particle physics and cosmology from almost commutative manifolds

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outline

- motivation
- almost commutative manifolds
  spectral triple
  spectral action principle

  noncommutative spectral geometry
- particle physics phenomenology
- physical implications of almost commutative manifolds
- cosmological consequences
- discussion (open questions, current progress, outlook)

how can one construct a quantum theory of gravity coupled to matter ?

purely gravitational theory without matter

gravity-matter interaction is the most important aspect of dynamics how can one construct a quantum theory of gravity coupled to matter ?

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gravity-matter interaction is the most important aspect of dynamics

what should be the (noncommutative) algebra of observables of a quantum theory of gravity?

much below planck scale: continuum fields and an effective action

$$\boxed{S_{\text{Einstein-Hilbert}} + S_{\text{Standard Model}}}$$

close to the planck scale: gravity and SM fields packaged into geometry and matter on a kaluza-klein noncommutative space goal: to answer the unification of the four fundamental forces

- NCG: bottom-up approach

guess small-scale structure of ST from knowledge at EW scale

goal: to answer the unification of the four fundamental forces

- string theory: top-down approach

derive standard model directly from planck scale physics

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QG $\longrightarrow$ ST is wildly noncommutative manifold at very high E

at an intermediate scale, the algebra of coordinates is only a mildly noncommutative algebra of matrix valued functions

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goal: to answer the unification of the four fundamental forces

QG ⟶ ST is wildly noncommutative manifold at very high E

at an intermediate scale, the algebra of coordinates is only a mildly noncommutative algebra of matrix valued functions

suitably chosen ⟶ standard model coupled to gravity

- string theory : top-down approach

derive standard model directly from planck scale physics

noncommutative spectral geometry postulates:

SM as a phenomenological model, which dictates geometry of spacetime, so that the maxwell-dirac action produces the SM

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geometric space: product of a compact riemannian manifold $\mathcal{M}$ for spacetime and a discrete finite NC space $\mathcal{F}$ for the SM

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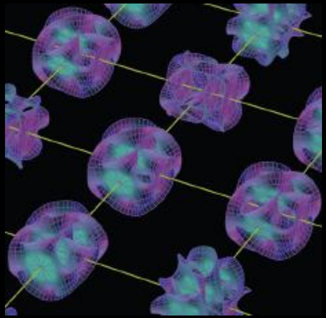

4-dim spacetime with an "internal" kaluza-klein space attached to each point; the "fifth" dim is a discrete, 0-dim space

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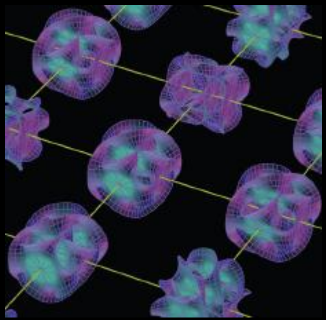
$\mathcal{M} \times \mathcal{F}$ almost commutative manifolds



the noncommutative algebra describing space is the algebra of functions over ordinary spacetime

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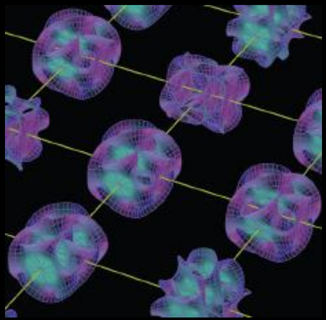
$\mathcal{M} \times \mathcal{F}$

almost commutative manifolds



in contrast with NC spaces such as the moyal plane for which $[\mathbf{x}^i, \mathbf{x}^j] = i\theta^{ij}$

central idea :

characterise ordinary riemannian manifolds by *spectral data*

extend it for noncommutative manifoldfs

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*can one hear the shape of a spectral triple (**spinorial drum**) ?*

spin manifolds in NCG

$\mathcal{M}$: compact 4dim riemannian spin manifold

connes (1996, 2008)

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$C^\infty(\mathcal{M})$: set of smooth infinitely (differentiable) functions;

$\mathcal{H} = L^2(\mathcal{M}, S)$: Hilbert space of square-integrable spinors S on $\mathcal{M}$

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algebra under pointwise multiplication

$\mathcal{H} = L^2(\mathcal{M}, S)$: Hilbert space of square-integrable spinors S on $\mathcal{M}$

$\mathcal{A} = C^\infty(\mathcal{M})$ acts on $\mathcal{H}$ as multiplication operators

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$\not{D}$: the Dirac operator $-i\gamma^\mu \nabla^s_\mu$

acts as first order differential operator on the spinors

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canonical triple $\boxed{(C^\infty(M), L^2(M, S), \not{D})}$

connes (1996, 2008)

- γ_5 operator $\mathbb{Z}_2$-grading $\gamma_5^2 = 1, \gamma_5^\star = \gamma_5$

$$L^2(M,S) = L^2(M,S)^+ \oplus L^2(M,S)^-$$

decomposes $\mathcal{H}$ into positive and negative eigenspace: chirality

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- antilinear isomorphism J_M

  charge conjugation operator on spinors

$$J_M{}^2 = -1,\ J_M \not{D} = \not{D} J_M\ ,\ J_M\gamma_5 = \gamma_5 J_M$$

connes (1996, 2008)

almost-commutative manifolds

almost-commutative manifolds

$$\mathcal{M} \times \mathcal{F}$$

the canonical triple encodes spacetime structure

encodes internal degrees of freedom at each point in spacetime

$$F := (\mathcal{A}_F, \mathcal{H}_F, D_F)$$

leads to the description of a gauge theory on spin manifold $\mathcal{M}$

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$A_{\mathcal{F}}$: matrix algebra, acting on hilbert space via matrix multiplication

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finite-dim (N) complex hilbert space $\mathcal{H}_\mathcal{F}$ upom which hermitian operator $D_\mathcal{F}$ acts

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$D_{\mathcal{F}}$ 96 x 96 matrix

in terms of 3x3 Yukawa mixing matrices and a real constant responsible for neutrino mass terms

$$D_F = \begin{pmatrix} S & T^* \\ T & \bar{S} \end{pmatrix}$$

$$S = \begin{pmatrix} & \begin{bmatrix} \Upsilon_v & \\ & \Upsilon_e \end{bmatrix} & & \\ \begin{bmatrix} \Upsilon_v^* & \\ & \Upsilon_e^* \end{bmatrix} & & & \\ & & & \begin{bmatrix} \Upsilon_u \otimes 1_3 & \\ & \Upsilon_d \otimes 1_3 \end{bmatrix} \\ & & \begin{bmatrix} \Upsilon_u^* \otimes 1_3 & \\ & \Upsilon_d^* \otimes 1_3 \end{bmatrix} & \end{pmatrix}$$

$$T = \begin{pmatrix} \begin{bmatrix} 0 & & & \\ & 0 & & \\ & & \Upsilon_R & \\ & & & 0 \end{bmatrix} & \\ & 0_4 \end{pmatrix}$$

