Standard Model Puzzles

The problem of flavour - the problem of the undetermined fermion masses and mixing angles (including neutrino masses and lepton mixing angles) together with the CP violating phases, in conjunction with the observed smallness of flavour changing neutral currents and very small strong CP violation.

The origin of mass after Higgs discovery - the origin of the weak scale, its stability under radiative corrections, and the TeV scale solution to the hierarchy problem or not (most urgent problem of LHC run 2).

The quest for unification - the question of whether the three known forces of the standard model may be related into a grand unified theory, and whether such a theory could also include a unification with gravity.
Why three families?
What is the origin of Quark and Lepton Masses?
What is the origin of Quark and Lepton Mixing?

1. Introduction: Reasons for physics beyond the Standard Model

Although the Standard Model (SM) of particle physics provides an excellent description of electroweak and strong interactions, there are many reasons that we expect to observe new forces giving rise to new particles at larger masses than the known fermions or bosons. One oft noted source of this belief is the observation of dark matter in the cosmos as evidenced by galactic angular velocity distributions \[^1\] , gravitational lensing \[^2\] , and galactic collisions \[^3\] . The existence of dark energy, believed to cause the accelerating expansion of the Universe, is another source of mystery \[^4\] . The fine tuning of quantum corrections needed to keep, for example, the Higgs boson mass at the electroweak scale rather than near the Planck scale is another reason habitually mentioned for new physics (NP) and is usually called "the hierarchy problem" \[^5\] .

It is interesting to note that the above cited reasons are all tied in one way or another to gravity. Dark matter may or may not have purely gravitational interactions, dark energy may be explained by a cosmological constant or at least be a purely general relativistic phenomena, and the Planck scale is defined by gravity; other scales may exist at much lower energies, so the quantum corrections could be much smaller. There are, however, many observations that are not explained by the SM, and have nothing to do with gravity, as far as we know. Consider the size of the quark mixing matrix (CKM) elements \[^6\] and also the neutrino mixing matrix (PMNS) elements \[^7\] .

These are shown pictorially in Fig. 1. We do not understand the relative sizes of these values or nor the relationship between quarks and neutrinos.

**Figure 1:**

(left) Sizes of the the CKM matrix elements for quark mixing, and (right) the PMNS matrix elements for neutrino mixing. The area of the squares represents the square of the matrix elements.

We also do not understand the masses of the fundamental matter constituents, the quarks and leptons. Not only are they not predicted, but also the relationships among them are not understood. These masses, shown in Fig. 2, span 12 orders of magnitude \[^7\] . There may be a connections between the mass values and the values of the mixing matrix elements, but thus far no connection besides simple numerology exists.

What we are seeking is a new theoretical explanation of the above mentioned facts. Of course, any new model must explain all the data, so that any one measurement could confound a model. It is not a good plan, however, to try and find only one discrepancy; experiment must determine a
The Large Lepton Mixing Angles

\[
\theta_{13} = 8.5^\circ \pm 0.2^\circ \\
\theta_{12} = 34^\circ \pm 1^\circ \\
\theta_{23} = 45^\circ \pm 3^\circ 
\]
Is CP violated in the lepton sector?

- What is the Dirac CP violating phase (neutrino oscillations)?
- What are the Majorana CP violating phases (NDBD)?
Neutrino mass model roadmap

- **Dirac or Majorana?**
  - Majorana

- **Extra Higgs @LHC?**
  - no

- **RPV SUSY@LHC?**
  - no

  - **See-saw models with right-handed neutrinos**

  - **Extra Dim models**
    - yes

  - **Higgs triplet or loop models**
    - yes

  - **RPV SUSY models**
    - yes

None of this has been seen at the LHC
Standard Model of Quarks and Leptons with three right-handed neutrinos

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

Left-handed quarks and leptons (active neutrinos)

Right-handed quarks and leptons (sterile neutrinos)
Sequential dominance (SD)

Flavour basis

\[
M_E = \begin{pmatrix}
    m_e & 0 & 0 \\
    0 & m_\mu & 0 \\
    0 & 0 & m_\tau
\end{pmatrix}, \quad M_R = \begin{pmatrix}
    M_{atm} & 0 & 0 \\
    0 & M_{sol} & 0 \\
    0 & 0 & [M_{dec}]
\end{pmatrix}
\]

