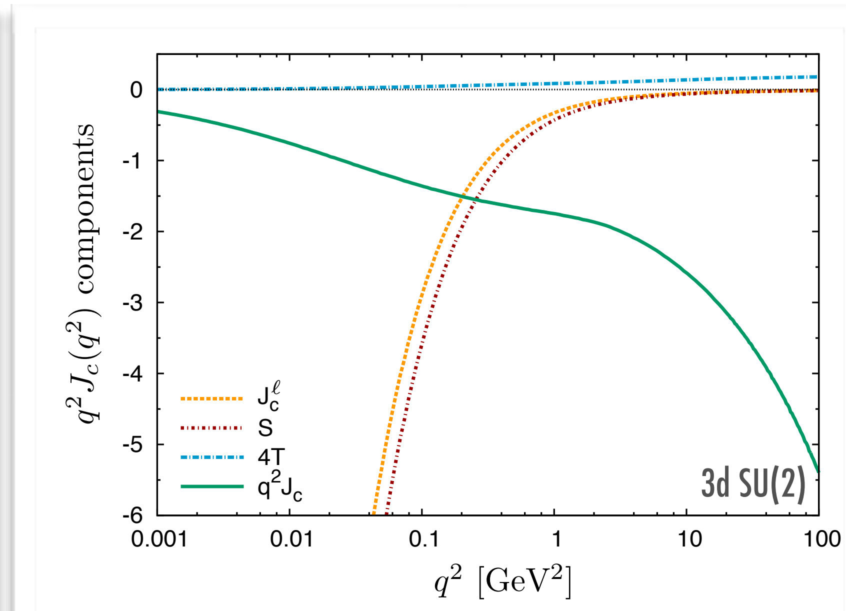
The ghost-gluon vertex is obtained by solving the WI neglecting the transverse part

- The IR behavior is similar to the one of the toy model

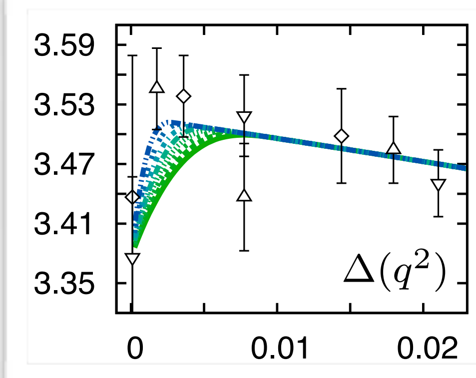
$$J_c(q^2) = J_c^{\ell}(q^2) + J_c^{s\ell}(q^2) \qquad J_c^{\ell}(q^2) \sim F(q^2)\int_k \frac{F(k)}{k^2(k+q)^2}$$

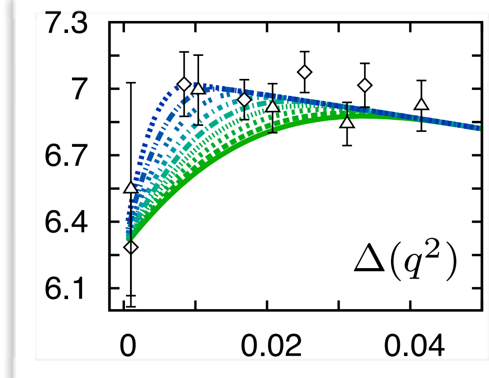
The IR leading contribution diverges in the IR



