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A 3d effective lattice theory for Yang-Mills and QCD thermodynamics

HIC for FAIR Helmholtz International Center

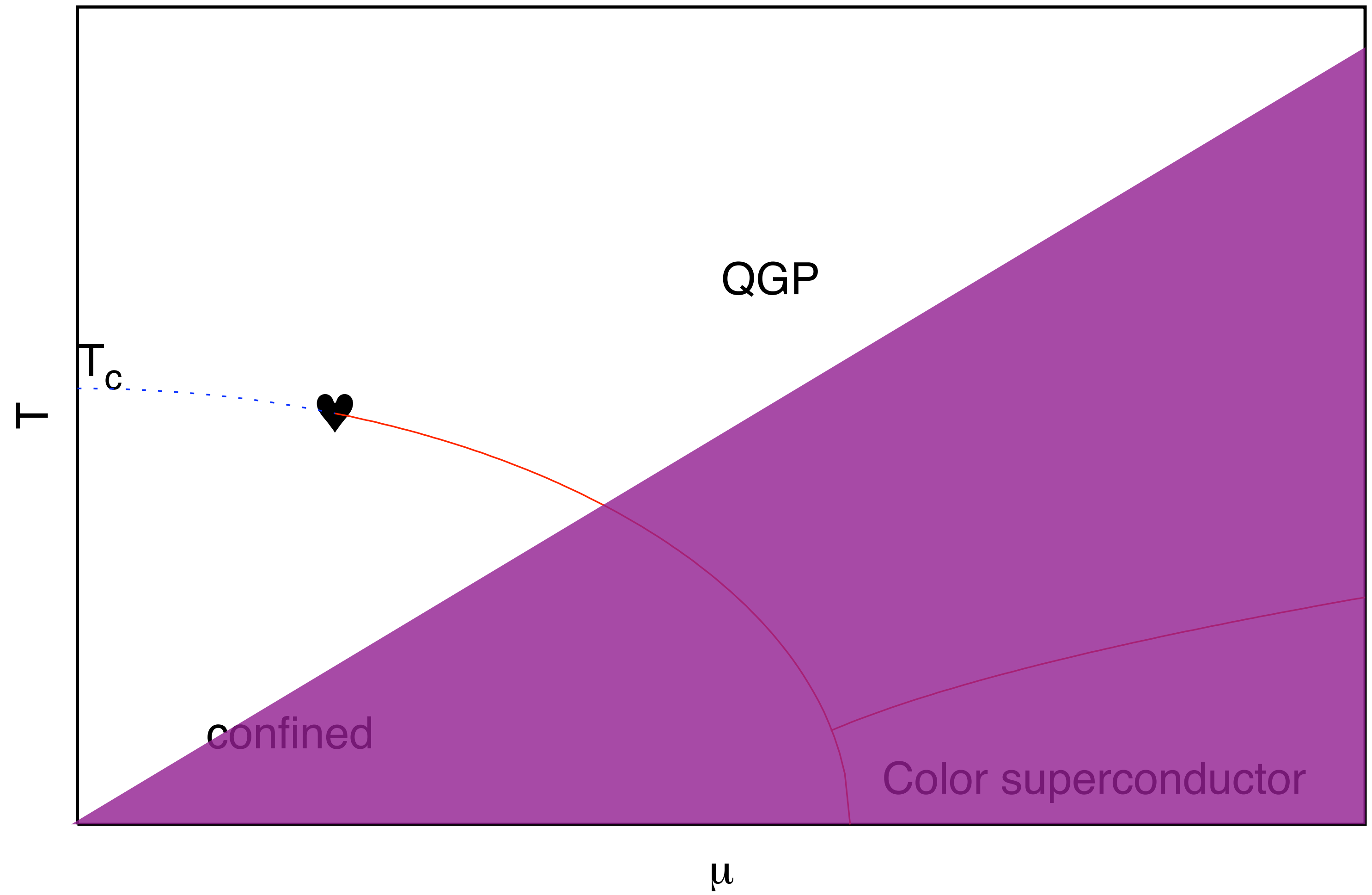
Owe Philipsen

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Collaborators: G. Bergner, M. Fromm, J. Langelage, S. Lottini, M. Neuman