- grading γ_F such that :

$$\gamma_F = +1 \quad \text{for left-handed fermions}$$

$$\gamma_F = -1 \quad \text{for right-handed fermions}$$

- a conjugation operator J_F for a finite space $\mathcal{F}$

real structure

$$J_F = \begin{pmatrix} & 1_{48} \\ 1_{48} & \end{pmatrix}$$

almost-commutative manifolds

$$\mathcal{M} \times \mathcal{F}$$

$$(\mathcal{A}, \mathcal{H}, D)$$

$$\boxed{M \times F := (\underbrace{C^\infty(M, \mathcal{A}_F)}_{\mathcal{A}}, \underbrace{L^2(M,S) \otimes \mathcal{H}_F}_{\mathcal{H}}, \underbrace{\not{\partial} \otimes \mathbb{I} + \gamma_5 \otimes D_F}_{D})}$$

D Dirac operator of the $\mathcal{M} \times \mathcal{F}$

almost-commutative manifolds

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D Dirac operator of the $\mathcal{M} \times \mathcal{F}$

the finite dimensional algebra $A_{\mathcal{F}}$ is (main input):

$$\boxed{\mathcal{A}_{\mathcal{F}} = M_a(\mathbb{H}) \oplus M_k(\mathbb{C})}$$

$k = 2a$

algebra of quaternions

algebra of complex $k \times k$ matrices

chamseddine and connes (2007)

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algebra of quaternions

algebra of complex $k \times k$ matrices

$k = 2a$

$k = 4$ first value that produces correct # of fermions per generation

prediction

- the number of fermions is the square of an even integer

- the existence of 3 generations is a physical input

chamseddine and connes (2007)

spectral action principle to derive a physical langrangian:

spectral action principle:

inner fluctuation $A = \sum_j a_j[\mathcal{D}, b_j]$, $a_j, b_j \in \mathcal{A}$

$\epsilon \in \{\pm 1\}$

$$\mathcal{D}_A = \mathcal{D} + A + \epsilon' J A J^{-1}$$

the action functional depends only on the spectrum of the *fluctuated Dirac operator* and is of the form:

$$Tr(f(D_A/\Lambda))$$

bosonic part

chamseddine and connes (1996, 1997)

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bosonic part

cut-off function

fixes the energy scale

positive function that falls to zero at large values of its argument

$$f(x) = 1; x \leq \Lambda$$

$$f(x) = e^{-x}$$

$$\int_0^\infty f(u)u\,du \quad , \quad \int_0^\infty f(u)\,du$$

finite

chamseddine and connes (1996, 1997)

spectral action principle:

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bosonic part

action sums up eigenvalues of $\mathcal{D}_A$ which are smaller than Λ

evaluate trace with heat kernel techniques (seeley-de witt coefficients)

$$\sum_{n=0}^{\infty} F_{4-n}\Lambda^{4-n} a_n$$

where

$$F(\mathcal{D}_A^2) = f(\mathcal{D}_A)$$

f : cut-off function ⟹ its taylor expansion at zero vanishes

⟹ the asymptotic expansion of the trace reduces to:

$$\mathrm{Tr}\left(f\left(\frac{D_A}{\Lambda}\right)\right) \sim 2f_4\Lambda^4 a_0({D_A}^2) + 2f_2\Lambda^2 a_2({D_A}^2) + f(0)a_4({D_A}^2) + \cancel{O(\Lambda^{-1})}$$

f plays a rôle through its momenta f_0, f_2, f_4

real parameters related to the coupling constants at unification, the gravitational constant, and the cosmological constant

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f plays a rôle through its momenta f_0, f_2, f_4

real parameters related to the coupling constants at unification, the gravitational constant, and the cosmological constant the spectral action accounts only for the bosonic part of the action

for the terms involving fermions and their coupling to the bosons we need the fermionic part (for KO-dim 2 almost commutative manifold)

$$Tr\left(f\left(D_A/\Lambda\right)\right) + \frac{1}{2}\langle J\Psi, \mathcal{D} \quad \Psi\rangle \ , \quad \Psi \in \mathcal{H}^{+}$$

the rest follows after a long calculation ...

⇓

the bosonic spectral action is modulo gravitational terms:

$$S_\Lambda = \frac{-2af_2\Lambda^2 + ef_0}{\pi^2}\int|\phi|^2 + \frac{f_0}{2\pi^2}\int a|D_\mu\phi|^2 - \frac{f_0}{12\pi^2}\int aR|\phi|^2$$
$$-\frac{f_0}{2\pi^2}\int\left(g_3^2 G^i_\mu G^{\mu i} + g_2^2 F^a_\mu F^{\mu\nu a} + \frac{5}{3}g_1^2 B_\mu B^\mu\right) + \frac{f_0}{2\pi^2}\int b|\phi|^4 + \mathcal{O}(\Lambda^{-2})$$

with a, b, c, d, e, constants depending on yukawa parameters

adding the fermionic term, leads to the SM lagrangian

the coefficients in this lagrangian are in terms of:

- moments $f(0), f_2, f_4$ of function f in spectral action
- cut-off scale Λ in spectral action
- vacuum expectation value of higgs field
- coefficients a, b, c, d, e determined by mass matrices in $D_{\mathcal{F}}$

chamseddine, connes, marcolli (2007)

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$\Rightarrow$ then use RGE to obtain predictions for the standard model

$m_{\mathrm{top}} \lesssim \sqrt{\frac{8}{3}} M_W$ $\quad$ $m_t \leq 180 \text{ GeV}$

chamseddine, connes, marcolli (2007)

higgs field as a gauge field associated to the finite space

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3 scalars: higgs, singlet, dilaton

real scalar singlet associated with the majorana mass of right-handed neutrino; it is nontrivally mixed with higgs

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1st approach: singlet integrated out and replaced by its vev

$$m_h{}^2 = \frac{4\lambda M_W{}^2}{3 g_2{}^2}$$

$$\boxed{167 \text{ GeV} \leq m_h \leq 176 \text{ GeV}}$$

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new approach: higgs doublet and singlet get mixed

