UNIVERSITY OF Southampton
School of Physics and Astronomy

invisibles
neutrinos, dark matter & dark energy physics

Discrete symmetries and models of flavour mixing

Steve King

DISCRETE 2014: Fourth Symposium on Prospects in the Physics of Discrete Symmetries

2-6 December 2014 King's College London, Strand Campus

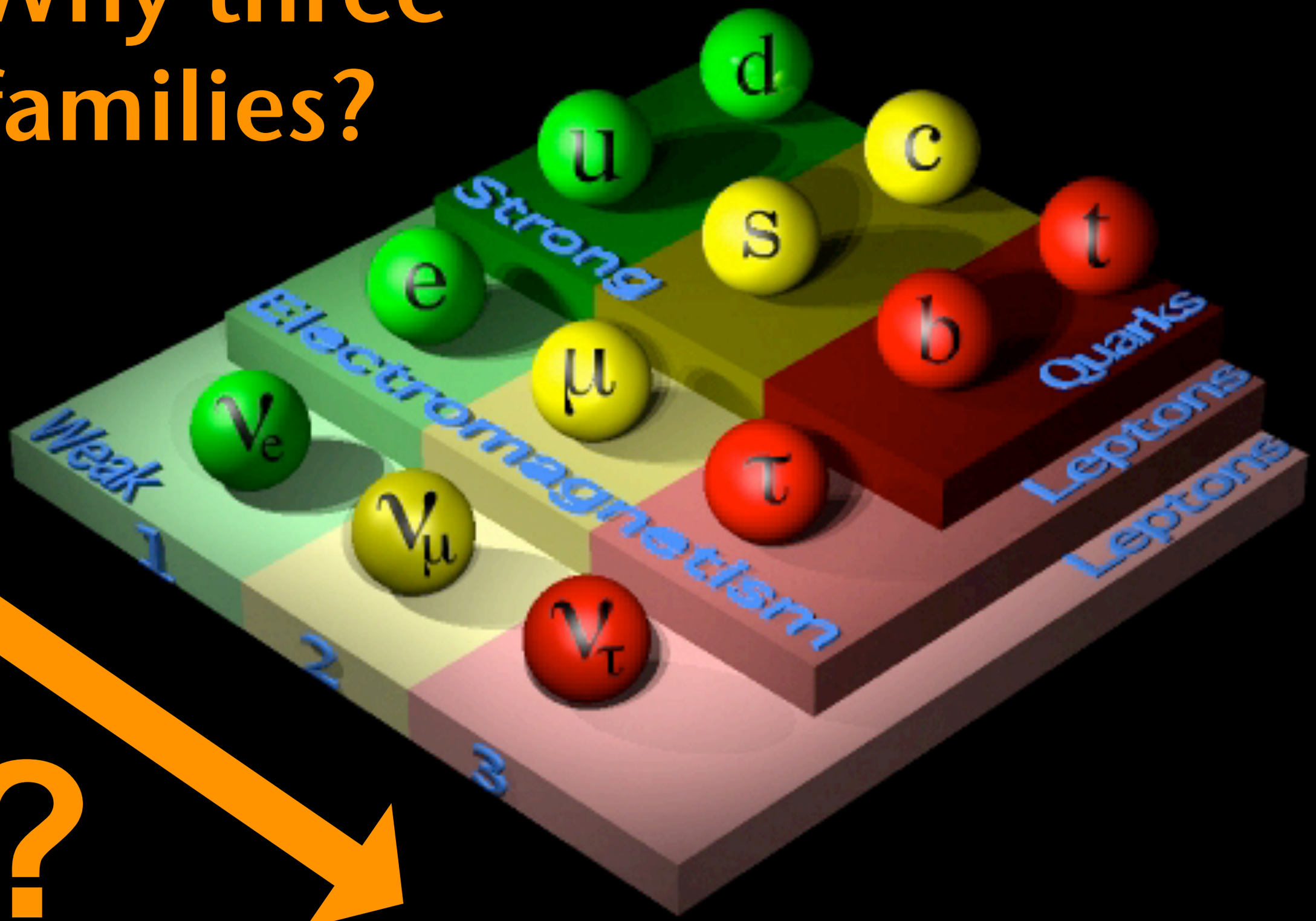
Standard Model Puzzles

The problem of flavour - the problem of the undetermined fermion masses and mixing angles (including neutrino masses and lepton mixing angles) together with the CP violating phases, in conjunction with the observed smallness of flavour changing neutral currents and very small strong CP violation.

The origin of mass after Higgs discovery - the origin of the weak scale, its stability under radiative corrections, and the TeV scale solution to the hierarchy problem or not (most urgent problem of LHC run 2).

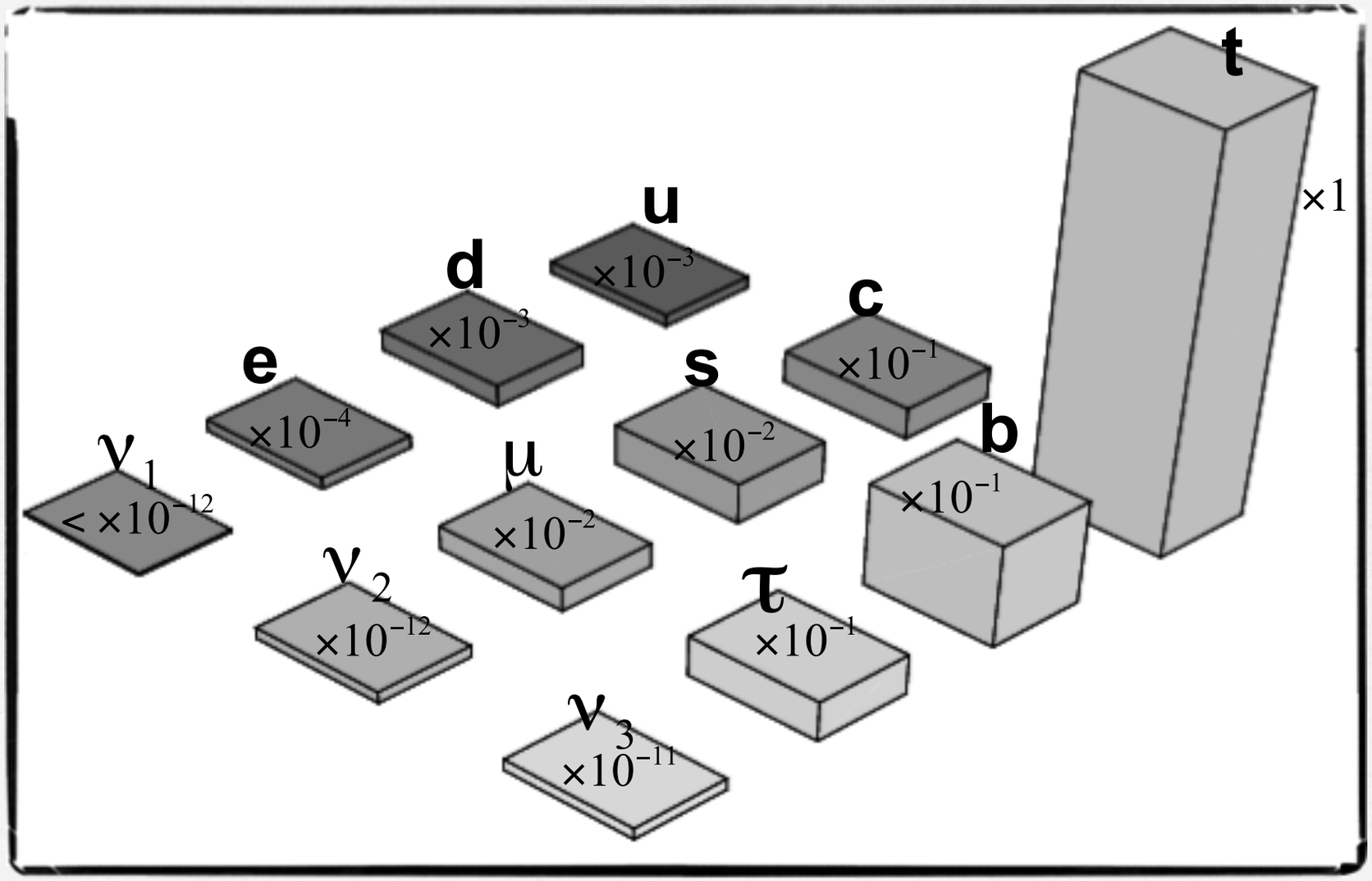
The quest for unification - the question of whether the three known forces of the standard model may be related into a grand unified theory, and whether such a theory could also include a unification with gravity.

Why three families?

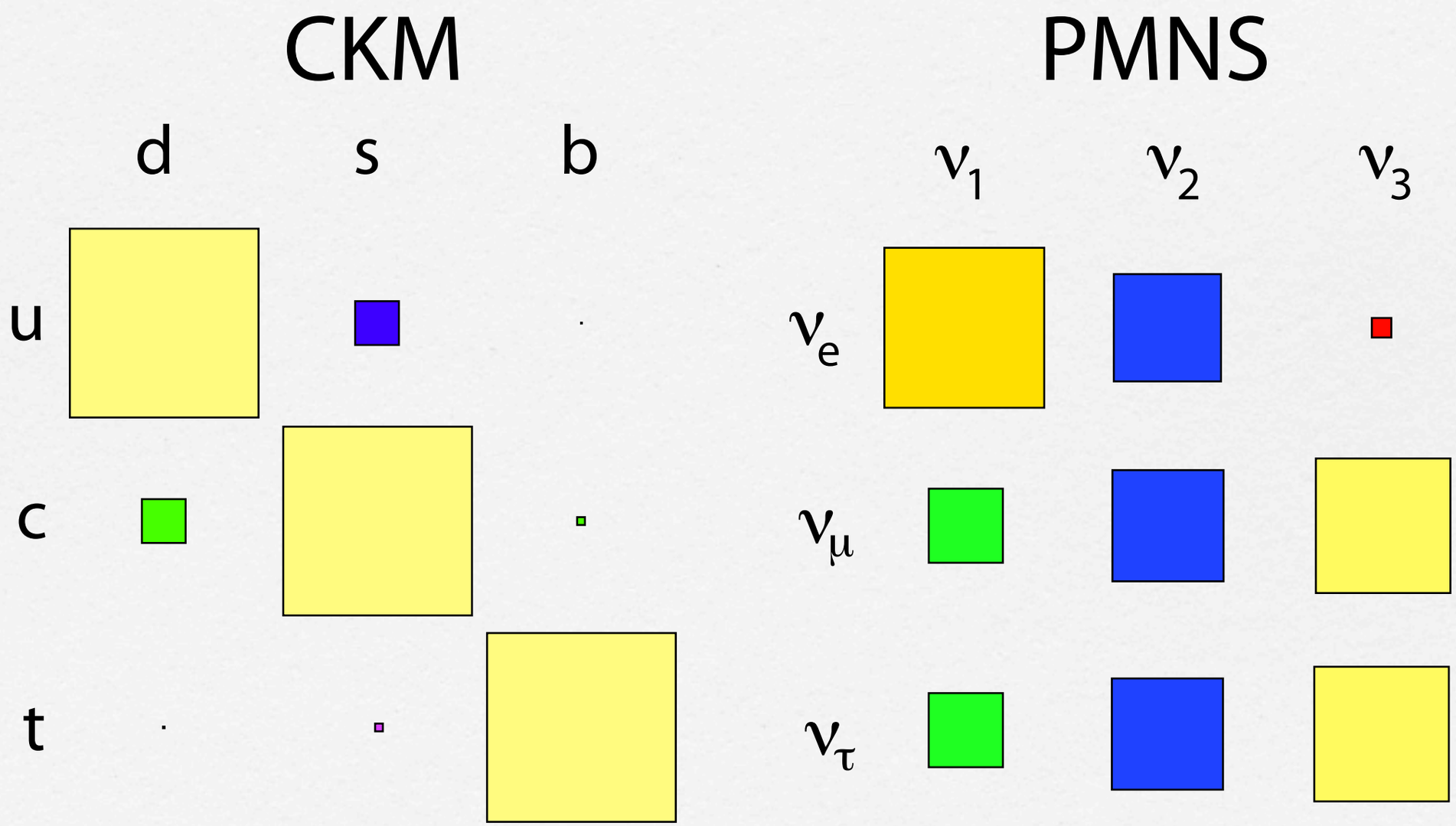


?

What is the origin of Quark and Lepton Masses?



What is the origin of Quark and Lepton Mixing?

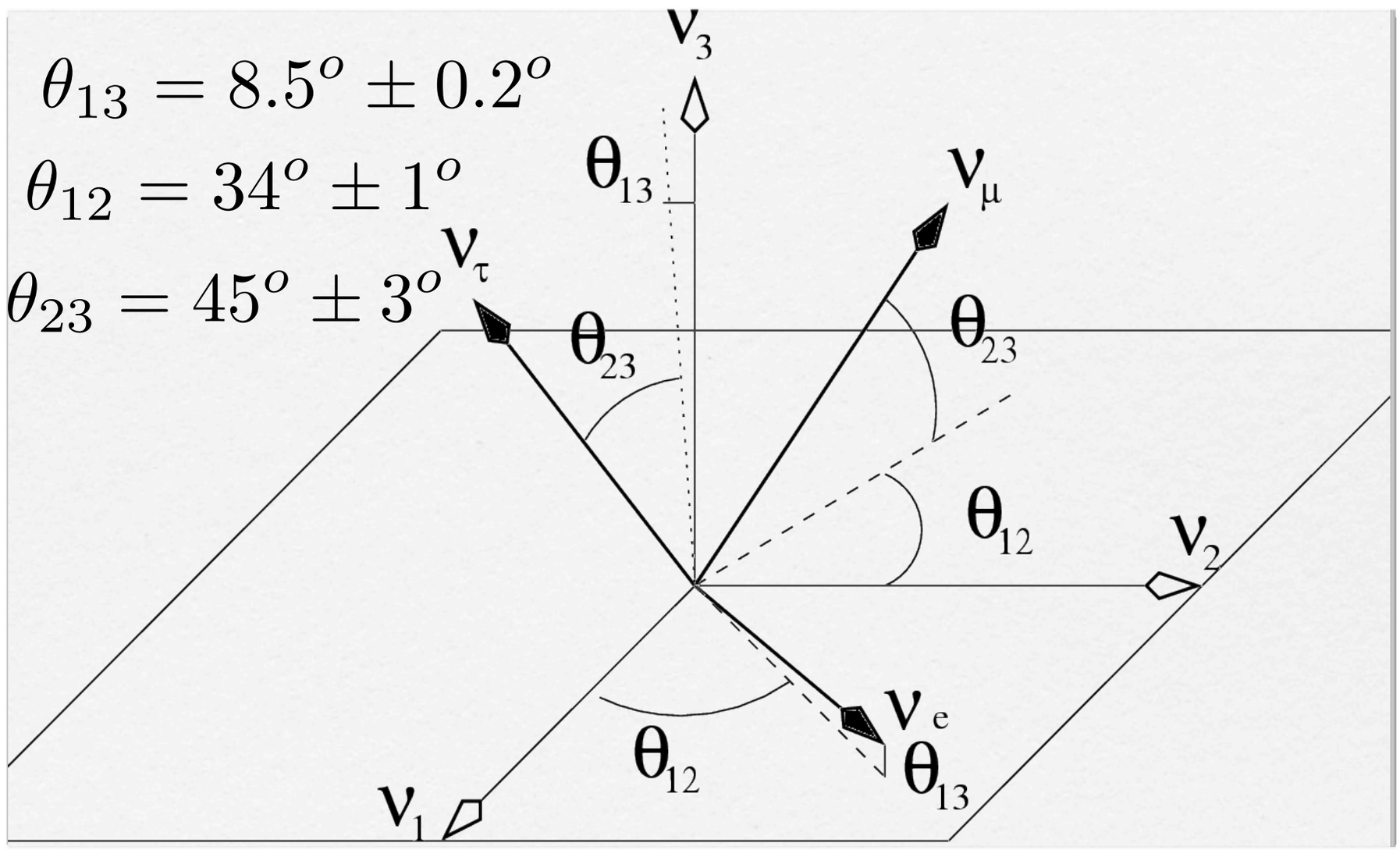


The Large Lepton Mixing Angles

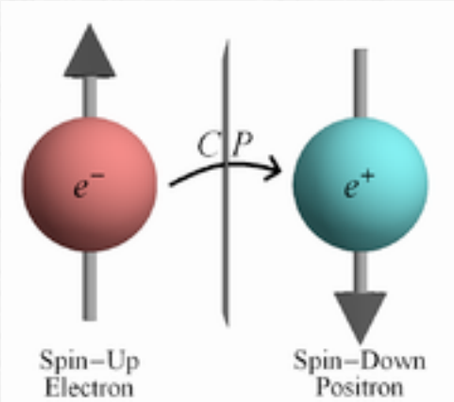
$$\theta_{13} = 8.5^o \pm 0.2^o$$

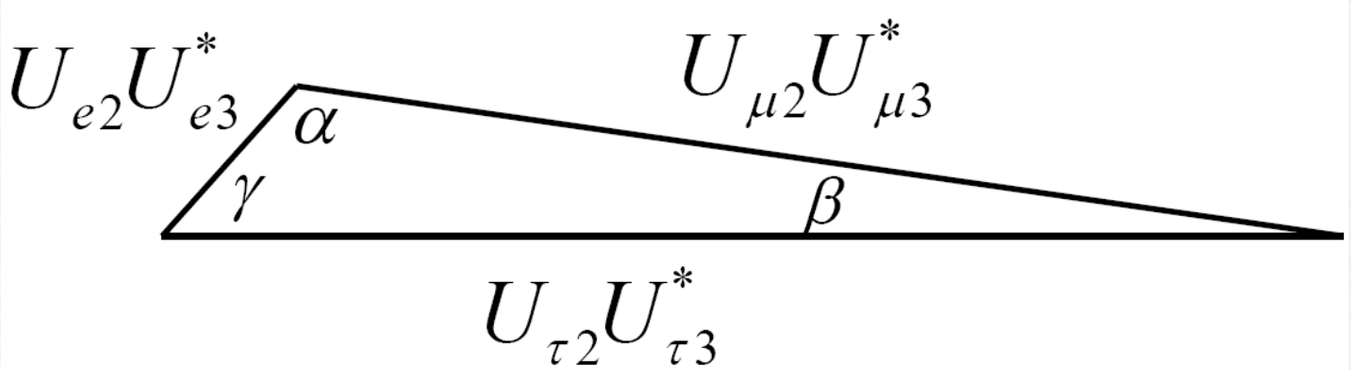
$$\theta_{12} = 34^o \pm 1^o$$

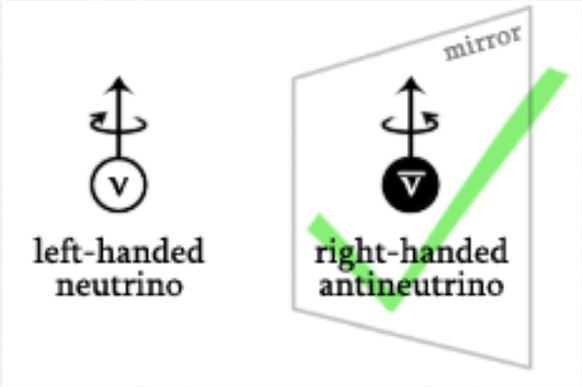
$$\theta_{23} = 45^o \pm 3^o$$



Is CP violated in the lepton sector?





