Discrete Symmetries in string/M theory and Grand Unification

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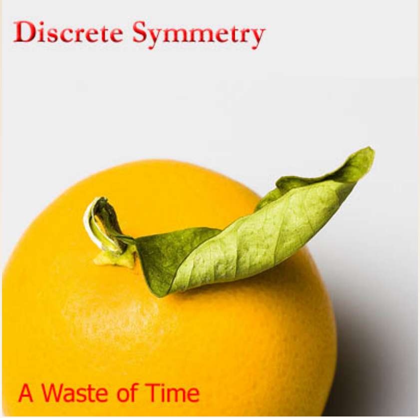
Perusing google images for "Discrete Symmetry" one comes up with fairly standard mathematics and physics links and pictures.
However, it also returned this:



which is a "randomly generated" band, album title and cover!

http://timdrussell.blogspot.co.uk/2008_01_01_archive.html

The Standard Model is a remarkable achievement of human thought
It is the most precisely tested and verified model in history.

BUT

THE UNIVERSE

IS DARK!!!

MOST OF THE MATTER IN OUR UNIVERSE IS **NOT** MADE OF STANDARD MODEL PARTICLES

The Standard Model doesn't describe Dark Matter

The Standard Model has an accidental Baryon number and Lepton Number symmetry

A generic extension of the Standard Model will not conserve B and L exactly.

What do we know about Dark Matter?

What do we know about Dark Matter?

NOT A LOT:

What do we know about Dark Matter?

DM properties

- Non-relativistic today
- Electrically neutral
- Has a long lifetime
- We have some knowledge of its distribution in the galaxy

WE HAVE NO CONSTRAINTS ON THE TYPE OF PARTICLE(S) THAT DARK MATTER IS COMPOSED OF, NOR THE MASSES OF THOSE PARTICLES

This talk

- Review some basic aspects of string/M theory and origin of discrete symmetries
- Review arguments that we generically expect "non-thermal" Dark Matter
- Review role of discrete symmetries in phenomenological string/M theory models
- Show that considerations of discrete symmetry breaking imply dark matter is "in the hidden sector"

This talk

Much of the work underlying this talk is reviewed in arXiv:1204.2795 The work on discrete symmetry is based on 1102.0556 and 1403.4948

My collaborators: Gordy Kane, Piyush Kumar, Konstantin Bobkov, Ran Lu, Scott Watson, Jing Shao, Bob Zheng, Diana Vaman

Also: some forthcoming work on discrete symmetries and SO10: with Steve King, Chakrit Pongkitivanichkul, Krzysek Bozek, Miguel Crispim Romao.

What we know about string/M theory

- There are six or seven extra dimensions of space
- The low energy fluctuations of these dimensions are moduli fields (φ)
- Couplings and masses in the Standard Model are determined by moduli VEVs
- The fact that α_{em} and other couplings are small implies the extra dimensions are NOT all Planckian!
- Moduli couple (semi) universally to all matter through m_{pl} suppressed interactions
- **They generically dominate the post-inflationary, pre BBN Universe!!**

What is "Generic"?

- We are interested in "generic" properties of string/M theory vacua with all moduli stabilised and a realistic observable sector
- We also, usually, assume Grand Unification and low energy supersymmetry breaking
- "Generic": not a theorem, might be avoided in "special cases"
- One would have to work fairly hard to construct a "non-generic" example.

What is "Generic?", an example

- Asking what the generic predictions of QFT are is **NOT** a good question (CPT, spin/stats aside).
- The Standard Model is one of infinitely many QFT's.
- The SM describes very precisely the results of all particle physics experiments and that is good enough
- A better question: what are the generic predictions of non-Abelian gauge theories with chiral fermions, hierarchical Yukawa couplings and spontaneous symmetry breaking?
- Answer: Charged currents, massive vector bosons and a rich spectrum of three-body decays of heavier fermions into lighter ones.
- The discovery of the muon, the tau, the W-boson and their decay properties represents a verification of these generic predictions.
- **We will elicit generic predictions about BSM physics from string/M theory.**

How can we justify our assumptions?

