Universality of radiative corrections to gauge couplings for strings with spontaneously broken supersymmetry

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Based on work with

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Phys. Lett. B 736 (2014)

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DISCRETE 2014 - King's College London

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Introduction

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- Over last 20 years : significant progress in String Phenomenology
- Semi-realistic vacua incorporating salient features of MSSM
- Reconstruction of low energy effective action with $\mathcal{N} = 1$ SUSY at tree level
- Quantitative comparison with low energy data : incorporate loop corrections
- Quantum & string length corrections are still the centre of attention

But what about SUSY breaking ?

Introduction

(stringy) Scherk-Schwarz mechanism

- Spontaneous breaking of SUSY with exactly tractable worldsheet description
- Worldsheet description in terms of freely-acting orbifolds
- Exponential growth of string states may de-stabilise classical vacuum : tachyons
- Related to Hagedorn problem : can be circumvented in special constructions

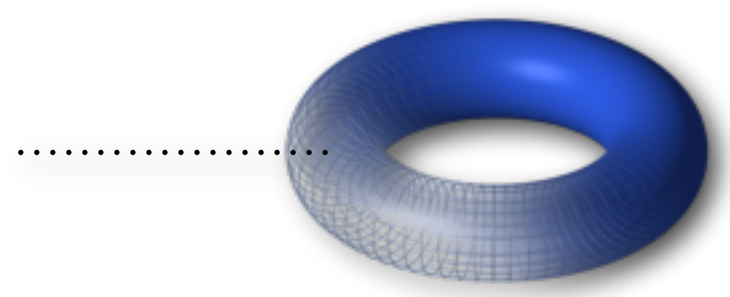
Scherk, Schwarz 1979
Rohm 1984
Kounnas, Porrati 1988
Atick, Witten 1988
Kounnas, Rostand 1990

Introduction

When SUSY is (spontaneously) broken but the vacuum is classically stable

it is meaningful and important to study one-loop radiative corrections to couplings in the low energy effective action

Higher loops : problematic due to one-loop tadpole that back-reacts on classical vacuum



$\neq 0$

Fischler, Susskind 1986

In this talk

Study one-loop threshold corrections to non-abelian gauge couplings in 4d heterotic vacua

supersymmetry is spontaneously broken : $\mathcal{N}=2 \longrightarrow \mathcal{N}=0$

This setup is relevant for more realistic constructions of $\mathcal{N}=1 \longrightarrow \mathcal{N}=0$ breaking

$\mathcal{N}=2$ sectors encode moduli dependence

universality ?

Vacuum configuration

Heterotic vacuum

Start with $\mathrm{E}_8 \times \mathrm{E}_8$ heterotic string in 10d

$\downarrow$ reduce on $\mathrm{K3} \sim T^4/\mathbb{Z}_N$, with N = 2, 3, 4, 6

$\mathrm{E}_8 \times \mathrm{E}_7$

$\mathcal{N} = 1$ heterotic vacuum in 6d

$\downarrow$ Scherk-Schwarz reduction on T^2 + trivial Wilson line background $\Big\}$ freely-acting $\mathbb{Z}_2'$ orbifold

$\mathrm{SO}(16) \times \mathrm{SO}(12)$ heterotic vacuum in 4d with $\mathcal{N} = 0$ SUSY

Heterotic vacuum

Consider orbifold $T^6/\mathbb{Z}_N \times \mathbb{Z}_2'$

$\mathbb{Z}_N$

$$Z^1 \rightarrow e^{2\pi i/N}\, Z^1$$
$$Z^2 \rightarrow e^{-2\pi i/N}\, Z^2$$

rotates complexified T^4 coordinates

realises singular limit of K3 surface (preserves 8 supercharges)

Alvarez-Gaumé, Ginsparg, Moore, Vafa 1986
Dixon, Harvey 1986
Itoyama, Taylor 1987

freely acting $\mathbb{Z}_2'$

$$v' = (-1)^{F_{\rm st}+F_1+F_2}\,\delta$$

spacetime fermion number

"fermion numbers" of original E_8's

order-2 shift along remaining T^2

responsible for spontaneous SUSY breaking down to $\mathcal{N}=0$

vacuum is classically stable (W=0)

Heterotic vacuum

One-loop partition function

$$
\begin{aligned}
\mathcal{Z} =& \frac{1}{2}\sum_{H,G=0}^{1} \frac{1}{N}\sum_{h,g=0}^{N-1} \left[\frac{1}{2}\sum_{a,b=0}^{1} (-)^{a+b} \vartheta\begin{bmatrix} a/2 \\ b/2 \end{bmatrix}^2 \vartheta\begin{bmatrix} a/2+h/N \\ b/2+g/N \end{bmatrix} \vartheta\begin{bmatrix} a/2-h/N \\ b/2-g/N \end{bmatrix} \right] && \text{RNS (SCFT)} \\
&\times \left[\frac{1}{2}\sum_{k,\ell=0}^{1} \bar\vartheta\begin{bmatrix} k/2 \\ \ell/2 \end{bmatrix}^6 \bar\vartheta\begin{bmatrix} k/2+h/N \\ \ell/2+g/N \end{bmatrix} \bar\vartheta\begin{bmatrix} k/2-h/N \\ \ell/2-g/N \end{bmatrix} \right] \left[\frac{1}{2}\sum_{r,s=0}^{1} \bar\vartheta\begin{bmatrix} r/2 \\ s/2 \end{bmatrix}^8 \right] && \text{gauge sector} \\
&\times \frac{1}{\eta^{12}\bar\eta^{24}} (-)^{H(b+\ell+s)+G(a+k+r)+HG}\, \Gamma_{2,2}\begin{bmatrix} H \\ G \end{bmatrix} \Lambda^{\mathrm{K3}}\begin{bmatrix} h \\ g \end{bmatrix} . && \text{lattices}
\end{aligned}
$$