- 3d effective lattice theories derived by strong coupling methods
- The deconfinement transition in Yang-Mills theory JHEP 1102 (2011) 057
- The deconfinement transition in QCD with heavy dynamical quarks JHEP 1201 (2012) 042
- Cold and dense QCD: transition to nuclear matter arXiv:1207.3005

The (lattice) calculable region of the QCD phase diagram



- Sign problem prohibits direct simulation, circumvented by approximate methods: reweigthing, Taylor expansion, imaginary chem. pot., need $\mu/T \lesssim 1 \quad (\mu = \mu_B/3)$
- No critical point in the controllable region
- Flux representations + worm algorithm, complex Langevin: only particular models

Large densities ?!?

Effective theories!

- Yang-Mills theory has a finite T phase tranisition, breaking of center symmetry (ZN)
- Dimensional reduction uses scale "hierarchy": $g^2T < gT < 2\pi T$

  Integrate hard scale perturbatively, treat eff. 3d theory on lattice,
  valid for weak coupling (deconfined phase)
- Does not work for transition, perturbative dim. red. breaks Z(N) of YM theory
- Bottom-up construction of Z(N)-invariant theory by matching:

  works for SU(2), unfinished for SU(3)
  Vuorinen, Yaffe; de Forcrand, Kurkela; Kurkela, Vuorinen; ....
- Here: solution for YM by strong coupling expansion (confined phase)!
- QCD with heavy fermions: sign problem of eff. theory mild, curable!

Starting point: Wilson's lattice Yang-Mills action

Partition function; link variables as degrees of freedom

$$Z = \int \prod_{x,\mu} dU(x;\mu) \exp(-S_{YM}) \equiv \int DU \exp(-S_{YM})$$

Wilson's gauge action

$$S_W = -\frac{\beta}{N} \sum_p \mathrm{ReTr}(U_p) = \sum_p S_p \qquad \beta = \frac{2N}{g^2}$$

Plaquette:

$$\Box \to 1 + ia^2 gF_{\mu\nu} - \frac{a^4 g^2}{2} F_{\mu\nu}F^{\mu\nu} + O(a^6) + \ldots$$

$U_\mu(x) = \mathrm{e}^{-\mathrm{i}agA_\mu(x)}$

$$T = \frac{1}{aN_t} \qquad \text{continuum limit} \quad a \to 0, N_t \to \infty$$

Small $\beta(a) \Rightarrow$ small T

The effective theory, Yang-Mills

- Split temporal and spatial link integration and use character expansion ($a_r(\beta)$: expansion parameter of representation r)

$$
\begin{aligned}
Z &= \int [dW] \exp\left\{ \ln \int [dU_i] \prod_p \left[1 + \sum_{r \neq 0} d_r a_r(\beta) \chi_r(U_p) \right] \right\} \\
&\equiv \int [dW] \exp\left[-S_{eff}\right] \qquad W(\vec{x}) = \prod_{\tau=1}^{N_\tau} U_0(\tau, \vec{x})
\end{aligned}
$$

Expansion parameter: $u = a_f(\beta) = \beta/18 + \cdots$

$$
-S_{eff} = \lambda_1 S_1 + \lambda_2 S_2 + \lambda_3 S_3 + \ldots
$$

- S_n depend only on Polyakov loops

- Leading order graph in case of $N_\tau = 4$:

Figure: 4 plaquettes in fundamental representation lead to a 2 Polyakov loop interaction term

