Symmetry Improved CJT Effective Potential

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Based on A.P. and D. Teresi, Nucl. Phys. B **874** (2013) 594
and `arXiv:1412.nnnn`

Motivation:

New Era of Precision QFT

- Improved Lattice Techniques
- Improved Solutions to Schwinger–Dyson Equations
- Geometric S-Matrix Approach in $N = 4$ SYM Theories

$\vdots$

- Other Analytical Non-Perturbative Approaches?

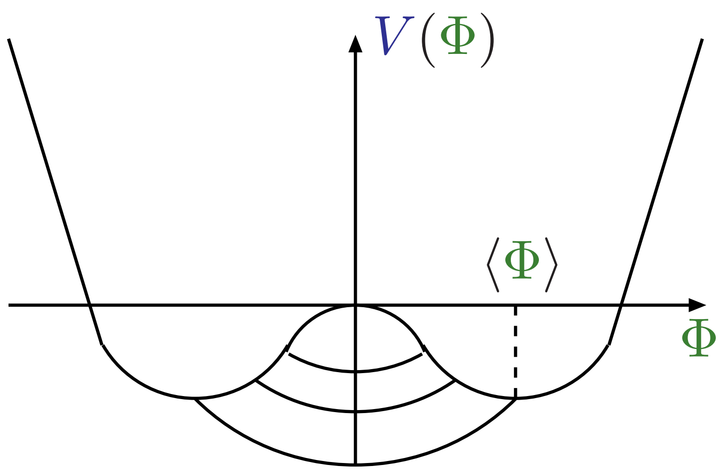
Outline:

- The Standard Theory of Electroweak Symmetry Breaking: SM
- The CJT Effective Action and Field-Theoretic Problems
- Symmetry-Improved CJT Formalism
- $\overline{\rm MS}$ Renormalization in CJT
- Finite-Width Effects within Quantum Loops
- Symmetry-Improved Effective Higgs Potential in CJT
- Conclusions and Future Directions

• The Standard Theory of Electroweak Symmetry Breaking

Higgs Mechanism in SM: $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \to \mathrm{SU}(3)_c \otimes \mathrm{U}(1)_{\mathrm{em}}$

[P. W. Higgs '64; F. Englert, R. Brout '64; G. S. Guralnik, C. R. Hagen, T. W. B. Kibble '64]



Higgs potential $V(\Phi)$

$$V(\Phi) \;=\; -m^2\Phi^\dagger\Phi \;+\; \lambda(\Phi^\dagger\Phi)^2\,.$$

Ground state:

$$\langle\Phi\rangle \;=\; \sqrt{\frac{m^2}{2\lambda}}\begin{pmatrix}0\\1\end{pmatrix}$$

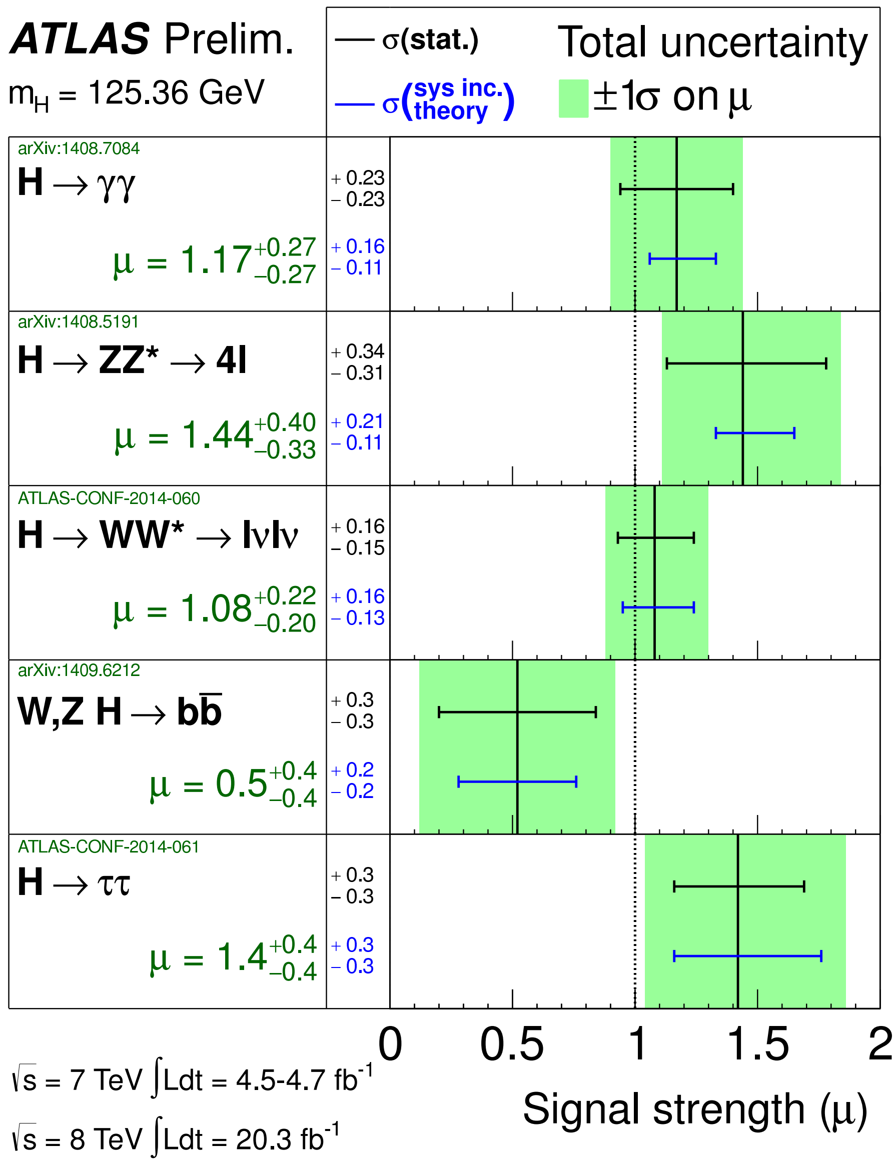
carries weak charge, but no electric charge and colour.

