On Supergravity Theories (after ~ 40 years)

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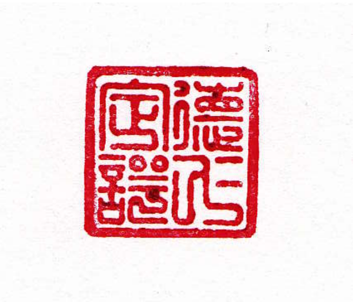

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Foreword

Supergravity is a fundamental proposal for a description of all interactions, at the classical field theory level or, as an effective description of a more fundamental quantum theory providing the UV completion (superstrings ?).

Maxwell, general relativity, supergravity, ...

Since ~ 40 years, it has induced a variety of theoretical developments and it is now a standard tool to study many physical systems and new ideas:

Generating symmetry breakings in the SSM, string compactifications, solutions of string theories, gauge-gravity (AdS/CFT) dualities, solutions of gravitation theories, black holes and attractors, cosmology, scattering amplitudes, sigma-models on particular geometries, ...

Some basic concepts and mathematical structures are at work in most of these applications.

Five (?) minutes lecture on global supersymmetry I

- An extension of Poincaré symmetry using anticommuting spinor generators (supercharges) with algebra $(i, j = 1, \ldots, N \leq 8)$

$$\{Q^i_\alpha, \overline{Q}^j_{\dot\alpha}\} = -2i\,\delta^{ij}(\sigma^\mu)_{\alpha\dot\alpha}\partial_\mu = 2\,\delta^{ij}(\sigma^\mu)_{\alpha\dot\alpha}P_\mu$$

$$[M^{\mu\nu}, Q] = \tfrac{i}{4}[\sigma^\mu, \overline{\sigma}^\nu]Q$$

“Square root of translations".

- Action on supermultiplets of fields: ϵ: spinorial (small) parameter.

$$\delta\Phi = i(\epsilon^\alpha Q_\alpha + \overline{\epsilon}_{\dot\alpha}\overline{Q}^{\dot\alpha})\Phi$$

Bosons $\Longleftrightarrow$ Fermions

- The simplest, smallest (off-shell) representation: chiral multiplet, $N = 1$

$$\delta z = \sqrt{2}\epsilon\psi \qquad \delta\psi_\alpha = -\sqrt{2}f\epsilon_\alpha - \sqrt{2}i\partial_\mu z(\sigma^\mu\overline{\epsilon})_\alpha$$

$$\delta f = -\sqrt{2}i\,\partial_\mu\psi\sigma^\mu\epsilon$$

(auxiliary field, algebraic field eq.)

Five (?) minutes lecture on global supersymmetry II

- Invariant actions systematically produced (*superspace techniques*).
- Compatible with quantum field theory (Haag, Lopuszanski, Sohnius)
- Auxiliary fields play a crucial role in supersymmetric theories:

$$\delta\psi_\alpha = -\sqrt{2} f \epsilon_\alpha - \sqrt{2} i \partial_\mu z (\sigma^\mu \overline{\epsilon})_\alpha$$

  f is a function of the (propagating) scalar fields z, by its algebraic field equation.
  If it gets a v.e.v. (from the scalar potential which defines $\langle z \rangle$) , then

$$\delta\psi_\alpha = -\sqrt{2}\langle f \rangle \epsilon_\alpha + \dots \qquad \langle f \rangle \sim \text{Susy breaking scale}^2$$

  and ψ is the goldstino spinor of spontaneously broken supersymmetry.
- Spontaneous breaking of a global symmetry leads to a massless state.
  Spontaneously broken supersymmetry $\Longrightarrow$ massless Goldstino spin1/2 state.
- Supersymmetry is at best a broken symmetry . . .