$\mathrm{SO}(12)$ $\quad\quad$ $\mathrm{U}(1)^2$ $\quad\quad$ $\mathrm{SO}(16)$

Heterotic vacuum

One-loop partition function

$$
\begin{aligned}
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& \times \left[\frac{1}{2}\sum_{k,\ell=0}^{1} \bar\vartheta \begin{bmatrix} k/2 \\ \ell/2 \end{bmatrix}^6 \bar\vartheta \begin{bmatrix} k/2+h/N \\ \ell/2+g/N \end{bmatrix} \bar\vartheta \begin{bmatrix} k/2-h/N \\ \ell/2-g/N \end{bmatrix} \right] \left[\frac{1}{2}\sum_{r,s=0}^{1} \bar\vartheta \begin{bmatrix} r/2 \\ s/2 \end{bmatrix}^8 \right] && \text{gauge sector} \\
& \times \frac{1}{\eta^{12}\bar\eta^{24}} (-)^{H(b+\ell+s)+G(a+k+r)+HG}\, \Gamma_{2,2} \begin{bmatrix} H \\ G \end{bmatrix} \Lambda^{\mathrm{K3}} \begin{bmatrix} h \\ g \end{bmatrix} . && \text{lattices}
\end{aligned}
$$

$\mathrm{SO}(12)$ $\quad$ $\cancel{\mathrm{U}(1)^2}$ $\quad$ $\mathrm{SO}(16)$

disregard

Friday, December 5, 14

Heterotic vacuum

One-loop partition function

$$\mathcal{Z} = \frac{1}{2}\sum_{H,G=0}^{1} \frac{1}{N}\sum_{h,g=0}^{N-1} \left[\frac{1}{2}\sum_{a,b=0}^{1} (-)^{a+b} \vartheta\begin{bmatrix} a/2 \\ b/2 \end{bmatrix}^2 \vartheta\begin{bmatrix} a/2+h/N \\ b/2+g/N \end{bmatrix} \vartheta\begin{bmatrix} a/2-h/N \\ b/2-g/N \end{bmatrix}\right]$$

RNS (SCFT)

$$\times \left[\frac{1}{2}\sum_{k,\ell=0}^{1} \bar\vartheta\begin{bmatrix} k/2 \\ \ell/2 \end{bmatrix}^6 \bar\vartheta\begin{bmatrix} k/2+h/N \\ \ell/2+g/N \end{bmatrix} \bar\vartheta\begin{bmatrix} k/2-h/N \\ \ell/2-g/N \end{bmatrix}\right] \left[\frac{1}{2}\sum_{r,s=0}^{1} \bar\vartheta\begin{bmatrix} r/2 \\ s/2 \end{bmatrix}^8\right]$$

gauge sector

$$\times \frac{1}{\eta^{12}\bar\eta^{24}} (-)^{H(b+\ell+s)+G(a+k+r)+HG}\, \varGamma_{2,2}\left[{}^{H}_{G}\right] \varLambda^{\mathrm{K3}}\left[{}^{h}_{g}\right] .$$

lattices

shifted Narain lattice $\varGamma_{2,2}[{}^{H}_{G}] = \tau_2 \sum_{m_i, n^i \in \mathbb{Z}} e^{i\pi G(\lambda_1\cdot m+\lambda_2\cdot n)}\, q^{\frac{1}{4}|P_L|^2}\, \bar q^{\frac{1}{4}|P_R|^2}$

T^2 lattice momenta

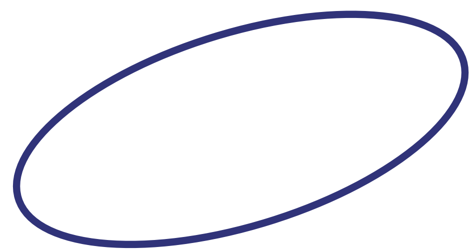
$$P_L = \frac{m_2 - U m_1 + \bar T(n^1 + U n^2)}{\sqrt{T_2 U_2}}$$

central charge

$$P_R = \frac{m_2 - U m_1 + T(n^1 + U n^2)}{\sqrt{T_2 U_2}}$$

Scherk Schwarz mechanism

Deformation of vertex operators / fields by symmetry Q

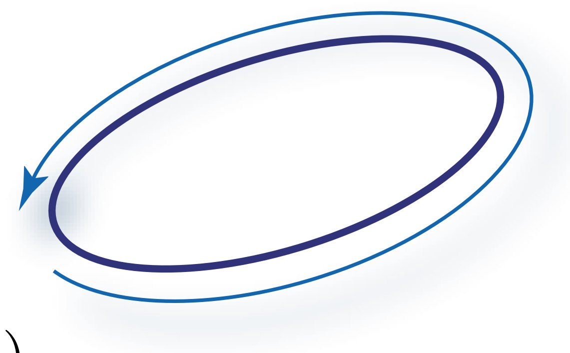


$\Phi(X_5)$

Scherk Schwarz mechanism

Deformation of vertex operators / fields by symmetry Q

$$\Phi(X_5 + 2\pi R) = e^{iQ}\, \Phi(X_5)$$



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Scherk Schwarz mechanism

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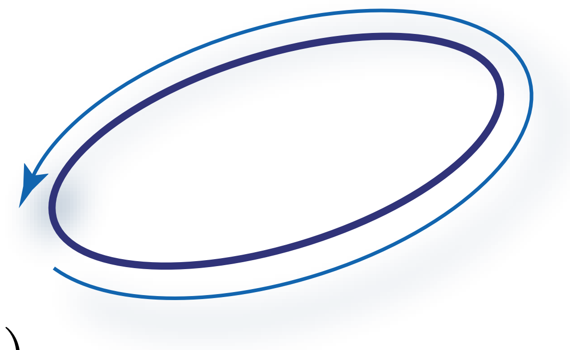


$$\Phi(X_5)$$

$$\Phi(X_5) = e^{iQX_5/2\pi R} \sum_{m\in\mathbb{Z}} \Phi_m\, e^{imX_5/R}$$

Kaluza-Klein spectrum of charged states is shifted $\quad m \to m + Q/2\pi$

$$M_{\rm KK} = \frac{|Q|}{2\pi R}$$

Scherk Schwarz mechanism

'Flat' gauging of $\mathcal{N}=2$ supergravity : completely fixed by string vacuum