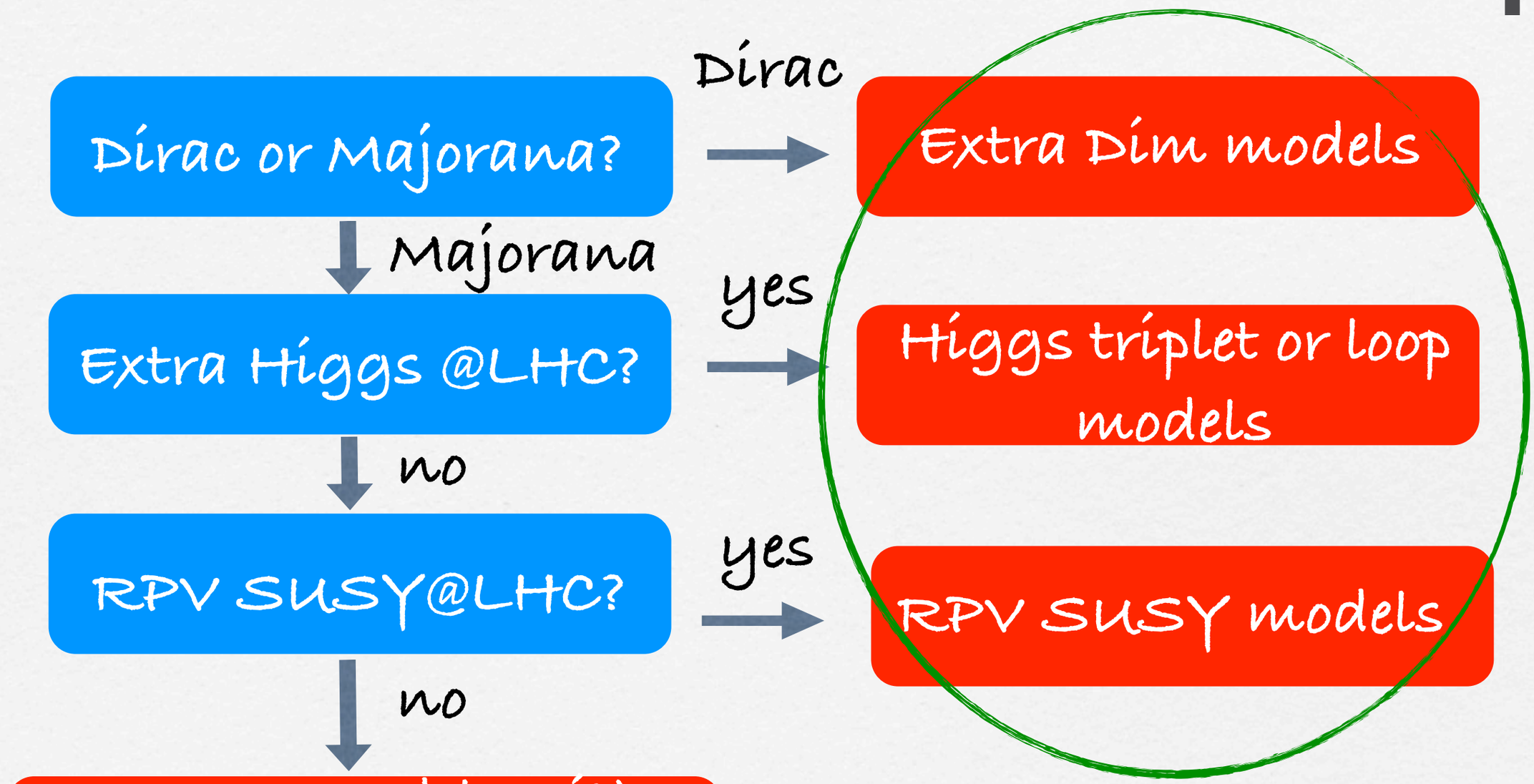


or

?!

- What is the Dirac CP violating phase (neutrino oscillations) ?
- What are the Majorana CP violating phases (NDBD)?

Neutrino mass model roadmap

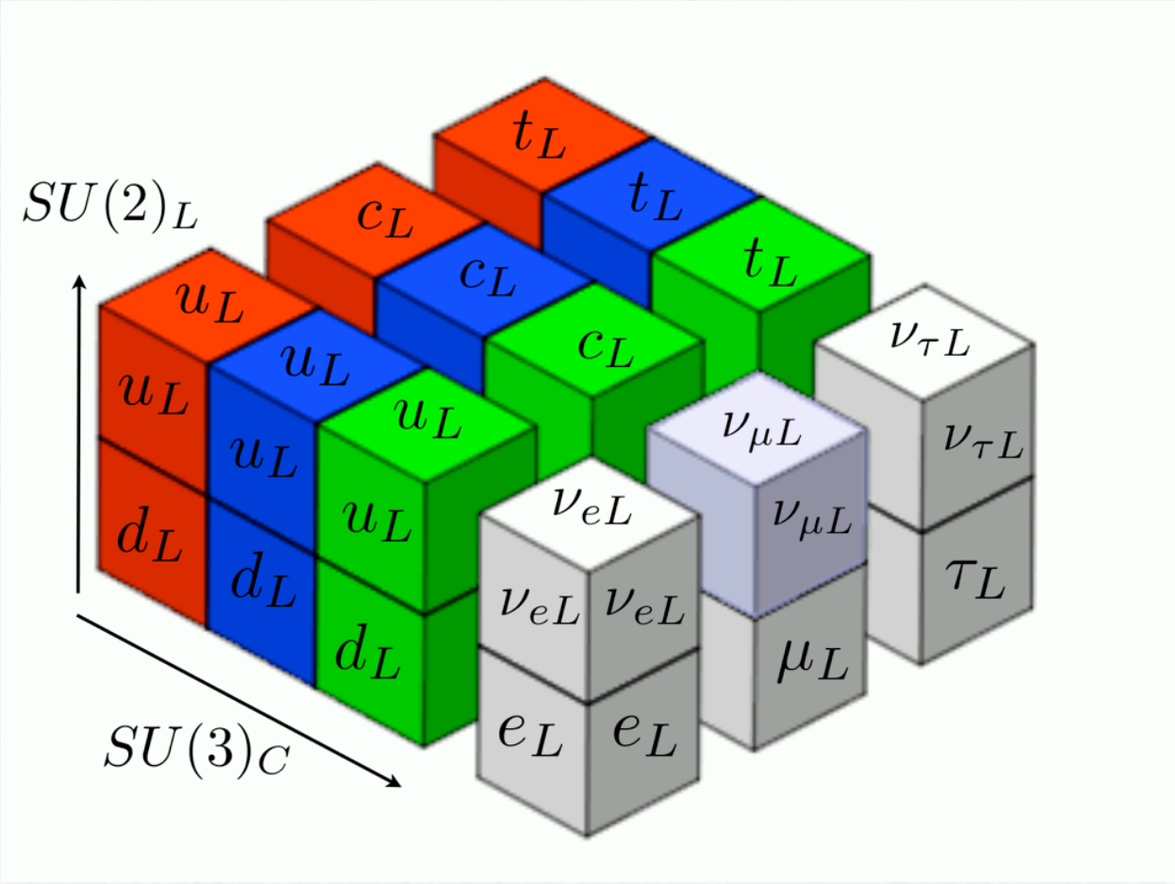


None of this has been seen at the LHC

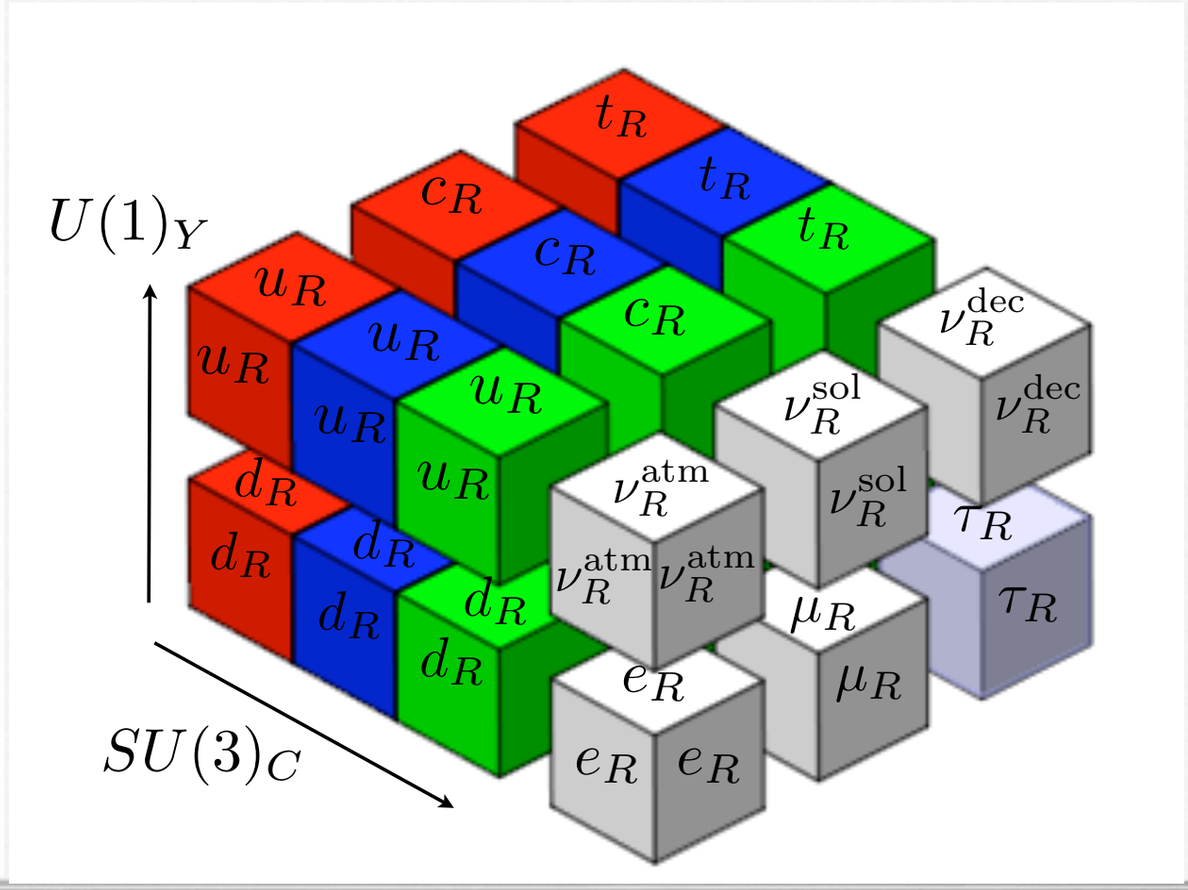
Standard Model of Quarks and Leptons with three right-handed neutrinos

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

Left-handed quarks and leptons (active neutrinos)



Right-handed quarks and leptons (sterile neutrinos)



Sequential dominance (SD)

hep-ph/9912492
hep-ph/0204360

Flavour basis

$$M_E = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \; M_R = \begin{pmatrix} M_{\rm atm} & 0 & 0 \\ 0 & M_{\rm sol} & 0 \\ 0 & 0 & [M_{\rm dec}] \end{pmatrix}$$

Dirac mass

$$m^D = \begin{pmatrix} m^D_{e,\rm atm} & m^D_{e,\rm sol} & [m^D_{e,\rm dec}] \\ m^D_{\mu,\rm atm} & m^D_{\mu,\rm sol} & [m^D_{\mu,\rm dec}] \\ m^D_{\tau,\rm atm} & m^D_{\tau,\rm sol} & [m^D_{\tau,\rm dec}] \end{pmatrix} \equiv \begin{pmatrix} m^D_{\rm atm} & m^D_{\rm sol} & [m^D_{\rm dec}] \end{pmatrix}$$

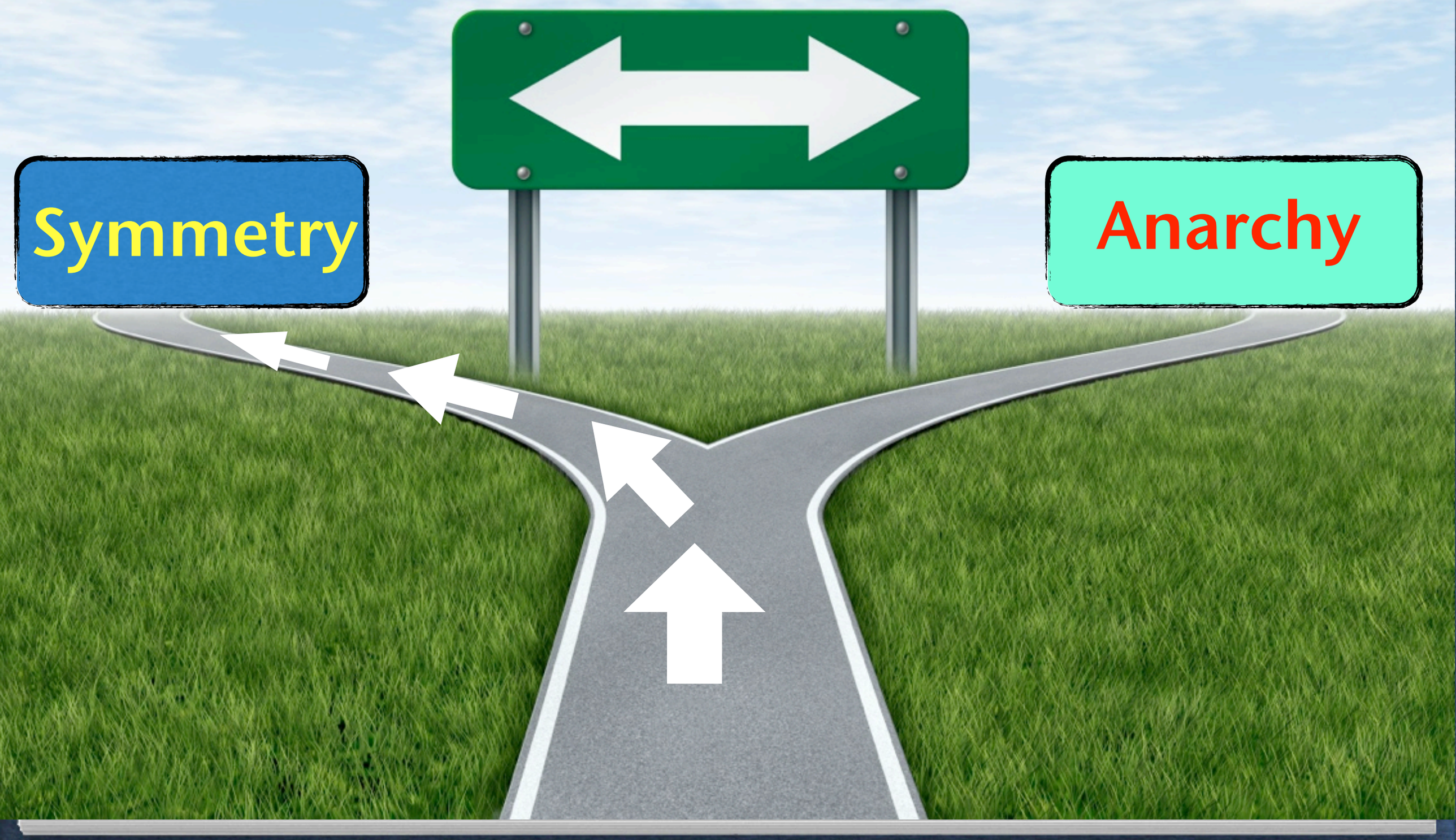
See-saw with
Sequential dominance

$$\frac{m^D_{\rm atm} {m^D_{\rm atm}}^T}{M_{\rm atm}} \gg \frac{m^D_{\rm sol} {m^D_{\rm sol}}^T}{M_{\rm sol}} \left[\gg \frac{m^D_{\rm dec} {m^D_{\rm dec}}^T}{M_{\rm dec}} \right]$$