⟹ masses of higgs and singlet get shifted

⟹ consistency with 125 GeV for higgs mass and 170 GeV for top quark mass

connes, chamseddine (2012)

higgs field as a gauge field associated to the finite space

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real scalar singlet associated with the majorana mass of right-handed neutrino; it is nontrivally mixed with higgs

stephan (2009)

alexander-nunneley, pilaftsis (2010)

other proposals

- build a model based on a larger symmetry

$$\mathcal{A} = M_a(\mathbb{H}) \oplus M_k(\mathbb{C}) \qquad k = 2a$$

up to now $a = 2$

consider $a = 4$ $\quad \mathcal{A}_G = M_4(\mathbb{H}) \oplus M_8(\mathbb{C})$

it explains the presence of the field necessary for a correct fit of the mass of the higgs around 126 GeV

devastato, lizzi, martinetti (2013)

- generalise inner fluctuations to real spectral triples that fail on the first order condition (i.e., the dirac operator is a differential operator of order 1):

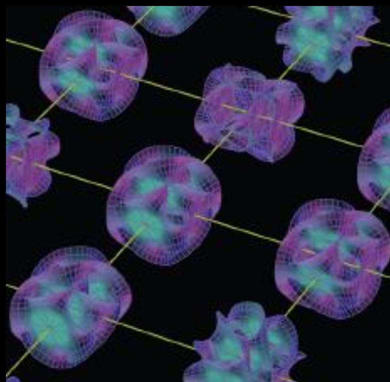
$$[[D,a],JbJ^{-1}]=0\,, \qquad \forall a,b\in\mathcal{A}.$$

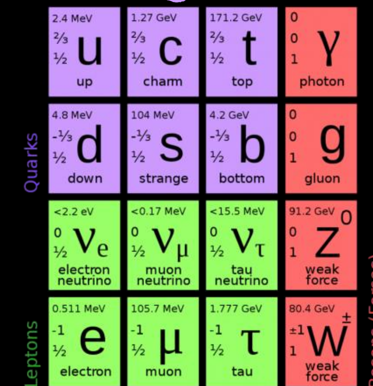
↓

pati-salam type model $SU(2)_R\times SU(2)_L\times SU(4)$

chamseddine, connes, van suijlekom (2013)

NCSG offers a geometric interpretation of the standard model coupled to gravity





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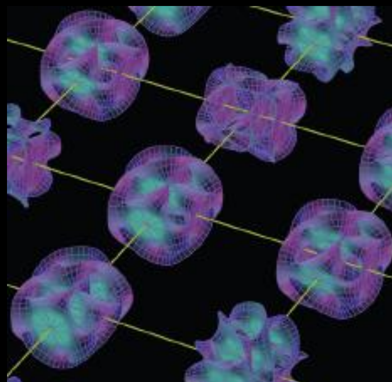
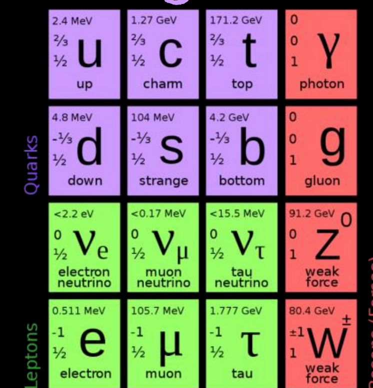


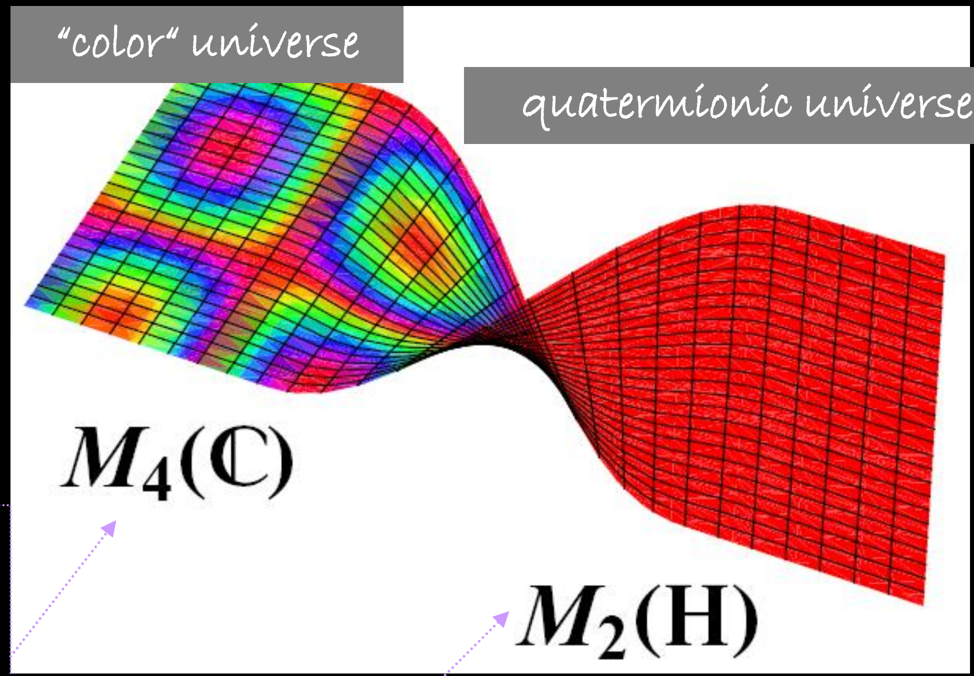


noncommutative geometry $\longrightarrow$?

almost commutative geometry $\xrightarrow{\text{connes}}$ gravity + yang mills higgs +constraints

riemannian geometry $\xrightarrow{\text{einstein}}$ gravity

ST is a parallel universe;
each copy is a 4dim manifold



algebra of 4x4 complex matrices,
decomposed into 1x1 and 3x3 matrices

→ split between leptons & quarks

algebra of 2x2 quatermionic matrices,
broken by chirality operator

fermions: live on both universes

higgs doublet: connects right to left sectors in quaternionic universe

→ this joining gives mass to quarks and leptons

physical meaning of the almost commutative spaces

doubling of the algebra

$\mathcal{M} \times \mathcal{F}$

$$(\mathcal{A},\mathcal{H},\mathcal{D},J,\gamma) = (C^\infty(\mathcal{M}), L^2(\mathcal{M},S), \partial\!\!\!/_{\mathcal{M}}, J_{\mathcal{M}}, \gamma_5) \otimes (\mathcal{A}_{\mathcal{F}}, \mathcal{H}_{\mathcal{F}}, D_{\mathcal{F}}, J_{\mathcal{F}}, \gamma_{\mathcal{F}})$$