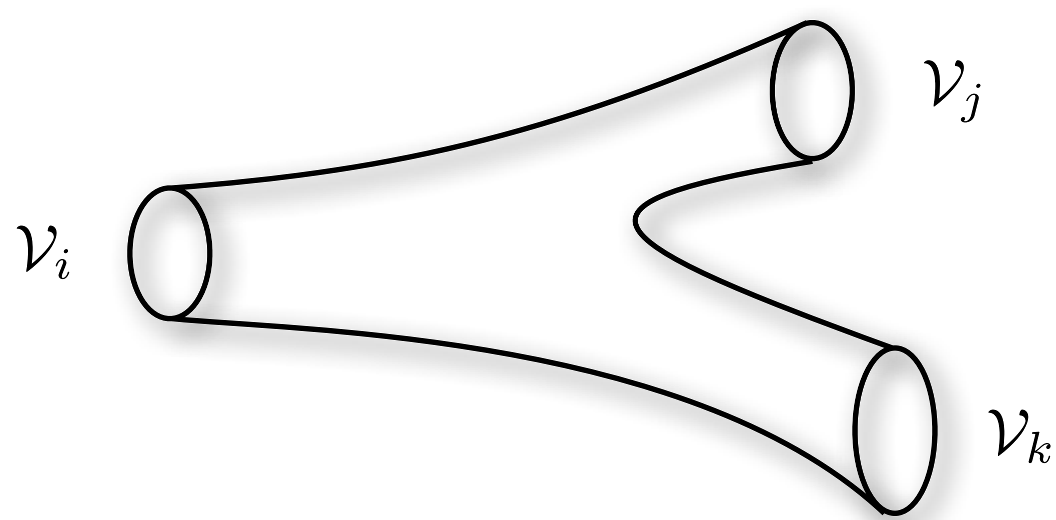
$$\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SO}(2,2)}{\mathrm{SO}(2)\times\mathrm{SO}(2)} \times \frac{\mathrm{SO}(4,4+n)}{\mathrm{SO}(4)\times\mathrm{SO}(4+n)}$$

S $\qquad T, U \qquad T_i,\ U_i,\ \Phi^A$

(BPS states $n=\infty$)

$$f_{ijk} = \langle\, \mathcal{V}_i\, \mathcal{V}_j\, \mathcal{V}_k\, \rangle_{\mathrm{string}}$$

action up to 2 derivatives fixed by 3-point correlators in string theory



Scherk Schwarz mechanism

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$$S \qquad\qquad T, U \qquad\qquad T_i,\ U_i,\ \Phi^A$$

gravitino mass $\quad m^2_{3/2} = \dfrac{|U|^2}{T_2 U_2} \sim (1/R_5)^2$

Scherk Schwarz mechanism

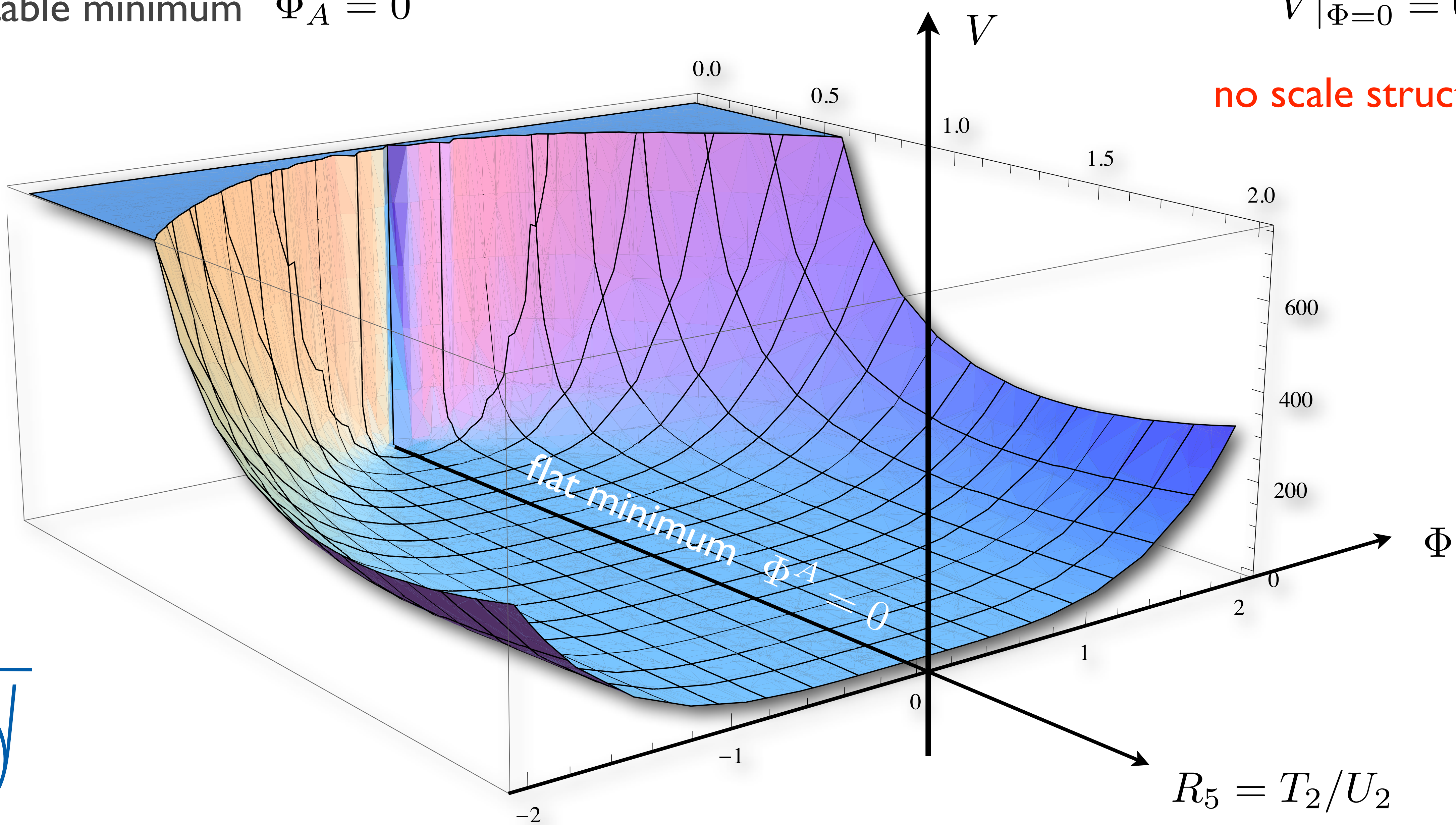
Keeping lightest BPS states $\Phi^A \in \mathbb{R}$