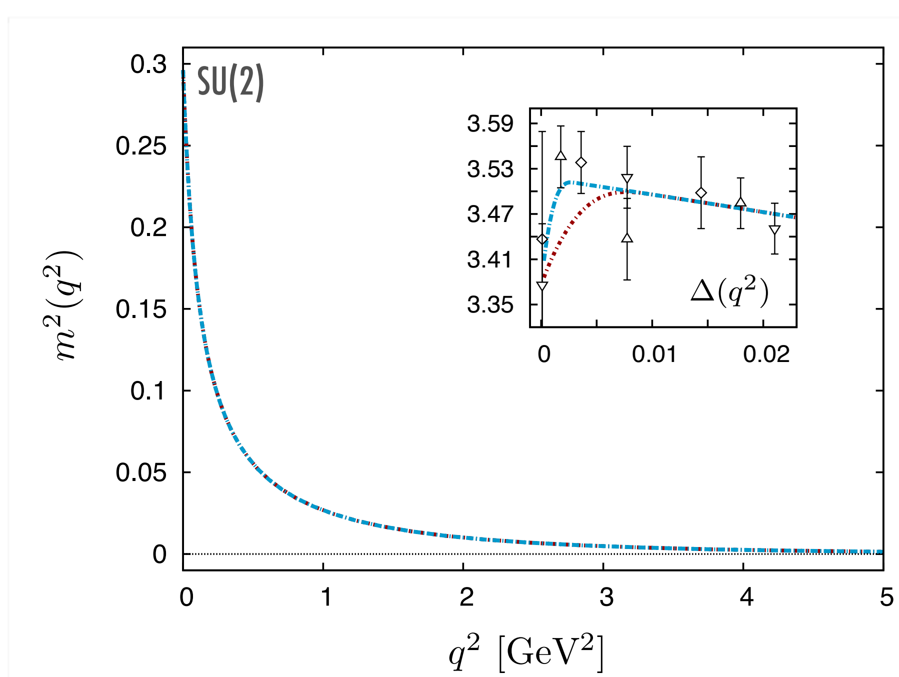


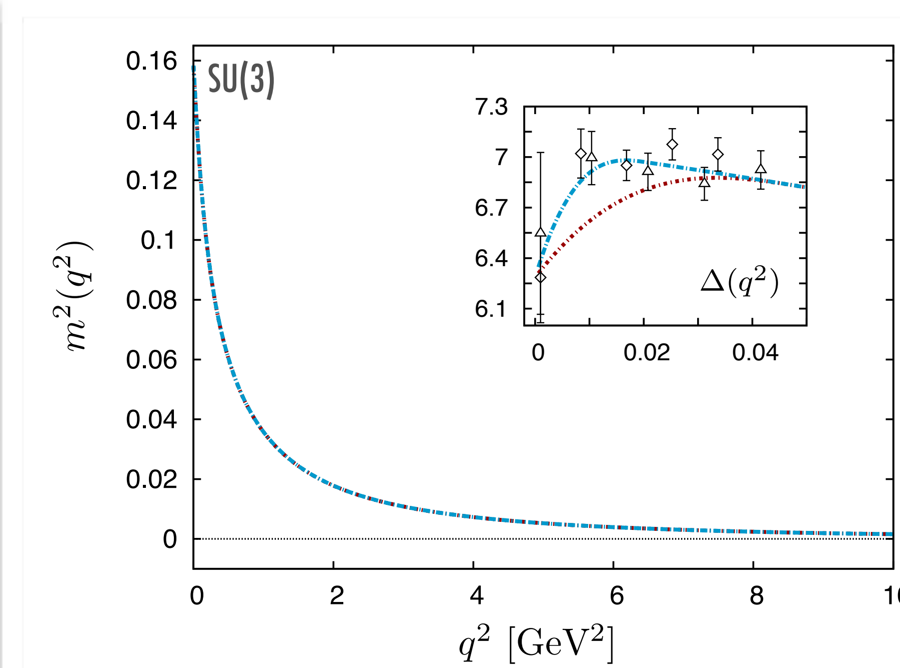
non perturbative
calculation-II





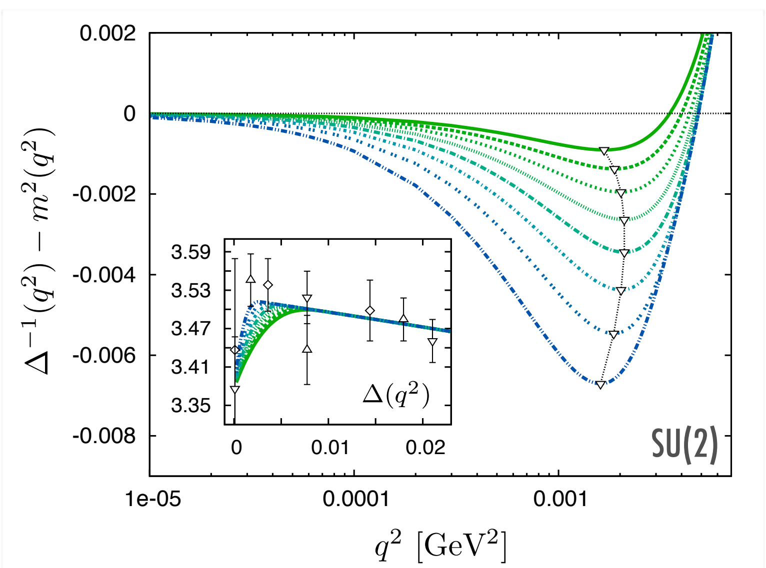
- The gluon propagator *must* show a maximum
- We are however unable to directly determine the gluonic contributions to $q^2 J(q^2)$
  - Two-loop gluon contribution are practically unknown (but see next talk by Joannis)
- Use an indirect method: $q^2 J(q^2) = \underbrace{\Delta^{-1}(q^2)}_{\text{lattice}} - \underbrace{m^2(q^2)}_{\text{mass eq.}}$
- The mass in insensitive to the presence of the IR maximum in the propagator