- Integration of spatial link variables leads to

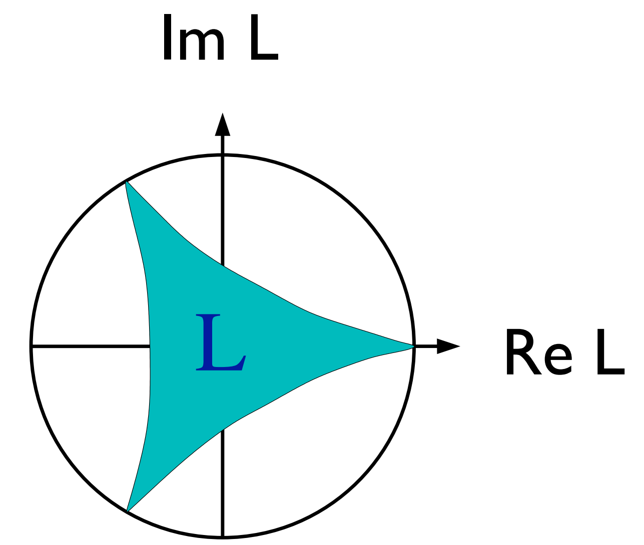
$$-S_1 = u^{N_\tau} \sum_{\langle ij \rangle} \mathrm{tr}\, W_i \, \mathrm{tr}\, W_j$$

- Possible generalizations: larger distance, higher dimensional representations, larger number of loops involved, ...
- *Here*: Decorate LO graph with additional spatial and temporal plaquettes

Effective one-coupling theory for SU(3) YM

(L= Tr W)

$$Z = \int [dL] \exp\left[-S_1 + V_{SU(3)}\right]$$
$$= \int [dL] \prod_{\langle ij \rangle} \left[1 + 2\lambda_1 \mathrm{Re}\left(L_i L_j^*\right)\right] *$$
$$* \prod_i \sqrt{27 - 18|L_i|^2 + 8\mathrm{Re}L_i^3 - |L_i|^4}$$



$$\lambda(u, N_\tau \geq 5) = u^{N_\tau} \exp\left[N_\tau\left(4u^4 + 12u^5 - 14u^6 - 36u^7 + \frac{295}{2}u^8 + \frac{1851}{10}u^9 + \frac{1055797}{5120}u^{10}\right)\right]$$

Subleading couplings

Subleading contributions for next-to-nearest neighbours:

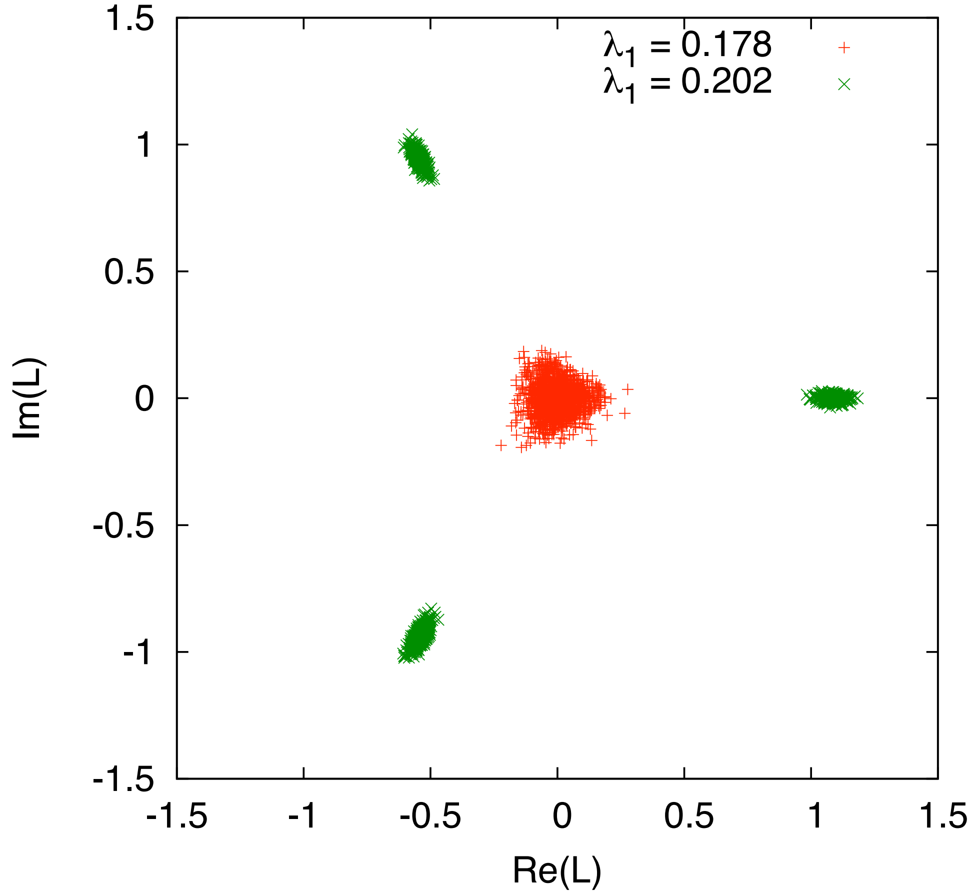
$$\lambda_2 S_2 \quad \propto \quad u^{2N_\tau+2} \sum_{[kl]}' 2\mathrm{Re}(L_k L_l^*) \quad \text{distance} \; = \sqrt{2}$$