scalar potential $$V = \frac{|T-2U|^2\,\Phi^A\Phi_A + 2\left(|T-2U|^2 + |T-2\bar{U}|^2\right)\,(\Phi^A\Phi_A)^2}{S_2 T_2 U_2}$$

stable minimum $\Phi_A = 0$

$V|_{\Phi=0} = 0$

no scale structure



Scherk Schwarz mechanism

Identify extra massless states in string spectrum

$$O_4\, O_4\, \bar V_{12}\, \bar O_4\, \bar V_{16} \;\; \times \tfrac{1}{2} \left(\varGamma_{2,2}[{}^1_0] + \varGamma_{2,2}[{}^1_1] \right)$$

bi-fundamental $(\mathbf{12}, \mathbf{16})$ of $\mathrm{SO}(12) \times \mathrm{SO}(16)$

$$m^2_{\rm BPS} = \frac{|T/2 - U|^2}{T_2 U_2} = |P_R|^2$$

no longer annihilating spacetime supercharges !

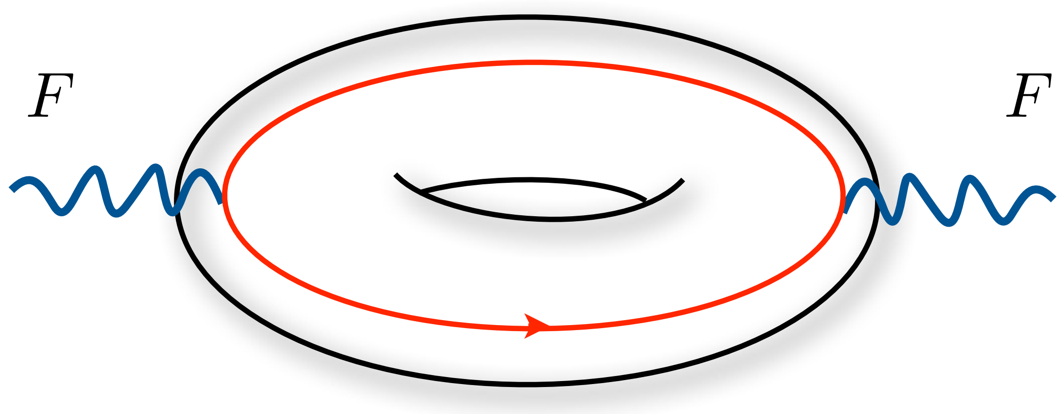
Gauge thresholds

One loop corrections

Running coupling associated to gauge group $\mathcal{G}$

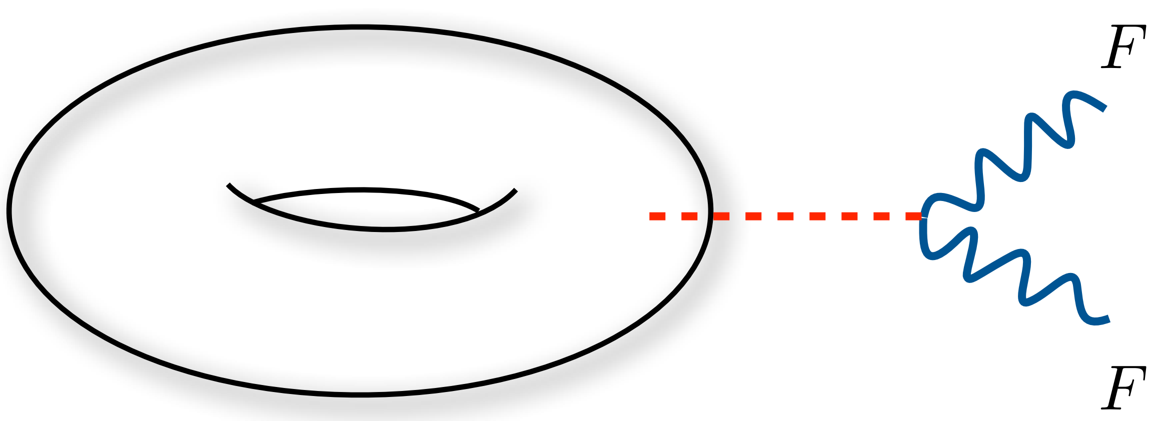
$$\frac{16\pi^2}{g^2_{\mathcal{G}}(\mu)} = \frac{16\pi^2}{g_s^2} + \beta_{\mathcal{G}} \log\frac{M_s^2}{\mu^2} + \Delta_{\mathcal{G}}$$