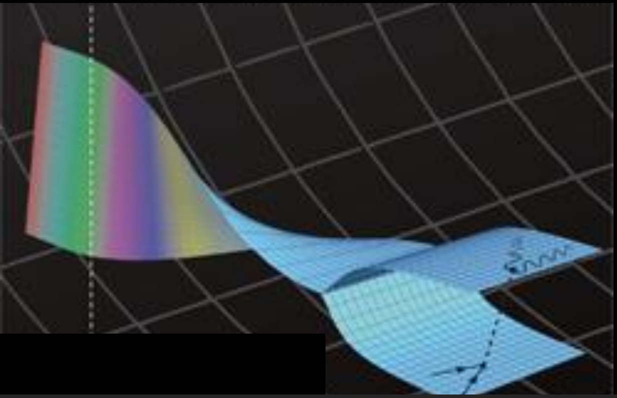
defined as

$$(\mathcal{A},\mathcal{H},\mathcal{D},J,\gamma) = (\mathcal{A}_1,\mathcal{H}_1,\mathcal{D}_1,J_1,\gamma_1)\otimes(\mathcal{A}_2,\mathcal{H}_2,\mathcal{D}_2,J_2,\gamma_2)$$

with

$$\mathcal{A} = \mathcal{A}_1 \otimes \mathcal{A}_2\ ,\ \ \mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2\ ,$$
$$\mathcal{D} = \mathcal{D}_1 \otimes 1 + \gamma_1 \otimes \mathcal{D}_2\ ,$$
$$\gamma = \gamma_1 \otimes \gamma_2\ , \quad J = J_1 \otimes J_2\ ,$$

$$J^2=-1,\quad [J,\mathcal{D}]=0, \qquad [J_1,\gamma_1]=0,\quad \{J,\gamma\}=0,$$



the doubling of the algebra is related to dissipation

sakellariadou, stabile, vitiello, PRD 84 (2011) 045026

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canonical formalism for dissipative systems

sakellariadou, stabile, vitiello, PRD 84 (2011) 045026

the doubling of the algebra is related to dissipation

brownian motion: $$m\ddot{x}(t) + \gamma\dot{x}(t) = f(t)$$

derived from a lagrangian
in a canonical procedure

the constraint condition at the classical level introduces a new coordinate y

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x-system: open (dissipating) system

the constraint condition at the classical level introduces a new coordinate y

$$\boxed{m\ddot{x} + \gamma\dot{x} = f \;, \quad m\ddot{y} - \gamma\dot{y} = 0}$$

{x – y} is a closed system

canonical formalism for dissipative systems

sakellariadou, stabile, vitiello, PRD 84 (2011) 045026

the doubling of the algebra is related to dissipation and the gauge field structure

$$L = \frac{m}{2}(\dot{x}_1^2 - \dot{x}_2^2) + \frac{e}{2}(\dot{x}_1 A_1 + \dot{x}_2 A_2) - e\Phi$$

$$A_i = \frac{B}{2}\epsilon_{ij} x_j \quad (i, j = 1, 2)$$

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$$A_i = \frac{B}{2}\epsilon_{ij} x_j \quad (i, j = 1, 2)$$

- doubled coordinate, e.g. x_2 acts as gauge field component A_1 to which x_1 coordinate is coupled
- energy dissipated by one system is gained by the other one
- gauge field as *bath/reservoir* in which the system is embedded

sakellariadou, stabile, vitiello, PRD 84 (2011) 045026

dissipation may lead to a quantum evolution

't hooft's conjecture: loss of information (dissipation) in a regime of deterministic dynamics may lead to QM evolution

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$$m\ddot{x} + \gamma\dot{x} + kx = 0 \qquad m\ddot{y} - \gamma\dot{y} + ky = 0$$

$$H = H_{\mathrm{I}} - H_{\mathrm{II}} \qquad H_{\mathrm{II}}|\psi\rangle = 0 \Longrightarrow \text{info loss}$$

to define physical states and guarantie that H is bounded from below

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to define physical states and guarantie that H is bounded from below

physical states are invariant under time reversal and periodical (τ)

$$_{H}\langle\psi(\tau)|\psi(0)\rangle_{H} = e^{i\phi} = e^{i\alpha\pi}$$

sakellariadou, stabile, vitiello, PRD 84 (2011) 045026

dissipation may lead to a quantum evolution

't hooft's conjecture: loss of information (dissipation) in a regime of deterministic dynamics may lead to QM evolution

$$m\ddot{x} + \gamma\dot{x} + kx = 0 \qquad m\ddot{y} - \gamma\dot{y} + ky = 0$$

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sakellariadou, stabile, vitiello, PRD 84 (2011) 045026

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sakellariadou, stabile, vitiello, PRD 84 (2011) 045026

algebra doubling

⇛ neutrino oscillations

gargiulo, sakellariadou, vitiello; EPJC 74 (2014) 2695

algebra doubling $\longrightarrow$ *deformed hopf algebra*

- define coproduct operators
- build bogogliubov operators as linear combinations of coproduct ones

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gargiulo, sakellariadou, vitiello; EPJC 74 (2014) 2695

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$\Rightarrow$ neutrino oscillations

mixing transformations connecting flavor fields ψ_f to fields ψ_m

$$\nu_e(x) = G_\theta^{-1}(t)\nu_1(x)G_\theta(t) \ ,$$
$$\nu_\mu(x) = G_\theta^{-1}(t)\nu_2(x)G_\theta(t) \ .$$

generator of field mixing transf. $G_\theta(t)$

gargiulo, sakellariadou, vitiello; EPJC 74 (2014) 2695 algebra doubling ⟶ deformed hopf algebra

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generator of field mixing transf. $G_\theta(t)$

contains rotation operator terms and bogogliubov transformation operator terms

gargiulo, sakellariadou, vitiello; EPJC 74 (2014) 2695

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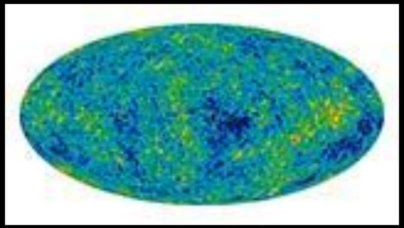
gargiulo, sakellariadou, vitiello; EPJC 74 (2014) 2695

cosmological consequences

early universe models tested with

- astrophysical data (CMB)
- high energy experiments (LHC)