$$\lambda_3 S_3 \quad \propto \quad u^{2N_\tau+6} \sum_{\{mn\}}'' 2\mathrm{Re}(L_m L_n^*) \quad \text{distance} \; = 2$$

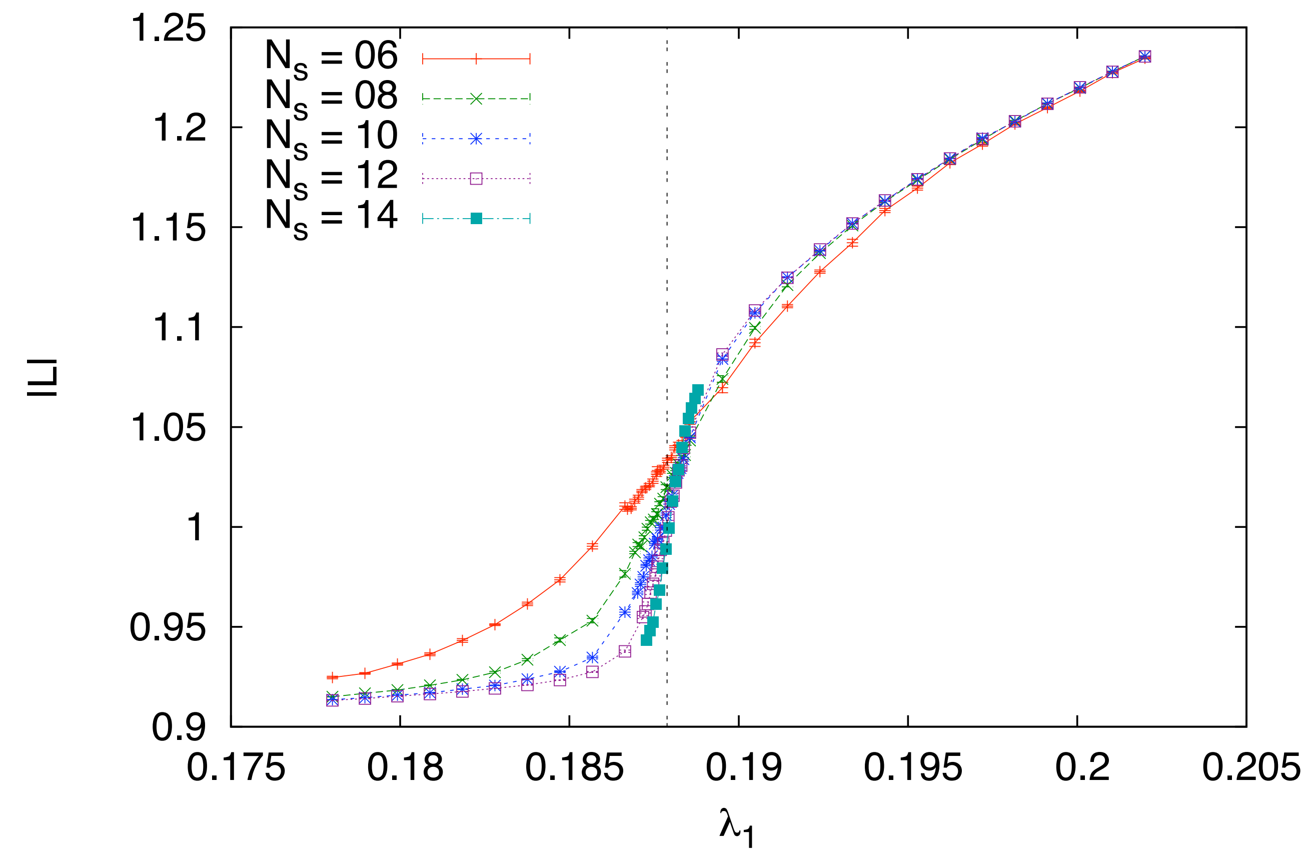
as well as terms from loops in the *adjoint* representation:

$$\lambda_a S_a \propto u^{2N_\tau} \sum_{<ij>} \mathrm{Tr}^{(a)} W_i \mathrm{Tr}^{(a)} W_j \quad ; \quad \mathrm{Tr}^{(a)} W = |L|^2 - 1$$

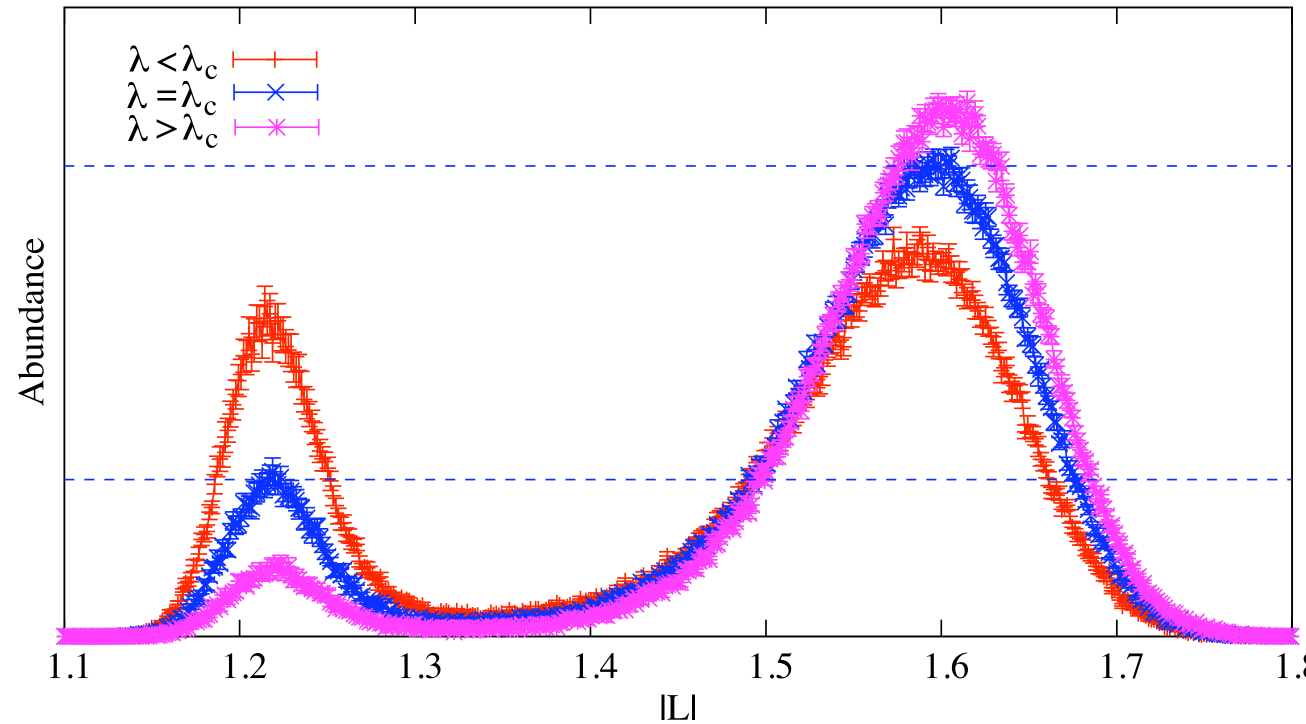
Numerical results for SU(3), one coupling