- How do we justify our assumption that solutions of string/M theory exist which reduce to the Standard Model at low energies?
- In 1984 (Candelas, Horowitz, Strominger and Witten) showed that a generic 4d solution of heterotic string theory is described by
  - A non-Abelian gauge theory with
  - Multiple generations of Chiral fermions
  - Hierarchical Yukawa couplings
- Similar statements are true in other perturbative limits eg Type IIA (Berkooz/Douglas/Leigh, Cvetic/Shiu et al), Type IIB (Ibanez et al, Blumenhagen/Lust) or M theory on a G_2-manifold (BSA/Witten).
- The assumptions about low energy supersymmetry and unification are well motivated.

Low energy effective theory from string/M theory

- For concreteness, assume the observable sector field content is that of the MSSM
- The LEFT is a 4d supergravity with (super) fields: the MSSM fields plus moduli and axion fields
- *ALL masses and couplings in the Lagrangian are functions of the moduli*
- On top of that the moduli and axions have kinetic and potential terms
- So it is crucial to understand the moduli and axion dynamics to understand the low energy phenomenology
- There has been substantial progress in this in the last 10 to 15 years.

Life with the Cosmological Moduli Problem

- The cosmological moduli problem plays a central role
- Goes back to Polonyi and (Coughlan, Ellis, Ross)
- In string theory: (de Carlos, Casas, Quevedo, Roulet) and (Banks, Kaplan, Nelson)
- When $H \sim m_\varphi$, φ starts oscillating and quickly dominates $\rho_{universe}$
- φ decays when $H \sim \Gamma_\varphi \sim \frac{m_\varphi^3}{m_{pl}^2}$
- $m_\varphi \geq 10$ TeV for this to happen before BBN
- Very difficult to avoid this unless you can arrange for H smaller than m_φ or for a late period of inflation
- **Generically expect moduli to dominate $\rho_{universe}$ up to BBN**

Life with the Cosmological Moduli Problem

- **Generically expect moduli to dominate $\rho_{universe}$ up to BBN**
- Now, unless you fine-tune, $m_{\varphi} \sim m_{3/2}$:*this is a fact about supergravity*
- This implies that ALL scalars in the theory have mass terms which are at least $m_{3/2}$
- Includes squarks and sleptons, $m_{scalar} \geq 10$ TeV
- In the MSSM this correlates beautifully with $m_{Higgs} \sim 125$ GeV!!
- Fermions like Higgsinos and gauginos can have smaller masses as their F-terms have a different origin
- So we expect a BSM spectrum with scalar masses of order 10 TeV and fermion masses of order TeV (or less).

Life with the Cosmological Moduli Problem

- **Generically expect moduli to dominate $\rho_{universe}$ up to BBN**
- The moduli decay releases a lot of entropy, $\frac{\Delta s}{s} \sim 10^{10}$
- Dilutes "prior relics" by ten orders of magnitude in density
- So thermal relics are not typically relevant here
- **Dark Matter generically predicted to be non-thermally produced**
- Decay products of moduli could be stable and possible dark matter candidates
- Decays could also include "dark radiation".

IMPORTANT

Generic prediction:

Early universe pre-BBN is MATTER dominated

Dark Matter non-thermally produced

Discrete Symmetries

- In models with low energy supersymmetry one needs a mechanism to prevent the proton from decaying too quickly
- This is usually accomplished with a discrete symmetry, R-parity being the most prominent example
- R-parity is closely related to B and L
- Such a symmetry will also make the lightest supersymmetric particle stable (DM candidate)
- To some, the introduction of R-parity seems ad hoc

Discrete Symmetries

- What is the origin of B and L symmetry?
- This becomes morally equivalent to:
- What is the origin of R-parity?
- What can give rise to R-parity breaking?
- We will discuss some of these in the string/M theory context
- Most of the discussion applies to other discrete symmetries also

Discrete Symmetries in String/M theory

- In string/M theory the gauge group and matter representations are determined by properties of the extra dimensions:
- E.g. in heterotic Calabi-Yau solutions it depends on the extra dimensional gauge field configuration [Candelas et al]
- E.g. In M theory on G_2 manifolds it is determined by specific kinds of singularities [BSA and E.Witten]
- If the shape of the extra dimensions has a symmetry, this can act on the matter fields and appears as a symmetry in the effective four dimensional Lagrangian
- **Discrete symmetries often arise as geometic symmetries of the extra dimensions**

Discrete Symmetries in String/M theory

- In fact, G_2-holonomy and Calabi-Yau manifolds used for string/M theory models have **zero Ricci tensor**
- Such *Ricci flat*, irreducible, compact manifolds can *only have discrete symmetries*
- If this is the case, the effective field theory will have a discrete symmetry.
- Examples of this kind have been known for thirty years, [Strominger/Witten]
- CP and discrete flavour symmetries can arise from this mechanism