Dirac mass

\[
m^D = \begin{pmatrix}
    m_{e,atm}^D & m_{e,sol}^D & [m_{e,dec}^D] \\
    m_{\mu,atm}^D & m_{\mu,sol}^D & [m_{\mu,dec}^D] \\
    m_{\tau,atm}^D & m_{\tau,sol}^D & [m_{\tau,dec}^D]
\end{pmatrix} = \begin{pmatrix}
    m_{atm}^D & m_{sol}^D & [m_{dec}^D]
\end{pmatrix}
\]

See-saw with Sequential dominance

These theories predict normal neutrino mass hierarchy

\[
\frac{m_{atm}^D m_{atm}^D}{M_{atm}} \gg \frac{m_{sol}^D m_{sol}^D}{M_{sol}} \gg \frac{m_{dec}^D m_{dec}^D}{M_{dec}} \gg m_3 \gg m_2 \gg m_1
\]
Towards a Theory of Flavour

Symmetry

Anarchy
Symmetry

GUTs

Family Symmetry
decompose into multiplets of the SM gauge group $SU(3) \times SU(2)_L \times U(1)_Y$ as

$$F = (d, c, L, e)_{\frac{1}{3}} \oplus (1, 2, -\frac{1}{2}), \quad (9.2)$$

and

$$T = (u, c, Q, e)_{\frac{2}{3}} \oplus (3, 1, -\frac{2}{3}) \oplus (3, 2, \frac{1}{6}), \quad (9.3)$$

Thus a complete quark and lepton SM family $(Q, u_c, d_c, L, e_c)$ is accommodated in the $F = 5$ and $T = 10$ representations, with right-handed neutrinos, whose CP conjugates are denoted as $\nu_c$, being singlet of $SU(5)$.

The Higgs doublets $H_u$ and $H_d$ which break electroweak symmetry in a two Higgs doublet model are contained in the $SU(5)$ multiplets $H_5$ and $H_1$.

The Yukawa couplings for one family of quarks and leptons are given by,

$$y_u H_5^i T^j T^k \epsilon^{ijklm} + y_\nu H_5^i F^i \nu_c + y_d H_5^i T^i j F^j, \quad (9.4)$$

where $\epsilon^{ijklm}$ is the totally antisymmetric tensor of $SU(5)$ with $i, j, k, l = 1, \ldots, 5$, which decompose into the SM Yukawa couplings

$$y_u H_u^i Qu_c + y_\nu H_u^i L \nu_c + y_d (H_d^i Qd_c + H_d^i e_c L), \quad (9.5)$$

–6 1–

GUTs

\[
 \begin{align*}
 E_6 & \quad \text{SU}(5) \times U(1) \\
 SO(10) & \quad \text{SU}(3)_C \times \text{SU}(3)_L \times \text{SU}(3)_R \\
 SU(4)_C \times SU(2)_L \times SU(2)_R & \\
 SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} & \\
 SU(3)_C \times SU(2)_L \times U(1)_Y & 
\end{align*}
\]
Discrete Family Symmetry

For string theory origin see talks on F-theory - talks by G.Leontaris and A.Meadowcroft
also see M-theory talk by M.Romao

Escobar, Luhn \((c \times d) \times (a, b)\)
\[\Delta(6n^2) \cong (\mathbb{Z}_n \times \mathbb{Z}_n) \rtimes S_3\]

\[\Delta(3n^2) \cong (\mathbb{Z}_n \times \mathbb{Z}_n) \rtimes \mathbb{Z}_3\]

Luhn, Nasri, Ramond

\[SU(3)\]
\[\Sigma(168)\]
\[\Delta(27)\]
\[T_7\]
\[\Delta(96)\]
\[SO(3)\]
\[A_5\]
\[S_4\]

\[S_4\] talk by M.Dimou
The Klein Symmetry

Phase symmetry of diagonal charged lepton mass matrix

\[ T^\dagger (M_e^\dagger M_e) T = M_e^\dagger M_e \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \]

\[ \omega = e^{2i\pi/n} \]