Five (?) minutes lecture on global supersymmetry III

- ... and spontaneous breaking of global supersymmetry is hard and phenomenologically problematic (scalars lighter than fermions, massless Goldstino, the wrong way to go), except if mediation is invoked.
- Motivations for local, gauged supersymmetry:
  - Curiosity and intellectual liberty, ...
  - Phenomenology/experimental requirements,
  - Including gravity.
- Since $$\{Q^i_\alpha, \overline{Q}^j_{\dot\alpha}\} = 2\,\delta^{ij}(\sigma^\mu)_{\alpha\dot\alpha}P_\mu$$

  local susy $\Rightarrow$ local Poincaré $\Rightarrow$ general coordinate transformations

  and local supersymmetry is a theory of gravitation: supergravity.
- Gauge field $\psi_{\mu\alpha}$, the gravitino field, helicities $\pm 3/2$, with supersymmetry variation
$$\delta\psi_{\mu\alpha} = D_\mu\epsilon(x)_\alpha + \ldots$$
  since it is a gauge field.

Five (?) minutes lecture on global supersymmetry IV

SSM = MSSM, NMSSM, . . . : $N = 1$ extensions of SM.

- Consistent scheme experimentally testable.
- Intrinsically related to supergravity: needs a source for supersymmetry breaking (to generate the necessary soft breaking terms in SSM).
- Needs a UV completion, including gravity.
- Perturbative quantum corrections logarithmic only: large ratio(s) of scale(s) stabilized.

  Also: provides a scheme for perturbative generation of a large scale ratio (M_W/M_P). But requires understanding of (soft) supersymmetry breaking terms, as the low-energy effect of the (spontaneous) breaking of supersymmetry in supergravity or (better) in a UV completion.

  Uses a particular class of supergravity models: no-scale models

Five (?) minutes lecture on global supersymmetry V

Dates:

- Field theories with linear supersymmetry, 1974 (Wess and Zumino).
- Soon found to have softer divergences than ordinary gauge theories (logarithmic renormalization only) and powerful all-order non-renormalization theorems (Iliopoulos, Zumino, Wess, Ferrara).
- Superspace techniques (Salam, Strathdee; Wess, Zumino, Ferrara).
- Spontaneous supersymmetry breaking (Fayet, Iliopoulos, 1974; O'Raifeartaigh, 1975).
- Currents and supercurrents, approaches to gravity coupling (Ferrara, Zumino, 1974).
- Supergravity was created in 1976 (Ferrara, Freedman and Van Nieuwenhuizen; Deser and Zumino).
- Matter and gauge couplings to $N = 1$ supergravity, 1982, Cremmer, Ferrara, Girardello, Van Proeyen

  (also Arnowitt, Chamseddine, Nath; Bagger, Witten)

Supergravity I

Pure $N = 1$ supergravity is *very simple*:

Einstein-Hilbert + Rarita-Schwinger

$$\mathcal{S}_{ERS}[e^a_\mu, \psi_\mu, \omega_{\mu\, ab}] = \frac{1}{2\kappa_D^2} \int d^D x\, e \left(R + \overline{\psi}_\mu \gamma^{\mu\nu\rho} \tilde{D}_\nu \psi_\rho \right)$$

But: local symmetries imply covariant derivatives

$$\begin{aligned} \tilde{D}_\mu \psi_\nu &= \partial_\mu \psi_\nu + \tfrac{1}{2} \omega_\mu{}^{ab} \sigma_{ab} \psi_\nu && \text{(spin connection)} \\ \omega_{\mu\, ab} &= \omega_{\mu\, ab}(e) + \kappa_{\mu\, ab} && \text{(contorsion tensor)} \\ \tilde{D}_\mu \psi_\nu - \tilde{D}_\nu \psi_\mu &= D_\mu \psi_\nu - D_\nu \psi_\mu + 2\, S^\lambda_{\mu\nu} \psi_\lambda && \text{(torsion tensor)} \end{aligned}$$

with gravitino torsion (for a $D = 4$ Majorana gravitino)

$$\begin{aligned} S^\lambda_{\mu\nu} &= -\tfrac{1}{4} \overline{\psi}_\mu \gamma^\lambda \psi_\nu \\ \kappa_{\mu\, ab} &= -\tfrac{1}{4} \left[\overline{\psi}_\mu \gamma_a \psi_b - \overline{\psi}_\mu \gamma_b \psi_a + \overline{\psi}_a \gamma_\mu \psi_b \right] \end{aligned}$$