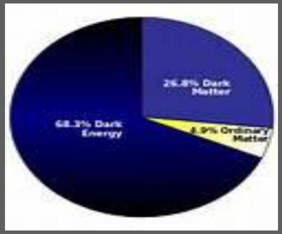


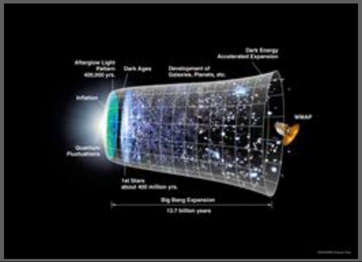
despite the golden era of cosmology, a number of questions:

- origin of dark matter and dark energy
- search for natural and well-motivated inflationary model (alternatives...)

. . .

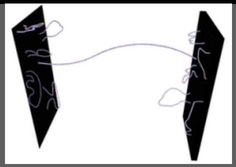
are still awaiting for a definite answer





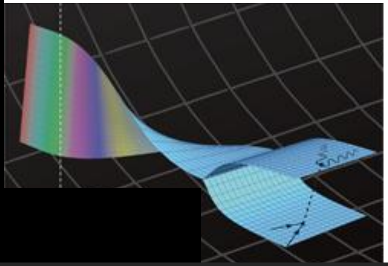
main approaches to build early universe cosmological models:

- string theory
- LQC, SF, WdW, CDT, CS,...



- noncommutative spectral geometry

$$\mathcal{S}^{\mathrm{E}} = \int \Big(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^\star R^\star$$
$$- \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha} + \frac{1}{4} B^{\mu\nu} B_{\mu\nu}$$
$$+ \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2$$
$$- \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \Big) \sqrt{g}\ d^4 x \ ,$$



corrections to einstein's equations

$$\mathcal{S}^{\mathrm{E}}_{\text{bosonic}} = \int \Big(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^\star R^\star + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha} + \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \Big) \sqrt{g}\, d^4 x \,,$$

$$\kappa_0^2 = \frac{12\pi^2}{96 f_2 \Lambda^2 - f_0 \mathfrak{c}} \,,$$

$$\alpha_0 = -\frac{3 f_0}{10\pi^2} \,,$$

$$\gamma_0 = \frac{1}{\pi^2} \left(48 f_4 \Lambda^4 - f_2 \Lambda^2 \mathfrak{c} + \frac{f_0}{4} \mathfrak{d} \right) \,,$$

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bare action à la wislon

$$\mathbf{H} = (\sqrt{\mathfrak{a} f_0} / \pi) \phi$$

$\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}$ describe possible choices of $D_{\mathcal{F}}$

yukawa parameters and majorana terms for ν_R

gravitational & coupling between higgs field and ricci curvature

equations of motion

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R + \frac{1}{\beta^2}\delta_{\rm cc}\left[2C^{\mu\lambda\nu\kappa}_{;\lambda;\kappa} + C^{\mu\lambda\nu\kappa}R_{\lambda\kappa}\right] = \kappa_0^2\delta_{\rm cc}T^{\mu\nu}_{\rm matter}$$

$$\beta^2 \equiv -\frac{1}{4\kappa_0^2\alpha_0}$$

$$\delta_{\rm cc} \equiv [1 - 2\kappa_0^2\xi_0\mathbf{H}^2]^{-1}$$

$$\alpha_0 = \frac{-3f_0}{10\pi^2}$$

nelson, sakellariadou, PRD 81 (2010) 085038

gravitational & coupling between higgs field and ricci curvature

equations of motion

neglect nonminimal coupling between geometry and higgs

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nelson, sakellariadou, PRD 81 (2010) 085038

gravitational & coupling between higgs field and ricci curvature

$\Longrightarrow$ equations of motion

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FLRW

weyl tensor vanishes $\Longrightarrow$

NCSG corrections to einstein equations vanish

nelson, sakellariadou, PRD <u>81</u> (2010) 085038

gravitational & coupling between higgs field and ricci curvature

⇛ equations of motion

neglect nonminimal coupling between geometry and higgs

⇒ corrections to einstein's eqs. will be apparent at leading order, only in anisotropic models

bianchi model: NCSG corrections to einstein's eqs. are present only in inhomogeneous and anisotropic spacetimes

nelson, sakellariadou, PRD 81 (2010) 085038

at energies approaching higgs scale, the nonminimal coupling of higgs field to curvature cannot be neglected

nelson, sakellariadou, PRD 81 (2010) 085038

at energies approaching higgs scale, the nonminimal coupling of higgs field to curvature cannot be neglected

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \kappa_0^2 \left[\frac{1}{1 - \kappa_0^2 |\mathbf{H}|^2 / 6} \right] T^{\mu\nu}_{\text{matter}}$$

$\Rightarrow$ effective gravitational constant

nelson, sakellariadou, PRD 81 (2010) 085038

at energies approaching higgs scale, the nonminimal coupling of higgs field to curvature cannot be neglected

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⇒ effective gravitational constant

$$\mathcal{L}_{|\mathbf{H}|} = -\frac{R}{12}|\mathbf{H}|^2 + \frac{1}{2}|D^\alpha \mathbf{H}||D^\beta \mathbf{H}|g_{\alpha\beta} - \mu_0|\mathbf{H}|^2 + \lambda_0|\mathbf{H}|^4$$

$$\Rightarrow \quad -\mu_0|\mathbf{H}|^2 \to -\left(\mu_0 + \frac{R}{12}\right)|\mathbf{H}|^2$$

⇒ increases the higgs mass

nelson, sakellariadou, PRD <u>81</u> (2010) 085038

$$\mathcal{L}_{|\mathbf{H}|} = -\frac{R}{12}|\mathbf{H}|^2 + \frac{1}{2}|D^{\alpha}\mathbf{H}||D^{\beta}\mathbf{H}|g_{\alpha\beta} - \mu_0|\mathbf{H}|^2 + \lambda_0|\mathbf{H}|^4$$

redefine higgs: $\tilde{\phi} = -\ln\left(|\mathbf{H}|/(2\sqrt{3})\right)$

$$\begin{aligned}\mathcal{L}_{\tilde{\phi}} = e^{-2\tilde{\phi}} \Big[& -R + 6D^{\alpha}\tilde{\phi}D^{\beta}\tilde{\phi}g_{\alpha\beta} \\ & -12\left(\mu_0 - 12\lambda_0 e^{-2\tilde{\phi}}\right)\Big]\end{aligned}$$

in the form of 4dim dilatonic gravity

link with compactifid string models

$$\mathcal{L}_{|\mathbf{H}|} = -\frac{R}{12}|\mathbf{H}|^2 + \frac{1}{2}|D^{\alpha}\mathbf{H}||D^{\beta}\mathbf{H}|g_{\alpha\beta} - \mu_0|\mathbf{H}|^2 + \lambda_0|\mathbf{H}|^4$$