Symmetry of Majorana matrix depends on PMNS

\[ m_\nu = S^T m_\nu S \quad m_\nu = U^T m_\nu U \]

\[ S = U^*_{\text{PMNS}} \text{ diag}(+1, -1, -1) \quad U^T_{\text{PMNS}} \]

\[ U = U^*_{\text{PMNS}} \text{ diag}(-1, +1, -1) \quad U^T_{\text{PMNS}} \]

\[ SU = U^*_{\text{PMNS}} \text{ diag}(-1, -1, +1) \quad U^T_{\text{PMNS}} \]

\[ \mathcal{K} = \{1, S, U, SU\} \]

\[ Z_2 \times Z_2 \]
Figure 8: A sketch of the direct model building approach. The charged lepton sector is (approximately) diagonal either due to a remnant (approximate) $T$ symmetry or simply by construction. The $Z_2$ factor arises accidentally. The flavons of semi-direct models appear linearly in the neutrino mass term, similar to Eq. (6.10), and break $G$ down to one of its $Z_2$ subgroups. An example of such a model is provided by the famous Altarelli-Feruglio $A_4$ model of tribimaximal mixing [30, 103].

$A_4$ is the group of even permutations on four objects, and as such a subgroup of $S_4$. It can be obtained from $S_4$ by simply dropping the $U$ generator. Not being a part of the underlying family symmetry, it is therefore evident that the $U$ symmetry of Eq. (6.8) must arise accidentally.

6.4 The indirect model building approach

In the class of indirect models, no $Z_2$ factor of the Klein symmetry of Eq. (6.6) forms a subgroup of $G$. Models of this class are typically based on the type I seesaw mechanism together with the assumption of sequential dominance, see Subsection 4.3. Here, the main role of the family symmetry consists in relating the Yukawa couplings $d, e, f$ of Eq. (4.21) as well as $a, b, c$ of Eq. (4.24) by introducing triplet flavon fields which acquire special vacuum configurations. The directions of the flavon alignments are determined by the $G$ symmetric operators of the flavon potential [101].

Working in a basis where both the charged leptons as well as the right-handed neutrinos are diagonal, the leptonic flavour structure is encoded in the direct neutrino Yukawa operator. The triplet flavons $\phi^\nu$ of indirect models enter linearly in this term, $L^\nu \sim \frac{\phi^\nu}{\Lambda^2} L H_u L H_u$, where $\Lambda$ is a cut-off scale and the sum is over the number of right-handed neutrinos. The lepton doublet $L$ with hypercharge $-1/2$ transforms as a triplet to $G$, while the right-handed neutrinos $\nu^c_i$ and the up-type Higgs doublet with hypercharge $+1/2$ are singlets of $G$.

Klein symmetry $S,U$ and $T$ are each identified as subgroups of some family symmetry

$\Delta(6n^2)$ is the only viable symmetry class

Holthausen,Lim, Lindner; SK,Neder,Stuart; Lavoura,Ludl; Fonseca,Grimus
\[ \Delta (6n^2) \text{ for leptons} \]

PMNS

\[ V = \begin{pmatrix} \frac{\sqrt{2}}{3} \cos(\vartheta) & \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{3} \sin(\vartheta) \\ -\frac{\sqrt{2}}{3} \sin\left(\frac{\pi}{6} + \vartheta\right) & \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{3} \cos\left(\frac{\pi}{6} + \vartheta\right) \\ \frac{\sqrt{2}}{3} \sin\left(\frac{\pi}{6} - \vartheta\right) & -\frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{3} \cos\left(\frac{\pi}{6} - \vartheta\right) \end{pmatrix} \]

\[ |V_{13}| \]

Predictions:

\[ \delta = 0, \pi \]

no CP violation in oscillations

\[ \theta_{23} = 45^\circ \pm \theta_{13}/\sqrt{2} \]

Atmospheric sum rule

TM2 mixing, reactor angle quantised

Talk by Neder

\[ T = a \]

\[ S = c \frac{n}{2} \]

\[ U = abc^\gamma \]
Semi-Direct Models

Family Symmetry

Generators $S, T, U$

$G$

$\Delta(96)$

$S4$

$A5$

$S, U$ broken

$T$ broken

$U$ broken

$A4$

BT mixing at LO

TB (BM) mixing at LO

GR mixing at LO

Charged Lepton corrections

TM1 or TM2 mixing

General HO corrections

Solar Sum Rules

Atmospheric Sum Rules

No Sum Rules

Klein symmetry and T are partly preserved as subgroups of some family symmetry

$TB = \text{tri-bimaximal}$

$BM = \text{bimaximal}$

$GR = \text{golden ratio}$

$BT = \text{bi-trimaximal}$

$TM = \text{trimaximal}$

| & $\theta_{13}$ & $\theta_{23}$ & $\theta_{12}$ |
|---|---|---|---|
| $TB$ | $0^\circ$ | $45^\circ$ | $35.3^\circ$ |
| $BM$ | $0^\circ$ | $45^\circ$ | $45^\circ$ |
| $GR$ | $0^\circ$ | $45^\circ$ | $31.7^\circ$ |
| $BT$ | $12.2^\circ$ | $36.2^\circ$ | $36.2^\circ$ |
| $TM$ | $\neq 0^\circ$ | $\neq 45^\circ$ | $35.3^\circ$ |