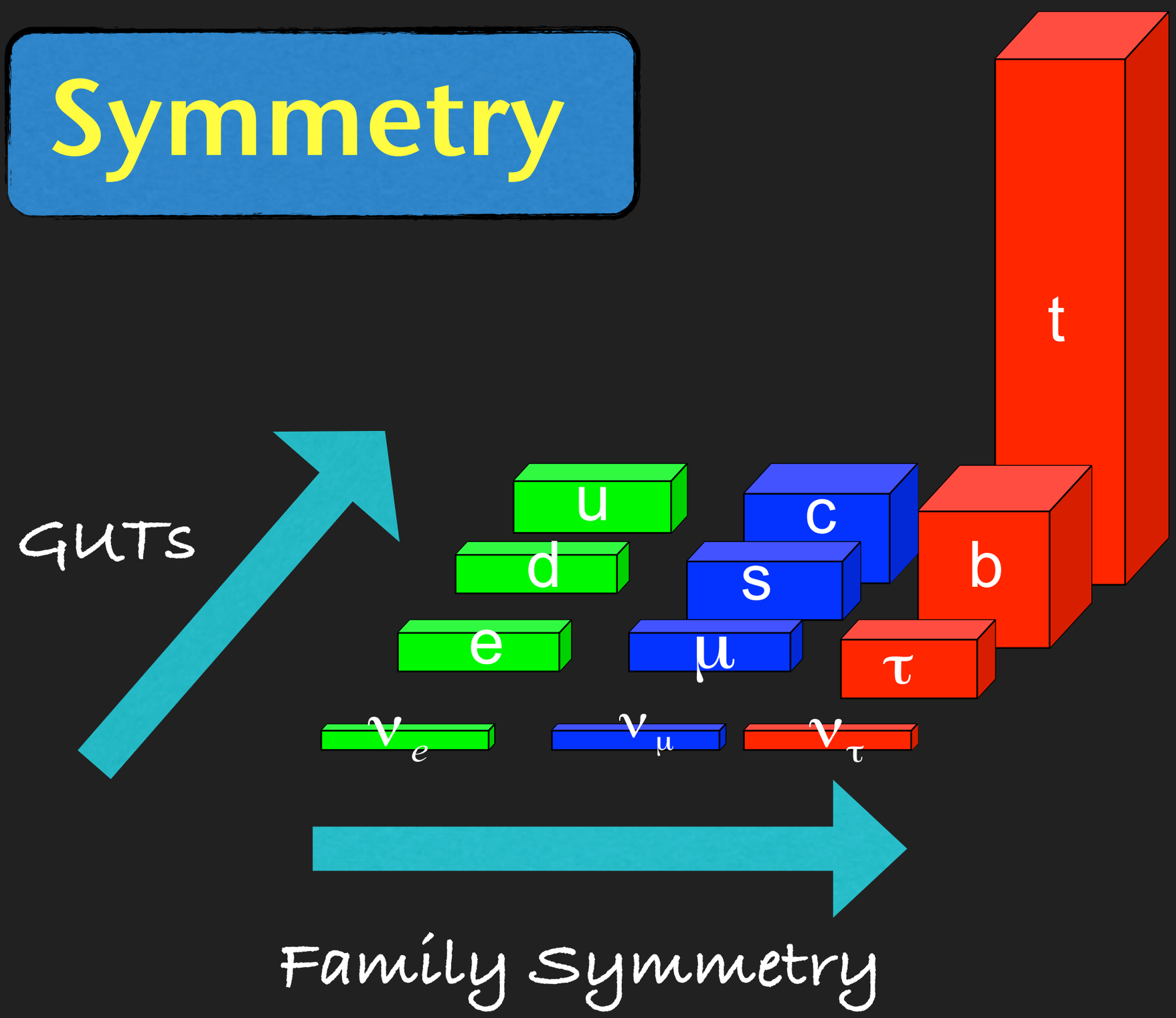
These theories predict
normal neutrino mass
hierarchy

$$m_3 \gg m_2 \; [\gg m_1]$$

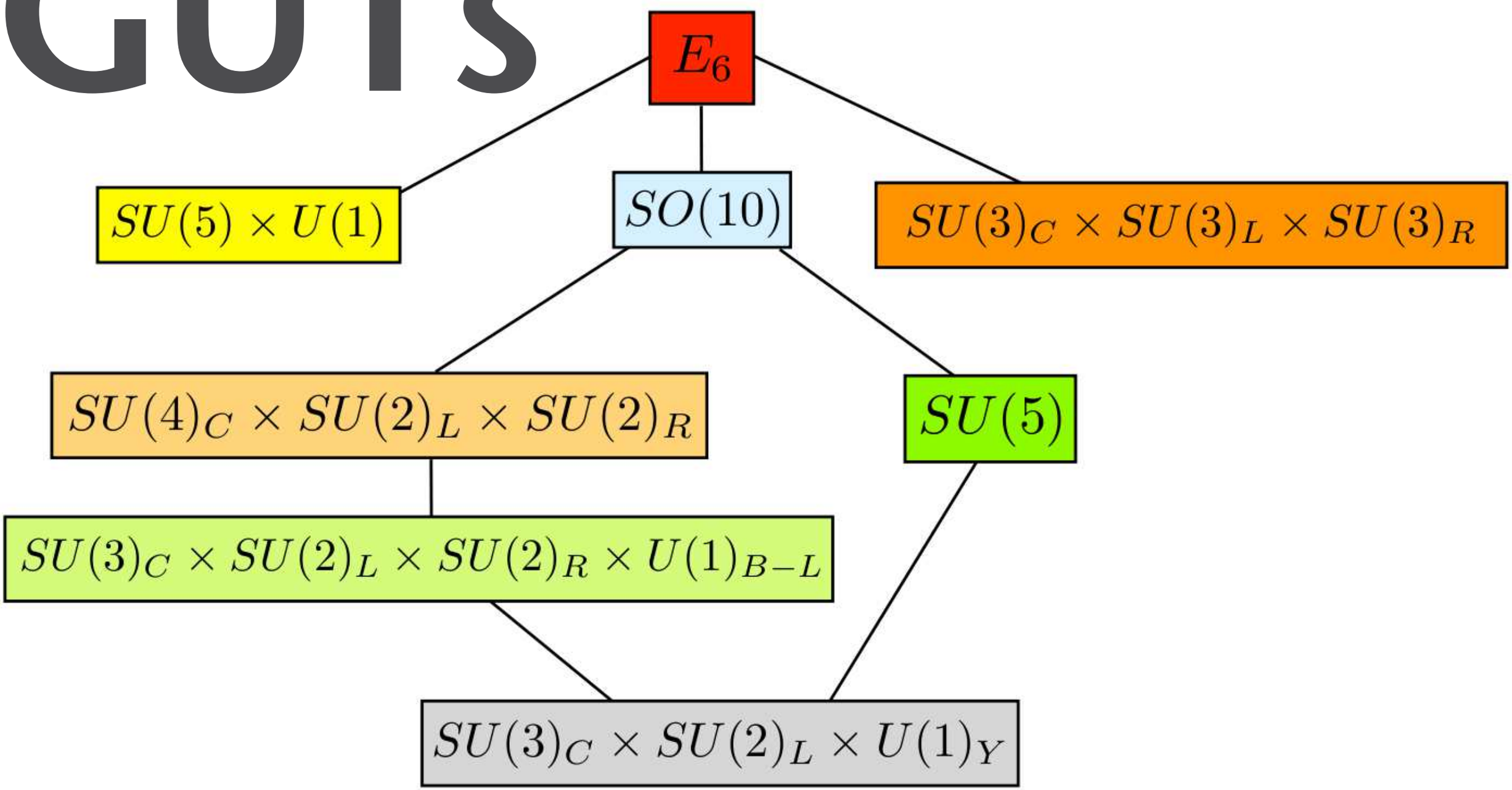
Towards a Theory of Flavour



Symmetry



GUTs

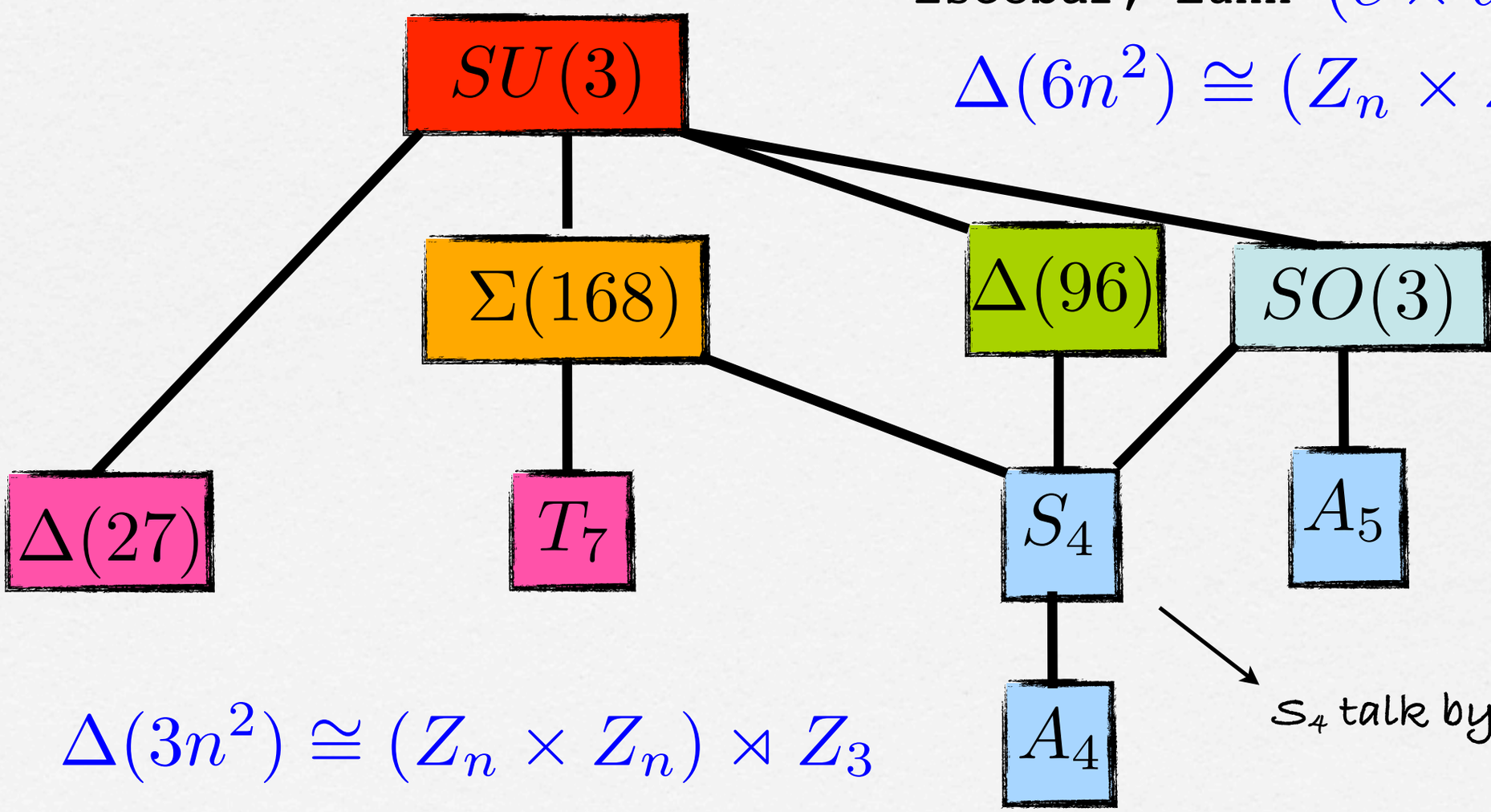

E_6
$SU(5) \times U(1)$
$SO(10)$
$SU(3)_C \times SU(3)_L \times SU(3)_R$
$SU(4)_C \times SU(2)_L \times SU(2)_R$
$SU(5)$
$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
$SU(3)_C \times SU(2)_L \times U(1)_Y$

Discrete Family Symmetry

For string theory origin see talks on F-theory - talks by G.Leontaris and A.Meadowcroft
also see M-theory talk by M.Romao

Escobar, Luhn $(c \times d) \rtimes (a, b)$

$$\Delta(6n^2) \cong (Z_n \times Z_n) \rtimes S_3$$



S_4 talk by M.Dimou

$$\Delta(3n^2) \cong (Z_n \times Z_n) \rtimes Z_3$$

Luhn, Nasri, Ramond

The Klein Symmetry

Felix Klein

Phase symmetry of diagonal charged lepton mass matrix

$$T^\dagger (M_e^\dagger M_e) T = M_e^\dagger M_e \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \qquad \omega = e^{2i\pi/n}$$

Symmetry of Majorana matrix depends on PMNS

$$m_\nu = S^T m_\nu S \qquad m_\nu = U^T m_\nu U$$

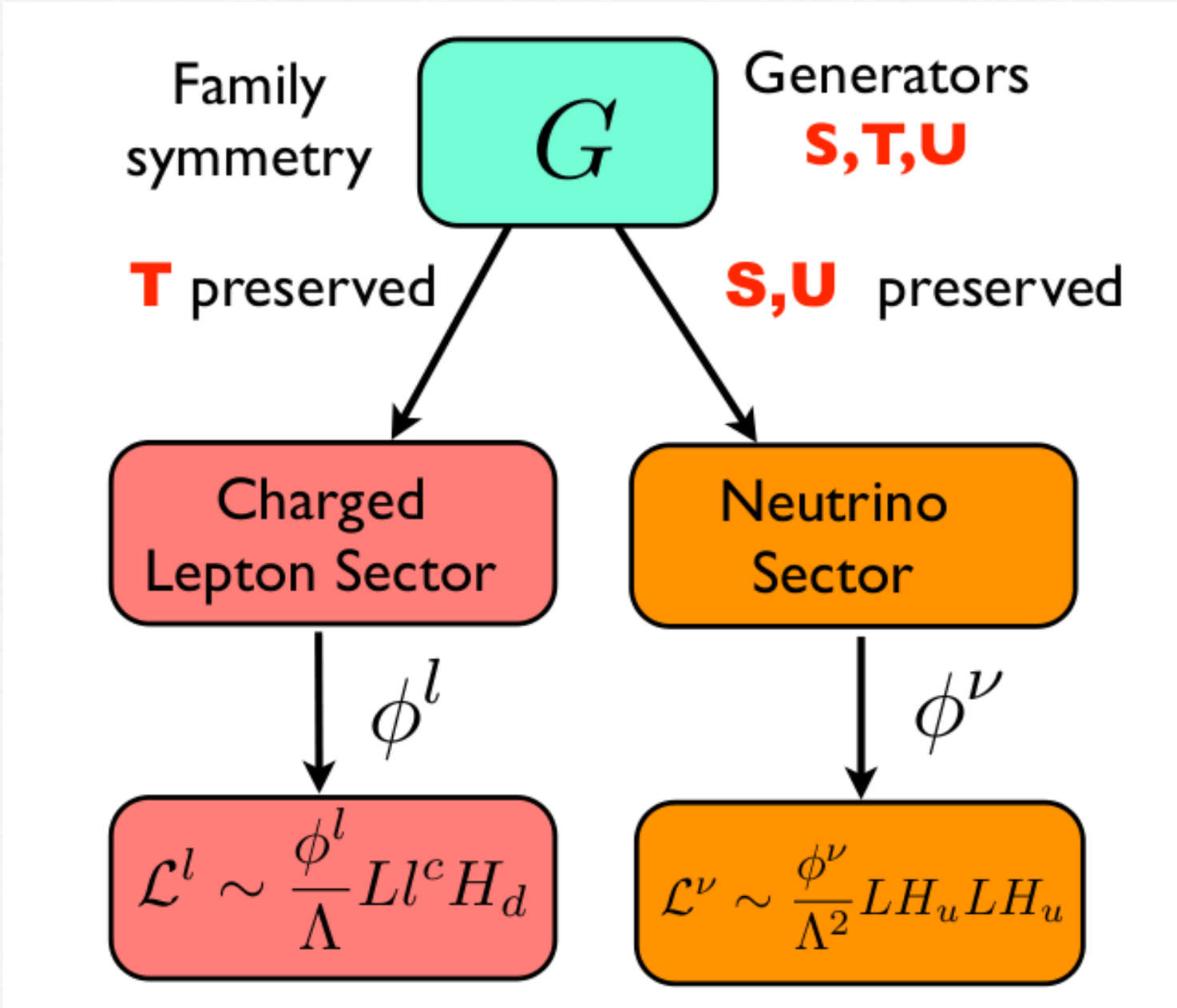
$$\left.\begin{aligned} S &= U^*_{\rm PMNS}\ {\rm diag}(+1,-1,-1)\ U^T_{\rm PMNS} \\ U &= U^*_{\rm PMNS}\ {\rm diag}(-1,+1,-1)\ U^T_{\rm PMNS} \\ SU &= U^*_{\rm PMNS}\ {\rm diag}(-1,-1,+1)\ U^T_{\rm PMNS} \end{aligned}\right\}$$