After Spontaneous Symmetry Breaking:

$\Rightarrow$ $W^\pm$, Z bosons and matter feel the presence of $\langle\Phi\rangle$ and become massive, but not γ and g^a, e.g. $M_W = g_w\langle\Phi\rangle$

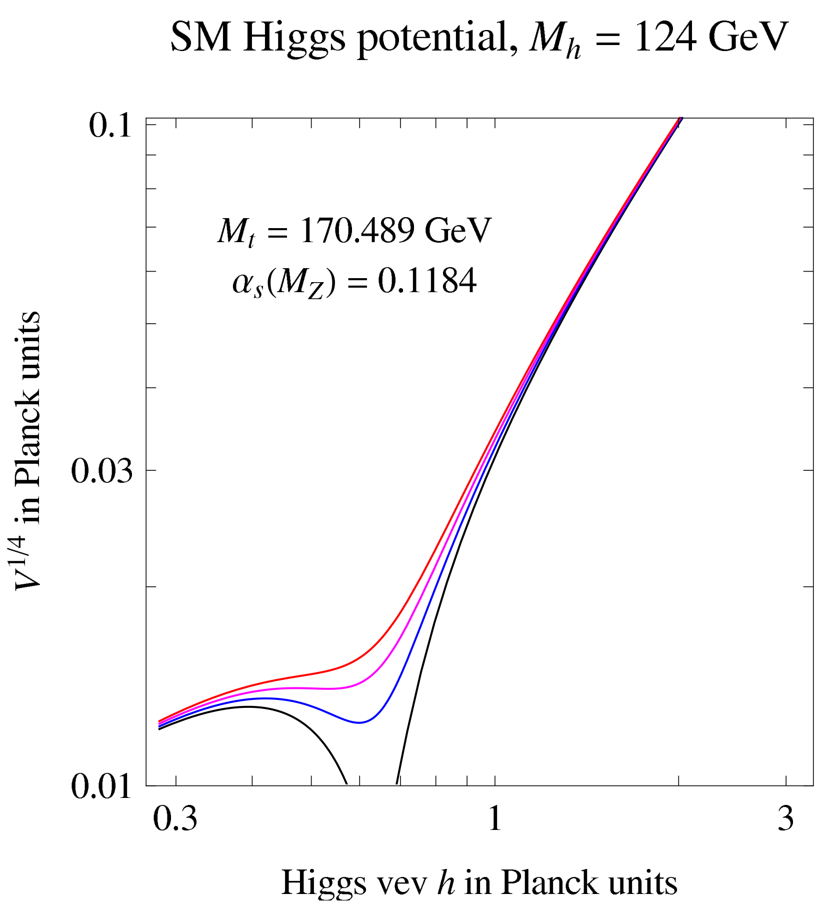
$\Rightarrow$ Quantum excitations of $\Phi = \langle\Phi\rangle + H\begin{pmatrix}0\\1\end{pmatrix}$; H is the Higgs boson.

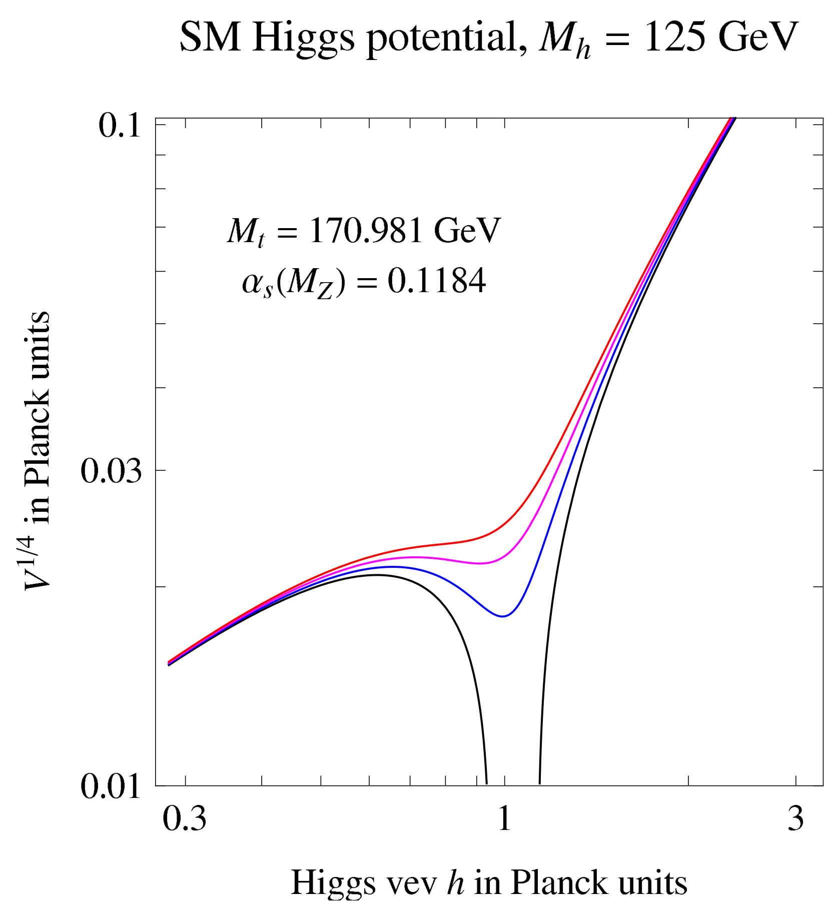
On the SM-like Higgs-Boson Discovery at the LHC:



Uncertainties of the SM Higgs potential at NNLO

[G. Degrassi *et al*, JHEP **1208** (2012) 098]





Higgs Potential versus Variations in Top Mass M_t by **0.1 MeV**

[Analysis includes the Multi-Critical scenario: D.L. Bennett, H.B. Nielsen, IJMA9 (1994) 5155]

• The CJT Effective Action and Field-Theoretic Problems

[J.M. Cornwall, R. Jackiw, E. Tomboulis, PRD10 (1974) 2428]

Connected Generating Functional of 2PI Effective Action:

$$W[J, K] \;=\; -i\,\ln \int \mathcal{D}\phi^i\, \exp\left[i \left(S[\phi] \;+\; J^i_x\, \phi^i_x \;+\; \frac{1}{2}\, K^{ij}_{xy}\, \phi^i_x\, \phi^j_y \right) \right] \;,$$

where $S[\phi] = \int_x \mathcal{L}[\phi]$ is the classical action of a $\mathbb{O}(N)$ theory.