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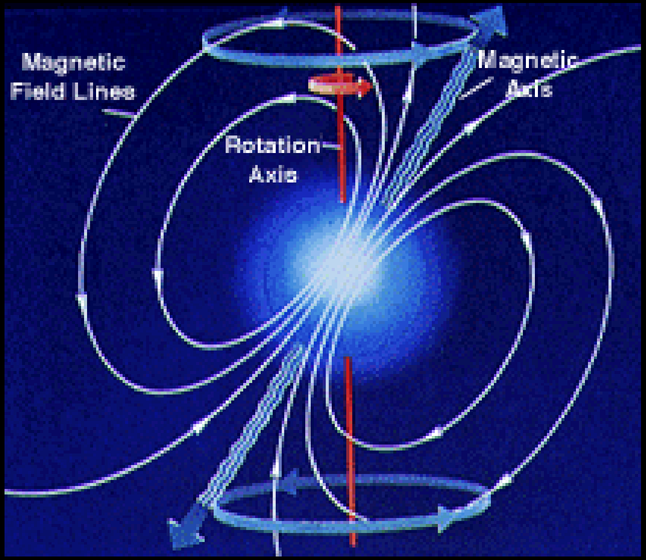
link with compactifid string models

scalar field (higgs) with nonzero coupling to background geometry and its mass /dynamics explicitly dependent of local matter content

link with chameleon cosmology

gravitational waves in NCSG



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yukawa parameters and majorana terms for ν_R

linear perturbations around minkowski background in synchronous gauge:

$$g_{\mu\nu} = \mathrm{diag}\left(\{a(t)\}^2\left[-1, \left(\delta_{ij} + h_{ij}\left(x\right)\right)\right]\right)$$

$a(t) = 1$ $\nabla_i h^{ij} = 0$

nelson, ochoa, sakellariadou, PRD 82 (2010) 085021

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$$\boxed{\left(\Box - \beta^2\right)\Box h^{\mu\nu} = \beta^2 \frac{16\pi G}{c^4} T^{\mu\nu}_{\text{matter}}}$$

with conservation eqs: $\dfrac{\partial}{\partial x^\mu} T^\mu_\nu = 0$

$$\beta^2 = -\frac{1}{32\pi G \alpha_0} \qquad \alpha_0 = \frac{-3f_0}{10\pi^2}$$

$$\frac{g_3^2 f_0}{2\pi^2} = \frac{1}{4} \qquad g_3^2 = g_2^2 = \tfrac{5}{3} g_1^2$$

nelson, ochoa, sakellariadou, PRD 82 (2010) 085021

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nelson, ochoa, sakellariadou, PRD 82 (2010) 085021 energy lost to gravitational radiation by orbiting binaries:

$$-\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} \approx \frac{c^2}{20G}|\mathbf{r}|^2 \dot{h}_{ij}\dot{h}^{ij}$$

nelson, ochoa, sakellariadou, PRD 82 (2010) 085021

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strong deviations from GR at frequency scale

$$2\omega_c \equiv \beta c \sim (f_0 G)^{-1/2} c$$

set by the moments of the test function f

scale at which NCSG effects become dominant

nelson, ochoa, sakellariadou, PRD 82 (2010) 085021 restrict β by requiring that the magnitude of deviations from general relativity must by less than the uncertainty

Binary	Distance (pc)	Orbital Period (hr)	Eccentricity	GR (%)
PSR J0737-3039	~ 500	2.454	0.088	0.2
PSR J1012-5307	~ 840	14.5	$< 10^{-6}$	10
PSR J1141-6545	> 3700	4.74	0.17	6
PSR B1916+16	~ 6400	7.752	0.617	0.1
PSR B1534+12	~ 1100	10.1	?	1
PSR B2127+11C	~ 9980	8.045	0.68	3

nelson, ochoa, sakellariadou, PRL 105 (2010) 101602

accuracy to which the rate of change of orbital period agrees with predictions of general relativity restrict β by requiring that the magnitude of deviations from general relativity must by less than the uncertainty

Binary	Distance (pc)	Orbital Period (hr)	Eccentricity	GR (%)
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PSR J1141-[illegible]	[illegible]	[illegible]	[illegible]	6
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PSR B2127+11C	~ 9980	8.045	0.68	3

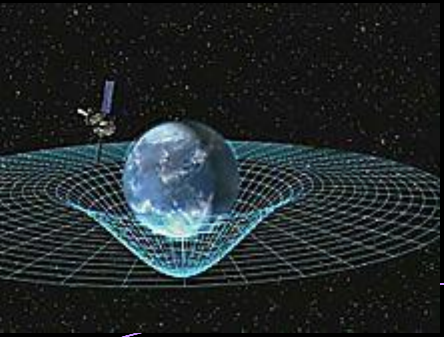
$$\beta > 7.55 \times 10^{-13}\,\mathrm{m}^{-1}$$

nelson, ochoa, sakellariadou, PRL 105 (2010) 101602

accuracy to which the rate of change of orbital period agrees with predictions of general relativity

gravity probe B

the satellite contains a set of gyroscopes in low circular polar orbit with altitude h=650 km



geodesic precession in the orbital plane
lense-thirring (frame dragging) precession in the plane of earth equator

GR

Effect	Measured	Predicted
Geodesic precession	6602 ± 18	6606
Lense-Thirring precession	37.2 ± 7.2	39.2

milliarcsec/yr

lambiase, sakellariadou, stabile, JCAP 12 (2013) 020

instantaneous geodesic precession

$$\Omega_{\rm geodesic} = \Omega_{\rm geodesic(GR)} + \Omega_{\rm geodesic(NCG)}$$

fixed to the GR predicted value

$$|\Omega_{\rm geodesic(NCG)}| \leq \delta\Omega_{\rm geodesic}$$

$$\delta\Omega_{\rm geodesic} = 18 \ \ {\rm mas/y}$$

$$\boxed{\beta > 7.1 \times 10^{-5}{\rm m}^{-1}}$$

lambiase, sakellariadou, stabile, JCAP 12 (2013) 020

instantaneous geodesic precession

$$\Omega_{\mathrm{geodesic}} = \Omega_{\mathrm{geodesic(GR)}} + \Omega_{\mathrm{geodesic(NCG)}}$$