Klein Symmetry

$$\mathcal{K} = \{1, S, U, SU\}$$

$$Z_2 \times Z_2$$

Direct Models



Klein symmetry S,U and T are each identified as subgroups of some family symmetry

$$\Delta(6n^2)$$

is the only viable symmetry class

Holthausen,Lim, Lindner; SK,Neder,Stuart; Lavoura,Ludl; Fonseca,Grimus

SK, Neder, Stuart 1305.3200

$\Delta(6n^2)$ for leptons

Talk by Neder

PMNS

$$V = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos(\vartheta) & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\sin(\vartheta) \\ -\sqrt{\frac{2}{3}}\sin\left(\frac{\pi}{6}+\vartheta\right) & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\cos\left(\frac{\pi}{6}+\vartheta\right) \\ \sqrt{\frac{2}{3}}\sin\left(\frac{\pi}{6}-\vartheta\right) & -\frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\cos\left(\frac{\pi}{6}-\vartheta\right) \end{pmatrix}$$

$$\vartheta = \pi\gamma'/n$$

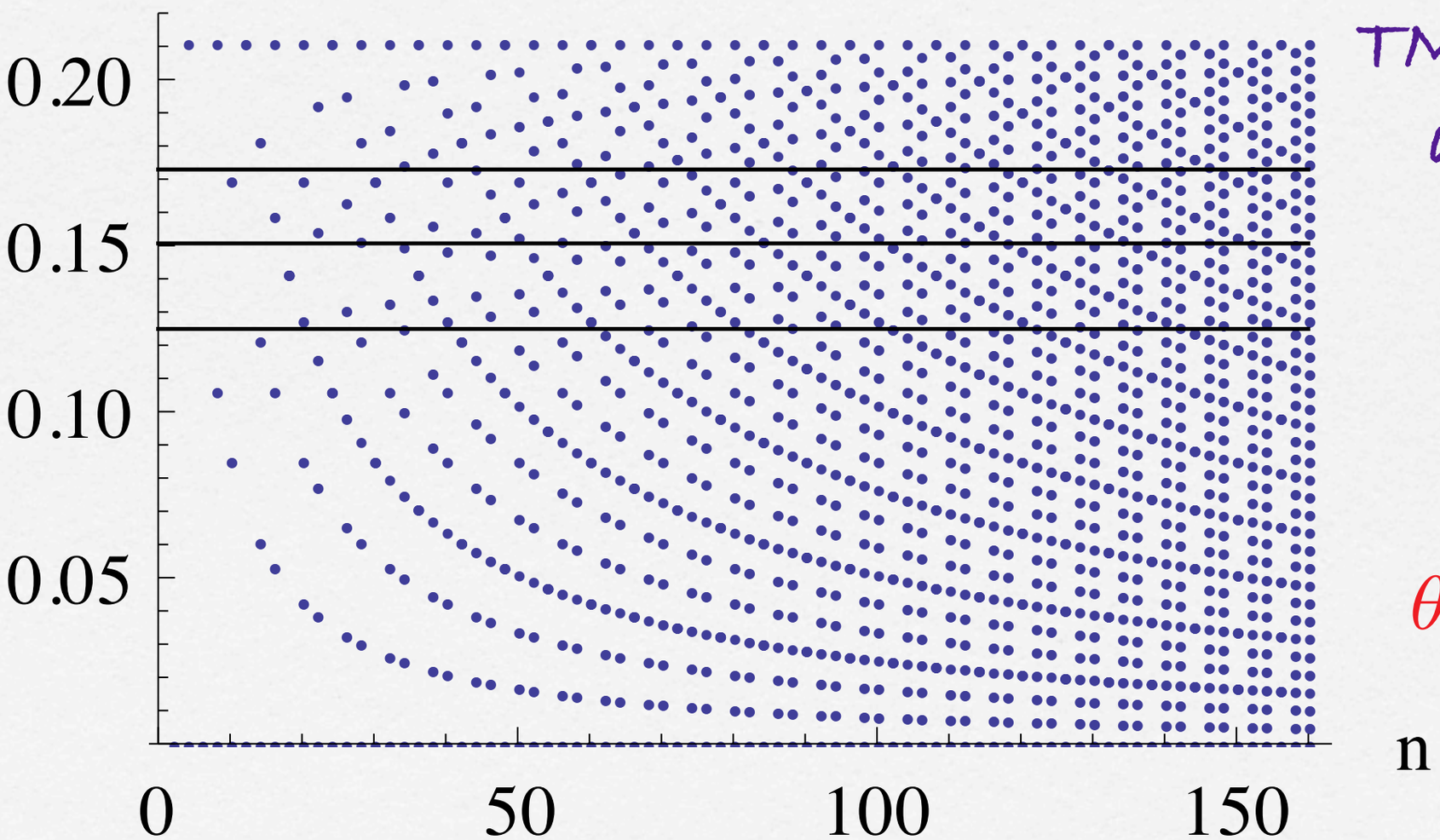
$$\gamma' = 1, \ldots, n/2.$$

$$T = a$$

$$S = c^{\frac{n}{2}}$$

$$U = abc^{\gamma}$$

$|V_{13}|$



TM2 mixing, reactor angle quantised

Predictions:

$$\delta = 0, \pi$$

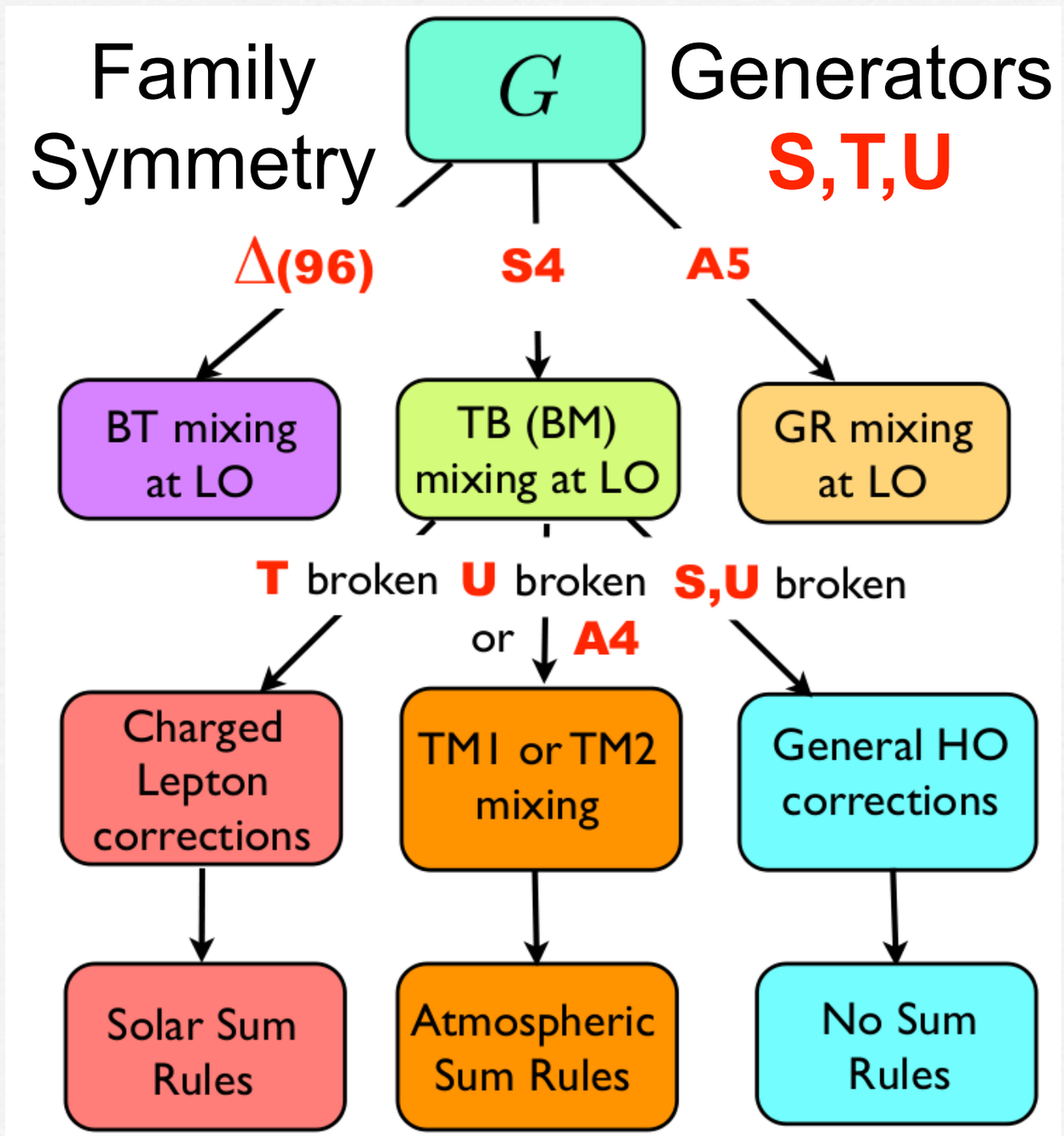
no CP violation in oscillations

$$\theta_{23} = 45^\circ \mp \theta_{13}/\sqrt{2}.$$

Atmospheric sum rule

Semi-Direct Models

SK,Luhn 1301.1340; SK,Merle, Morisi, Shimizu, Tanimoto, 1402.4271



Klein symmetry and T are partly preserved as subgroups of some family symmetry

$$\begin{aligned} \mathrm{TB} &= \text{tri-bimaximal} \\ \mathrm{BM} &= \text{bimaximal} \\ \mathrm{GR} &= \text{golden ratio} \\ \mathrm{BT} &= \text{bi-trimaximal} \\ \mathrm{TM} &= \text{trimaximal} \end{aligned}$$

	θ_{13}	θ_{23}	θ_{12}
TB	$0°$	$45°$	$35.3°$
BM	$0°$	$45°$	$45°$
GR	$0°$	$45°$	$31.7°$
BT	$12.2°$	$36.2°$	$36.2°$
TM	$\neq 0°$	$\neq 45°$	$35.3°$

Neutrino Mixing at LO

$$\theta_{13} = 0^o \quad \theta_{23} = 45^o$$

- **Bimaximal**

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} P \quad \theta_{12} = 45^o$$

V. Barger, S. Pakvasa, T. Weiler and K. Whisnant, hep-ph/9806387;
A. Baltz, A.S. Goldhaber, M. Goldhaber, hep-ph/9806540;
Y. Nomura and T. Yanagida, hep-ph/9807325;
H. Georgi and S. Glashow, hep-ph/9808293;
S. Davidson and S.F. King, hep-ph/9808296

- **Tri-bimaximal**

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P \quad \theta_{12} = 35.26^o$$

P.F. Harrison, D.H. Perkins and W.G. Scott, hep-ph/0202074.

- **Golden ratio**

$$U_{GR} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} P \quad \theta_{12} = 31.7^o$$

Y. Kajiyama, M. Raidal and A. Strumia, [arXiv:0705.4559];
L. L. Everett and A. J. Stuart, [arXiv:0812.1057]

$$\theta_{12}^{\nu} = \frac{\pi}{5} \quad \text{(GR2)} \qquad \text{and} \qquad \theta_{12}^{\nu} = \frac{\pi}{6} \quad \text{(HEX)}$$

$$\tan\theta_{12} = \frac{1}{\phi} \qquad \phi = \frac{1+\sqrt{5}}{2}$$

W. Rodejohann, [arXiv:0810.5239]

T breaking

hep-ph/0506297
hep-ph/0508031
hep-ph/0702286

$$U_{PMNS} = U_e U_\nu^\dagger$$

Charged lepton correction

Simple mixing for neutrinos (Klein sym.)

$$U = U_{12}^{e\dagger} U_{23}^{e\dagger} R_{23}^\nu R_{12}^\nu$$

Assuming $\theta_{13}^\nu = \theta_{13}^e = 0$

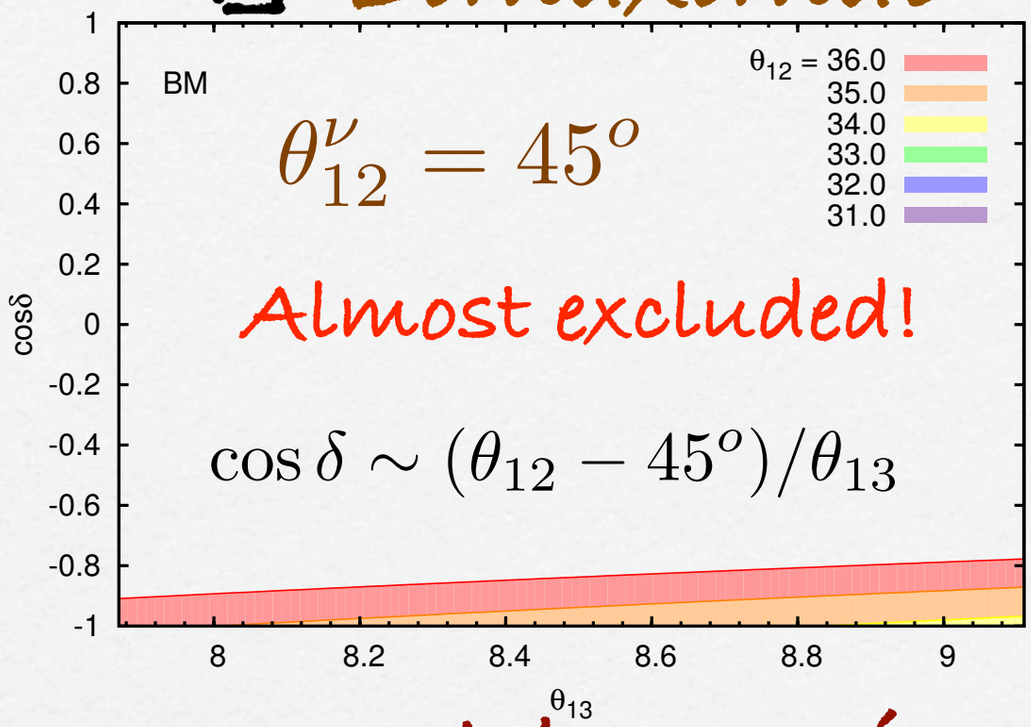
Ballett,SK,Luhn,Pascoli, Schmidt 1410.7573

Solar Sum Rule

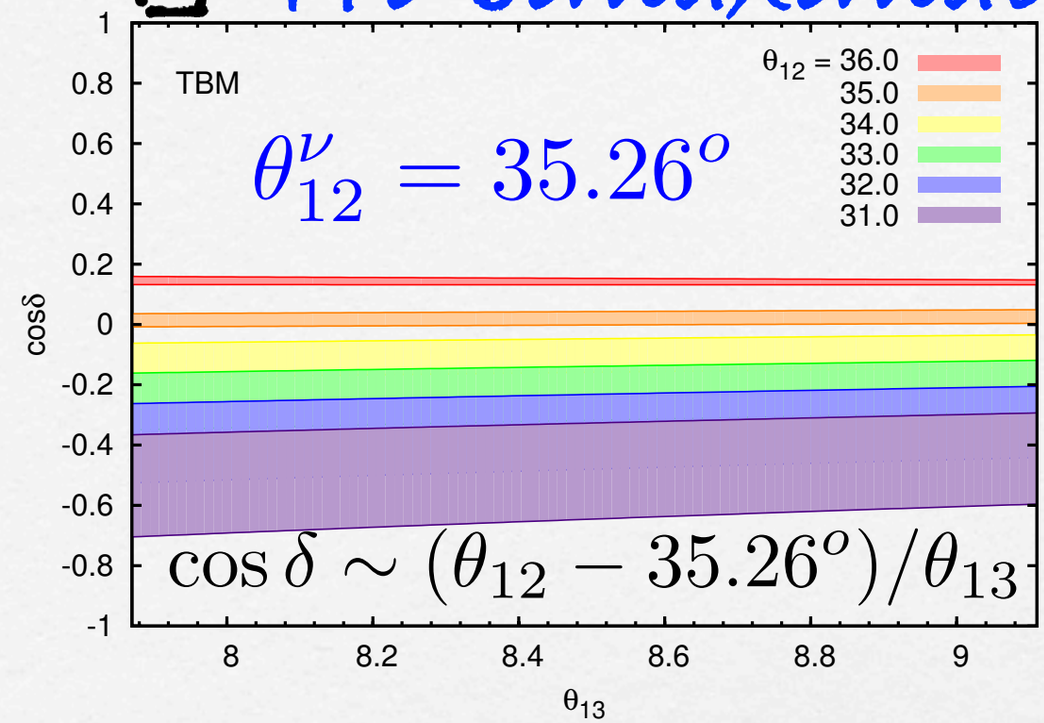
$$\frac{|U_{\tau 1}|}{|U_{\tau 2}|} = \frac{|s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta}|}{|s_{23}c_{12} + s_{13}c_{23}s_{12}e^{i\delta}|} = t_{12}^\nu$$