Supergravity II

Covariantization $\Longrightarrow$ four-gravitino interaction, and then:

Four-dimensional pure $N = 1$ supergravity is *not so simple*:

$$\begin{aligned}\mathcal{L} \;=\; & \frac{1}{2\kappa_4^2} eR\,(\omega(e)) + \frac{1}{2\kappa_4^2} e\overline{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu(\omega(e))\psi_\rho \\ & + \frac{e}{32\kappa_4^2}\Big[4(\overline{\psi}^\mu \gamma_\mu \psi_\rho)(\overline{\psi}^\nu \gamma_\nu \psi^\rho) - (\overline{\psi}_\mu \gamma_\nu \psi_\rho)(\overline{\psi}^\mu \gamma^\nu \psi^\rho) \\ & - 2(\overline{\psi}_\mu \gamma_\nu \psi_\rho)(\overline{\psi}^\mu \gamma^\rho \psi^\nu)\Big]\end{aligned}$$

with now $\tilde{D}_\nu \psi_\rho = \partial_\nu \psi_\rho + \frac{1}{2}\omega_{\nu\,ab}(e)\sigma^{ab}\psi_\rho$, the usual spin connection of pure gravitation theory.

- In four space-time dimensions:
  - Gravitino: $4 \times 4 - 4 = 12_F$ off-shell. 2_F with helicities $\pm 3/2$ on-shell.
  - Graviton: $10 - 4 = 6_B$ on-shell. 2_B with helicities ± 2 on-shell.

Supergravity III

More complications: auxiliary fields of $N = 1$ supergravity

The $N = 1$ supergravity action is invariant under local susy variations

$$\begin{aligned} \delta e^a_\mu &= -\tfrac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu & \qquad \delta e^\mu_a &= \tfrac{1}{2}\bar{\epsilon}\gamma^\mu\psi_a \\ \delta\psi_\mu &= D_\mu\epsilon & \qquad \delta\overline{\psi}_\mu &= D_\mu\bar{\epsilon} \end{aligned}$$

With standard covariant derivative $\quad D_\mu\epsilon \;=\; \partial_\mu\epsilon + \tfrac{1}{2}\omega_{\mu\,ab}\sigma^{ab}\epsilon$

But it is not an off-shell representation of the supersymmetry algebra:
$[\delta_1, \delta_2]$ is a diffeomorphism only for fields solving the field equations

Another sign is the number of off-shell field components: $12_F \neq 6_B$.

More (auxiliary) fields needed for a linear off-shell representation, with
$N^{aux}_B - N^{aux}_F = 6$

Vanish for pure supergravity, produce interactions when coupled to matter or gauge multiplets.

Supergravity IV

Several choices of auxiliary fields:

- Minimal schemes with $\mathbf{12}_B + \mathbf{12}_F$ $\qquad (N_B^{aux} = 6, \quad N_F^{aux} = 0)$
  - Old minimal: A^μ, not a gauge field ($\mathbf{4}_B$), couples to a non-conserved (in general) current and f_0, complex scalar ($\mathbf{2}_B$)
  - New minimal: A^μ with gauge symmetry ($\mathbf{3}_B$), couples to a conserved current, antisymmetric tensor $B_{\mu\nu}$ with gauge symmetry ($\mathbf{3}_B$).
- Non minimal schemes have $\mathbf{16}_B + \mathbf{16}_F$ (relevant to superstring theories), $\mathbf{20}_B + \mathbf{20}_F, \ldots$
- Each scheme generates a particular class of interactions. For instance: R–symmetric with new minimal.