$SK, Luhn$

1301.1340;

$SK, Merle,$

$Morisi,$

$Shimizu,$

$Tanimoto,$

$1402.4271$
Neutrino Mixing at LO

\[ \theta_{13} = 0^\circ \quad \theta_{23} = 45^\circ \]

\[ U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} P \quad \theta_{12} = 45^\circ \]

\[ U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P \quad \theta_{12} = 35.26^\circ \]

\[ U_{GR} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} P \quad \theta_{12} = 31.7^\circ \]

\[ \tan \theta_{12} = \frac{1}{\phi} \quad \phi = \frac{1 + \sqrt{5}}{2} \]

\[ \theta_{12} = \frac{\pi}{5} \quad \text{(GR2)} \quad \text{and} \quad \theta_{12} = \frac{\pi}{6} \quad \text{(HEX)} \]

V. Barger, S. Pakvasa, T. Weiler and K. Whisnant, hep-ph/9806387;
A. Baltz, A.S. Goldhaber, M. Goldhaber, hep-ph/9806540;
Y. Nomura and T. Yanagida, hep-ph/9807325;
H. Georgi and S. Glashow, hep-ph/9808293;
S. Davidson and S.F. King, hep-ph/9808296


Y. Kajiyama, M. Raidal and A. Strumia, [arXiv:0705.4559];
T breaking

\[ U_{PMNS} = U_e U_\nu^\dagger \]

Charged lepton correction

Simple mixing for neutrinos (Klein sym.)

\[ U = U_{12}^{e\dagger} U_{23}^{e\dagger} R_{23}^{\nu} R_{12}^{\nu} \]

Assuming \( \theta_\nu^{13} = \theta_e^{13} = 0 \)

Solar Sum Rule

\[ \frac{|U_{\tau 1}|}{|U_{\tau 2}|} = \frac{|s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta}|}{|s_{23}c_{12} + s_{13}c_{23}s_{12}e^{i\delta}|} = t_{12}^{\nu} \]

RHS only depends on \( \theta_{12}^{\nu} \)

Ballett, SK, Luhn, Pascoli, Schmidt 1410.7573

### Bimaximal

\[ \theta^\nu_{12} = 45^\circ \]

Almost excluded!

\[ \cos \delta \sim (\theta_{12} - 45^\circ)/\theta_{13} \]

### Golden ratio

\[ \theta^\nu_{12} = 31.7^\circ \]

\[ \cos \delta \sim (\theta_{12} - 31.7^\circ)/\theta_{13} \]

### Tri-bimaximal

\[ \theta^\nu_{12} = 35.26^\circ \]

\[ \cos \delta \sim (\theta_{12} - 35.26^\circ)/\theta_{13} \]

### Solar Sum Rule

\[
\cos \delta = \frac{t_{23}s^2_{12} + s^2_{13}c^2_{12}/t_{23} - s^\nu_{12}^2(t_{23} + s^2_{13}/t_{23})}{\sin 2\theta_{12}s_{13}}.
\]

Ballett, SK, Luhn, Pascoli, Schmidt

1410.7573

**useful approximation**

\[ \cos \delta \sim (\theta_{12} - \theta^\nu_{12})/\theta_{13} \]
Tri-maximal Deviations

\[ \sin \theta_{12} = \frac{1}{\sqrt{3}}(1 + s), \quad \sin \theta_{23} = \frac{1}{\sqrt{2}}(1 + a), \quad \sin \theta_{13} = \frac{r}{\sqrt{2}} \]