RHS only depends on θ_{12}^ν

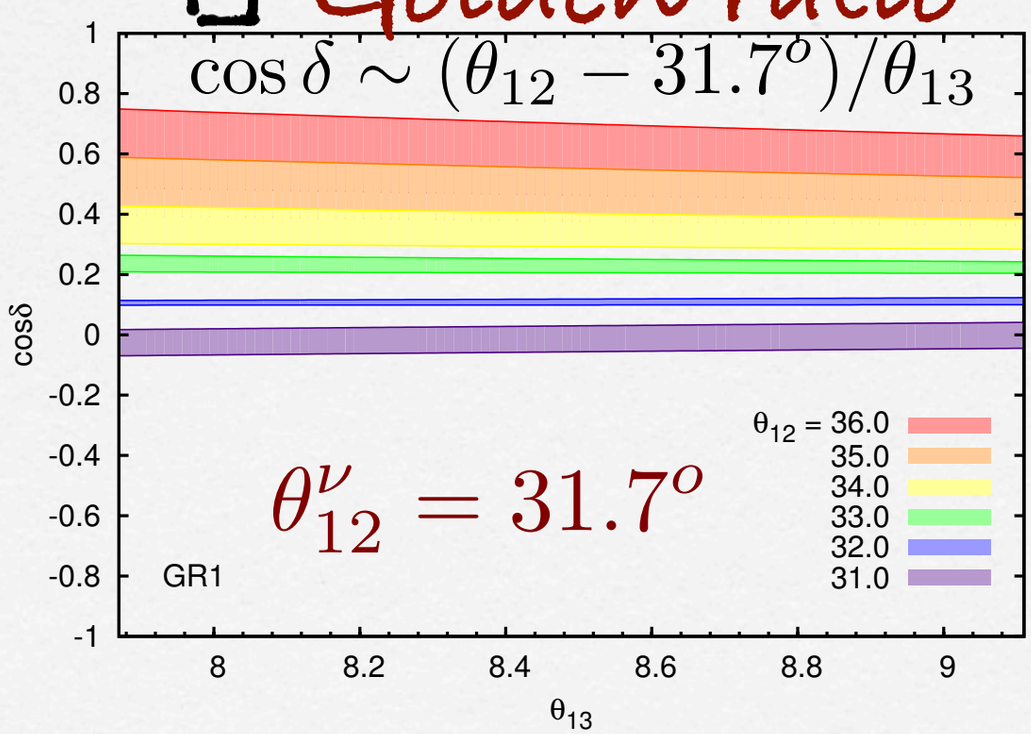
Bimaximal



Tri-bimaximal



Golden ratio



Solar Sum Rule

$$\cos\delta = \frac{t_{23}s_{12}^2 + s_{13}^2 c_{12}^2/t_{23} - s_{12}^{\nu 2}(t_{23} + s_{13}^2/t_{23})}{\sin 2\theta_{12} s_{13}}.$$

Ballett,SK,Luhn,Pascoli,Schmidt
1410.7573

useful approximation

$$\cos\delta \sim (\theta_{12} - \theta_{12}^{\nu})/\theta_{13}$$

SK,0710.0530; Pakvasa,Rodejohann,Weiler,0711.0052

Tri-maximal Deviations

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}(1+s)\,, \qquad \sin\theta_{23} = \frac{1}{\sqrt{2}}(1+a)\,, \qquad \sin\theta_{13} = \frac{r}{\sqrt{2}}$$

s = solar a = atmospheric r = reactor

$$U \approx \begin{pmatrix} \sqrt{\frac{2}{3}}(1-\frac{1}{2}s) & \frac{1}{\sqrt{3}}(1+s) & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+s-a+r\cos\delta) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}s-a-\frac{1}{2}r\cos\delta) & \frac{1}{\sqrt{2}}(1+a) \\ \frac{1}{\sqrt{6}}(1+s+a-r\cos\delta) & -\frac{1}{\sqrt{3}}(1-\frac{1}{2}s+a+\frac{1}{2}r\cos\delta) & \frac{1}{\sqrt{2}}(1-a) \end{pmatrix}$$

$$U_{\rm TM1} \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ \frac{1}{\sqrt{6}} & - & - \end{pmatrix} \qquad U_{\rm TM2} \approx \begin{pmatrix} - & \frac{1}{\sqrt{3}} & - \\ - & \frac{1}{\sqrt{3}} & - \\ - & -\frac{1}{\sqrt{3}} & - \end{pmatrix}$$

Preserves SU or S only

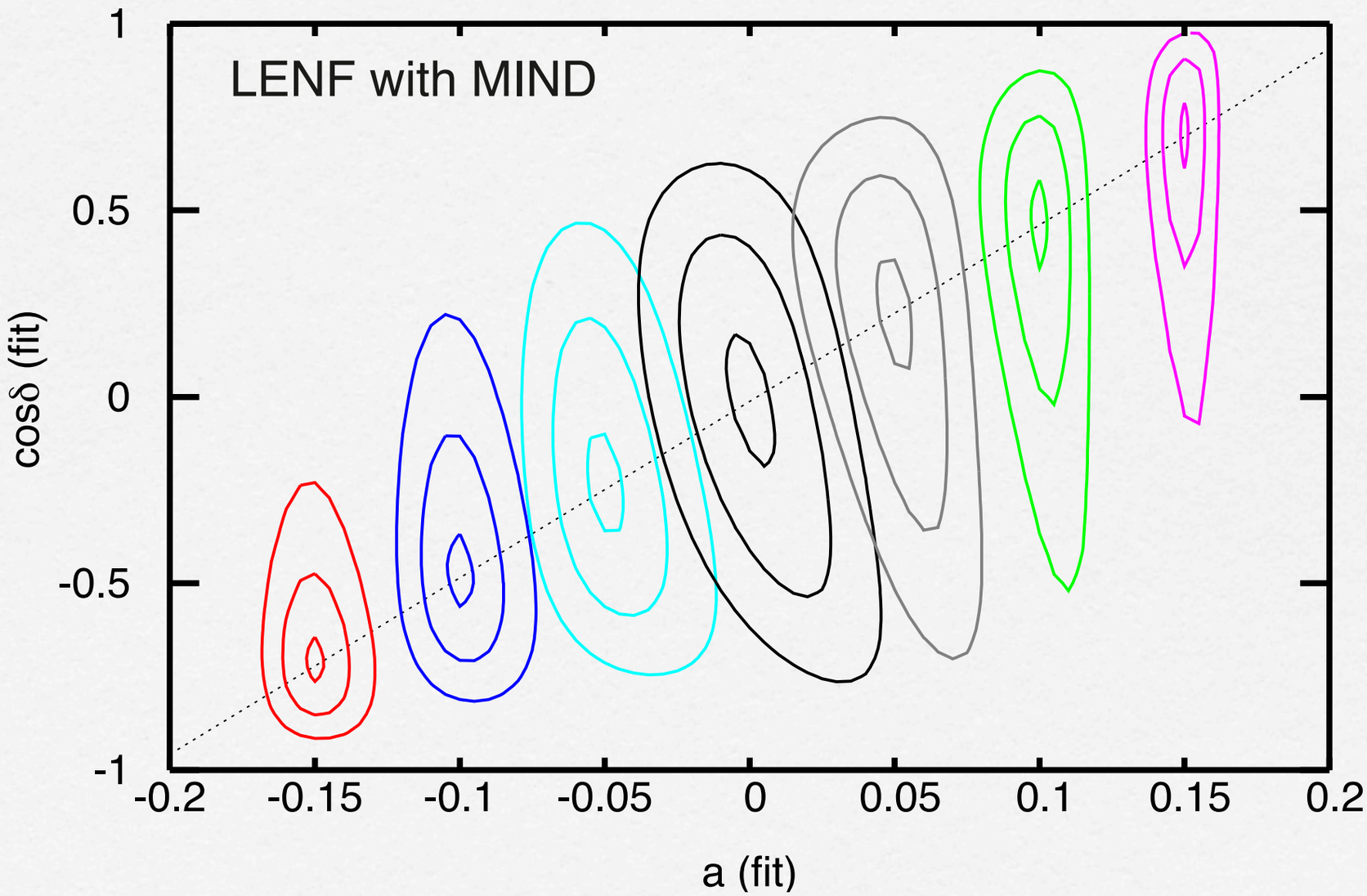
- Tri-maximal 1 $s \approx 0,\ a \approx r\cos\delta$
- Tri-maximal 2 $s \approx 0,\ a \approx -\frac{1}{2}r\cos\delta$

Atmospheric Sum Rule

$$\theta_{23} \approx 45^o + \sqrt{2}\theta_{13}\cos\delta$$

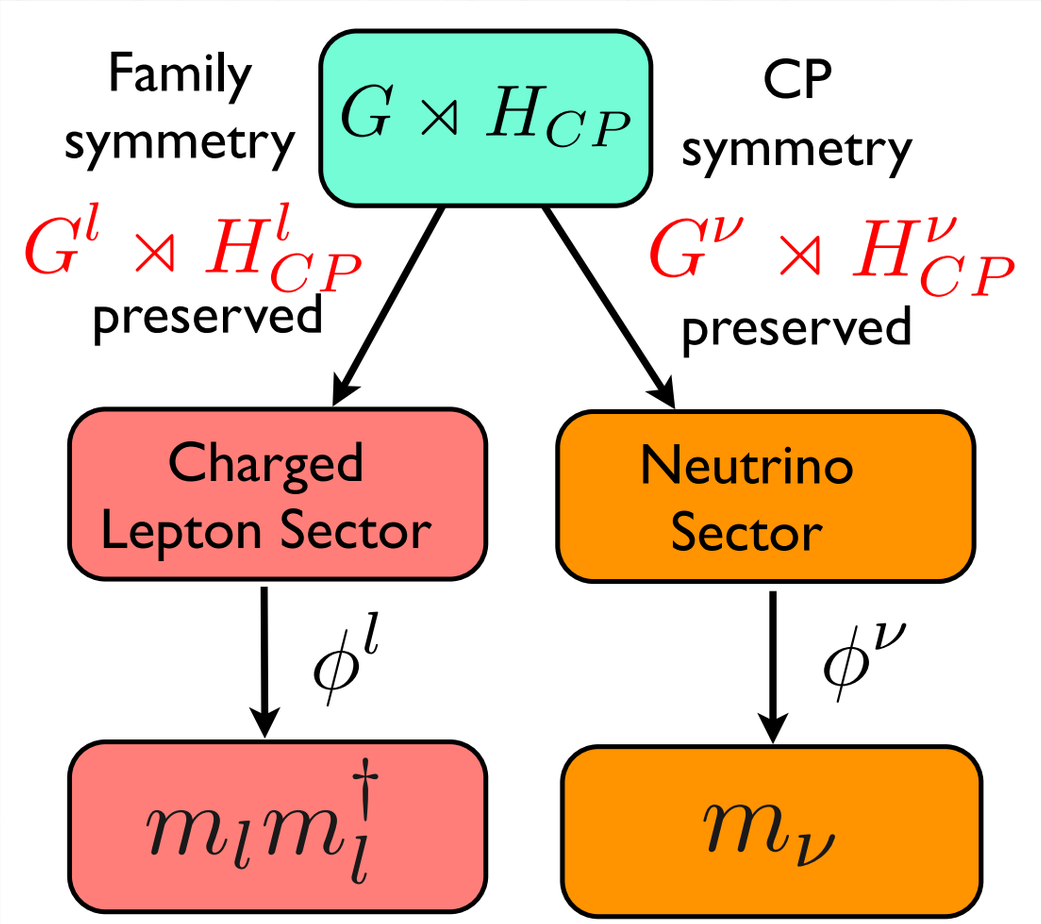
$$s \approx 0, \;\; a \approx r\cos\delta$$

Ballett,SK,Luhn, Pascoli,Schmidt, 1308.4314

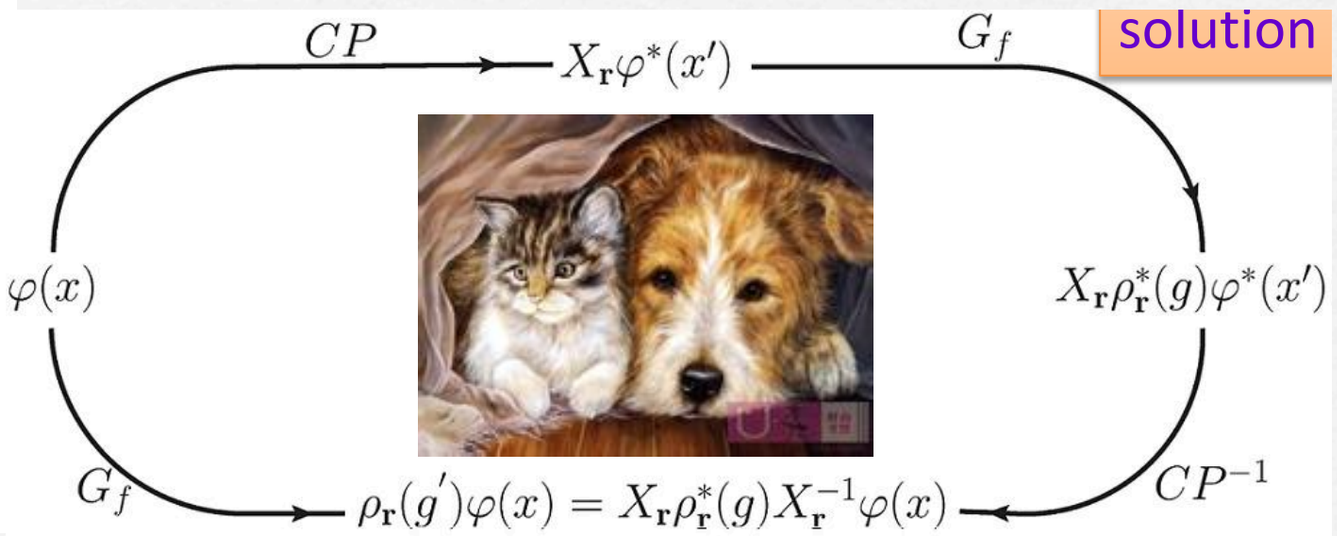


CP violation

Talks by Branco, Chen, Hagedorn, Neder, de Medeiros Varzielas



Feruglio,Hagedorn; Holthausen,Lindner,Schmidt; Ding,SK,Luhn,Stuart; Nishi,Xing; Hagedorn,Meroni,Molinaro;Ding,SK,Neder, Branco, de Medeiros Varzielas...

