SO(10) Grand Unification from $M$-theory on a $G_2$-manifold

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Discrete 2014
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1. Introduction and Motivation
2. Model building in $M$-Theory
   - The framework
   - The $G_2$-MSSM
3. $SO(10)$ models from $M$-Theory
4. Conclusions and future work
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4 Conclusions and future work
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In this case we also find all the realistic scenario ingredients: $N = 1$ SUSY, Chiral fermions, Non-abelian gauge theories, etc.
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The compactified manifold, $K$, plays a crucial role in defining the 4D QFT:

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\lambda_{ijk} \sim \exp(-\text{vol}_{ijk})
\]

The unified gauge coupling is a function of the volume of $K$:

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This breaks the gauge group to $H = \{g \in G, [g, \mathcal{W}] = 0\}$ as $\Psi^w$ transforms under the elements of $G$ that commute with $\mathcal{W}$. 
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This provides a mechanism with a solution for the doublet-triplet splitting problem.
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Consider the $SU(5)$ GUT with the MSSM spectrum, plus the geometric symmetry $Z_n$. Let $\overline{5}^w \supset (H_d, \overline{D})$ be localised along the Wilson line. Under the discrete symmetry $Z_n$ The GUT multiplets transform

\begin{align*}
\overline{5}^w &\rightarrow \eta^\omega \left( \eta^\delta H_d^w \oplus \eta^\gamma \overline{D}^w \right), \\
5^h &\rightarrow \eta^\chi 5^h, \\
\overline{5}^m &\rightarrow \eta^\tau \overline{5}^m, \\
10^m &\rightarrow \eta^\sigma 10^m,
\end{align*}

where $\eta \equiv e^{2\pi i/n}$, $2\delta + 3\gamma = 0 \text{ mod } n$ (for simplicity we will set $\overline{5}^w$ to $\omega = 0$).
The $G_2$-MSSM

<table>
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<tr>
<th>Description</th>
<th>Coupling</th>
<th>Constraint</th>
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<td>Down-type Yukawas</td>
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<td>Neutrino Majorana-masses</td>
<td>$H_d^w H_d^w \bar{5}^m \bar{5}^m$</td>
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A solution exists while **forbidding a tree-level $\mu$-term and dimension four and five proton decay operators**.

An effective $\mu$-term is generated by moduli vev

$$K \supset \frac{s}{M_{Pl}} H_u H_d + \text{h.c.},$$

$$\mu = \langle m_{3/2} K_{H_u H_d} - F^k K_{H_u H_d} k \rangle \sim 0.1 m_{3/2} \sim \mathcal{O}(\text{TeV}).$$
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In minimal \( SO(10) \) the \( \mu \)-term is contained in

\[
\mathcal{W} \ni 10^w 10^w = \mu H_u H_d + m_D D \overline{D},
\]

but as \( \mathcal{W} \in SO(10) \), both terms are invariant under its action.
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$$W \supset H_d^T \cdot \mu_H \cdot H_u + \overline{D}^T \cdot M_D \cdot D,$$

where $\mu_H$ and $M_D$ are two $2 \times 2$ superpotential mass parameters matrices, $H_{u,d}^T = (H_{u,d}^w, H_{u,d}^h)$, $\overline{D}^T = (\overline{D}^w, \overline{D}^h)$, and $D^T = (D^w, D^h)$. 

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- Moduli vevs break the discrete symmetry, leading to to proton-decay interactions.
- The presence of light coloured states will ruin unification.
Consider the relevant Kahler potential operators

\[
K \supset \frac{s}{M^2_{Pl}} DQQ + \frac{s}{M^2_{Pl}} De^c u^c + \frac{s}{M^2_{Pl}} DNd^c + \\
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The effective potential may be calculated a la Giudice-Masiero to be

\[ W_{\text{eff}} \supset \lambda DQQ + \lambda D\bar{c}u^c + \lambda DNd^c + \]
\[ + \lambda D\bar{d}u^c + \lambda DQL, \]

where

\[ \lambda \approx \frac{1}{M^2_{Pl}} (\langle s \rangle m_{3/2} + \langle F_s \rangle) \sim 10^{-14}. \]
The proton-decay rate can be estimated by

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- The \( D \) triplet decay rate can also be estimated
\[ \tau_D = \Gamma_D^{-1} \sim (\lambda^2 m_D)^{-1} \sim 0.1 \text{ sec}, \]
which is consistent with BBN constraint.
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- Consider an addition vector-like family $16_X, \overline{16}_X$.
- Using the discrete symmetry, we find a solution where $d^c_X, \overline{d}^c_X$ are split from the rest of the family.
- The resulting spectrum is effectively the same as the MSSM with additional vector-like family.
Such solution exists if either $16_X$ or $\overline{16}_X$ absorb Wilson line phases. Considering

$$16_X \rightarrow \eta^x \left( \eta^{-3\gamma} L \oplus \eta^{3\gamma+\delta} e^c \oplus \eta^{3\gamma-\delta} N \oplus \eta^{-\gamma-\delta} u^c \oplus \eta^{-\gamma+\delta} d^c \oplus \eta^\gamma Q \right).$$

and $16_X \rightarrow \eta^x \overline{16}_X$,

the splitting condition is given by

$$\overline{d^c}_X d^c_X : x - \gamma + \delta + \overline{x} = 0 \mod n,$$

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Unification scale is found to be $M_{\text{GUT}} \sim 10^{16}$ GeV, with $\alpha_u^{-1} \sim 9.6$. 
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One can perform an RGE analysis of the spectrum assuming general values for the mass parameters at the GUT scale.
The addition of an extra family has other benefits for model building: $N_X, \overline{N}_X$ vevs ($\nu_X$) break the rank of the gauge group and generate Right-handed neutrinos Majorana masses.

$$\frac{1}{M_{Pl}} \overline{N}_X \overline{N}_X N^m N^m : M_{\text{Majorana}} \sim \frac{\nu_X^2}{M_{Pl}}$$
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- R-Parity violating (RPV) interactions through the Kahler terms:

$$K_{RPV} \supset \alpha \frac{s}{M_{Pl}^3} 16_X 16^m 16^m 16^m + \beta \frac{s}{M_{Pl}^2} 10^w 16_X 16^m.$$
Outline

1 Introduction and Motivation

2 Model building in $M$-Theory
   - The framework
   - The $G_2$-MSSM

3 $SO(10)$ models from $M$-Theory

4 Conclusions and future work
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  - Consistent unification scenario.

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Thank you!