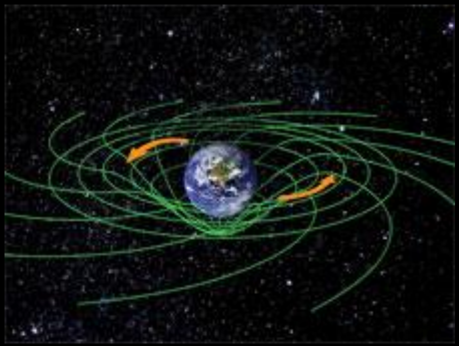
fixed to the GR predicted value

$$|\Omega_{\mathrm{geodesic(NCG)}}| \leq \delta\Omega_{\mathrm{geodesic}}$$

$$\delta\Omega_{\mathrm{geodesic}} = 18 \ \mathrm{mas/y}$$

$$\boxed{\beta > 7.1 \times 10^{-5}\mathrm{m}^{-1}}$$

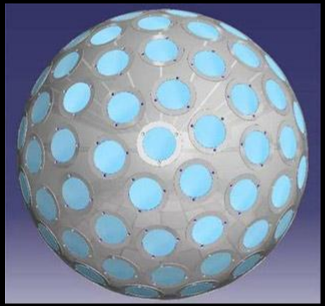
instantaneous lense-thirring precession



lambiase, sakellariadou, stabile, JCAP 12 (2013) 020

LARES (laser relativity satellite)

accuracy of ~1% in the measurement of the lense-thirring effect



LARES was inserted in an orbit with 1450 km of perigee, an inclination of 69.5 degrees and eccentricity 9.54×10^{-4}

$$\boxed{\beta > 1.2 \times 10^{-6} \mathrm{m}^{-1}}$$

capozziello, lambiase, sakellariadou, an. stabile, a. Stabile (arXiv:1410.8316)

constraints from torsion balance

lambiase, sakellariadou, stabile, JCAP 12 (2013) 020

constraints from torsion balance

the modifications by NCSG action to the newtonian potentials

Φ , Ψ $$ds^2 = -(1+2\Phi)dt^2 + 2\mathbf{A}\cdot d\mathbf{x}dt + (1+2\Psi)d\mathbf{x}^2$$

are similar to those by a fifth-force through a potential

$$V(r) = -\frac{GMm}{r}\left(1+\alpha e^{-r/\lambda}\right)$$

$$\lambda = \beta^{-1} \qquad \alpha \sim \mathcal{O}(1)$$

eot-wash and irvine experiments

$$\lambda \lesssim 10^{-4}\mathrm{m} \Longrightarrow \boxed{\beta \geq 10^4 \ \mathrm{m}^{-1}}$$

..

lambiase, sakellariadou, stabile, JCAP 12 (2013) 020

the rôle of scalar fields

inflation through the nonminimal coupling between the geometry and the higgs field

inflation through the nonminimal coupling between the geometry and the higgs field

<u>proposal</u>: the higgs field, could play the rôle of the inflaton

but

GR: to get the amplitude of density perturbations, the higgs mass would have to be 11 orders of magnitude **higher**

re-examine the validity of this statement within NCSG

nelson, sakellariadou, PLB <u>680</u> (2009) 263

buck, fairbairn, sakellariadou, PRD <u>82</u> (2010) 043509 effective potential at high energies: $V(H) = \lambda(H) H^4$

running of the higgs self-coupling at two-loops:

$\Rightarrow$ slow-roll conditions satisfied

buck, fairbairn, sakellariadou, PRD 82 (2010) 043509

effective potential at high energies: $V(H) = \lambda(H) H^4$

running of the higgs self-coupling at two-loops:

$\Rightarrow$ slow-roll conditions satisfied but...

ϵ needs to be too small to allow for sufficient e-folds $N \sim \epsilon^{-1/2} \mathrm{d}\phi$

$\Rightarrow$ $(V_\star/\epsilon_\star)^{1/4}$ becomes too large to fit the CMB constraint

buck, fairbairn, sakellariadou, PRD 82 (2010) 043509

effective potential at high energies: $V(H) = \lambda(H)H^4$

running of the higgs self-coupling at two-loops:

⇒ slow-roll conditions satisfied but...
CMB constraints cannot be satisfied
(incompatibility with top quark mass)

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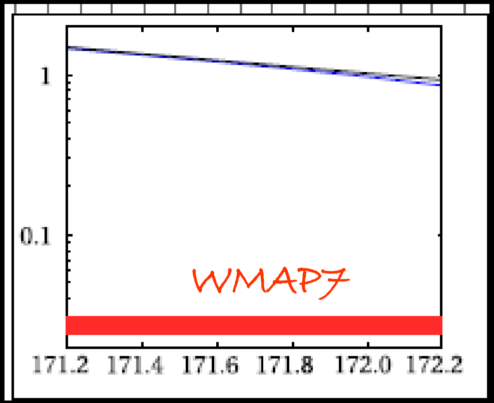

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discussion

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bosonic spectral action

- description of geometry in terms of spectral properties of operators

⇒ model of particle interactions very close to real phenomenology, compatible with measured higgs at 125 GeV

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- - possibility to infer quantities related to the boson based on input of only fermionic parameters in fluctuated Dirac operator
  - the Dirac operator defines also the fermionic part of the action

open questions/current progress:

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- dependence on cutoff function
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define the bosonic spectral action via the zeta function

kurkov, lizzi, sakellariadou, watcharangkool (in progress)

NCSG: a geometrical theory that unifies gravity and the theory of SM on a classical level

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- almost commutative manifolds are based on riemannian ST

⟹ to do physics we need a generalisation to psuedo-riemannian ST (e.g. lorentzian or minkowskian)

hawkins (1997)
moretti (2003)
pashke, verch (2004)
van suijlekom (2004)
paschke, sitarz (2006)
strohmaier (2006)
franco (2010)
van den dungen, pashke, rennie (2012)

NCSG: a geometrical theory that unifies gravity and the theory of SM on a classical level