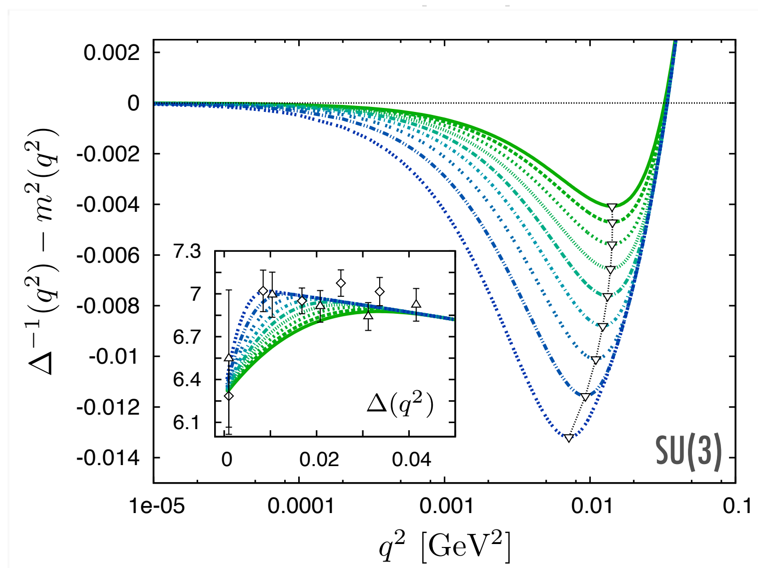


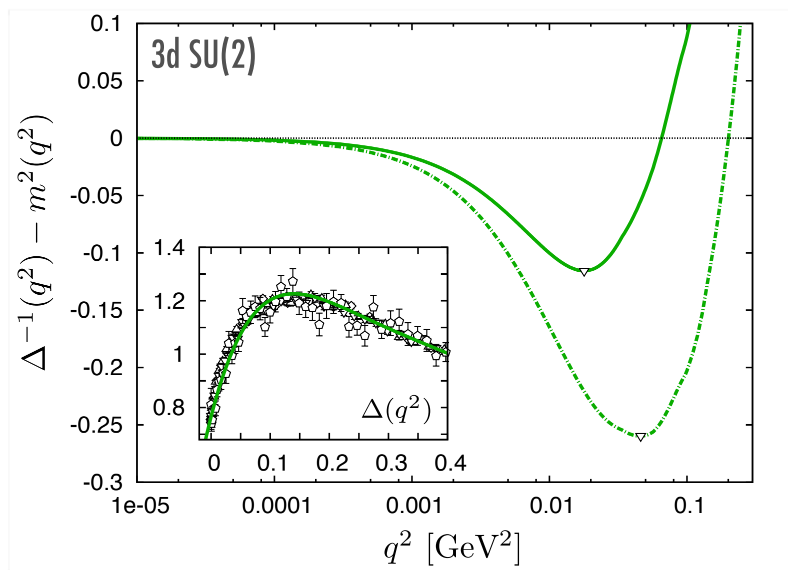


non perturbative calculation-III

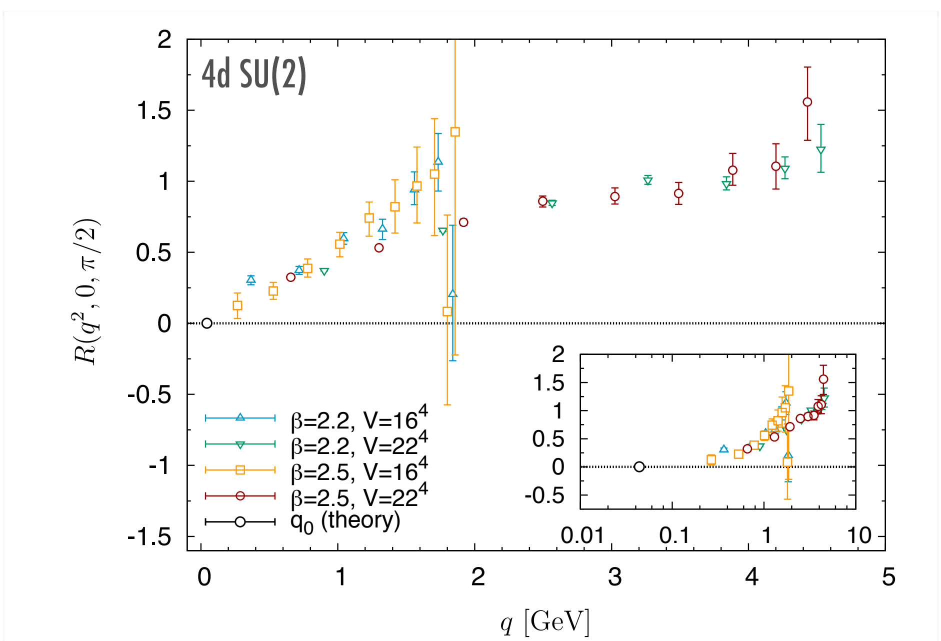
- The full gluon inverse dressing function displays a **minimum**