Legendre transform of $W[J, K]$ with respect to J and K:

$$\frac{\delta W[J, K]}{\delta J^i_x} \;\equiv\; \phi^i_x \;, \qquad \frac{\delta W[J, K]}{\delta K^{ij}_{xy}} \;=\; \frac{1}{2}\left(i\Delta^{ij}_{xy} \;+\; \phi^i_x\, \phi^j_y \right) \;,$$

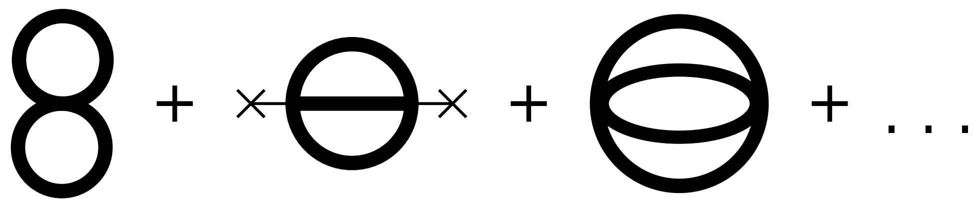
to get the 2PI effective action

$$\Gamma[\phi, \Delta] \;=\; W[J, K] \;-\; J^i_x\, \phi^i_x \;-\; \frac{1}{2}\, K^{ij}_{xy}\, \left(i\Delta^{ij}_{xy} \;+\; \phi^i_x \phi^j_y \right) \;.$$

The 2PI CJT Effective Action:

$$\Gamma[\phi,\Delta] \;=\; S[\phi] + \frac{i}{2}\,\mathrm{Tr}\,\ln\Delta^{-1} + \frac{i}{2}\,\mathrm{Tr}\,({\Delta^{0}}^{-1}\,\Delta) \;-\; i\,\Gamma^{(2)}_{\mathrm{2PI}}[\phi,\Delta]\,,$$

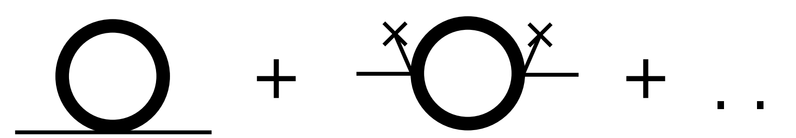
where $\Gamma^{(2)}_{\mathrm{2PI}}[\phi,\Delta] \;=$



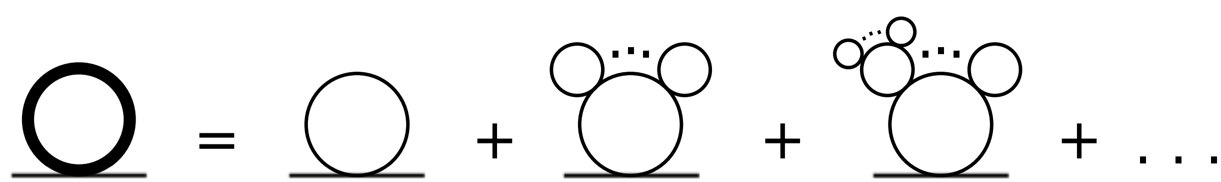
Equations of Motion:

- $$\frac{\delta\Gamma[\phi,\Delta]}{\delta\phi} \;=\; 0$$

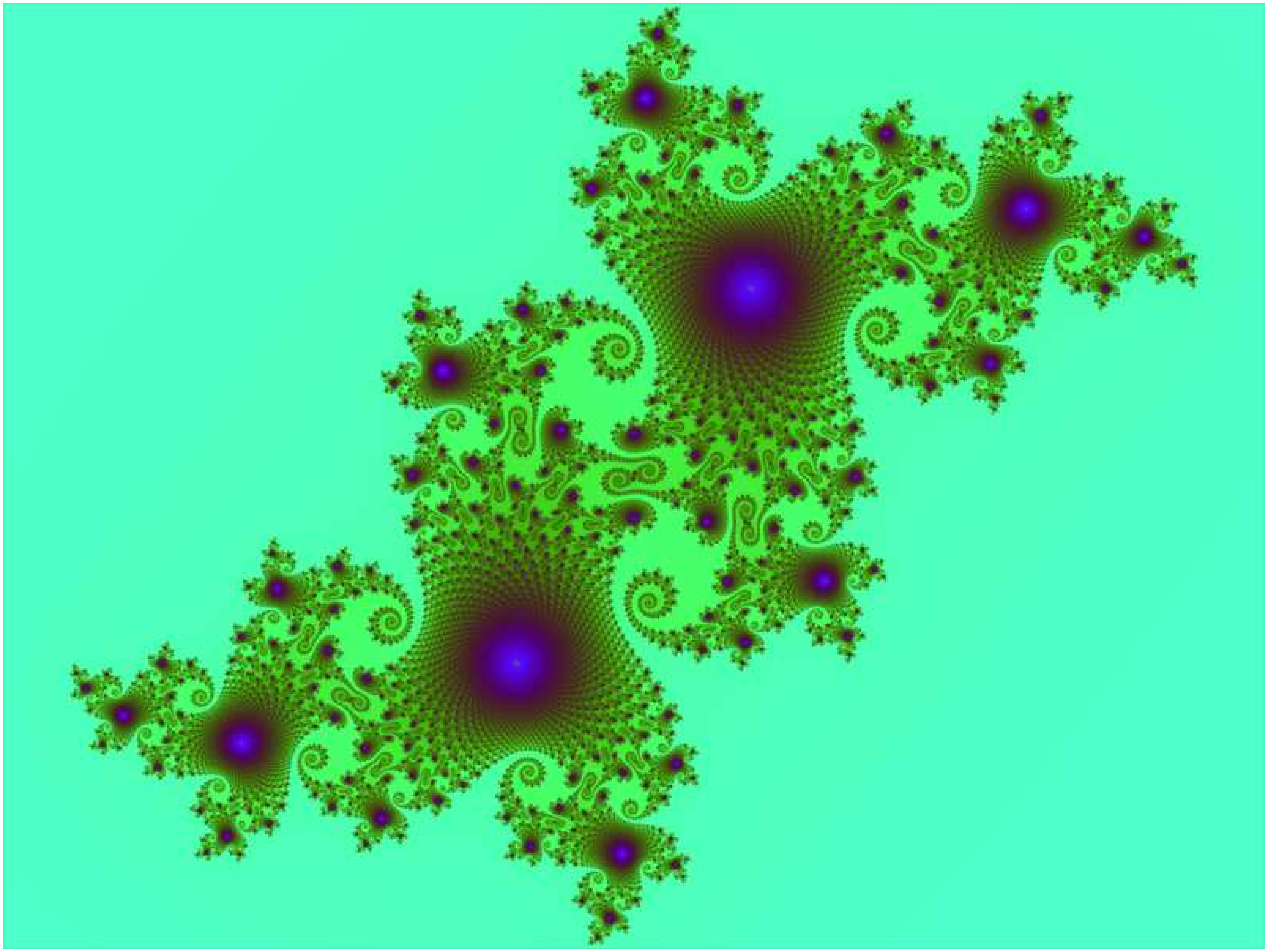
- $$\frac{\delta\Gamma[\phi,\Delta]}{\delta\Delta} \;=\; 0 \;\Rightarrow\; \Delta^{-1} \;=\; {\Delta^{0}}^{-1} \;+$$



Hartree-Fock:



Artist's **impression** of the **infinite** **HF-term**!