Discrete Symmetries and Moduli fields

- Is there a mechanism which fixes the extra dimensions to have a discrete symmetry?
- **NO!** in general not
- As discussed above, the effective theory contains moduli fields (φ)
- These are zero modes of natural Laplacians on the extra dimensions so are naturally charged under any discrete symmetry Γ.
- For *generic* moduli vevs, the symmetry will be broken
- This is because *for generic vevs, the manifold has no particular shape*

Discrete Symmetries in String/M theory

- So if we have a good understanding of moduli dynamics and potentials we have a framework for addressing the origin of B, L etc
- There has been a lot of progress in understanding moduli stabilisation and supersymmetry breaking in the past decade – see next slide

One very quick slide on Moduli stabilisation and supersymmetry breaking

- Lots of progress in last 15 years:
- Using fluxes: (Becker/Becker'96;Dasgupta/Rajesh/Sethi'99; Giddings/Kachru/Polchinski'01;BSA'02;KKLT'03)
- Large Volume IIB: (Balasubramanian, Berglund, Conlon, Quevedo, Cicoli, Suruliz, Valandro....hep-th/0502058....)
- M theory on G_2 manifolds: (BSA, Bobkov, Kane, Kumar, Vaman, Shao: hep-th: 0606262, 0701034, arxiv:0810.3285)
- *We now have fairly well understood models for moduli stabilisation*
- Bonus: the moduli potential often spontaneously breaks supersymmetry as well (as was expected)

Discrete Symmetries in String/M theory: Applications to particle phenomenology

- We will consider supersymmetric models, motivated by the hierarchy problem and grand unification
- In supersymmetric GUTs, one not only enocunters the "proton stability problem", but also the doublet-triplet splitting problem
- These are related to each other and are often solved with discrete symmetries
- In the remainder we will illustrate with an example originating in M theory on a G_2-manifold

Simple SU(5) model in M theory [Witten 2002]

- Observable sector field content is that of minimal $SU(5)$:
- three $\mathbf{10} \oplus \bar{\mathbf{5}}$ families and one $\mathbf{5} \oplus \bar{\mathbf{5}}$ Higgs sector
- $SU(5) \longrightarrow SU(3) \times SU(2) \times U(1)$ by Wilson lines at GUT scale
- $\Gamma = \mathbf{Z_N}$ prevents μ-term and RPV operators
- So proton decay is okay with $\mathbf{Z_N}$ conserved
- But $\mathbf{Z_N}$ must be broken as charged Higgsino's must have masses bigger than O(100)GeV
- In arXiv 1102.0556 and 1403.3948 we explored this using our understanding of moduli potentials

A (string/M) Theory of R-parity Breaking

- The low energy theory is basically "the MSSM coupled to moduli and axions"
- *All masses and couplings in the MSSM are functions of moduli VEVs*
- e.g. $\frac{1}{g^2}F_{\mu\nu}^2 \longrightarrow \frac{\varphi}{m_{pl}}F_{\mu\nu}^2$
- The "offending" operators are in the superpotential (flavour indices suppressed)
- $W_{RPV} \sim \lambda' LLE^c + \lambda'' LQD^c + \lambda''' U^c U^c D^c$
- $W_{RPV} \sim \epsilon\mu L H_u$
- With understanding of moduli potentials we can calculate the couplings generated via the moduli vevs

The G_2-MSSM

- The observable sector consists of the $MSSM$ particle content plus moduli and axions
- All scalar masses at the GUT scale are order $m_{3/2} \sim 10$TeV except the axions (shift symmetry)
- This includes the moduli and squarks and sleptons
- Gauginos and Higgsinos have suppressed masses
- When Witten's $\mathbf{Z_N}$ symmetry is exact the LSP is stable
- Moduli dominate the early Universe and decay just before BBN
- There is a non-thermal DM candidate: a 100 to 200 GeV mass neutral W-ino

Life with the Cosmological Moduli Problem

- A neutral W-ino with mass of order 120 - 250 GeV has correct relic abundance to be a non-thermal DM particle
- This is basically ruled out now from the continuum photon spectrum measurements of Fermi (e.g. Cohen/Lisanti/Pierce/Slatyer and Fan/Reece)
- *But, it does not seem generically possible to make the W-ino LSP decay without fine-tuning and violating constraints on neutrino masses*
- THUS MUST LOOK BEYOND THE MSSM for Dark Matter!!