- The most general is the simplest, old minimal (describes all classes).
- Each scheme can be constructed from superconformal theories with various compensating fields to gauge-fix dilatation, special conformal and R symmetries

Supergravity V

Four-dimensional supersymmetry (linear) representations

SUSY	Supergravity	$\lvert$Hel.$\rvert \le 1$	$\lvert$Hel.$\rvert \le 1/2$	Chirality	
$N=1$	$2_B + 2_F$	$\checkmark$	$\checkmark$ *	$\checkmark$	
$N=2$	$4_B + 4_F$	$\checkmark$ *	$\checkmark$ *	-	$D=6$
$N=3$	$8_B + 8_F$	$\checkmark$ *	-	-	
$N=4$	$16_B + 16_F$ *	$\checkmark$ *	-	-	$D=10$
$N=5$	$32_B + 32_F$ *	-	-	-	
$N=6$	$64_B + 64_F$ *	-	-	-	
$N=8$	$128_B + 128_F$ *	-	-	-	$D=11$

- *: scalar fields in supermultiplet
- Number of supercharges is $4N$
- 16 supercharges ($N=4$): type I, heterotic strings
- 32 supercharges ($N=8$): type IIA, IIB strings, M–“theory"
- $N=7$ does not exist (it is the $N=8$ theory)
- $N=0,1$ only for realistic models, or nonlinear, (or truncated. . .)

$N = 1$ supergravity and matter couplings

$N = 1$ supergravity couples to:

all gauge groups (gauge superfield A_μ, λ, helicities $\pm 1, \pm 1/2$)

all representations for chiral multiplets (ψ and z, helicities $\pm 1/2, 0, 0$)

and allows chirality of fermion representations.

The idea is then:

- Couple the SSM to supergravity, add a “hidden" sector to break supersymmetry.
- Generate a susy breaking scale $m_{3/2}$ and scalar vev’s in the hidden sector $\langle \phi \rangle$
- Decouple gravity: expand, take $M_P \longrightarrow \infty$, keep $m_{3/2}$ fixed, . . .
- The result is a global $N = 1$ theory with soft breaking terms.
- However: $N = 1$ Poincaré only if the cosmological constant at the breaking point is zero. In general, AdS global $N = 1$ . . .
- A severe constraint on the hidden sector . . .

$N = 1$ supergravity and matter couplings

The complete Lagrangian has been obtained by Cremmer, Ferrara, Girardello and Van Proeyen (1982). It needs 1.5 pages in Nucl. Phys. B212.

In the superconformal formulation, it is symbolically

$$\mathcal{L} = -\frac{3}{2}\Big[S_0\overline{S}_0\, e^{-\mathcal{K}/3}\Big]_D + \Big[S_0^3 W + \frac{1}{4}f(\Phi)\,\mathrm{Tr}\,\mathcal{W}\mathcal{W}\Big]_F$$

where S_0 is the chiral compensating multiplet of the old minimal formalism. Similar to the global superspace Lagrangian

$$\mathcal{L} = \int d^2\theta d^2\overline{\theta}\,\mathcal{K}(\overline{\Phi}^{\mathcal{A}}, \Phi) + \int d^2\theta\,\Big[W(\Phi) + \frac{1}{4}f(\Phi)\,\mathrm{Tr}\,\mathcal{W}\mathcal{W}\Big] + \mathrm{h.c.}$$

Superconformal and superspace calculus turn these symbolic expression into Lagrangians . . . .

Curiously, most applications use only the scalar potential and some fermion mass terms.

The scalar potential

$$V = \frac{1}{\kappa^4}\Big[e^{\mathcal{K}}\mathcal{K}^{-1}{}^i_j(W_i + \mathcal{K}_i W)(W^j + \mathcal{K}^j W) - 3\, e^{\mathcal{K}}\,\overline{W}W\Big]$$
$$+\frac{1}{2} f(\Phi)^{-1}\mathcal{K}_i (T^A z)^i \mathcal{K}_j (T^A z)^j$$

- Blue terms are positive or zero: they are generated by auxiliary fields. Susy breaks if they are not zero.
- The red term is negative or zero, it is generated by a supergravity auxiliary field. Unbroken susy: Anti-de Sitter or Minkowski ($W = 0$).
- Supergravity and supersymmetry are actually extensions of Anti-de Sitter symmetry: $SO(2,3) \sim Sp(4,\mathbb{R}) \longrightarrow OSp(n|4)$
  Poincaré is obtained in the large AdS radius limit only. (Fine-tuning ?)
- A de Sitter ground state can only be generated with broken supersymmetry.
- (Good point for AdS/CFT)