– Field-Theoretic Problems in CJT

- Systematic formal resummation of high-order graphs:
  - Rigorous Derivation of Schwinger–Dyson Equations
  - Thermal Masses in the high-T Regime
  - Finite-Width Effects within Quantum Loops [this talk]

- Non-Equilibrium QFT, through Kadanoff–Baym equations.

  [For instance, P. Millington, AP, PRD88 (2013) 8, 085009;
  B.P.S. Dev, P. Millington, AP, D. Teresi, arXiv:1410.6434.]

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BUT

- Truncations of CJT lead to residual violations of symmetries,
  e.g. global or local symmetries.

$\longrightarrow$ Erroneous First-Order Phase Transition in $\mathbb{O}(N)$ Theories
$\longrightarrow$ Goldstone Bosons become Massive
$\longrightarrow$ Erroneous Thresholds for the Resummed Higgs-Boson Propagator.
$\longrightarrow$ $\cdots$

Pertinent Literature to the Goldstone-Symmetry Problem

- G. Baym, G. Grinstein, Phys. Rev. D **15** (1977) 2897.

$\vdots$

- G. Amelino-Camelia, Phys. Lett. B **407** (1997) 268.
- N. Petropoulos, J. Phys. G **25** (1999) 2225.
- Y. Nemoto, K. Naito, M. Oka, Eur. Phys. J. A **9** (2000) 245.
- J. T. Lenaghan, D. H. Rischke, J. Phys. G **26** (2000) 431.
- H. van Hees, J. Knoll, Phys. Rev. D **66** (2002) 025028.
- J. Baacke, S. Michalski, Phys. Rev. D **67** (2003) 085006.
- Y. Ivanov, F. Riek, H. van Hees, J. Knoll, Phys. Rev. D **72** (2005) 036008.
- E. Seel, S. Struber, F. Giacosa, D. H. Rischke, Phys. Rev. D **86** (2012) 125010.
- G. Markó, U. Reinosa, Z. Szép, Phys. Rev. D **87** (2013) 105001.

• Symmetry-Improved CJT Formalism

[AP, D. Teresi, NPB874 (2013) 594]

Equivalence between 1PI and 2PI Effective Actions to All Orders:

$$\Gamma^{1\mathrm{PI}}[\phi] \;=\; \Gamma[\phi\,,\,\Delta(\phi)]\,, \qquad \text{with} \quad \frac{\delta\Gamma[\phi\,,\,\Delta(\phi)]}{\delta\Delta} \;=\; 0\,.$$

1PI Ward Identity (e.g. for $\mathbb{O}(2)$):

$$\frac{\delta\Gamma^{1\mathrm{PI}}[\phi]}{\delta\phi^i_x}\,T^a_{ij}\,\phi^j_x \;=\; 0 \quad\Longrightarrow\quad v\int_x \frac{\delta^2\Gamma^{1\mathrm{PI}}[\phi]}{\delta G_y\,\delta G_x} \;=\; \frac{\delta\Gamma^{1\mathrm{PI}}[\phi]}{\delta H} \;\to\; 0\,.$$

Replace:

$$\frac{\delta^2\Gamma^{1\mathrm{PI}}[\phi]}{\delta G_y\,\delta G_x} \;=\; \Delta^{-1,G}_{xy}\,,$$

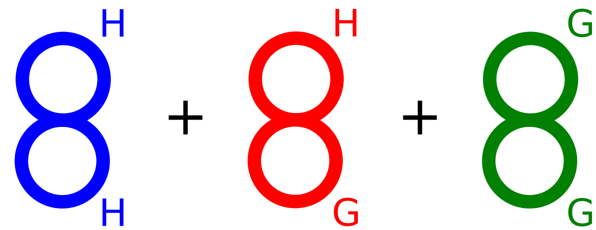
to obtain the Symmetry-Improved Equations of Motion:

$$\frac{\delta\Gamma[v\,,\,\Delta]}{\delta\Delta_{H/G}} \;=\; 0\,,$$

$$v\,\Delta^{-1}_G(k=0,v) \;=\; 0\,.$$

– $\mathbb{O}$(2) Hartree–Fock Equations of Motion:

HF Approximation: $\Gamma^{(2)}_{\rm HF}[\Delta_H\,,\,\Delta_G] \quad = $



<u>*Ansatz:*</u> $\Delta^{-1}_{H/G}(k) \;=\; k^2 \,-\, M^2_{H/G} \,+\, i\varepsilon$

Equations of Motion:

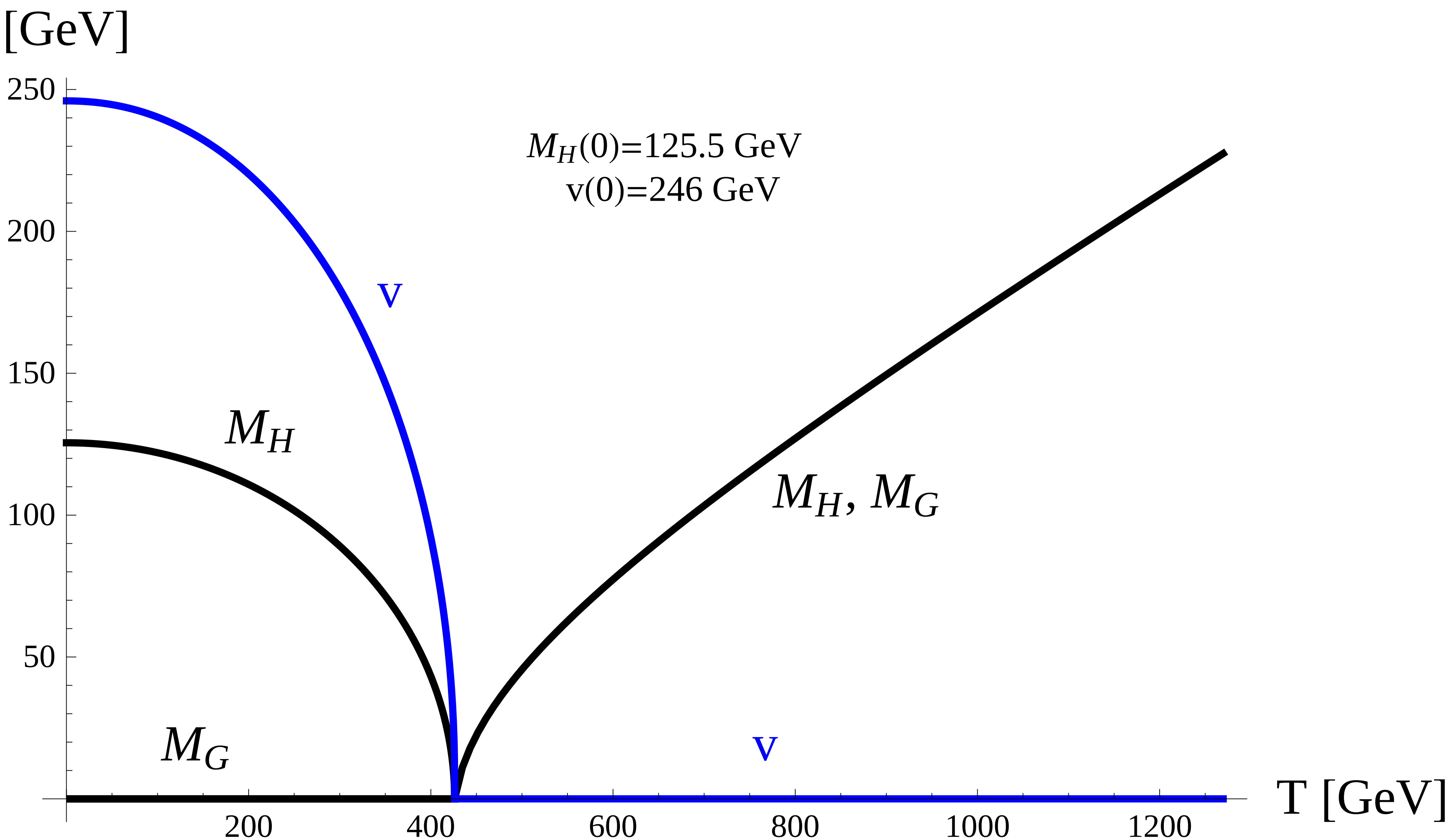
$$
\begin{aligned}
M_H^2 \;&=\; 3\lambda v^2 - m^2 \;+\; (\delta\lambda_1^A + 2\delta\lambda_1^B)v^2 \;-\; \delta m_1^2 \\
&\quad + (3\lambda + \delta\lambda_2^A + 2\delta\lambda_2^B)\int_k i\Delta_H(k) \;+\; (\lambda + \delta\lambda_2^A)\int_k i\Delta_G(k)\;,
\end{aligned}
$$

$$
\begin{aligned}
M_G^2 \;&=\; \lambda v^2 - m^2 \;+\; \delta\lambda_1^A v^2 \;-\; \delta m_1^2 \\
&\quad + (\lambda + \delta\lambda_2^A)\int_k i\Delta_H(k) \;+\; (3\lambda + \delta\lambda_2^A + 2\delta\lambda_2^B)\int_k i\Delta_G(k)\;,
\end{aligned}
$$

$$
v\, M_G^2 \;=\; 0\;.
$$

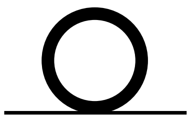
– Second-Order Phase Transition in the HF Approximation

[AP, D. Teresi, NPB874 (2013) 594]



• $\overline{\mathrm{MS}}$ Renormalization in CJT

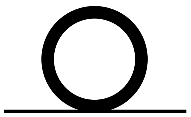
Naive Renormalization:

 $= \int_k i\Delta(k) \;\sim\; M^2 \frac{1}{\epsilon}\,,$ Naive CT: $\delta m^2 \overset{?}{=} M^2 \frac{1}{\epsilon}\,.$

But, $M^2 = M^2(T) \quad \Longrightarrow \quad T$emperature-dependent CT?!

• $\overline{\mathrm{MS}}$ Renormalization in CJT

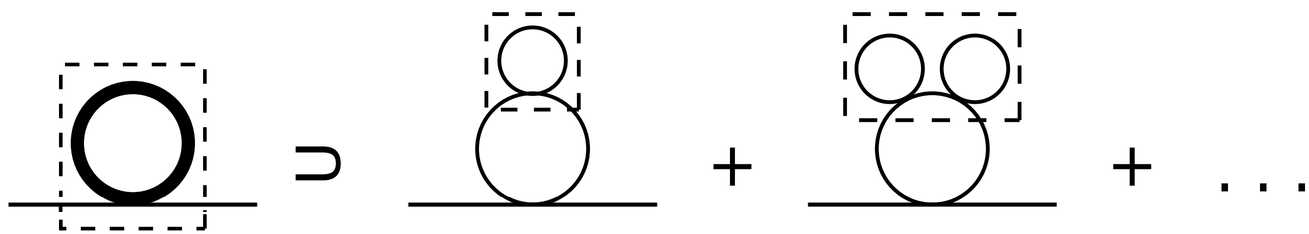
Naive Renormalization:

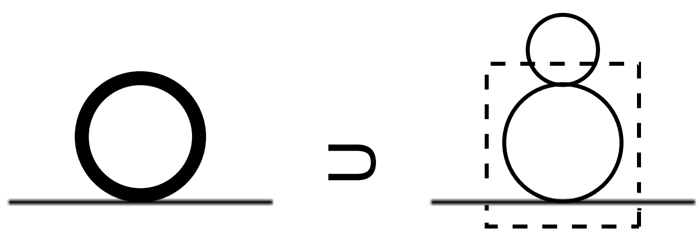
 $= \int_k i\Delta(k) \sim M^2 \frac{1}{\epsilon}$,   Naive CT: $\delta m^2 \stackrel{?}{=} M^2 \frac{1}{\epsilon}$.