threshold correction



massive charged string states running in the loop

$$\sim \mathrm{Tr}\, Q^2_{\mathcal{G}}$$



universal contribution due to dilaton exchange

IR finite

One loop corrections

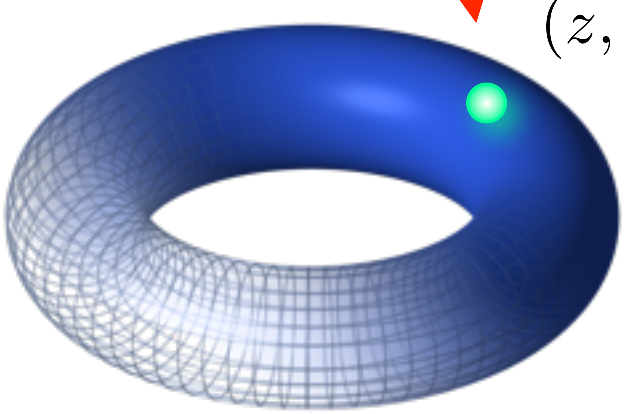
Running coupling associated to gauge group $\mathcal{G}$

$$\frac{16\pi^2}{g_{\mathcal{G}}^2(\mu)} = \frac{16\pi^2}{g_s^2} + \beta_{\mathcal{G}} \log \frac{M_s^2}{\mu^2} + \Delta_{\mathcal{G}}$$

threshold correction

$$\frac{16\pi^2}{g_{\mathcal{G}}^2} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \int_{\text{torus}} d^2z \, \langle \mathcal{V}^a(z,\bar{z}) \mathcal{V}^b(0) \rangle_{\text{CFT}}$$

one loop correction to gauge coupling



$(z,\bar{z}) \in \Sigma_1$

One loop corrections

Running coupling associated to gauge group $\mathcal{G}$

$$\frac{16\pi^2}{g_{\mathcal{G}}^2(\mu)} = \frac{16\pi^2}{g_s^2} + \beta_{\mathcal{G}} \log \frac{M_s^2}{\mu^2} + \Delta_{\mathcal{G}}$$

threshold correction

$$\frac{16\pi^2}{g_{\mathcal{G}}^2} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \int_{\text{torus}} d^2z \, \langle \mathcal{V}^a(z,\bar{z}) \mathcal{V}^b(0) \rangle_{\text{CFT}}$$

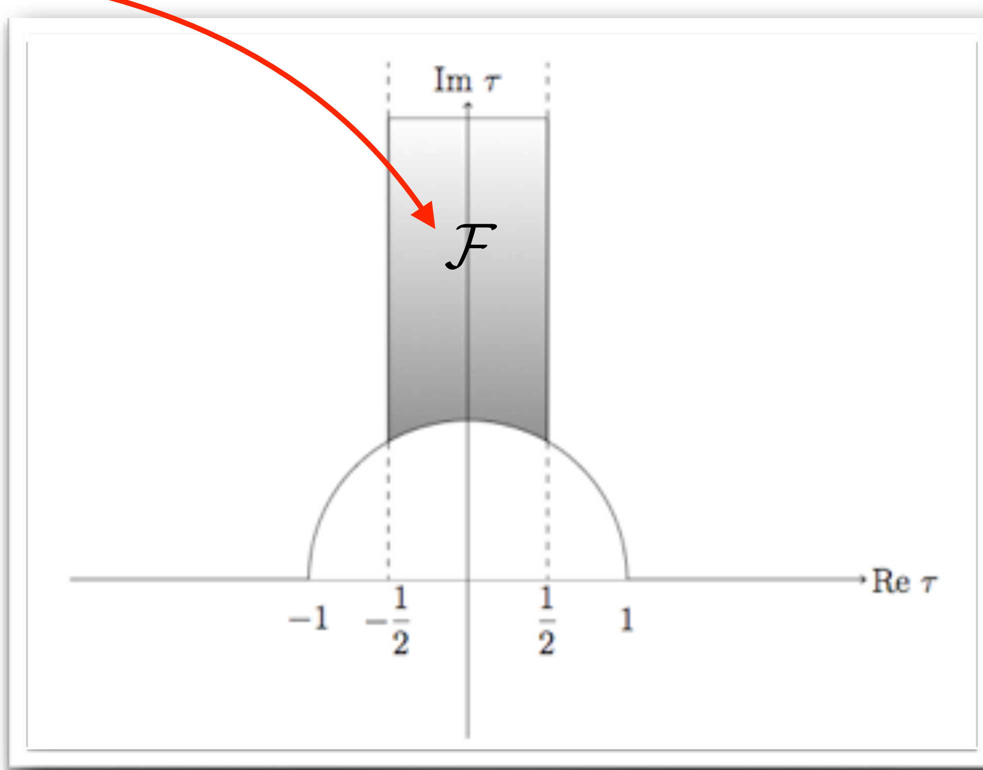
one loop correction to gauge coupling

large reparametrizations

$$\tau \to \frac{a\tau + b}{c\tau + d}$$

$$ad - bc = 1$$

$$a, b, c, d \in \mathbb{Z}$$



equivalent WS tori

$PSL(2;\mathbb{Z})$

Modular Invariance

Supersymmetric Universality

If supersymmetry is unbroken

Difference of thresholds for gauge group factors $\mathcal{G}_1$, $\mathcal{G}_2$

$$\Delta_{\mathcal{G}_1} - \Delta_{\mathcal{G}_2} = \delta\beta_{12} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\, \Gamma_{2,2}(T,U)$$

$$= -\delta\beta_{12}\ \log\, \left(T_2 U_2\, |\eta(T)\, \eta(U)|^4 \right) + \text{constant}$$