Just to show "some" calculations:

$$\begin{aligned}\mu_\alpha &= m_{3/2}\, K_{H_u \alpha} - F^{\bar{k}}\, K_{H_u\, \alpha\, \bar{k}} \\ B\mu_\alpha &= (2\, m_{3/2}^2\, K_{H_u \alpha} - m_{3/2}\, F^{\bar{k}}\, K_{H_u\, \alpha\, \bar{k}} + m_{3/2}\, F^k\, K_{H_u\, \alpha\, k}) - \\ &\quad (m_{3/2} F^m\, K^{n\bar{l}} K_{\bar{l} m H_u} K_{n\alpha} + (H_u \leftrightarrow L_\alpha)) - \\ &\quad F^n\, F^{\bar{m}} (\frac{1}{2} K_{H_u \alpha n \bar{m}} - K^{j\bar{l}} K_{\bar{l} n H_u} K_{j \bar{m} \alpha} + (H_u \leftrightarrow L_\alpha)) \end{aligned} \quad (1)$$

From the leading contribution to the Kahler potential terms:

$$K \supset P_\alpha \frac{(\varphi^1)^\dagger}{m_{pl}}\, H_u\, L_\alpha + Q_\alpha \frac{(\varphi^2)}{m_{pl}}\, H_u\, L_\alpha, \quad (2)$$

where P_α, Q_α are $\mathcal{O}(1)$ coefficients in general, one finds :

$$\begin{aligned} \mu_\alpha &= P_\alpha \frac{(\varphi^1)^\dagger}{m_{pl}} m_{3/2} + P_\alpha \frac{\langle F^{\varphi_1} \rangle}{m_{pl}} \\ B\mu_\alpha &= 2P_\alpha \frac{(\varphi^1)^\dagger}{m_{pl}} m_{3/2}^2 + P_\alpha \frac{(\varphi^1)^\dagger}{m_{pl}} m_{3/2} + Q_\alpha \frac{\langle F^{\varphi_2} \rangle}{m_{pl}} m_{3/2} \end{aligned} \quad (3)$$

Then, using the result that $\langle F^{\varphi_i} \rangle \ll \langle \varphi_i \rangle m_{3/2}$ [1102.0556] so that the terms proportional to $\langle F^{\varphi_1} \rangle$ and $\langle F^{\varphi_2} \rangle$ are negligible to a very good approximation, one gets:

$$\mu \sim \mu_\alpha \sim \langle \frac{\varphi_i}{m_{pl}} \rangle m_{3/2} \sim 0.1 m_{3/2} \quad (4)$$

$$B\mu_\alpha \simeq 2\, m_{3/2}\, \mu_\alpha \quad (5)$$

Fate of Discrete Symmetry?

- The fact that μ_α is generically of order $\mu \geq 100$ GeV is ruled out by neutrino masses (Hall/Suzuki'84)
- One possibility is that $\mathbf{Z_N}$ is only partially broken to a subgroup which prevents μ_α.
- Another is that, since the $\mathbf{Z_N}$ charges of L_α and H_d are different, if the moduli have only certain charges, a product of moduli is required to generate $L_\alpha H_u$ e.g. $\mu_\alpha \sim \langle \varphi_1 \varphi_2 \varphi_3 .. \rangle$
- Either way, in the MSSM case, the LSP still has to decay to avoid the Fermi constraints
- **Dark Matter is in the Hidden Sector!**

Summary

- In supersymmetric GUTS one often uses discrete symmetries to solve doublet-triplet splitting problem
- In string/M theory these symmetries are often geometric symmetries of extra dimensions
- The non-zero Higgsino mass requires the symmetry to be broken
- Generic moduli vevs do break the symmetry
- Generically though problems with a) neutrino masses and b) Fermi continuum gamma ray spectrum
- These tend to imply that dark matter must be in the "hidden sector" i.e. not charged under $SU(5)$

Take Home Messages

Dark Matter is non-thermally produced

Dark Matter is in a hidden sector and is unlikely to be a WIMP

THANK YOU!!