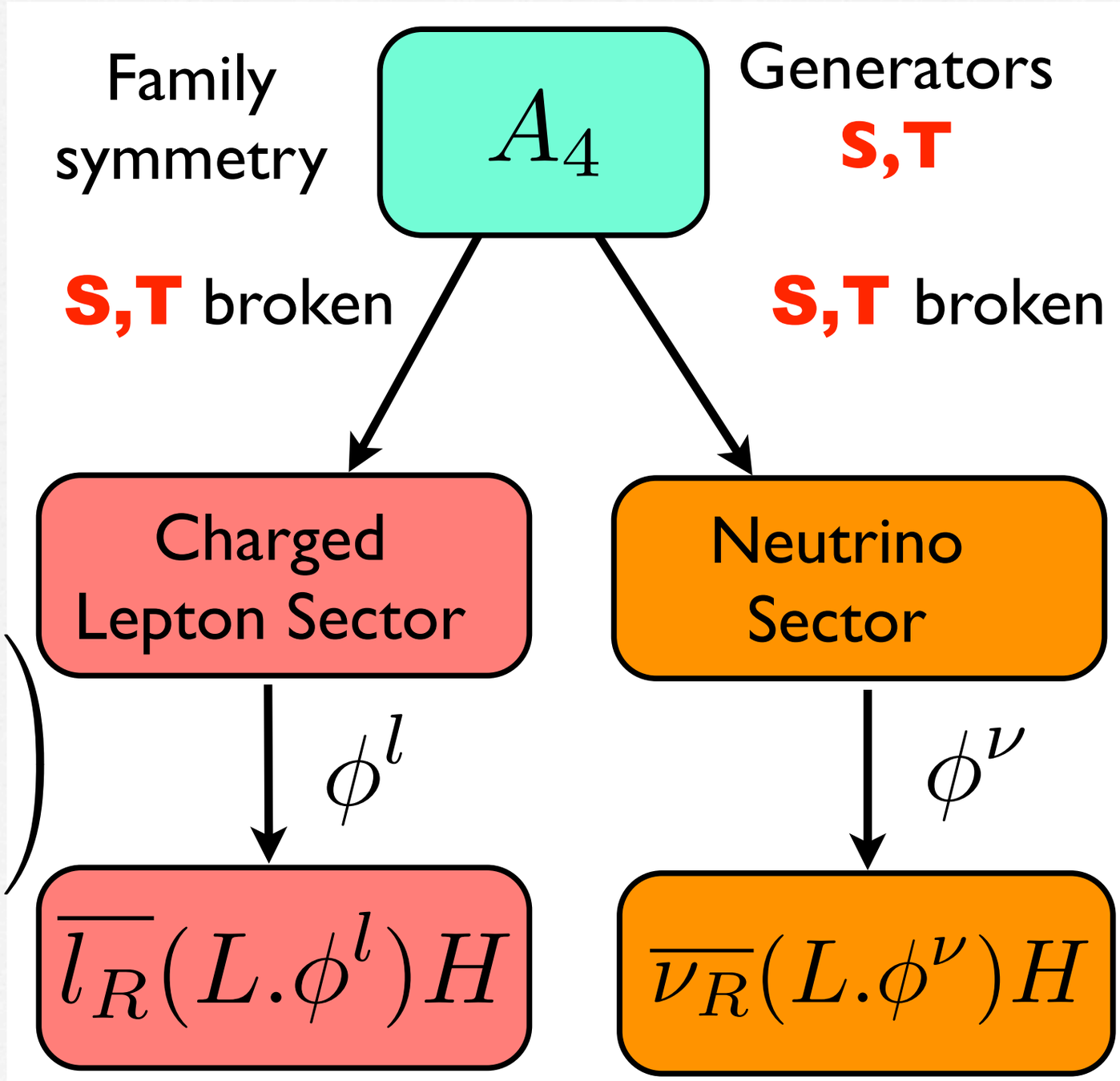


- S_4 and A_4 models with CP symmetry are constructed,all the possible cases following from the model-independent analysis can be realized. **Dirac CP phase is predicted to be trivial or maximal**.

$\Delta(96), \Delta(6n^2), \Delta(3n^2)$ allows many possible predictions for Dirac CP phase

Indirect Models

Family symmetry is fully broken in each sector by orthogonal alignments



Diagonal

$$\begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$$

A_4 Vacuum alignments

Symmetry preserving eigenvectors of group elements

Orthogonal alignments

$$\langle\phi\rangle = \begin{pmatrix}0\\0\\v\end{pmatrix} \begin{pmatrix}0\\v\\0\end{pmatrix} \begin{pmatrix}v\\0\\0\end{pmatrix} \begin{pmatrix}\pm v\\\pm v\\\pm v\end{pmatrix} \qquad \langle\phi\rangle = \begin{pmatrix}0\\e\\e\end{pmatrix} \perp \begin{pmatrix}v\\v\\-v\end{pmatrix}, \begin{pmatrix}v\\0\\0\end{pmatrix}$$

Direct

Indirect

CSD(n)

Orthogonal alignments

$$\langle\phi\rangle = \begin{pmatrix}2v\\-v\\v\end{pmatrix} \perp \begin{pmatrix}v\\v\\-v\end{pmatrix}, \begin{pmatrix}0\\e\\e\end{pmatrix} \qquad \langle\phi\rangle = \begin{pmatrix}a\\na\\(n-2)a\end{pmatrix} \perp \begin{pmatrix}2v\\-v\\v\end{pmatrix}$$

Fredrik Björkeroth* and Stephen F. King†

Constrained sequential dominance

$$\frac{1}{\Lambda}H_u(L\cdot\phi_{\rm atm})N_1^c+\frac{1}{\Lambda}H_u(L\cdot\phi_{\rm sol})N_2^c+\frac{1}{\Lambda}H_u(L\cdot\phi_{\rm dec})N_3^c$$

$$\langle\phi_{\rm atm}\rangle\propto\begin{pmatrix}0\\1\\1\end{pmatrix}\qquad\langle\phi_{\rm sol}\rangle\propto\begin{pmatrix}1\\n\\n-2\end{pmatrix}\qquad\langle\phi_{\rm dec}\rangle\propto\begin{pmatrix}0\\0\\1\end{pmatrix}$$

CSD(n)

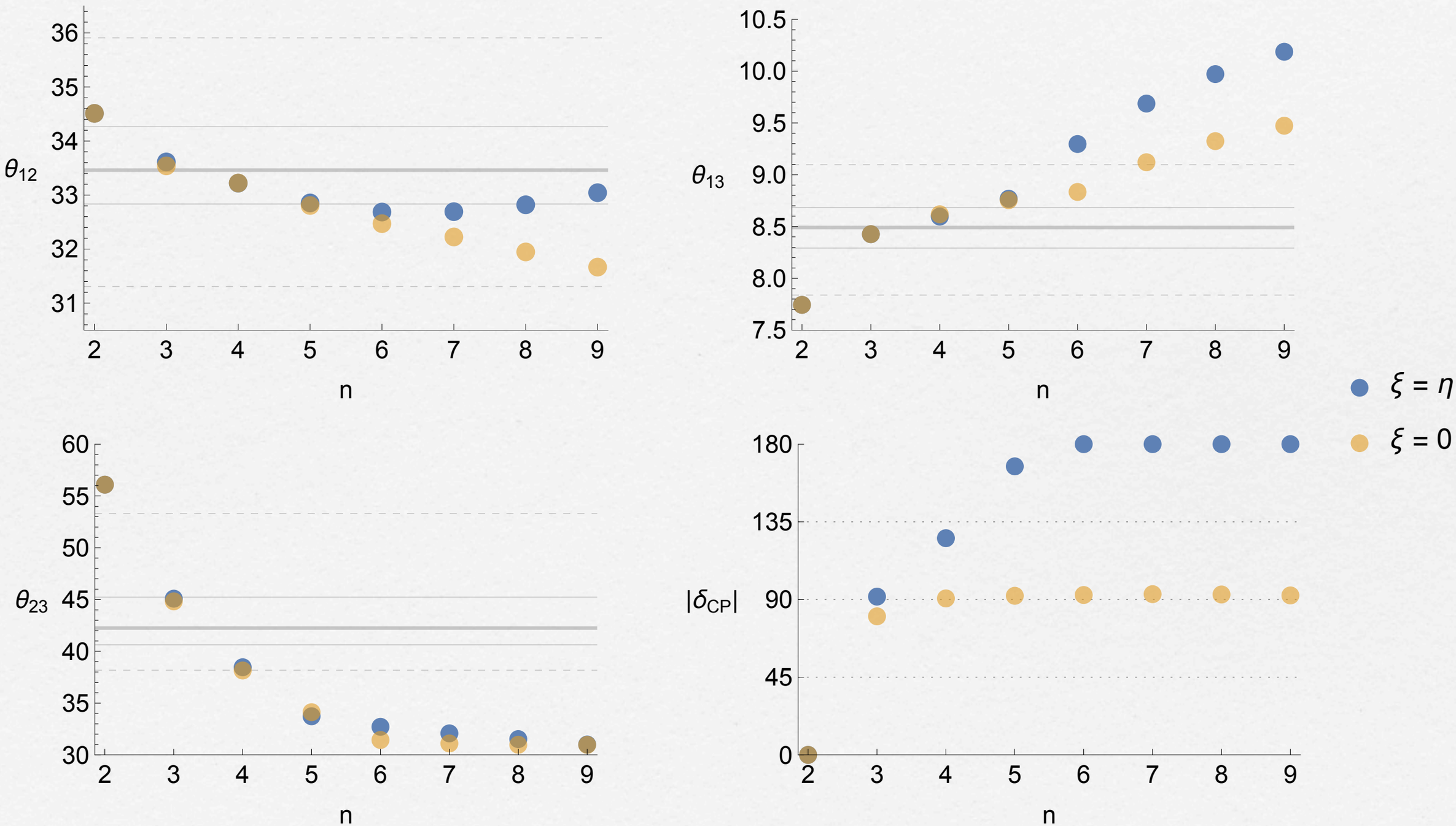
$$Y^\nu=\begin{pmatrix}0&b&0\\a&nb&0\\a&(n-2)b&c\end{pmatrix}\qquad M_{RR}\sim{\rm diag}(M_1,M_2,M_3)$$

$$m^\nu_{(n)}=m_ae^{i\alpha}\begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix}+m_be^{i\beta}\begin{pmatrix}1&n&n-2\\n&n^2&n(n-2)\\n-2&n(n-2)&(n-2)^2\end{pmatrix}+m_ce^{i\gamma}\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}$$

$$\eta=\beta-\alpha\qquad\qquad\xi=\gamma-\alpha$$

CSD(n) predictions

Fredrik Björkeroth* and Stephen F. King†



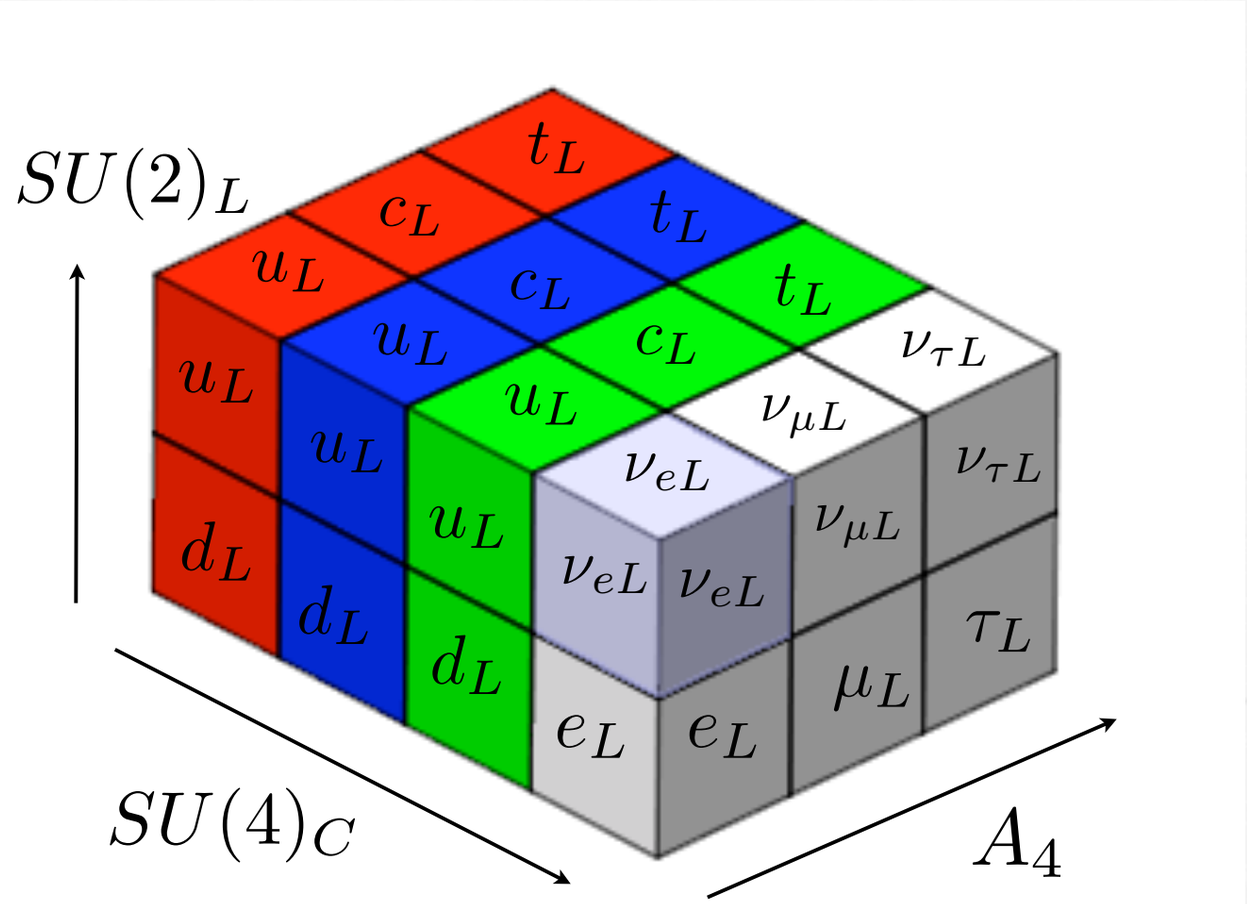
1406.7005

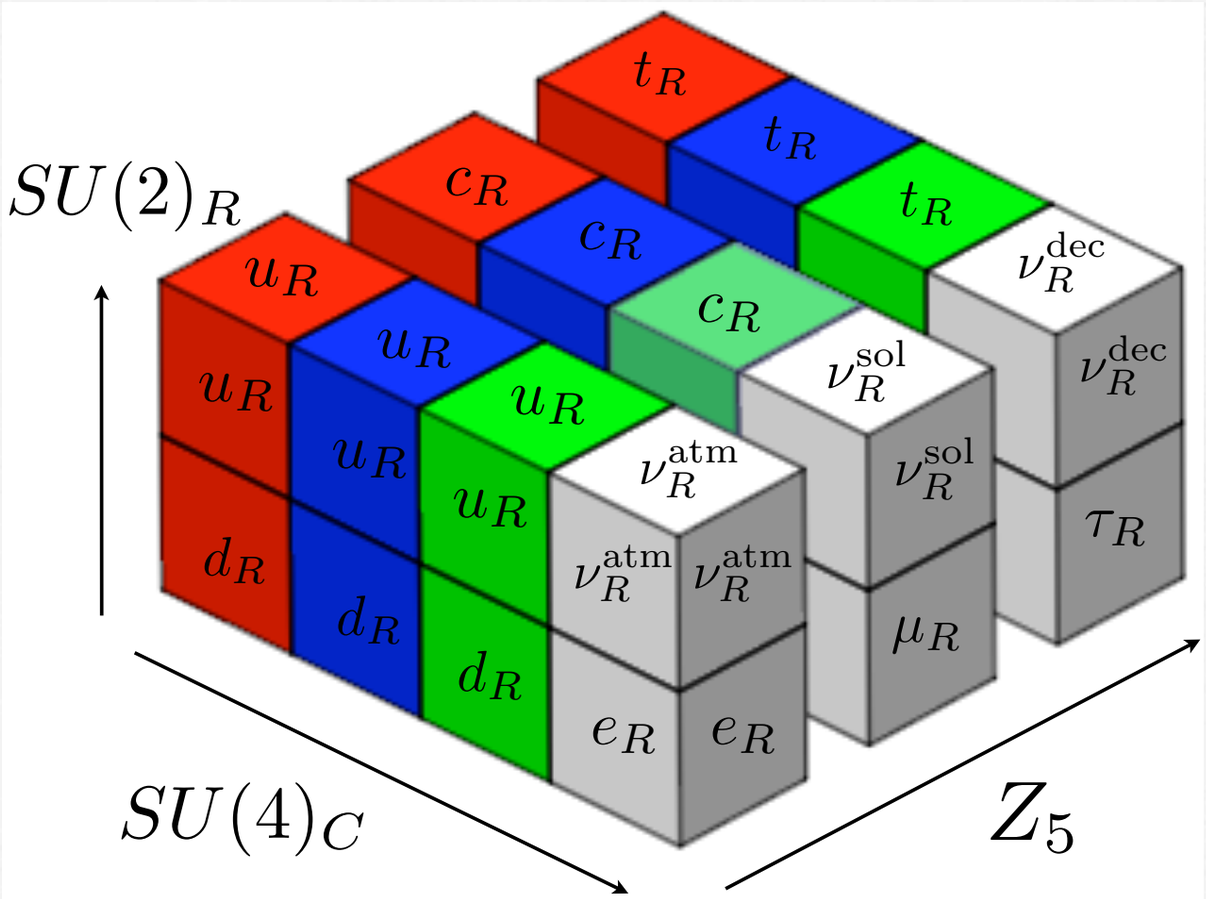
A to Z of Flavour with Pati-Salam

$$A_4 \times Z_5 \times SU(4)_C \times SU(2)_L \times SU(2)_R$$

Left-handed quarks and leptons triplets of A_4

Right-handed quarks and leptons distinguished by Z_5





name	field	$SU(4)_C \times SU(2)_L \times SU(2)_R$	A_4	Z_5	R
Quarks	F	$(4,2,1)$	3	1	1
and leptons	$F^c_{1,2,3}$	$(\overline{4},1,2)$	1	α,α^3,1	1
PS Higgs	$\overline{H^c}$, H^c	$(4,1,2)$, $(\overline{4},1,2)$	1	1	0
A_4 triplet	$\phi^u_{1,2}$	$(1,1,1)$	3	α^4, α^2	0
flavons	$\phi^d_{1,2}$	$(1,1,1)$	3	α^3, α	0
	h_3	$(1,2,2)$	3	1	0
Higgs	h_u	$(1,2,2)$	$1''$	α	0
bidoublets	h_d,h^d_{15}	$(1,2,2)$, $(15,2,2)$	$1'$	α^3,α^4	0
	h^u_{15}	$(15,2,2)$	1	α	0
Dynamical	Σ_u	$(1,1,1)$	$1''$	α	0
masses	Σ_d,Σ^d_{15}	$(1,1,1)$, $(15,1,1)$	$1'$	α^3,α^2	0
Majoron	ξ	$(1,1,1)$	1	α^4	0
	$X_{F''_{1,3}}$	$(4,2,1)$	$1''$	α,α^3	1
Fermion	$X_{F'_{1,3}}$	$(4,2,1)$	$1'$	α,α^3	1
Messengers	$X_{\overline{F_i}}$	$(\overline{4},2,1)$	1	α^i	1
	X_{ξ_i}	$(1,1,1)$	1	α^i	1