Dixon, Kaplunovsky, Louis '91

Independently of the details of the vacuum (almost)

$\boxed{\text{universality}}$

+ exceptions

Mayr, Stieberger '93

Non-supersymmetric
Universality

Non-supersymmetric Universality

Supersymmetry is spontaneously broken

$$-\frac{1}{4g_{\mathcal{G}}^2}\,F_{\mu\nu}\,F^{\mu\nu}$$

no longer BPS saturated

ALL states run in the loop

$$\Delta_{\mathcal{G}} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\,\Gamma_{2,2}(T,U)\,\Phi_{\mathcal{G}}(\bar\tau)$$

previous simple expression is no longer valid

$$\Delta_{\mathcal{G}} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\,\frac{1}{2}\sum_{H,G=0}^{1}\Gamma_{2,2}\left[{H \atop G}\right]\Phi_{\mathcal{G}}\left[{H \atop G}\right](\tau,\bar\tau)$$

explicitly non-holomorphic

Non-supersymmetric Universality

Threshold for $\mathrm{SO}(16)$ gauge group with $\mathrm{K3}$ realized as $T^4/\mathbb{Z}_2$ orbifold

$$\Delta_{\mathrm{SO}(16)} = \int_{\mathcal{F}} d\mu \left\{ -\frac{1}{48}\, \varGamma_{2,2}\big[{}^0_0\big]\, \frac{\hat{\bar{E}}_2\, \bar{E}_4\, \bar{E}_6 - \bar{E}_6^2}{\bar{\eta}^{24}} \right.$$

BPS subsector

$$+\Gamma_{2,2}\big[{}^0_1\big] \left[-\frac{1}{4N \times 144}\, \frac{\varLambda^{\mathrm{K3}}\big[{}^0_0\big]}{\eta^{12}\, \bar{\eta}^{24}} (\vartheta_3^8 - \vartheta_4^8)\, \bar{\vartheta}_3^4\, \bar{\vartheta}_4^4 \left((\hat{\bar{E}}_2 - \bar{\vartheta}_3^4)\, \bar{\vartheta}_3^4\, \bar{\vartheta}_4^4 + 8\bar{\eta}^{12} \right) \right]$$

dependence on hypers

$$+\Gamma_{2,2}\big[{}^0_1\big] \left[-\frac{1}{96}\, \frac{\bar{\vartheta}_3^4\, \bar{\vartheta}_4^4 (\bar{\vartheta}_3^4 + \bar{\vartheta}_4^4) \left[(\hat{\bar{E}}_2 - \bar{\vartheta}_3^4)\, \bar{\vartheta}_3^4\, \bar{\vartheta}_4^4 + 8\bar{\eta}^{12} \right]}{\bar{\eta}^{24}} \right.$$

BPS subsector (exceptionally for $\mathbb{Z}_2$)

$$\left. -\frac{1}{144}\, \frac{\vartheta_2^4\, (\vartheta_3^8 - \vartheta_4^8)}{\eta^{12}}\, \frac{(\hat{\bar{E}}_2 - \bar{\vartheta}_3^4)\, \bar{\vartheta}_3^4\, \bar{\vartheta}_4^4 + 8\bar{\eta}^{12}}{\bar{\eta}^{12}} \right] + (S \cdot \tau) + (ST \cdot \tau) \Bigg\}$$

non-holomorphic

Non-supersymmetric Universality

These expressions do not look very friendly...

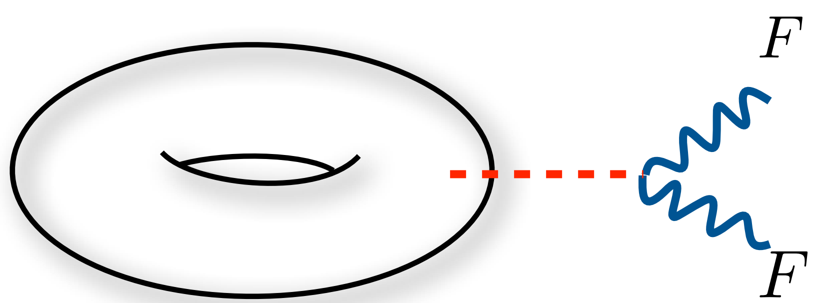
BUT may still be computed explicitly!

However, not in this talk...

CERN

Non-supersymmetric Universality

Focus on the difference of thresholds for $\mathrm{SO}(12)$ and $\mathrm{SO}(16)$



diagrams due to dilaton exchange cancel (as before)

eliminates $\hat{\hat{E}}_2$, or equivalently $\dfrac{1}{\tau_2}$ terms

dependence on K3 moduli also cancels $\sim \Lambda^{\mathrm{K3}}\big[{}^{0}_{0}\big]$