Dilaton supergravity, no-scale models

For a single chiral superfield S and a constant superpotential W,

$$V = \frac{1}{\kappa^4} e^{\mathcal{K}} \Big[K^{-1}_{S\overline{S}} \mathcal{K}_S \mathcal{K}_{\overline{S}} - 3 \Big] \overline{W} W$$

is identically zero if

$$\mathcal{K} = -3 \ln(S + \overline{S}) \qquad\qquad \forall\, W$$

but the auxiliary field f_S and the gravitino mass are

$$f_S = \overline{W}(S+\overline{S})^{-1/2} \neq 0 \qquad\qquad m_{3/2} = W(S+\overline{S})^{-3/2}$$

Hence, W induces supersymmetry breaking in Minkowski space, to obtain:

Broken supersymmetry in Minkowski space with a free scale $\langle S + \overline{S} \rangle$

The prototype of no-scale models:

Tree-level susy breaking scale arbitrary, radiative corrections may define it with some logarithmic factor and then with an induced scale hierarchy.

(Cremmer, Ferrara, Kounnas, Nanopoulos, 1983)

Dilaton supergravity, no-scale models

Consider now a string compactification:

In general, it produces a real dilaton scalar and an antisymmetric tensor $B_{\mu\nu}$ with gauge invariance $\delta B_{\mu\nu} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu$ in the universal gravitation sector (in type II, in NS–NS sector).

The antisymmetric tensor is equivalent to a real scalar with shift symmetry:

$$\partial_{[\mu} B_{\nu\rho]} \quad \leftrightarrow \quad \partial_\mu \, \mathrm{Im}\, s \qquad\qquad C \quad \leftrightarrow \quad \mathrm{Re}\, s$$

and there should be a description in terms of a chiral multiplet S, with however an auxiliary field f_S which could be a source of supersymmetry breaking.

The relation is a Legendre transformation between supermultiplets.

The behaviour of the dilaton scalar in the effective supergravity Lagrangian is important: its value is the string coupling. Does it stabilize, does it slide to zero (run away), are further moduli fields needed ?

Dilaton supergravity, no-scale models

Within supergravity, two descriptions and a duality generated by a Legendre transformation:

- Description with $B_{\mu\nu}$: (The superpotential is constant)

$$\mathcal{L} = -\frac{3}{2}\left[S_0\overline{S}_0\mathcal{H}(X)\right]_D + \left[S_0^3\,W\right]_F \qquad X = \frac{L}{S_0\overline{S}_0}$$

- Description with chiral multiplet S:

$$\widetilde{\mathcal{L}} = -\frac{3}{2}\left[S_0\overline{S}_0 e^{-\frac{1}{3}\mathcal{K}(S+\overline{S})/3}\right]_D + \left[S_0^3 W\right]_F$$

- Legendre transformation: $e^{-\frac{1}{3}\mathcal{K}(S+\overline{S})} = \mathcal{H}(X) - X(S+\overline{S})$

- Dilaton supergravities:

Heteroric: $\mathcal{H} \sim X^{-1/2} \qquad \mathcal{K} = -ln(S+\overline{S})$

Type II: $\mathcal{H} \sim X^{4} \qquad \mathcal{K} = -4\,ln(S+\overline{S})$

Dilaton supergravity, no-scale models

- The Legendre transformation implies:

$$f_S = -C\mathcal{H}_{CC}\overline{z}_0 f_0$$

  and f_S is not an independent auxiliary field. Generalization to many fields:

  The auxiliary field f_S of a chiral multiplet dual to a linear superfield with an antisymmetric rensor is a linear combination of other auxiliary fields.

$$f_S \sim \frac{\partial}{\partial z^i}\mathcal{H}_C f^i$$

- The dilaton is not stabilized. More fields and interactions required.
- The single field no-scale model with $\mathcal{K} = -3\ln(S+\overline{S})$ does not describe a $B_{\mu\nu}$ + dilaton sector.
- Hence, low-energy scenarios in which supersymmetry breaking is induced by the dilaton superfield S only are forbidden by supergravity arguments.