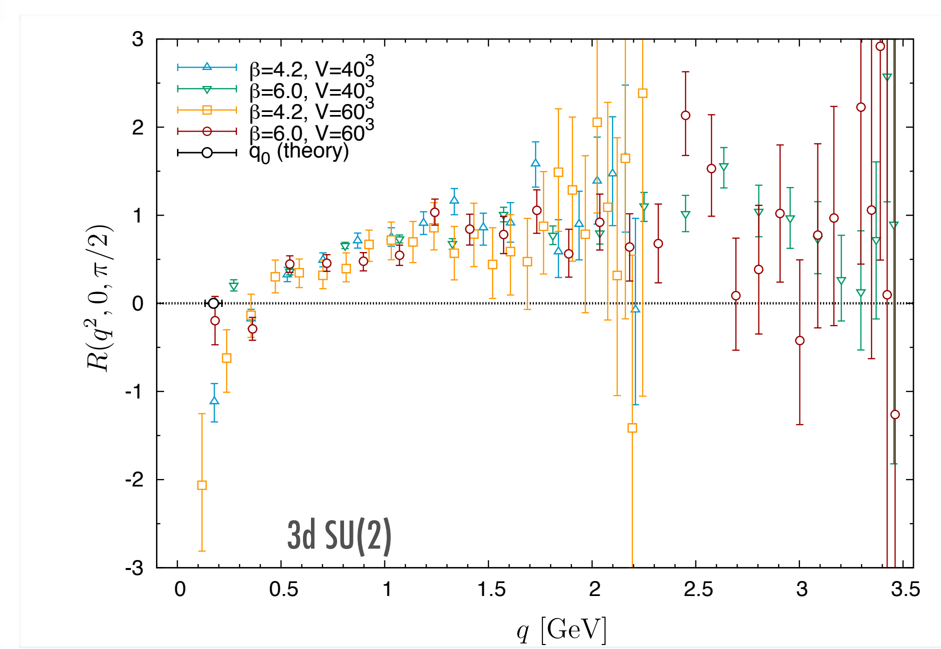






- The **position** of the **minimum** determines the **zero crossing** of the R projector





- d=3
  - SU(2): $q_0 \sim 200$ MeV
- d=4
  - SU(2): $q_0 \sim 44$ MeV
    L~130 (22)
  - SU(3): $q_0 \sim 132$ MeV
    L~60 (0)

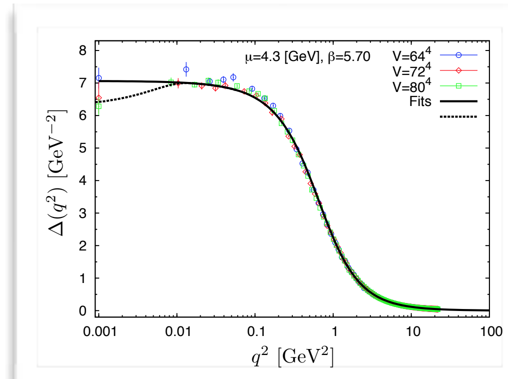
Epilogue

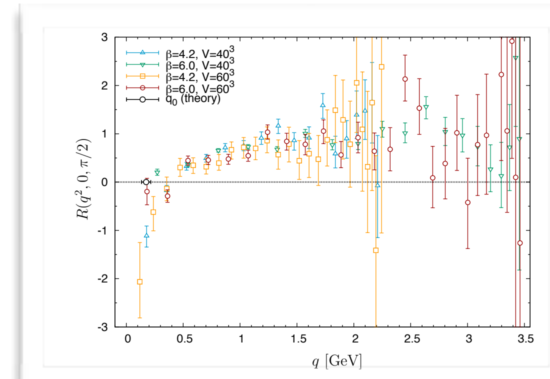
conclusions & outlook

- The concept of a gluon mass gives a precise description of YM theories in the nonperturbative regime

- Massless ghosts imply a negative IR divergence in the kinetic part of the gluon propagator

  - **Maximum** in the **gluon propagator**
  - **Divergence** in the **three gluon vertex**





- Estimates of where the divergence of the vertex starts to take over (zero-crossing) can be given without computing the full vertex

- **Same behavior** expected for all Green's functions containing a 'primary' **ghost loop**