BACKUP

Z_{18} example

Q_L	U_R	D_R	L	E	H_u	H_d	T_u	T_d
4	15	7	10	1	17	7	9	9

The anomaly coefficients are:

$$A_3 = \frac{3}{2}\left(2q_{Q_L} + q_{U_R} + q_{D_R}\right) + \frac{1}{2}\left(q_{T_u} + q_{T_d}\right) = 54 = 0 \bmod 9$$

$$A_2 = \frac{3}{2}\left(3q_{Q_L} + q_L\right) + \frac{1}{2}\left(q_{H_u} + q_{H_d}\right) = 45 = 0 \bmod 9$$

$$A_1 = \frac{9}{5}\left(\frac{1}{6}q_{Q_L} + \frac{4}{3}q_{U_R} + \frac{1}{3}q_{D_R} + \frac{1}{2}q_L + q_E\right) + \frac{3}{10}\left(q_{H_u} + q_{H_d}\right) + \frac{1}{5}\left(q_{T_u} + q_{T_d}\right) = 63 = 0 \bmod 9$$

Z_{18} example

Q_L	U_R	D_R	L	E	H_u	H_d	T_u	T_d
4	15	7	10	1	17	7	9	9

NOTE: the charges of different elements of GUT matter multiplets are different!

This does not disturb gauge coupling unification

So can have anomaly free *non-R*-symmetries.

Hidden Sector Dark Matter

- What forms can Hidden Sector Dark Matter take?
- Hidden Sector = not charged under Standard Model Gauge Group
- String/M theory can provide some guidance here
- In the remainder we will see how this works for M theory on a G_2-manifold
- These are potentially the largest class of "geometric" solutions in the string/M theory landscape
- More than heterotic and Type IIA.
- Not known if there are more G_2 solutions compared to F-theory.

M theory on a G_2-manifold

- Seven Extra dimensions form a manifold X with a metric of holonomy G_2
- X is like a 7d version of a Calabi-Yau space
- If X is smooth, low energy physics is 4d supergravity with an Abelian gauge group and light, uncharged chiral superfields.
- The chiral superfields contain scalars which are the moduli of X and axions
- In general there can be lots of $U(1)$'s and moduli and axions

M theory on a G_2-manifold

- Light charged particles are **localised** at particular singularities of X
- Non-Abelian gauge fields localised on 3d submanifolds $Q_i \subset X$
- Chiral fermions localised at points $p_A \subset Q_i \subset X$
- Generically the Q_i's do NOT intersect $\rightarrow$ NO light bi-fundamental matter
- So, e.g. gauge mediation is disfavored *geometrically*

Light particles from M theory on a G_2-manifold

- Can have lots of very light axions with a range of masses (Axiverse)
- Can have free $U(1)$'s, but their number could be anything from zero to thousands(?)
- Can have light (chiral) fermions and scalars charged under $U(1)$'s
- Can have simple gauge groups G_i along Q_i
- Light charged fermions and scalars can be charged under G_i in any representation smaller than Adjoint(G_i)
- This latter matter is localised at points on the Q_i's
- Generically no matter is charged under more than one G_i
- Wilson lines can break the G_i's into subgroups e.g. $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$

Dark Matter from M theory on a G_2-manifold

- Light Axions are always present and typically have GUT scale decay constants
- Dark matter will generically have axionic components
- Stable Hidden sector particles might be dark matter also if produced through moduli decays: BUT one typically requires an efficient enough mechanism to reduce their abundance
- One important constraint here is to not produce too much dark radiation
- Lets discuss mixing with the Standard Model particles

Hidden to Observable interactions

- Hidden $U(1)$'s will generically mix with $U(1)_Y$ since generically there will be massive, GUT scale states charged under both (e.g. strings/branes stretched from Q_1 to Q_2.
- Integrating these out gives mixing parameters ϵ at one loop
- So, $\epsilon \sim 10^{-3}$-ish
- One could also have a similar sized "Higgs portal"
- These small mixings suppress annihilations of Hidden Dark Matter into SM particles
- Can we classify or categorize the Hidden DM models that actually work this way?
- I hope we can discuss this further during the workshop

Dark Matter from M theory on a G_2-manifold

- Some of the G_i's will be confining at low energies
- The lightest G_i hadron may often be stable
- Can G_i hadrons be dark matter candidates in string/M theory?
- This will be partially answered in (BSA, M. Fairbairn) to appear.