Supergravity scalar potentials again

For all N, the typical supergravity potential is the sum of three terms:

$$V = V_m + V_g + V_0$$

- $V_m \geq 0$: generated when matter fermions are present.
  Exists for $N = 1, 2$ (chiral and hyper multiplets).

  $$\delta\psi = \mathcal{A}\epsilon \quad \longrightarrow \quad V_m \sim +|\mathcal{A}|^2$$

- $V_g \geq 0$: generated when gauginos are present.
  Exists for all $N = 1, \ldots, 8$.

  $$\delta\lambda = \mathcal{B}\epsilon \quad \longrightarrow \quad V_m \sim +|\mathcal{B}|^2$$

- $V_0 \leq 0$: generated by the gravitinos (helicities $\pm 2, \pm 1/2$):
  Exists for all $N = 1, \ldots, 8$.

  $$\delta\lambda = \mathcal{C}\epsilon \quad \longrightarrow \quad V_m \sim -|\mathcal{C}|^2$$

- Some or all supersymmetries break if V_m and/or V_g are positive on the vacuum. (*Information is in the gravitino mass matrix*).

Supergravity scalar potentials again

- The gauge potential plays a fundamental role in the vacuum structure of supergravities, and in superstring compactifications.
- Produced by gauging a symmetry of the theory: abelian (for instance, R–symmetry in $N = 1$) or non-abelian. Compact or non-compact.
- Flat directions in the potential, with no-scale behaviour and Minkowski space, algebraically characterized: a condition on the gauged algebra.

$(\ldots)$: number of abelian, ungauged, gauge fields in the supermultiplet:

SUSY	Supergravity	|Hel.|≤ 1	|Hel.|$\leq 1/2$	
$N = 1$	$2_B + 2_F \quad (0)$	$\checkmark \quad (N)$	$\checkmark$	
$N = 2$	$4_B + 4_F \quad (1)$	$\checkmark \quad (N)$	$\checkmark$	$D = 6$
$N = 3$	$8_B + 8_F \quad (3)$	$\checkmark \quad (N)$	-	
$N = 4$	$16_B + 16_F \quad (6)$	$\checkmark \quad (N)$	-	$D = 10$
$N = 5$	$32_B + 32_F \quad (10)$	-	-	
$N = 6$	$64_B + 64_F \quad (15)$	-	-	
$N = 8$	$128_B + 128_F \quad (28)$	-	-	$D = 11$

Relation to superstring compactifications

String compactifications ($\mathbf{16}$ or $\mathbf{32}$ supercharges) include various background quantities in their compact geometry. These include background values of:

- tensor fields (in the supergravity multiplet)
- dual tensors (of the tensor hierarchy) and branes
- geometric fluxes (spin connection fluxes)
- various “non-geometric" fluxes (condensates for instance) . . .

These background quantities can be associated with the generalized structure constants of a gauged supergravity, and the vacuum structure of the string compactification can be studied directly in the supergravity (numerical methods help).

This holds for large classes of string compactifications (but not for all) with a large variety of breaking patterns and low-energy structure.

The idea is to develop a bottom-up approach to the vast problem of string compactifications with fluxes.

Gauged supergravities

- All *ungauged* supergravities have been constructed long ago. They depend on the abelian field strengths $F_{\mu\nu}$ only and have then (in four dimensions) electric-magnetic duality.
- A symmetry of an ungauged theory can be gauged using the abelian gauge fields of the theory. One selects an algebra and associates a (electric or magnetic) gauge field A_μ^M of the theory with each generator

$$[T_A, T_B] = f_{AB}{}^C \qquad X_M = \Theta_M{}^A T_A \qquad \Theta_M{}^A\text{: embedding tensor}$$

- The consistency conditions for the procedure have been established for a generic field theory in a fundamental paper by de Wit, Samtleben and Trigiante (hep-th/0507289).
- Large classes of gauged supergravities have been constructed, large classes are missing.
- Particularly interesting for 16 ($N = 4$) and 32 ($N = 8$) supercharges related to superstrings and M theories.