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Yukawa operators

Third family renormalisable

$$F.h_3F_3^c \to Q_3H_u u_3^c + Q_3H_d d_3^c + L_3H_u\nu_3^c + L_3H_d e_3^c$$

First and second family involve flavons

$$F.\phi_i^u h_u F_i^c \to Q.\langle\phi_i^u\rangle H_u u_i^c + L.\langle\phi_i^u\rangle H_u \nu_i^c,$$

$$F.\phi_i^d h_d F_i^c \to Q.\langle\phi_i^d\rangle H_d d_i^c + L.\langle\phi_i^d\rangle H_d e_i^c,$$

Flavons control first two columns

Cabibbo~1/4

CSD4 vacuum alignment

$$\langle\phi_1^u\rangle = \frac{V_1^u}{\sqrt{2}}e^{im\pi/5}\begin{pmatrix}0\\1\\1\end{pmatrix}, \quad \langle\phi_2^u\rangle = \frac{V_2^u}{\sqrt{21}}e^{im\pi/5}\begin{pmatrix}1\\4\\2\end{pmatrix}$$

"$\langle\phi_{\rm atm}\rangle$" "$\langle\phi_{\rm sol}\rangle$"

$$\langle\phi_1^d\rangle = V_1^d e^{in\pi/5}\begin{pmatrix}1\\0\\0\end{pmatrix}, \quad \langle\phi_2^d\rangle = V_2^d e^{in\pi/5}\begin{pmatrix}0\\1\\0\end{pmatrix}$$

"$\langle\phi_e\rangle$" "$\langle\phi_\mu\rangle$"

$$Y^u = Y^\nu = \begin{pmatrix}0 & b & 0\\ a & 4b & 0\\ a & 2b & c\end{pmatrix}$$

$$Y^d \sim Y^e \sim \begin{pmatrix}y_d^0 & 0 & 0\\ 0 & y_s^0 & 0\\ 0 & 0 & y_b^0\end{pmatrix}$$

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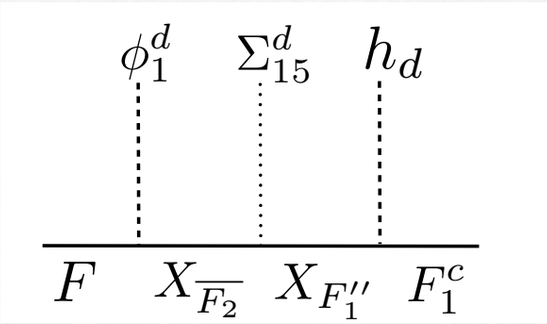
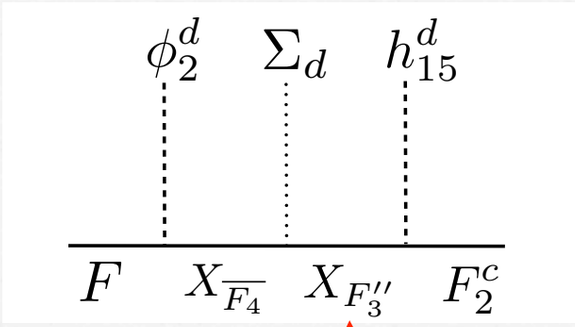
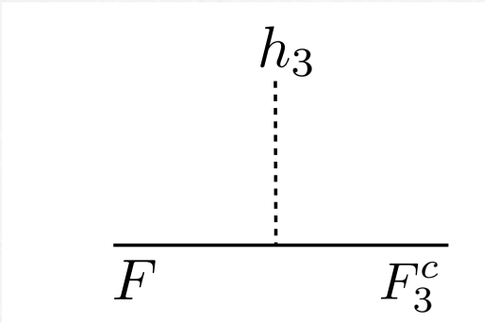
Yukawa operators



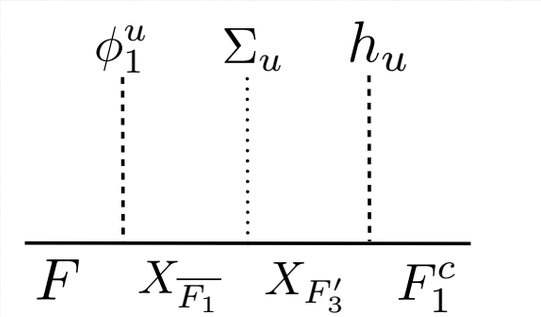


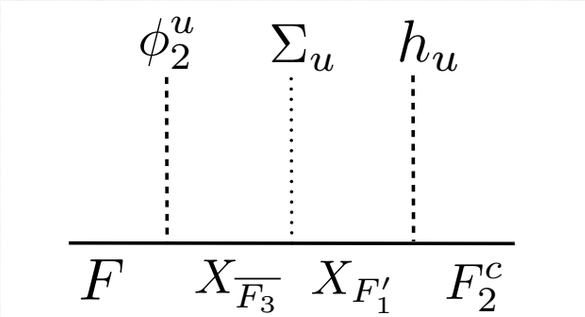


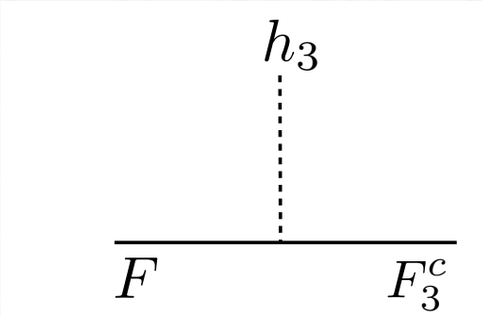
Down type quarks and charged leptons

$$F.\frac{\phi_1^d}{\Sigma_{15}^d}h_dF_1^c + F.\frac{\phi_2^d}{\Sigma_d}h_{15}^dF_2^c + F.h_3F_3^c$$

$$F.\frac{\phi_1^u}{\Sigma_u}h_uF_1^c + F.\frac{\phi_2^u}{\Sigma_u}h_uF_2^c + F.h_3F_3^c$$







up type quarks and neutrinos

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Yukawa and Mass Matrices

$$Y^u = Y^\nu = \begin{pmatrix} 0 & be^{-i3\pi/5} & \epsilon c \\ ae^{-i3\pi/5} & 4be^{-i3\pi/5} & 0 \\ ae^{-i3\pi/5} & 2be^{-i3\pi/5} & c \end{pmatrix} \qquad Y^d = \begin{pmatrix} y_d^0 e^{-i2\pi/5} & 0 & Ay_d^0 e^{-i2\pi/5} \\ By_d^0 e^{-i3\pi/5} & y_s^0 e^{-i2\pi/5} & Cy_d^0 e^{-i3\pi/5} \\ By_d^0 e^{-i3\pi/5} & 0 & y_b^0 + Cy_d^0 e^{-i3\pi/5} \end{pmatrix}$$

$$M_R \approx \begin{pmatrix} M_1 e^{8i\pi/5} & 0 & 0 \\ 0 & M_2 e^{4i\pi/5} & 0 \\ 0 & 0 & M_3 \end{pmatrix} \qquad Y^e = \begin{pmatrix} -(y_d^0/3)e^{-i2\pi/5} & 0 & Ay_d^0 e^{-i2\pi/5} \\ By_d^0 e^{-i3\pi/5} & -4.5y_s^0 e^{-i2\pi/5} & -3Cy_d^0 e^{-i3\pi/5} \\ By_d^0 e^{-i3\pi/5} & 0 & y_b^0 - 3Cy_d^0 e^{-i3\pi/5} \end{pmatrix}$$

SO(10)-like diagonal RHN masses $M_1 : M_2 : M_3 \sim m_u^2 : m_c^2 : m_t^2$

Physical neutrino masses in a normal hierarchy (CSD)

CSD(4) + PS gives Cabibbo connection $\theta_C \approx 1/4$ or $\theta_C \approx 14^\circ$

All CP phases are fifth roots of unity due to Z_5

CKM parameters

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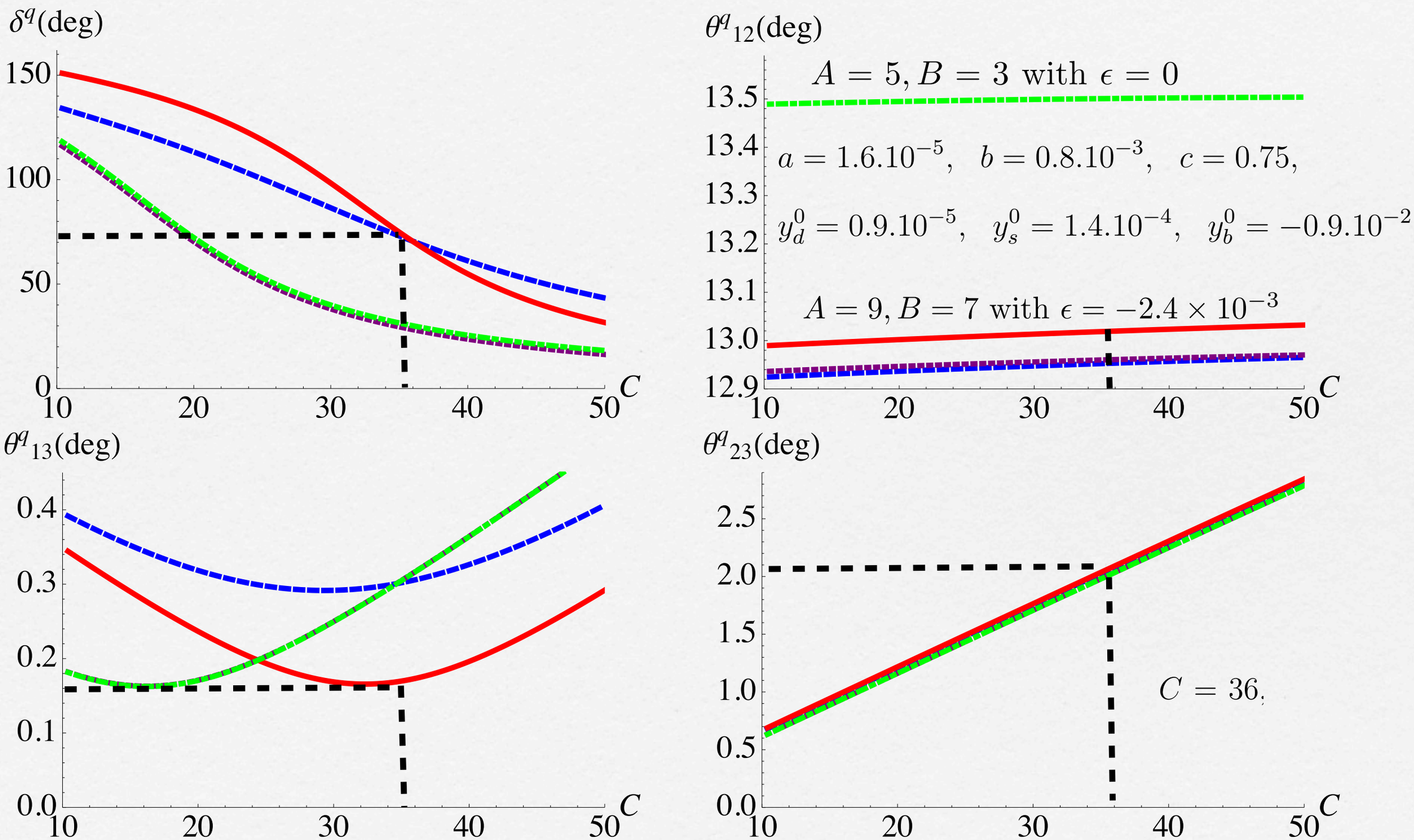

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See-Saw mass matrix

$$m^\nu = -v_u^2\, Y^\nu M_{\mathrm{R}}^{-1} Y^{\nu T}$$

Neutrino mass matrix only depends on m_a, m_b, m_c

$$m^\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{2i\eta} \begin{pmatrix} 1 & 4 & 2 \\ 4 & 16 & 8 \\ 2 & 8 & 4 \end{pmatrix} + m_c e^{2i\eta} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

CSD(4)

$$m^\nu \sim \frac{\langle\phi_{\mathrm{atm}}\rangle\langle\phi_{\mathrm{atm}}\rangle^T}{\langle\xi\rangle^2} + \frac{\langle\phi_{\mathrm{sol}}\rangle\langle\phi_{\mathrm{sol}}\rangle^T}{\langle\xi\rangle} \Rightarrow \eta = 2\pi/5$$

CP phase from Z_5

Z_5 invariant potential

$$\left|\frac{\langle\xi\rangle^5}{\Lambda^3} - m^2\right|^2 = 0 \qquad \langle\xi\rangle = |(\Lambda^3 m^2)^{1/5}| e^{-4i\pi/5}$$

Neutrino mass variables

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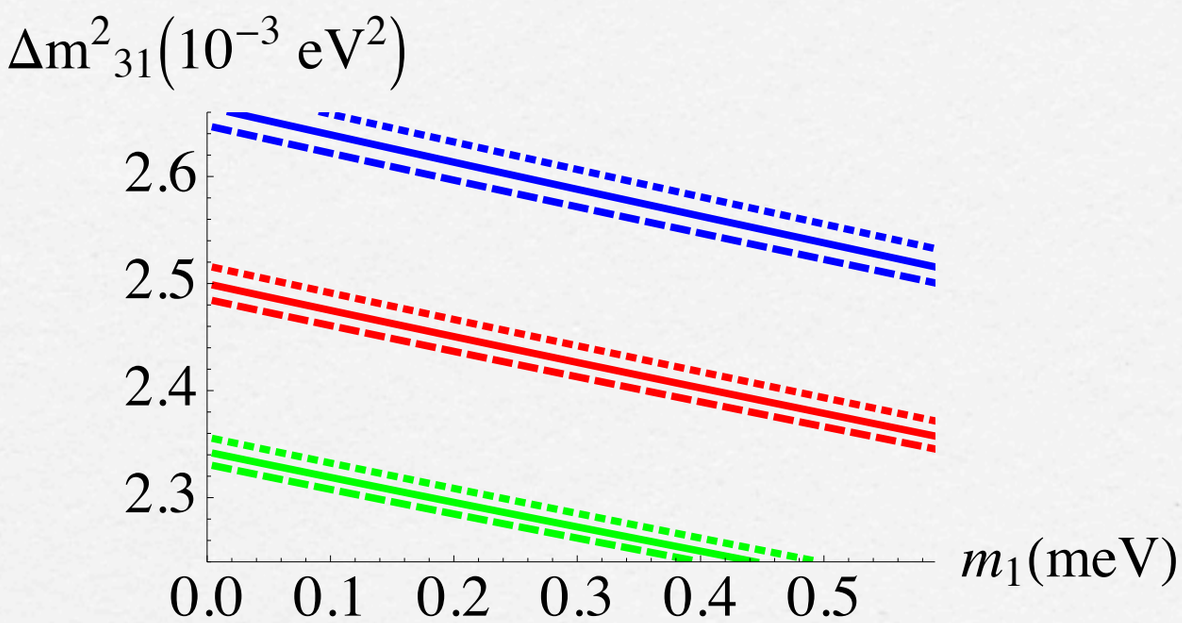