\(s = \text{solar}\)  \(a = \text{atmospheric}\)  \(r = \text{reactor}\)

\[
\begin{pmatrix}
\sqrt{\frac{2}{3}}(1 - \frac{1}{2}s) & \frac{1}{\sqrt{3}}(1 + s) & \frac{1}{\sqrt{2}}re^{-i\delta} \\
-\frac{1}{\sqrt{6}}(1 + s - a + r \cos \delta) & \frac{1}{\sqrt{3}}(1 - \frac{1}{2}s - a - \frac{1}{2}r \cos \delta) & \frac{1}{\sqrt{2}}(1 + a) \\
\frac{1}{\sqrt{6}}(1 + s + a - r \cos \delta) & -\frac{1}{\sqrt{3}}(1 - \frac{1}{2}s + a + \frac{1}{2}r \cos \delta) & \frac{1}{\sqrt{2}}(1 - a)
\end{pmatrix}
\]

\[
\begin{pmatrix}
\sqrt{\frac{2}{3}} & - & - \\
-\frac{1}{\sqrt{6}} & - & - \\
\frac{1}{\sqrt{6}} & - & -
\end{pmatrix}
\]

\[
\begin{pmatrix}
- & \frac{1}{\sqrt{3}} & - \\
- & \frac{1}{\sqrt{3}} & - \\
- & - & \frac{1}{\sqrt{3}}
\end{pmatrix}
\]

Preserves SU or S only

- **Tri-maximal 1** \(s \approx 0, a \approx r \cos \delta\)
- **Tri-maximal 2** \(s \approx 0, a \approx -\frac{1}{2}r \cos \delta\)
Atmospheric Sum Rule

\[ \theta_{23} \approx 45^\circ + \sqrt{2} \theta_{13} \cos \delta \]

\[ s \approx 0, \ a \approx r \cos \delta \]

Ballett, SK, Luhn, Pascoli, Schmidt, 1308.4314

![Diagram showing LENF with MIND and parameter correlations.](image)
CP violation

\[ G \times H_{CP} \]

- **CP symmetry**
- **Flavour symmetry**

\[ G^l \times H_{CP}^l \]

preserved

\[ G^\nu \times H_{CP}^\nu \]

preserved

- **Charged Lepton Sector**
- **Neutrino Sector**

\[ m_l m_l^{\dagger} \]

\[ m_\nu \]

\[ \Delta(96), \Delta(6n^2), \Delta(3n^2) \]

allows many possible predictions for Dirac CP phase

**S\text{\textsubscript{4}}** and **A\text{\textsubscript{4}}** models with CP symmetry are constructed, all the possible cases following from the model-independent analysis can be realized. **Dirac CP phase is predicted to be trivial or maximal.**
Indirect Models

Family symmetry is fully broken in each sector by orthogonal alignments.

Family symmetry

Generators $S,T$

$S,T$ broken

Diagonal

$\begin{pmatrix}
\nu \\
0 \\
0 \\
\nu
\end{pmatrix}$

Charged Lepton Sector

$\begin{pmatrix}
\phi^l \\
\nu^R (L.\phi^l) H
\end{pmatrix}$

Neutrino Sector

$\begin{pmatrix}
\phi^\nu \\
\nu^R (L.\phi^\nu) H
\end{pmatrix}$
A4 Vacuum alignments

Symmetry preserving eigenvectors of group elements

Orthogonal alignments

Indirect CSD(n)

Direct

\[ \langle \phi \rangle = \begin{pmatrix} 0 \\ 0 \\ v \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ v \\ 0 \\ \pm v \end{pmatrix} \]

\[ \langle \phi \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} e \\ v \end{pmatrix} , \begin{pmatrix} 0 \\ e \end{pmatrix} \begin{pmatrix} v \\ -v \end{pmatrix} \]

\[ \langle \phi \rangle = \begin{pmatrix} 2v \\ -v \\ v \\ -v \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} e \\ e \end{pmatrix} \begin{pmatrix} v \\ v \end{pmatrix} \]

\[ \langle \phi \rangle = \begin{pmatrix} a \\ na \\ (n-2)a \end{pmatrix} \begin{pmatrix} 2v \\ -v \\ v \end{pmatrix} \]
Constrained sequential dominance

\[
\frac{1}{\Lambda} H_u (L \cdot \phi_{\text{atm}}) N_1^c + \frac{1}{\Lambda} H_u (L \cdot \phi_{\text{sol}}) N_2^c + \frac{1}{\Lambda} H_u (L \cdot \phi_{\text{dec}}) N_3^c
\]

\[
\langle \phi_{\text{atm}} \rangle \propto \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad \langle \phi_{\text{sol}} \rangle \propto \begin{pmatrix} 1 \\ n \\ n-2 \end{pmatrix} \quad \langle \phi_{\text{dec}} \rangle \propto \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}
\]