An example, maximal supergravity with $SO(8)$

- Can be obtained by S_7 sphere compactification of 11-dimensional supergravity. (de Wit, Nicolai)
- $N = 8$ supergravity has 28 abelian gauge fields $F^I_{\mu\nu}$ and then 28 duals $\widetilde{F}^I_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{I\rho\sigma}$ and 70 scalars..

  Obvious gauging: the 28 gauge fields in the adjoint of $SO(8)$: electric gauging.
- Is the gauging unique ?
- Starting point:
  - The electric-magnetic duality group is $Sp(56,\mathbb{R})$ (Gaillard, Zumino).
  - The 70 scalar fields are in $E_{7,7}/SU(8)$ with $E_{7,7} \subset Sp(56,\mathbb{R})$.
  - Fermions reduce the symmetry to $SU(8)$
  - Gauge group $SO(8) \subset SU(8)$, $28 = 28$.

An example, maximal supergravity with $SO(8)$

Group theory:

First embedding chain, relevant to gauge fields:

$$\boldsymbol{Sp(56,\mathbb{R}) \supset SU(28) \times U(1) \supset SU(8) \times U(1)}$$

$$\begin{aligned}
\mathbf{56} &= \mathbf{28_1} + \overline{\mathbf{28}}_{\mathbf{-1}} = \mathbf{28_1} + \overline{\mathbf{28}}_{\mathbf{-1}} \\
\mathbf{1596} &= \mathbf{783_0} + \mathbf{1_0} + \mathbf{406_2} + \overline{\mathbf{406}}_{\mathbf{-2}} \\
&= \mathbf{63_0} + \mathbf{1_0} + \mathbf{720_0} + \mathbf{336_2} + \overline{\mathbf{336}}_{\mathbf{-2}} + \mathbf{70_2} + \mathbf{70_{-2}}
\end{aligned}$$

Second embedding chain, relevant to scalar fields:

$$\boldsymbol{Sp(56,\mathbb{R}) \supset E_{7,7} \supset SU(8)}$$

$$\begin{aligned}
\mathbf{56} &= \mathbf{56} = \mathbf{28} + \overline{\mathbf{28}} \\
\mathbf{1596} &= \mathbf{133} + \ldots = \mathbf{63} + \mathbf{70} + \ldots
\end{aligned}$$

$\boldsymbol{E_{7,7}}$ is not unique in $\boldsymbol{Sp(56,\mathbb{R})}$: for a given $\boldsymbol{SU(8)}$, the $\mathbf{70}$ component is complex with a $\boldsymbol{U(1)}$ charge: a phase choice to adapt the $\boldsymbol{E_{7,7}}$ of the scalars inside the electric-magnetic duality group.

An example, maximal supergravity with $SO(8)$

- Leads to a one-parameter family of $SO(8)$, $N = 8$ gauged supergravity.
(Dall'Agata, Inverso, Trigiante; Borghese, Guarino, Roest)
- Invisible at the $SO(8)$ level: there is only one $N = 8$, $SO(8)$ theory, a different definition of electric/magnetic.
- But visible when a second parameter is introduced in the embedding tensor, reducing the gauged algebra.

A very simple (but surprising) example of the gauging procedure in extended supergravities, with the largest compact gauging $SO(8)$.

Conclusion

- After (almost) 40 years, supergravity has found its way into the toolbox of theorists.
- More developments are needed, work is in progress (classification of gauged theories, for instance, flux orbits, relation with string compactifications, breaking patterns).
- Gravitation theory evolves, supergravity evolves.
- Gauge–gravity duality ?
- It cannot claim to be “the" fundamental theory, but it is the “supersymmetric way" to a coherent description of quantum gravity.
- Stands at the interface between M_P physics and accessible low energies: if LHC experiments see supersymmetry, they would also see supergravity soft terms . . .