But, $M^2 = M^2(T) \quad \Longrightarrow \quad T$emperature-dependent CT?!

What has gone wrong?

[J.-P. Blaizot, E. Iancu, U. Reinosa, NPA736 (2004) 149; J. Berges *et al*, AP320 (2005) 344]

$\delta m^2 \sim$ 

 $\sim \delta\lambda$

Is there any systematic renormalization, e.g. in the $\overline{\mathrm{MS}}$ scheme?

[W.A. Bardeen, A. Buras, D. Duke, T. Muta, PRD18 (1978) 3998]

– $\overline{\mathrm{MS}}$ Renormalization in the CJT Formalism

Procedure:

- Isolate UV infinities in EoMs, e.g. by Dimensional Regularization.

- Require that the UV-finite part of EoMs **be** UV finite:

$$(\dots)_{\mathrm{UV}}\, \mathcal{T}_H^{\mathrm{fin}} \;+\; (\dots)_{\mathrm{UV}}\, \mathcal{T}_G^{\mathrm{fin}} \;+\; (\dots)_{\mathrm{UV}}\, v^2 \;+\; (\dots)_{\mathrm{UV}}\, 1 \;\stackrel{!}{=}\; 0$$

- Cancel seperately the UV infinities $\propto$ $\mathcal{T}_H^{\mathrm{fin}}(T)$, $\mathcal{T}_G^{\mathrm{fin}}(T)$, $v^2(T)$, 1.

- Check UV consistency:

$4 \times 2 = 8$ Constraints, for 5 CTs : $\delta m_1^2\,,\ \delta\lambda_1^A\,,\ \delta\lambda_1^B\,,\ \delta\lambda_2^A\,,\ \delta\lambda_2^B\ .$

This is a non-trivial check!

– T-independent Resummed Counterterms in the HF approximation:

[AP, D. Teresi, NPB874 (2013) 594]

$$
\begin{aligned}
\delta\lambda_1^A &= \delta\lambda_2^A = \frac{2\lambda^2}{16\pi^2\epsilon}\,\frac{3-\dfrac{4\lambda}{16\pi^2\epsilon}}{1-\dfrac{6\lambda}{16\pi^2\epsilon}+\dfrac{8\lambda^2}{(16\pi^2\epsilon)^2}} \\
&= -\lambda + \frac{(16\pi^2\epsilon)^2}{8\lambda} + O(\epsilon^3)\,,
\end{aligned}
$$

$$
\begin{aligned}
\delta\lambda_1^B &= \delta\lambda_2^B = \frac{2\lambda^2}{16\pi^2\epsilon}\,\frac{1}{1-\dfrac{2\lambda}{16\pi^2\epsilon}} \\
&= -\lambda - \frac{16\pi^2\epsilon}{2} - \frac{(16\pi^2\epsilon)^2}{4\lambda} + O(\epsilon^3)\,,
\end{aligned}
$$

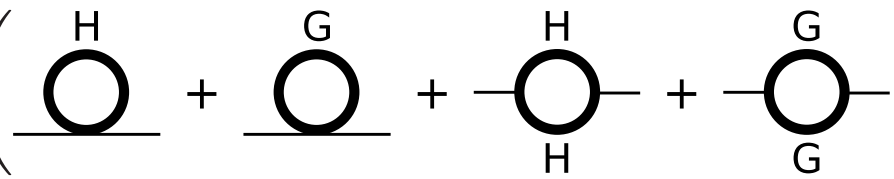
$$
\delta m_1^2 = \frac{4\lambda m^2}{16\pi^2\epsilon}\,\frac{1}{1-\dfrac{4\lambda}{16\pi^2\epsilon}} = -m^2 - m^2\frac{16\pi^2\epsilon}{4\lambda} + O(\epsilon^2)\,.
$$

• Finite-Width Effects within Quantum Loops

Equations of Motion including Sunset Diagrams:

- $$\Delta_H^{-1}(p) \;=\; p^2 \;-\; (3\lambda + \delta\lambda_1^A + 2\delta\lambda_1^B)v^2 \;+\; m^2 \;+\; \delta m_1^2$$

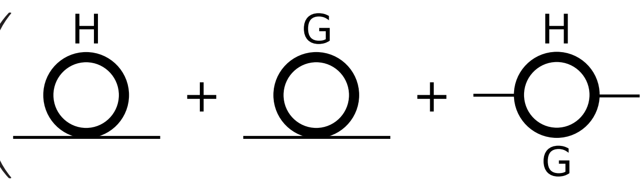
  $-i\,\Big($

  $\Big)\;,$

- $$\Delta_G^{-1}(p) \;=\; p^2 \;-\; (\lambda + \delta\lambda_1^A)v^2 \;+\; m^2 \;+\; \delta m_1^2$$

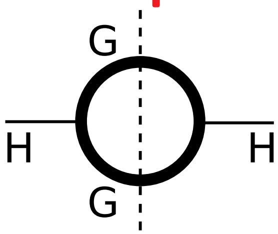
  $-i\,\Big($

  $\Big)\;,$

- $$v\,\Delta_G^{-1}(0) \;=\; 0\;.$$

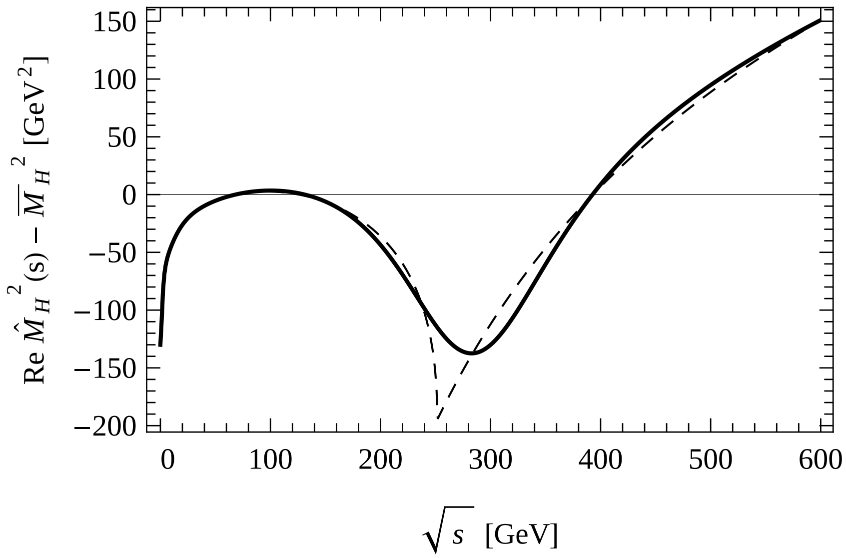
Absorptive Effects:

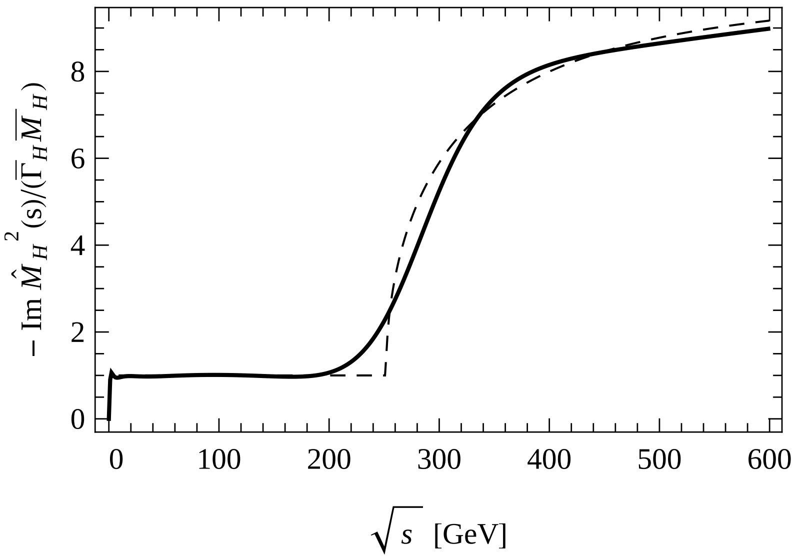


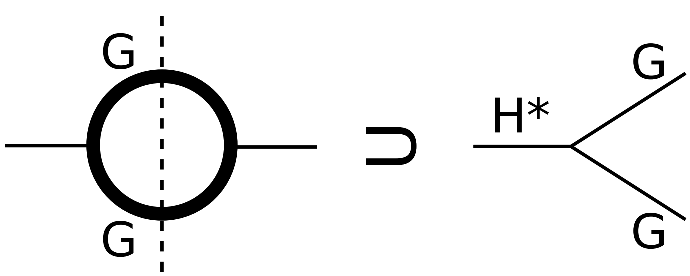
G consistently massless $\Longleftrightarrow$ threshold at $s \equiv p^2 = 0$

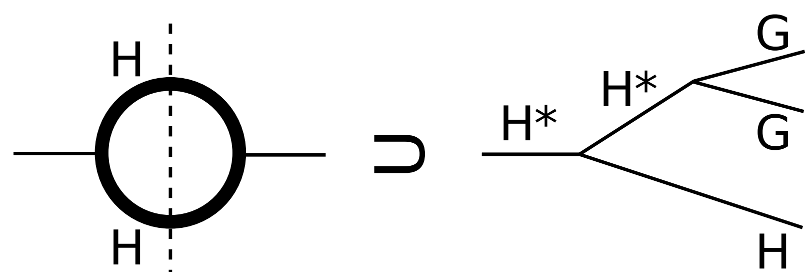
– Higgs Selfenergy in CJT

[AP, D. Teresi, NPB874 (2013) 594]



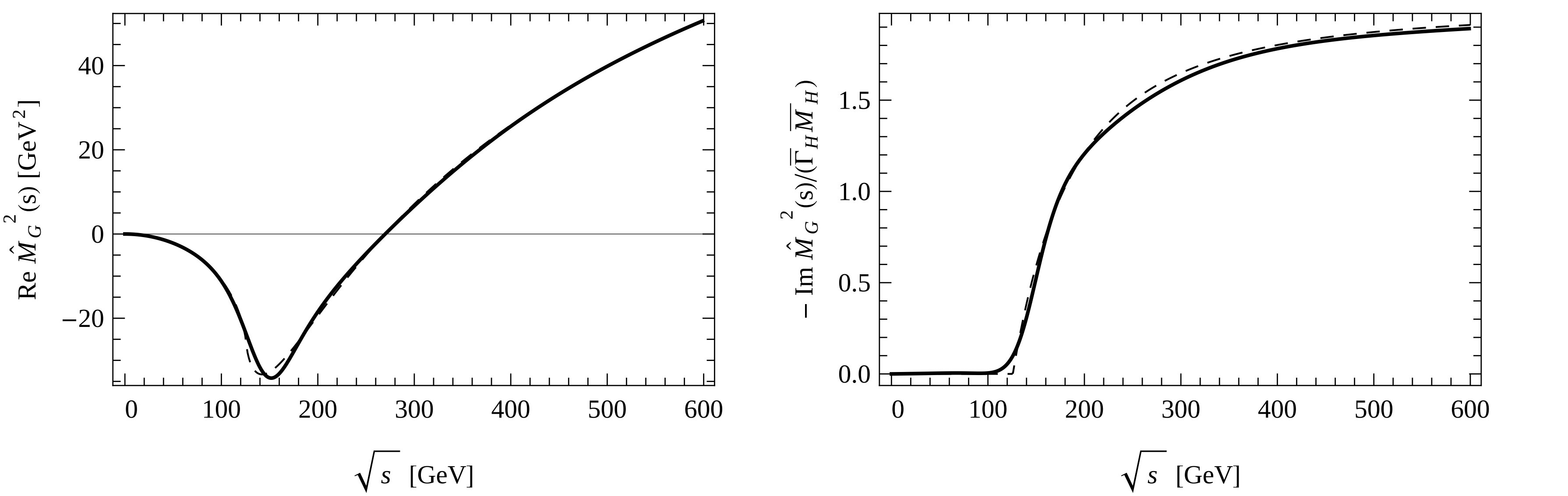


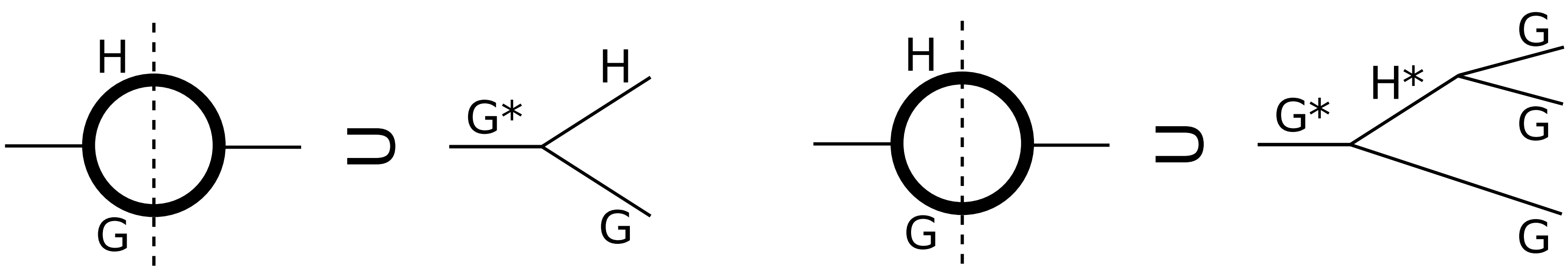




– Goldstone Selfenergy in CJT

[AP, D. Teresi, NPB874 (2013) 594]