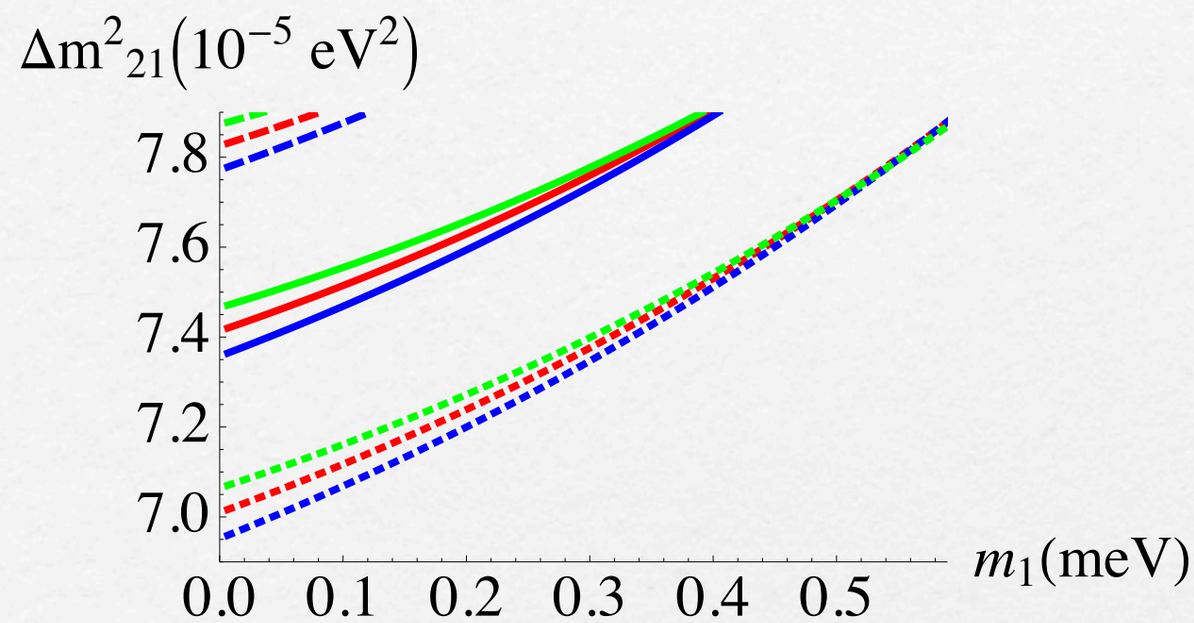

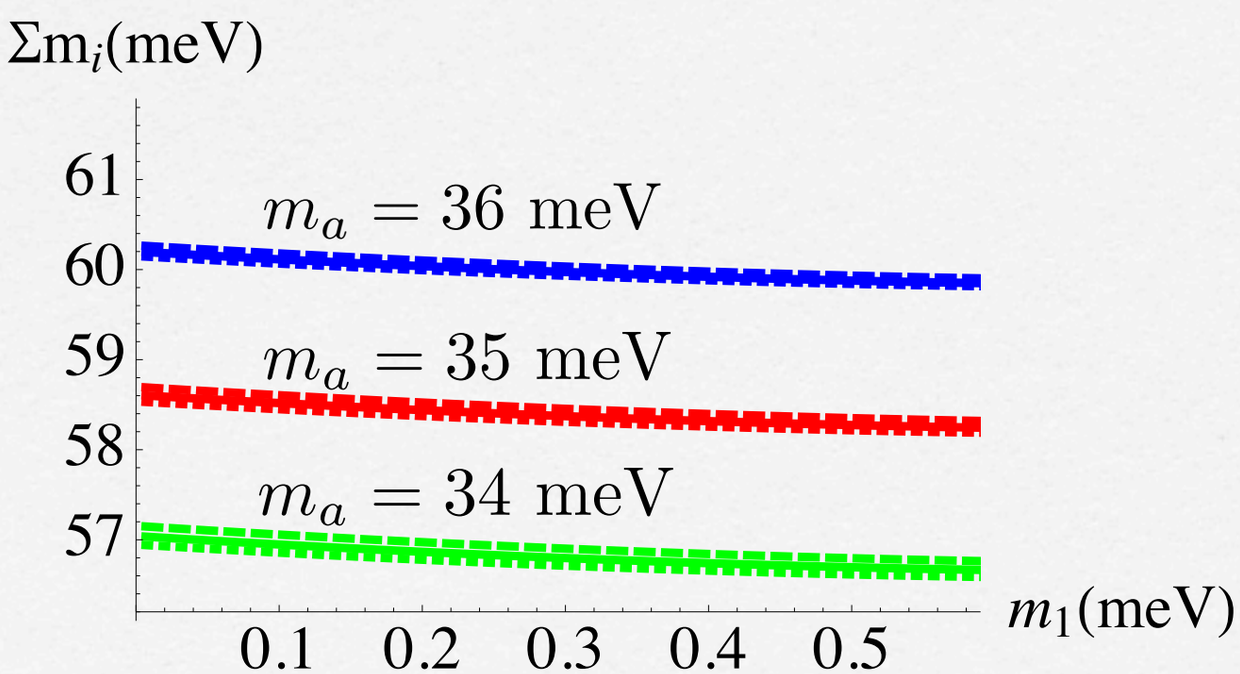

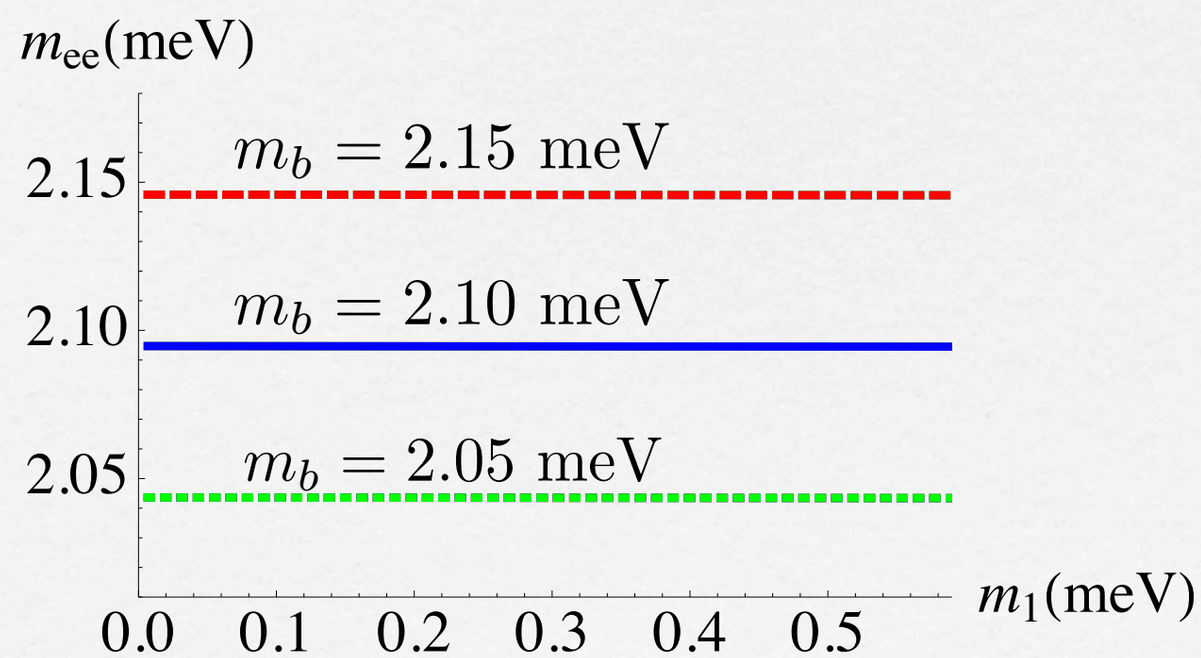

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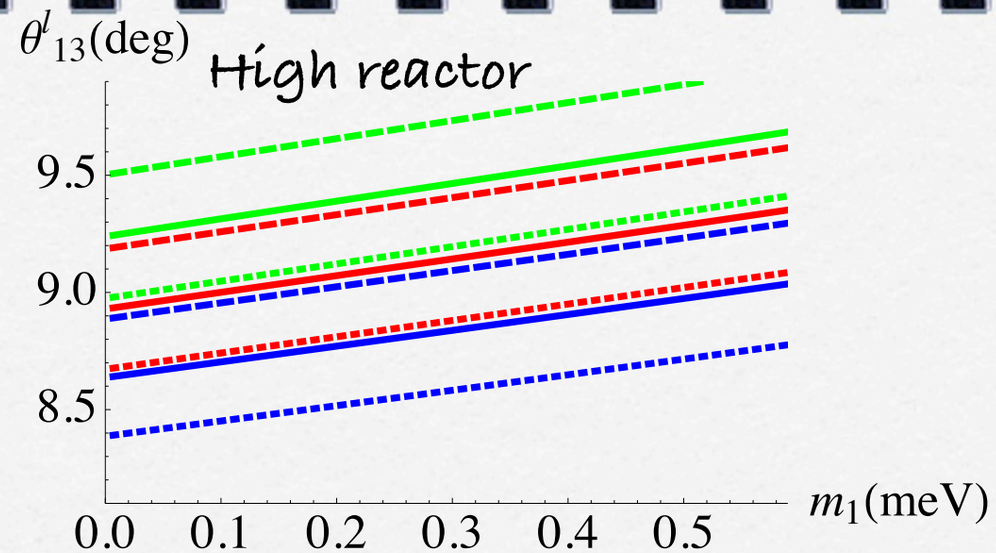
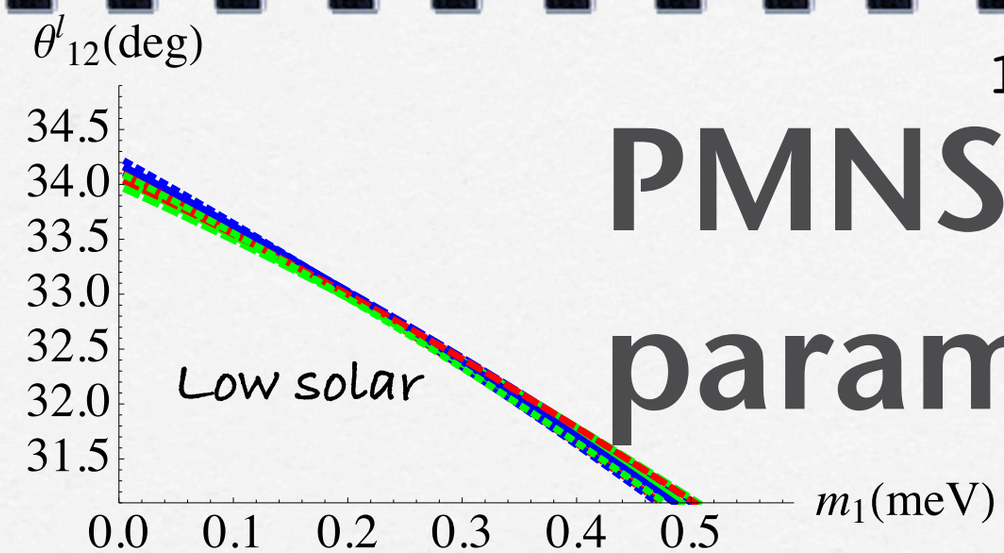
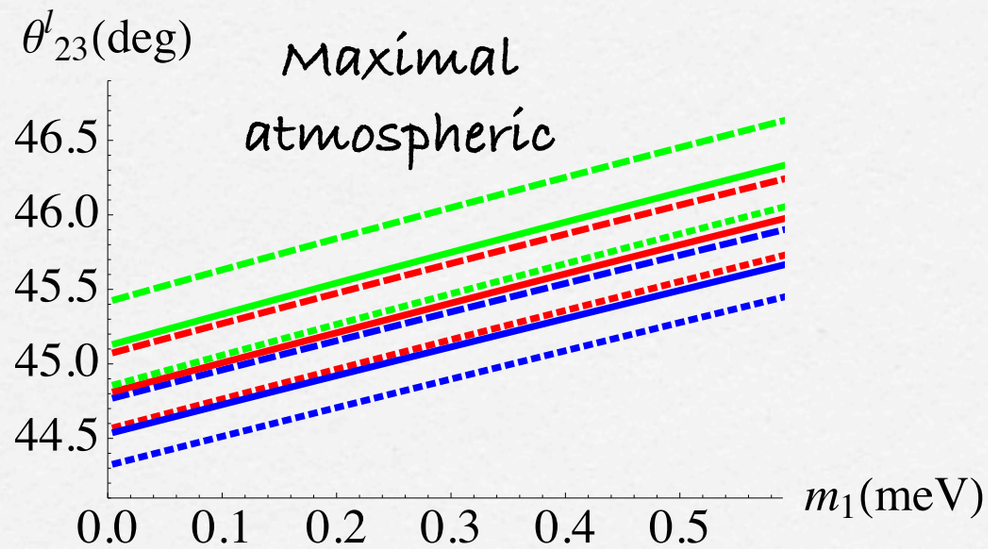
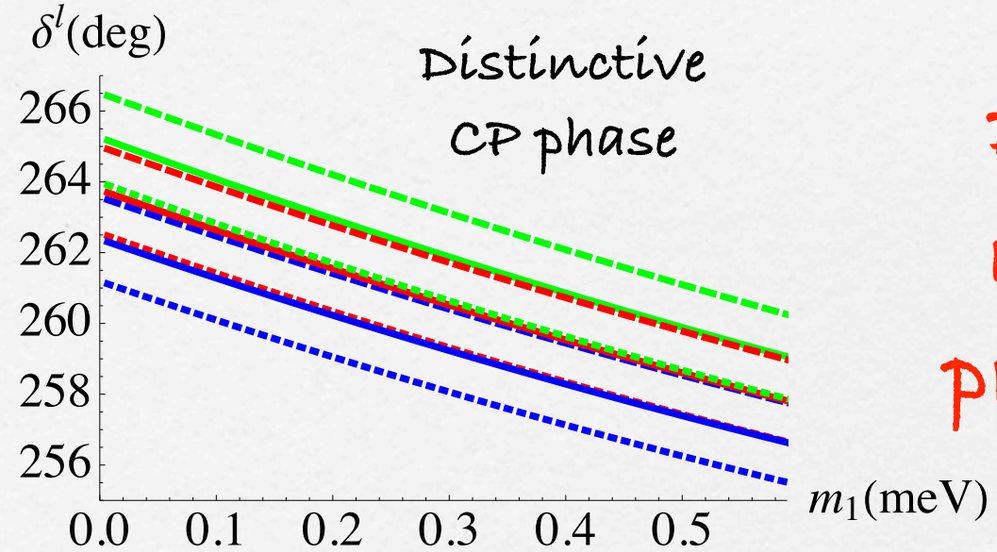
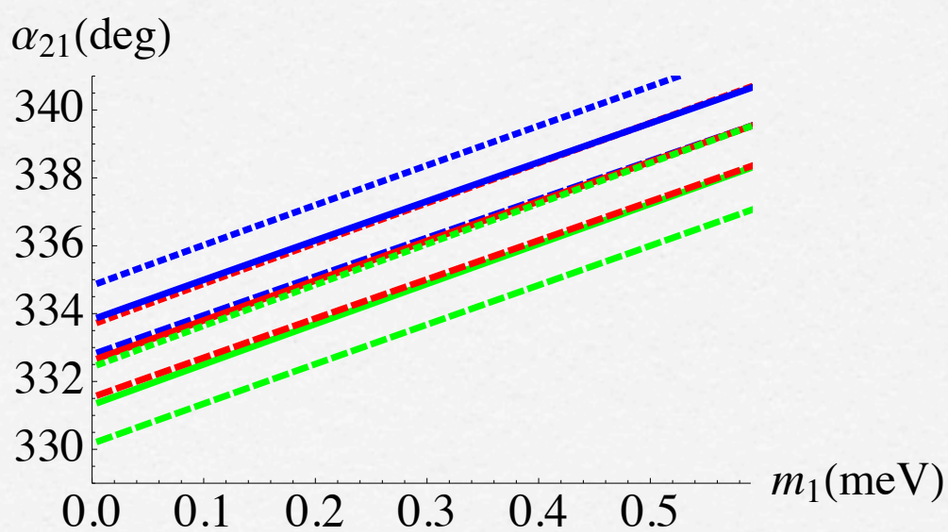
PMNS parameters

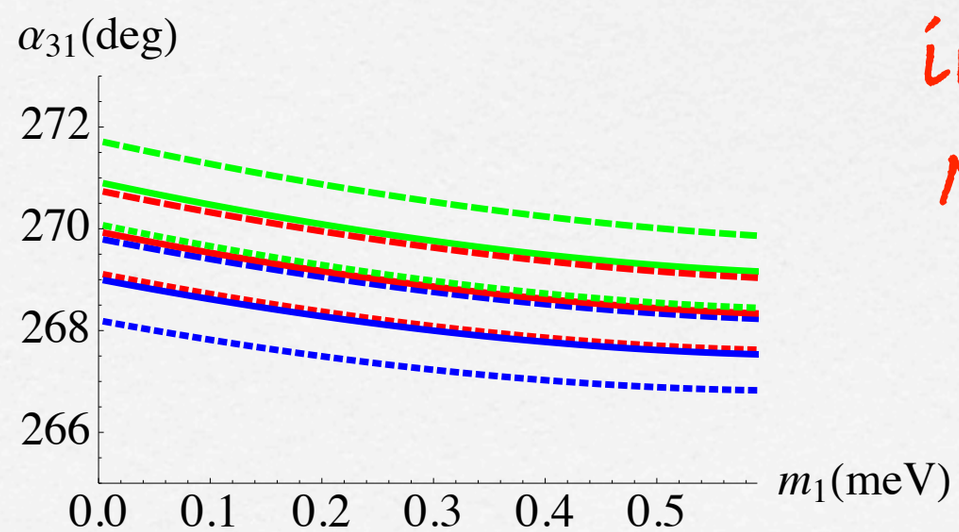












Entire PMNS matrix predicted ... including Majorana phases!

Conclusions

- GUT x Discrete Family Symmetry is very predictive (could originate from string theory)
- Direct models: Klein and T from Delta($6n^2$), predicts zero Dirac phase, quantised reactor angle, possible CP phase predictions
- Semi-direct models: partial surviving symmetry S or SU, smaller family symmetry, lepton mixing sum rules, possible CP phase predictions
- Indirect models: no surviving symmetry, allows A_4 broken by orthogonal CSD(n) vacuum alignment, very predictive scheme
- A to Z Pati-Salam predicts PMNS matrix entirely
- Discrete symmetries could originate from string theory