\[
Y^\nu = \begin{pmatrix} 0 & b & 0 \\ a & nb & 0 \\ a & (n-2)b & c \end{pmatrix} \quad M_{RR} \sim \text{diag}(M_1, M_2, M_3)
\]

\[
m_\nu(n) = m_a e^{i\alpha} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{i\beta} \begin{pmatrix} 1 & n & n-2 \\ n & n^2 & n(n-2) \\ n-2 & n(n-2) & (n-2)^2 \end{pmatrix} + m_c e^{i\gamma} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]

\[
\eta = \beta - \alpha \quad \xi = \gamma - \alpha
\]
5.2. Variation with $n$

Fig. 2 and Fig. 3 show the variation of physical parameters – mixing angles and neutrino masses – with vacuum alignment $n$. In Fig. 2 we see that the reactor angle increases with $n$ while the atmospheric and solar angles decrease. Examining the 3 ranges (dashed lines) we also see that $\theta_{23}$ is typically worst fit (only CSD(3) lies within the 1 bounds), and is also least sensitive to the choice of sub-subdominant phase $\theta$. The best fit of $m_1$ in Fig. 3 indicates it can vary greatly with $n$ for some phase choices. It is however unlikely that this can be used to constrain models in the near future, as the mass scale for CSD neutrinos is well below current experimental bounds of $\mu < 0.044$ eV [28].
A to Z of Flavour with Pati-Salam

\[ A_4 \times Z_5 \times SU(4)_C \times SU(2)_L \times SU(2)_R \]

Left-handed quarks and leptons triplets of \( A_4 \)

Right-handed quarks and leptons distinguished by \( Z_5 \)

\[ SU(2)_L \]

\[ SU(2)_R \]

\[ SU(4)_C \]

\[ A_4 \]

\[ Z_5 \]
Table 1: The basic Higgs, matter, flavon and messenger content of the model, where $e_i = \frac{e}{5}$ under $Z_5$. $R$ is a supersymmetric R-symmetry.

<table>
<thead>
<tr>
<th>name</th>
<th>field</th>
<th>$SU(4)_C \times SU(2)_L \times SU(2)_R$</th>
<th>$A_4$</th>
<th>$Z_5$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quarks and leptons</strong></td>
<td>$F$</td>
<td>$3$</td>
<td>1</td>
<td>$\alpha, \alpha^3, 1$</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>$F_{1,2,3}^c$</td>
<td>$1$</td>
<td>$\alpha^4, \alpha^2$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td><strong>PS Higgs</strong></td>
<td>$H^{c}$, $H^c$</td>
<td>$1$</td>
<td>$\alpha^3, \alpha$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td><strong>$A_4$ triplet flavons</strong></td>
<td>$\phi^u_{1,2}$</td>
<td>$3$</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$\phi^d_{1,2}$</td>
<td>$3$</td>
<td>$\alpha, \alpha^3, 1$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>Higgs bidoublets</strong></td>
<td>$h_3$</td>
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<td>1</td>
<td>$\alpha$</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$h_u$</td>
<td>$1''$</td>
<td>$\alpha^3, \alpha^4$</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>$h_d, h_{15}^d$</td>
<td>$1'$</td>
<td>$\alpha$</td>
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<tr>
<td></td>
<td>$h_{15}^u$</td>
<td>$1$</td>
<td>$\alpha^3, \alpha^2$</td>
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<tr>
<td><strong>Dynamical masses</strong></td>
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<td>$\alpha$</td>
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<td></td>
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<tr>
<td></td>
<td>$\Sigma_d, \Sigma_{15}^d$</td>
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<td>$\alpha^3, \alpha^2$</td>
<td>0</td>
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<tr>
<td><strong>Majoron</strong></td>
<td>$\xi$</td>
<td>$1$</td>
<td>$\alpha^4$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td><strong>Fermion Messengers</strong></td>
<td>$X_{F''_{1,3}}$</td>
<td>$1''$</td>
<td>$\alpha, \alpha^3$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$X'<em>{F</em>{1,3}}$</td>
<td>$1'$</td>
<td>$\alpha, \alpha^3$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$X_{F_{3i}}$</td>
<td>$1$</td>
<td>$\alpha^i$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$X_{\xi_i}$</td>
<td>$1$</td>
<td>$\alpha^i$</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
Yukawa operators