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aastrup, grimstrup (2006, 2007)
marcolli, zainy, yasry (2008)
aastrup, grimstrup, nest (2009)

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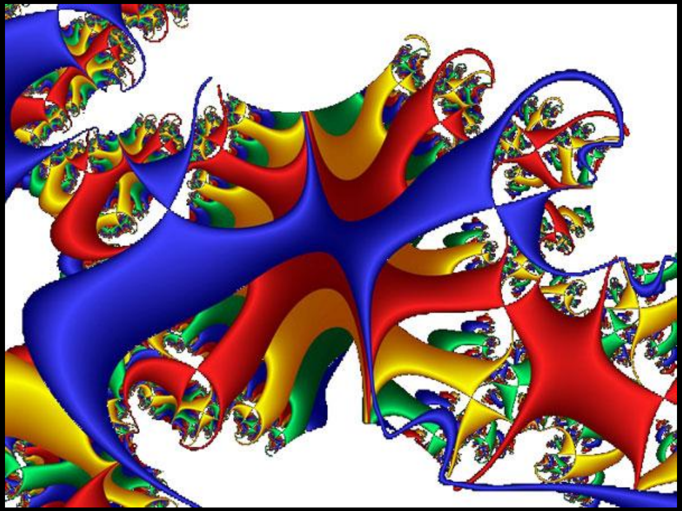
get within NCG setup a unified and geometrical formulation of QG with matter

NCG: duality between geometry and algebra, with a striking coincidence between the algebraic rules and the linguistic ones



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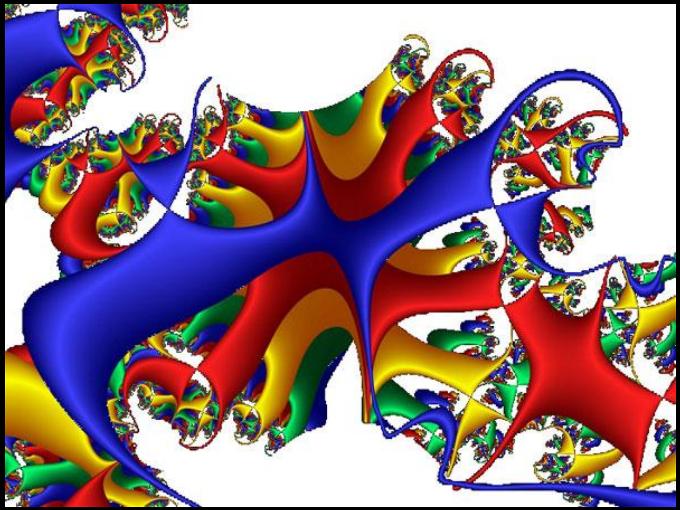

spectral info on distant universe : red shift is scaling of frequencies

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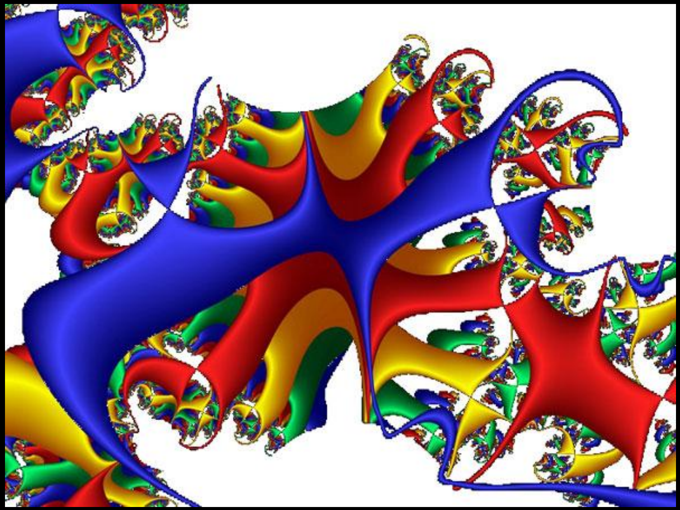

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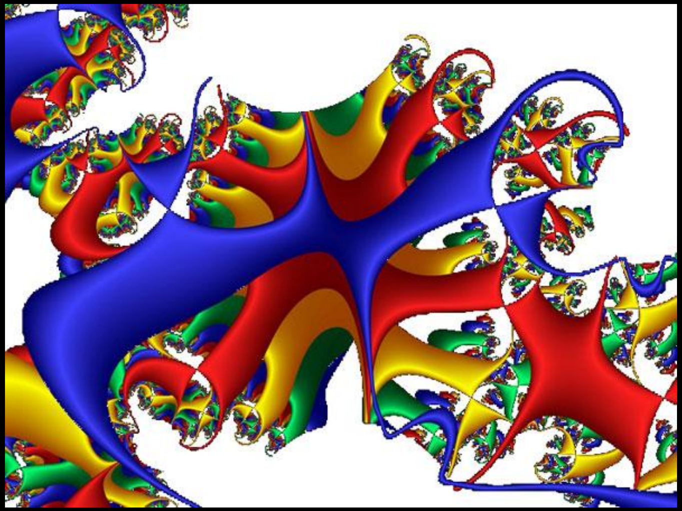

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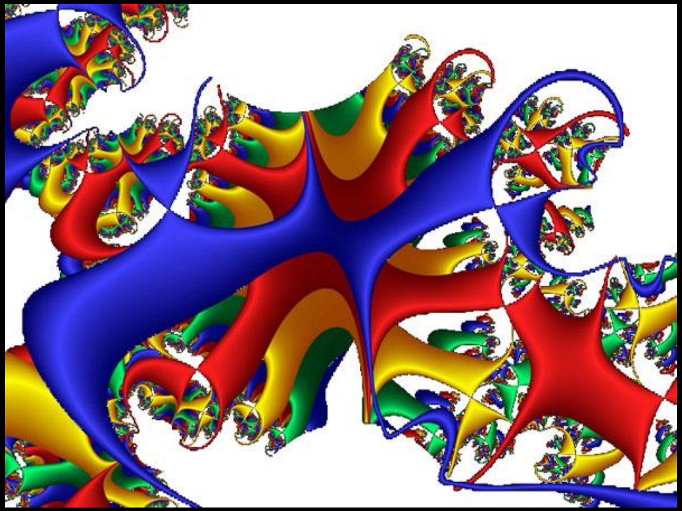
at Planck's temperature, there was no geometry at all and only after a phase transition a SSB selected a particular geometry and hence the particular universe in which we live



thank you