Left with $(\beta_1 - \beta_2)\displaystyle\int_{\mathcal{F}} d\mu\, \Gamma_{2,2}\big[{}^{0}_{0}\big]$ BPS sub-sector (unbroken $\mathcal{N}=2$)

$$+ \int_{\mathcal{F}_0(2)} d\mu\, \Gamma_{2,2}\big[{}^{0}_{1}\big] \sum_i (\mathrm{holom.})_i \times (\mathrm{anti-holom.})_i$$

manifestly non-BPS saturated

Non-supersymmetric Universality

Focus on the difference of thresholds for $\mathrm{SO}(12)$ and $\mathrm{SO}(16)$

$$\# \int_{\mathcal{F}_0(2)} d\mu \;\, \mathit{\Gamma}_{2,2}\big[{}^{0}_{1}\big] \left\{ -\frac{\vartheta_2^8\,|\vartheta_3^4+\vartheta_4^4|^2\,\bar\vartheta_3^4\,\bar\vartheta_4^4}{\eta^{12}\,\bar\eta^{12}} - \frac{\vartheta_2^4\,\vartheta_4^4\,|\vartheta_2^4-\vartheta_4^4|^2\,\bar\vartheta_3^4\,\bar\vartheta_4^4}{\eta^{12}\,\bar\eta^{12}} + \frac{\vartheta_2^4\,\vartheta_3^4\,|\vartheta_2^4+\vartheta_3^4|^2\,\bar\vartheta_3^4\,\bar\vartheta_4^4}{\eta^{12}\,\bar\eta^{12}} \right\}$$

seems very non-holomorphic...

write in terms of Kac-Moody characters

$$12\,(O_8^2\,V_8 + 3V_8^3)\,(\bar O_8^2\,\bar V_8 - \bar V_8^3)$$

factorizes

Non-supersymmetric Universality

Focus on the difference of thresholds for $\mathrm{SO}(12)$ and $\mathrm{SO}(16)$

$$\# \int_{\mathcal{F}_0(2)} d\mu \ \Gamma_{2,2}\left[{0 \atop 1}\right] \left\{ - \frac{\vartheta_2^8\, |\vartheta_3^4 + \vartheta_4^4|^2\, \bar\vartheta_3^4\, \bar\vartheta_4^4}{\eta^{12}\, \bar\eta^{12}} - \frac{\vartheta_2^4\, \vartheta_4^4\, |\vartheta_2^4 - \vartheta_4^4|^2\, \bar\vartheta_3^4\, \bar\vartheta_4^4}{\eta^{12}\, \bar\eta^{12}} + \frac{\vartheta_2^4\, \vartheta_3^4\, |\vartheta_2^4 + \vartheta_3^4|^2\, \bar\vartheta_3^4\, \bar\vartheta_4^4}{\eta^{12}\, \bar\eta^{12}} \right\}$$

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write in terms of Kac-Moody characters

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factorizes

Non-supersymmetric Universality

Focus on the difference of thresholds for $\mathrm{SO}(12)$ and $\mathrm{SO}(16)$

$$\# \int_{\mathcal{F}_0(2)} d\mu \; \Gamma_{2,2}\left[{0 \atop 1}\right] \left\{ -\frac{\vartheta_2^8 \, |\vartheta_3^4 + \vartheta_4^4|^2 \, \bar\vartheta_3^4 \, \bar\vartheta_4^4}{\eta^{12}\, \bar\eta^{12}} - \frac{\vartheta_2^4 \, \vartheta_4^4 \, |\vartheta_2^4 - \vartheta_4^4|^2 \, \bar\vartheta_3^4 \, \bar\vartheta_4^4}{\eta^{12}\, \bar\eta^{12}} + \frac{\vartheta_2^4 \, \vartheta_3^4 \, |\vartheta_2^4 + \vartheta_3^4|^2 \, \bar\vartheta_3^4 \, \bar\vartheta_4^4}{\eta^{12}\, \bar\eta^{12}} \right\}$$

seems very non-holomorphic...

write in terms of Kac-Moody characters

$$12\,(O_8^2\, V_8 + 3 V_8^3)\,\underbrace{\boxed{(\bar O_8^2\, \bar V_8 - \bar V_8^3)}}_{=\,8}$$

factorizes

Non-supersymmetric Universality

Focus on the difference of thresholds for $\mathrm{SO}(12)$ and $\mathrm{SO}(16)$

$$\# \int_{\mathcal{F}_0(2)} d\mu \ \Gamma_{2,2}\left[{}^{0}_{1}\right] \left\{ -\frac{\vartheta_2^8\,|\vartheta_3^4+\vartheta_4^4|^2\,\bar\vartheta_3^4\,\bar\vartheta_4^4}{\eta^{12}\,\bar\eta^{12}} - \frac{\vartheta_2^4\,\vartheta_4^4\,|\vartheta_2^4-\vartheta_4^4|^2\,\bar\vartheta_3^4\,\bar\vartheta_4^4}{\eta^{12}\,\bar\eta^{12}} + \frac{\vartheta_2^4\,\vartheta_3^4\,|\vartheta_2^4+\vartheta_3^4|^2\,\bar\vartheta_3^4\,\bar\vartheta_4^4}{\eta^{12}\,\bar\eta^{12}} \right\}$$

seems very non-holomorphic...

write in terms of Kac-Moody characters

$$12\,(O_8^2\,V_8 + 3V_8^3)\boxed{(\bar O_8^2\,\bar V_8 - \bar V_8^3)}$$

factorizes

$$= 8$$

MSDS identities

I.F., Kounnas 2009

Non-supersymmetric Universality

Focus on the difference of thresholds for $\mathrm{SO}(12)$ and $\mathrm{SO}(16)$

$$\# \int_{\mathcal{F}_0(2)} d\mu\ \Gamma_{2,2}\left[{}^0_1\right] \left\{ -\frac{\vartheta_2^8\, |\vartheta_3^4 + \vartheta_4^4|^2\, \bar\vartheta_3^4\, \bar\vartheta_4^4}{\eta^{12}\, \bar\eta^{12}} - \frac{\vartheta_2^4\, \vartheta_4^4\, |\vartheta_2^4 - \vartheta_4^4|^2\, \bar\vartheta_3^4\, \bar\vartheta_4^4}{\eta^{12}\, \bar\eta^{12}} + \frac{\vartheta_2^4\, \vartheta_3^4\, |\vartheta_2^4 + \vartheta_3^4|^2\, \bar\vartheta_3^4\, \bar\vartheta_4^4}{\eta^{12}\, \bar\eta^{12}} \right\}$$

seems very non-holomorphic...

$$= \frac{1}{6} \int_{\mathcal{F}_0(2)} d\mu\ \Gamma_{2,2}\left[{}^0_1\right] \left(8 - \frac{\vartheta_2^{12}}{\eta^{12}} \right)$$

BPS-like !

simple closed form expression !