Third family renormalisable

\[ F.h_3F_3^c \rightarrow Q_3 H_u \upsilon_3^c + Q_3 H_d \upsilon_3^c + L_3 H_u \nu_3^c + L_3 H_d \epsilon_3^c. \]

First and second family involve flavons

\[ F.\phi_i^u h_u F_i^c \rightarrow Q.\langle \phi_i^u \rangle H_u \upsilon_i^c + L.\langle \phi_i^u \rangle H_u \nu_i^c, \]
\[ F.\phi_i^d h_d F_i^c \rightarrow Q.\langle \phi_i^d \rangle H_d \upsilon_i^c + L.\langle \phi_i^d \rangle H_d \epsilon_i^c, \]

CSD4 vacuum alignment

\[ \langle \phi_1^u \rangle = \frac{V_1^u}{\sqrt{2}} e^{i \pi/5} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \langle \phi_2^u \rangle = \frac{V_2^u}{\sqrt{21}} e^{i 4\pi/5} \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}, \]

Flavons control first two columns

\[ Y^u = Y^\nu = \begin{pmatrix} 0 & b & 0 \\ a & 4b & 0 \\ a & 2b & c \end{pmatrix}, \]

\[ Y^d \sim Y^e \sim \begin{pmatrix} y_d^0 & 0 & 0 \\ 0 & y_s^0 & 0 \\ 0 & 0 & y_b^0 \end{pmatrix}, \]
Yukawa operators

\[
\begin{align*}
F & \times X_{F_2} X_{F_1'} F_1^c \\
F & \times X_{F_1} X_{F_3'} F_2^c \\
F & \times F_3^c \\
\frac{\phi^d_1}{\Sigma_{15}} h_d F_1^c + \frac{\phi^d_2}{\Sigma_d} h^d_{15} F_2^c + h_3 F_3^c \\
\frac{\phi^u_1}{\Sigma_u} h_u F_1^c + \frac{\phi^u_2}{\Sigma_u} h_u F_2^c + h_3 F_3^c \\
\phi^u_1 & \times X_{F_1} X_{F_3'} F_1^c \\
\phi^u_2 & \times X_{F_3} X_{F_1'} F_2^c \\
h_3 & \times F_3^c
\end{align*}
\]

Down type quarks and charged leptons

up type quarks and neutrinos
Yukawa and Mass Matrices

\[ Y^u = Y^\nu = \begin{pmatrix} 0 & b e^{-i3\pi/5} & \epsilon c \\ ae^{-i3\pi/5} & 4be^{-i3\pi/5} & 0 \\ ae^{-i3\pi/5} & 2be^{-i3\pi/5} & c \end{pmatrix} \]

\[ M_R \approx \begin{pmatrix} M_1 e^{8i\pi/5} & 0 & 0 \\ 0 & M_2 e^{4i\pi/5} & 0 \\ 0 & 0 & M_3 \end{pmatrix} \]

\[ Y^d = \begin{pmatrix} y_d^0 e^{-i2\pi/5} & 0 & Ay_d^0 e^{-i2\pi/5} \\ By_d^0 e^{-i3\pi/5} & y_s^0 e^{-i2\pi/5} & Cy_d^0 e^{-i3\pi/5} \\ By_d^0 e^{-i3\pi/5} & 0 & y_b^0 + Cy_d^0 e^{-i3\pi/5} \end{pmatrix} \]

\[ Y^e = \begin{pmatrix} -(y_d^0/3)e^{-i2\pi/5} & 0 & Ay_d^0 e^{-i2\pi/5} \\ By_d^0 e^{-i3\pi/5} & -4.5y_s^0 e^{-i2\pi/5} & -3Cy_d^0 e^{-i3\pi/5} \\ By_d^0 e^{-i3\pi/5} & 0 & y_b^0 - 3Cy_d^0 e^{-i3\pi/5} \end{pmatrix} \]

SO(10)-like diagonal RHN masses  \( M_1 : M_2 : M_3 \sim m_u^2 : m_c^2 : m_t^2 \)

Physical neutrino masses in a normal hierarchy (CSD)  \( \theta_C \approx 1/4 \) or \( \theta_C \approx 14^\circ \)

All CP phases are fifth roots of unity due to \( Z_5 \)
CKM parameters

\[ \delta^q(\text{deg}) \]
\[ \theta^q_{12}(\text{deg}) \]
\[ \theta^q_{13}(\text{deg}) \]
\[ \theta^q_{23}(\text{deg}) \]