MCR thanks the support from the FCT under the grant SFRH/BD/84234/2012.
Tree-level terms in the superpotential are functions of distances between the supermultiplets in $K$.

The superpotential mass parameters, the $\mu$-terms, values are suppressed Planck masses.

By considering $\overline{5}^w$ and $5^h$ to be near in $K$ then the $D^h\overline{D}^w$ mass can be GUT/Planck valued while the Higgses don’t have a $\mu$-term $\Rightarrow$ we split the $H$ and $D$ masses.

On the other hand, direct gaugino searches tell us $\mu \geq 100$ GeV.

Fortunately there is a natural way of generating an effective $\mu$-term of $O(\text{TeV})$ in $M$-Theory!

Discrete symmetry is a geometric symmetry of the extra dimensions, therefore the moduli fields are naturally charged under it. As moduli acquire vevs they break the discrete symmetry.
The moduli vev generate an effective $\mu$-term a la Giudice-Masiero from Kahler potential operators of the form

$$K \supset \frac{s}{M_{Pl}} H_u H_d + \text{h.c.}.$$ 

From standard supergravity calculations we know that

$$\mu = \langle m_{3/2} K_{H_u H_d} - F^k K_{H_u H_d} \rangle,$$

which leads to

$$\mu \sim \frac{\langle s \rangle}{M_{Pl}} m_{3/2} + \frac{\langle F_s \rangle}{M_{Pl}}.$$ 

Developments in M-Theory have shown that the moduli vevs are approximately $\langle s \rangle \sim 0.1 M_{Pl}$, $\langle F_s \rangle \sim m_{1/2} M_{Pl}$, $m_{3/2} \sim \mathcal{O}(10 \text{ TeV})$.

In $M$-Theory the gaugino masses are suppressed, so the F-term is subleading and we conclude

$$\mu \sim 0.1 m_{3/2} \sim \mathcal{O}(\text{TeV}).$$
To study the potential mixing we focus on the superpotential contributions to the up-type quarks mass matrix.

Schematically, \( W \supset \overline{U} \cdot M_U \cdot U \), where \( U^T = (u_i, \overline{u}^c X, u_X) \), \( \overline{U}^T = (u^c_i, \overline{u}_X, u^c_X) \), with \( i = 1, 2, 3 \), and

\[
M_U = \begin{pmatrix}
y_u^{ij} H_u & \mu_{iX} & \lambda_{iX} \\
\mu_{Xj} & \lambda_{XX} & \mu_{XX} \\
\lambda_{Xj} & \mu_{XX} & \lambda_{XX}
\end{pmatrix}
\]

where \( y_u \) are EWS Yukawas, \( \mu \)-terms are of order TeV, and \( \lambda \) couplings are of order \( 10^{-14} \).

In the limit \( \lambda_{iX}, \lambda_{Xj}, \lambda_{XX} \rightarrow 0 \) one finds that the determinant of \( M_U \) is independent of \( \mu_{iX}, \mu_{Xj} \), i.e. of the mixing masses.

To leading order in \( \lambda \) the mass eigenstates do not mix matter with \( 16_X \).
RPV interactions arise from the Kahler potential terms and are effectively described by the superpotential

\[ W_{\text{eff}}^{\text{RPV}} \supset \alpha \lambda \frac{v_X}{M_{Pl}} LLe^c + \alpha \lambda \frac{v_X}{M_{Pl}} QLe^c + \alpha \lambda \frac{v_X}{M_{Pl}} u^c d^c d^c + \beta \lambda v_X LH_u. \]

Due to the nature of $SO(10)$, neutrinos have the same Dirac mass as the up-type quarks. A realistic $\tau$-neutrino physical mass requires

\[ M_{\text{Majorana}} \gtrsim 10^{14} \text{ GeV} \Rightarrow v_X \sim 10^{16} \text{ GeV}. \]

The effective strength of the first terms is $\lambda v_X / M_{Pl} \sim \mathcal{O}(10^{-16})$ (taking $\alpha \sim \mathcal{O}(1)$). The resulting LSP lifetime can be estimated as

\[ \tau_{\text{LSP}} \sim \frac{10^{-13} \text{ sec}}{(v_X / M_{Pl})^2} \left( \frac{m_0}{10 \text{ TeV}} \right)^4 \left( \frac{100 \text{ GeV}}{m_{\text{LSP}}} \right)^5 \sim 10^{-13} \text{ sec}, \]

for $m_0 \sim 10 \text{ TeV}$, $m_{\text{LSP}} \sim 100 \text{ GeV}$. 
The last term in the effective superpotential

$$\beta \lambda v_x L H_u,$$

is constrained by neutrino mass limit exclusions.

The bilinear contribution to the mixing to be at most of

$$\beta \lambda v_x \lesssim \mathcal{O}(10^{-3} \text{ TeV}),$$

which means leads to the upper bound

$$v_x \lesssim 10^{14}/\beta \text{ GeV},$$

and one has to consider there is some suppression by $\beta$ of order $\mathcal{O}(10^{-2})$. 