Angelantonj, I.F., Pioline, *to appear*

Non-supersymmetric Universality

Full Result for $\mathrm{K3} \sim T^4/\mathbb{Z}_2$

$$\Delta_{\mathrm{SO}(16)} - \Delta_{\mathrm{SO}(12)} = 36 \log\left[T_2 U_2 \left|\eta(T)\,\eta(U)\right|^4\right] - \frac{4}{3}\log\left[T_2 U_2 \left|\vartheta_4(T)\,\vartheta_2(U)\right|^4\right]$$
$$+\frac{1}{3}\log\left|\hat{j}_2\left(T/2\right) - \hat{j}_2\left(U\right)\right|^4 \left|j_2(U) - 24\right|^4$$

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$$\hat{j}_2(\tau) = \left(\frac{\vartheta_2(\tau)}{\eta(\tau)}\right)^{12} + 24 \qquad j_2(\tau) = \left(\frac{\eta(\tau)}{\eta(2\tau)}\right)^{24} + 24$$

$$\hat{j}_2(\tau) = j_2\left(-\frac{1}{2\tau}\right)$$

Non-supersymmetric Universality

Full Result for $\mathrm{K3} \sim T^4/\mathbb{Z}_N$

$$\Delta_{\mathrm{SO}(16)} - \Delta_{\mathrm{SO}(12)} = \alpha \, \log\left[T_2 U_2 \, |\eta(T)\,\eta(U)|^4\right] + \beta \, \log\left[T_2 U_2 \, |\vartheta_4(T)\,\vartheta_2(U)|^4\right] \\ +\gamma \, \log \left|\hat{j}_2\left(T/2\right) - \hat{j}_2\left(U\right)\right|^4 |j_2(U) - 24|^4$$

$$(\alpha,\beta,\gamma) = (36, -\tfrac{4}{3}, \tfrac{1}{3}) \qquad \mathbb{Z}_2 \ \& \ \mathbb{Z}_3$$

$$(\alpha,\beta,\gamma) = \tfrac{5}{8}(36, -\tfrac{4}{3}, \tfrac{8}{15}) \qquad \mathbb{Z}_4$$

$$(\alpha,\beta,\gamma) = \tfrac{35}{144}(36, -\tfrac{4}{3}, \tfrac{1}{3}) \qquad \mathbb{Z}_6$$

Independently of the details of the vacuum (almost)

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Scherk Schwarz deformation breaks T-duality group

$$\mathrm{SL}(2;\mathbb{Z})_T \times \mathrm{SL}(2;\mathbb{Z})_U \quad \longrightarrow \quad \Gamma^0(2)_T \times \Gamma_0(2)_U$$

Non-supersymmetric Universality

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logarithmic singularity at $T/2 = U$ (plus images)

extra charged massless states

Drastically different than the supersymmetric case

Non-supersymmetric Universality

Full Result for $\mathrm{K3} \sim T^4/\mathbb{Z}_N$

difference of beta function coefficients

$$\Delta_{\mathrm{SO}(16)} - \Delta_{\mathrm{SO}(12)} = \alpha \log\left[T_2 U_2 \left|\eta(T)\,\eta(U)\right|^4\right] + \beta \log\left[T_2 U_2 \left|\vartheta_4(T)\,\vartheta_2(U)\right|^4\right]$$
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jump in beta function coefficients

Non-supersymmetric Universality

This simple and beautiful universal result is based on the "reduced MSDS identity"

$$\bar{O}_8^2\,\bar{V}_8 - \bar{V}_8^3 = 8$$

Massive
Spectral boson-fermion
Degeneracy
Symmetry

I.F., Kounnas 2009

Indicates presence of hidden spectral flow in the right-moving sector of heterotic theory

Faraggi, I.F., Mohaupt, Tsulaia 2010

Vacuum Energy

For large T^2 volume, the one-loop effective potential is given by

$$V_{\text{eff}} = -\frac{1}{T_2^2}\left[2N_{(Q=1)}\,E^\star(3;U) + 2^3\left(\frac{N_{(Q=0)}}{2} - N_{(Q=1)}\right)\,E^\star(3;2U)\right] + \mathcal{O}(e^{-|\alpha|\sqrt{T_2}})$$

N_Q # massless bosons minus fermions with charge $Q = (F_{\text{s.t.}} + F_1 + F_2) \bmod 2$

$$E^\star(s;U) = \frac{\Gamma(s)}{2\pi^s}\sum_{(m,n)\neq(0,0)}\frac{U_2^s}{|m+Un|^{2s}}$$

$V_{\text{eff}} \sim -\beta(U)\,m_{\frac{3}{2}}^4 + \mathcal{O}(e^{-|\gamma|m_{3/2}})$ absence of corrections $M_s^2\,m_{\frac{3}{2}}^2$

Determination via radiative corrections of the VEV of the "no scale modulus" at the ~TeV scale requires the absence of such terms

CERN

Ferrara , Kounnas, Zwirner 1994

Conclusions

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☑ One-loop radiative corrections to gauge couplings in heterotic strings

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- ☑ Moduli dependence encoded in N=2 sectors
- ☑ Opens possibilities for string model building

Outlook

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Faraggi, Kounnas, Partouche 2014

Thank you !