- \( A = 5, B = 3 \) with \( \epsilon = 0 \)
- \( a = 1.6 \times 10^{-5}, \ b = 0.8 \times 10^{-3}, \ c = 0.75, \)
- \( y^0_d = 0.9 \times 10^{-5}, \ y^0_s = 1.4 \times 10^{-4}, \ y^0_b = -0.9 \times 10^{-2} \)
- \( A = 9, B = 7 \) with \( \epsilon = -2.4 \times 10^{-3} \)
- \( C = 36 \)
See-Saw mass matrix

\[ m^\nu = -\nu_u^2 Y^\nu M_R^{-1} Y^\nu T \]

Neutrino mass matrix only depends on \( m_a, m_b, m_c \)

\[ m^\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{2i\eta} \begin{pmatrix} 1 & 4 & 2 \\ 4 & 16 & 8 \\ 2 & 8 & 4 \end{pmatrix} + m_c e^{2i\eta} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

CSD(4)

\[ m^\nu \sim \frac{\langle \phi_{\text{atm}} \rangle^T \langle \phi_{\text{atm}} \rangle}{\langle \xi \rangle^2} + \frac{\langle \phi_{\text{sol}} \rangle^T \langle \phi_{\text{sol}} \rangle}{\langle \xi \rangle} \]

\[ \eta = \frac{2\pi}{5} \]

\( \mathbb{Z}_5 \) invariant potential

\[ \left| \frac{\langle \xi \rangle^5}{\Lambda^3} - m^2 \right|^2 = 0 \]

\[ \langle \xi \rangle = |(\Lambda^3 m^2)^{1/5}| e^{-4i\pi/5} \]
Neutrino mass variables

$\Delta m^2_{31}(10^{-3} \text{eV}^2)$

$\Sigma m_i(\text{meV})$

$m_{ee}(\text{meV})$

$0.0 \ 0.1 \ 0.2 \ 0.3 \ 0.4 \ 0.5 \ m_1(\text{meV})$

$0.0 \ 0.1 \ 0.2 \ 0.3 \ 0.4 \ 0.5 \ m_1(\text{meV})$

$0.0 \ 0.1 \ 0.2 \ 0.3 \ 0.4 \ 0.5 \ m_1(\text{meV})$
to varying squared di parameters are predicted as a function of parameters responsible for the lightest neutrino mass and hence to showing results where we keep the parameters appearing in the same real parameters. Here we shall show results for the negative sign of the sign of values as in Fig. 6, with the colour coding and line styles as before. Figure 7: PMNS predictions of the model, resulting from Eqs. 101, 102, plotted as a function of the lightest neutrino mass. Figure 8: Majorana phases (in PDG convention defined below Eq. 78) as predicted by the model, for the other plots. Note that the model are displayed in Fig. 8, using the same parameter sets and colour coding as for the other plots. The parameter sets and colour coding as in Eq. 78, the entire PMNS matrix containing 3 mixing angles and 3 CP phases in Eq. 101, the entire PMNS matrix containing 3 mixing angles and 3 CP phases. As discussed previously (c.f. Eqs. 87, 91, 92) the e double beta decay parameter given by, resulting from Eqs. 101, 102, plotted as a function of the lightest neutrino mass. The model may be tested most readily by its prediction of maximal atmospheric mixing. It would be interesting to perform a measure for a normal neutrino mass hierarchy, so we have not plotted their predictions. As discussed previously (c.f. Eqs. 87, 91, 92) the e double beta decay parameter given by, resulting from Eqs. 101, 102, plotted as a function of the lightest neutrino mass. The model may be tested most readily by its prediction of maximal atmospheric mixing. It would be interesting to perform a measure for a normal neutrino mass hierarchy, so we have not plotted their predictions.
Conclusions

- GUT x Discrete Family Symmetry is very predictive (could originate from string theory)

- **Direct models:** Klein and T from Delta($6n^2$), predicts zero Dirac phase, quantised reactor angle, possible CP phase predictions

- **Semi-direct models:** partial surviving symmetry S or SU, smaller family symmetry, lepton mixing sum rules, possible CP phase predictions

- **Indirect models:** no surviving symmetry, allows $A_4$ broken by orthogonal CSD(n) vacuum alignment, very predictive scheme

- A to Z Pati-Salam predicts PMNS matrix entirely

- Discrete symmetries could originate from string theory