Dark Radiation in the Axiverse (with Chakrit Pongkitivanichkul

- String/M theory Axiverse – spectrum of lots very light axions with potentially interesting phenomenology (Arvinitaki et al)
- Computed in string/M theory in vacua with broken supersymmetry and stable moduli (BSA, K. Bobkov, P. Kumar; Cicoli, Conlon, Quevedo)
- When moduli decay, these light axions will be produced and be relativistic i.e. dark radiation $\rightarrow N_{eff}$
- This has been computed for simple cases with one or two axions (Cicoli et al, Higaki et al)
- **In general though you expect $N_{eff} \propto N$ the number of axions**
- **This is the Axiverse induced dark radiation problem.**

Dark Radiation in the Axiverse (with Chakrit Pongkitivanichkul

$$moduli - axion : \mathcal{L} = \sum_{i=1}^{N} \sum_{j=1}^{N} C_{ij} U_{ik} \varphi_k \partial_\mu t_j \partial^\mu t_j \qquad (6)$$

$$moduli - gaugeboson : \mathcal{L} = \sum_{i=1}^{N} B_i U_{ik} \varphi_k F_{\mu\nu} F^{\mu\nu} \qquad (7)$$

$$moduli - scalar : \mathcal{L} = \sum_{i=1}^{N} D_i U_{ik} \varphi_k D_\mu f D^\mu f \qquad (8)$$

the C, B's and D's are coefficients and U is the moduli mixing matrix

Dark Radiation in the Axiverse (with Chakrit Pongkitivanichkul

Decay widths:

$$
\begin{aligned}
\Gamma_{\text{axions}} &= \sum_{j=1}^{N} \Gamma(\varphi_k \to t_j t_j) \\
&= \sum_{j=1}^{N} \left(\sum_{i=1}^{N} C_{ij} U_{ik} \right)^2 \frac{m_{\varphi_k}^3}{M_{PL}^2} \\
\Gamma_{\text{gauge particles}} &= n_G \left(\sum_{i=1}^{N} B_i U_{ik} \right)^2 \frac{m_{\varphi_k}^3}{M_{PL}^2} \\
\Gamma_{\text{fermions/sfermions}} &= n_f \left(\sum D_i U_{ik} \right)^2 \frac{m_{\varphi_k}^3}{M_{PL}^2} \qquad (9)
\end{aligned}
$$

Dark Radiation in the Axiverse (with Chakrit Pongkitivanichkul)

$$\langle \Delta N_{eff} \rangle \propto \frac{\Gamma_{axions}}{\Gamma_{visible}} \propto \frac{N \langle C \rangle^2}{n_G \langle B \rangle^2 + n_f \langle D \rangle^2} \propto N \qquad (10)$$

This is the Axiverse induced Dark Radiation problem.

Solutions to the problem

The special Kahler potentials for moduli which can arise from string/M theory can actually solve the problem!

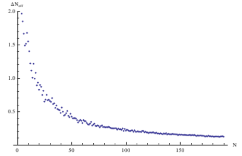


Figure 1: Result from geometric sequence configurations showing ΔN_{eff} as a function of N, where $r = 2$

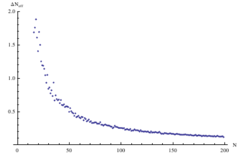


Figure 2: Result from double moduli dominated configurations showing ΔN_{eff} as a function of N, where $\epsilon N = 0.1$

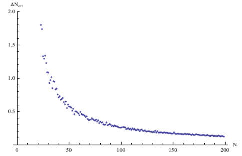


Figure 3: Result from triple moduli dominated configurations showing ΔN_{eff} as a function of N, where $\epsilon N = 0.1$

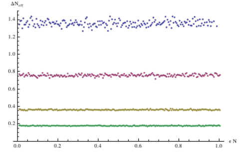


Figure 4: Result from double moduli dominated configurations showing ΔN_{eff} as a function of ϵN, where points in blue, red, yellow, green are $N = 30, 50, 100, 200$ respectively

Additional Conclusions

- DM will include axions with GUT scale decay constants. More effort should go into ideas for detecting these
- It could also include Hidden Matter originating from G_i and $U(1)$ gauge theories
- This opens up a lot of possibilities and I hope we can start to classify them
- In this case, the only way to have "direct detection" is if there is a low energy $U(1)'$ in the gauge theory or a suitable Higgs portal.
- In this sense, we would have to be "lucky" for it to work out that way, but it might be possible.