• Symmetry-Improved Effective Higgs Potential in CJT

Symmetry-Improved Effective Potential $\widetilde{V}_{\rm eff}(\phi)$ from 1PI Ward Identity:

$$\phi\, \Delta_G^{-1}(k=0;\phi) \;=\; -\,\frac{d\widetilde{V}_{\rm eff}(\phi)}{d\phi}\ .$$

Solution:

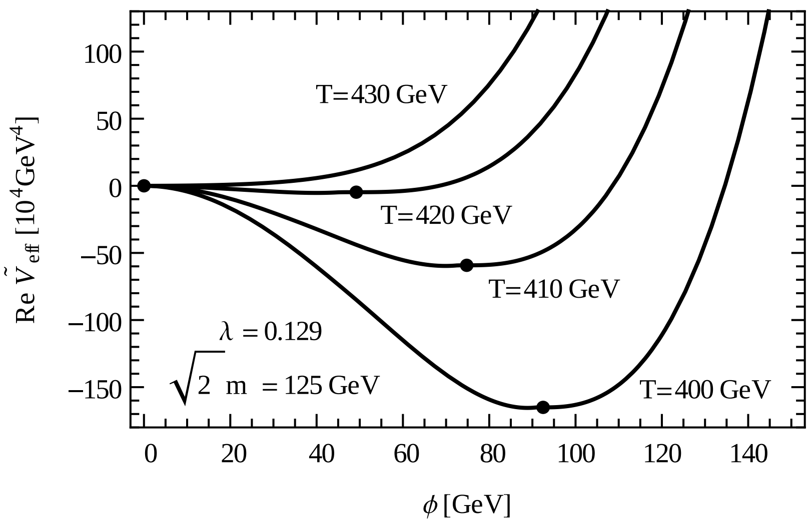
$$\begin{aligned} \widetilde{V}_{\rm eff}(\phi) \;&=\; -\int_0^{\phi} d\phi\, \phi\, \Delta_G^{-1}(k=0;\phi) \;+\; \widetilde{V}_{\rm eff}(\phi=0) \\ &=\; -\int_v^{\phi} d\phi\, \phi\, \Delta_G^{-1}(k=0;\phi) \;+\; P(T,\mu)\ , \end{aligned}$$

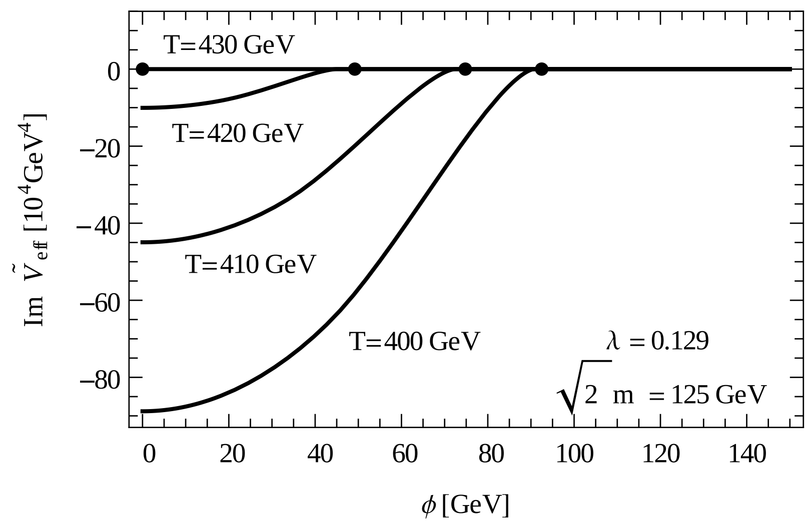
where $P(T,\mu)$ is the thermodynamic pressure $=$ hydrostatic pressure, i.e. it satisfies Baym's thermodynamic consistency.

[G. Baym, PR127 (1962) 1391]

– CJT Effective Higgs Potential

[AP, D. Teresi, NPB874 (2013) 594]





Note: $\mathrm{Im}\,\widetilde{V}_{\rm eff}(\phi) < 0$, for $0 < \phi < v$

$\Longrightarrow$ Vacuum instability for the concave part of the potential.

[E.J. Weinberg, A.-Q. Wu, PRD36 (1987) 2474]

• Conclusions

- **Maintaining symmetries in CJT loopwise is a long-standing problem**

- **Novel Approach to Global Symmetries:**

  $\Longrightarrow$ **Symmetry-Improved CJT Effective Action**

  $\longrightarrow$ Massless Goldstone Bosons
  $\longrightarrow$ 2nd-Order Phase Transition in the HF Approximation

- $\overline{\text{MS}}$ **Renormalization with T-independent Resummed Counterterms**

- **Absorptive Effects properly described:**

  $\longrightarrow$ Smooth thresholds consistent with massless Goldstone bosons
  $\longrightarrow$ Consistent resummation within Quantum Loops.

- **Symmetry-Improved Effective Higgs Potential is unique, with proper thermodynamic properties**

• Future Directions

- **Solve the IR Problem of the SM Effective Potential**

  [D. Teresi, parallel talk]

- **Extension: 2PI $\longrightarrow$ nPI Effective Actions**

- **Extension to Local Symmetries: $\mathbb{U}(1)$, $\mathbb{SU}(N)$**

- **Spontaneous Breaking of Local Gauge Symmetries**

- **Higher Precision Predictions in the 2PI Formalism**

$\vdots$

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[D. Teresi, parallel talk]

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$\vdots$

New Era of Analytical Non-Perturbative QFT?