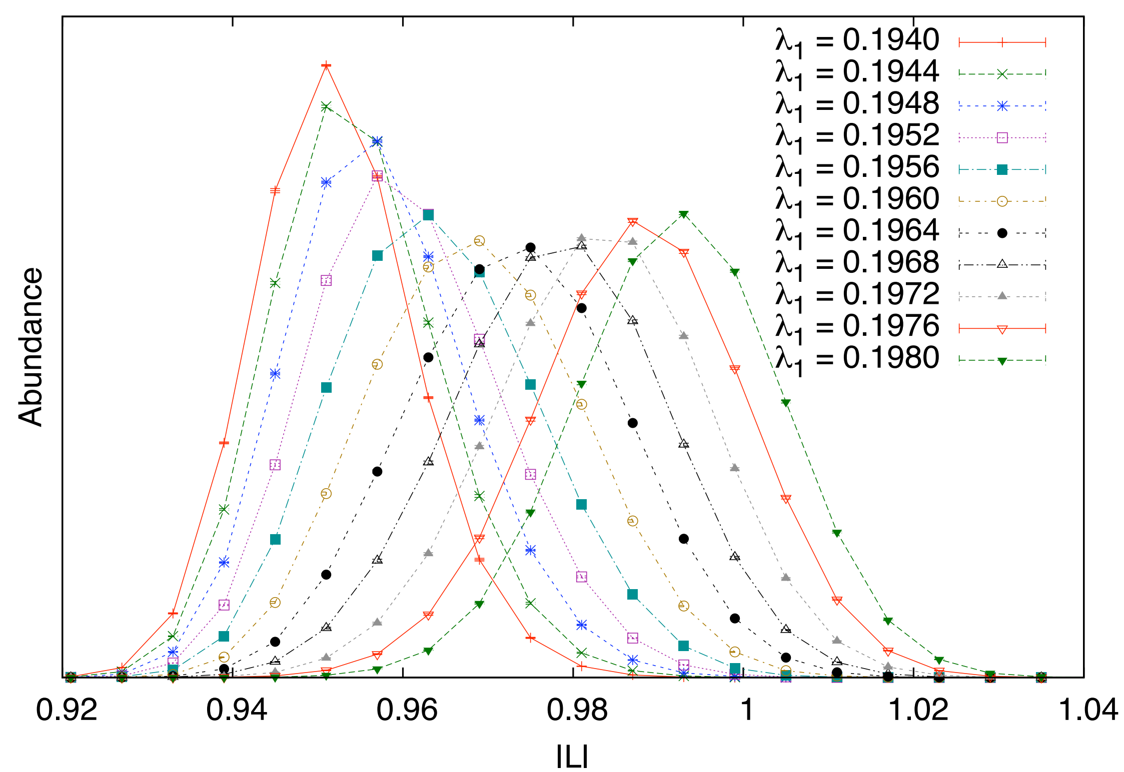
Order-disorder transition
=Z(3) breaking



First order phase transition for SU(3):

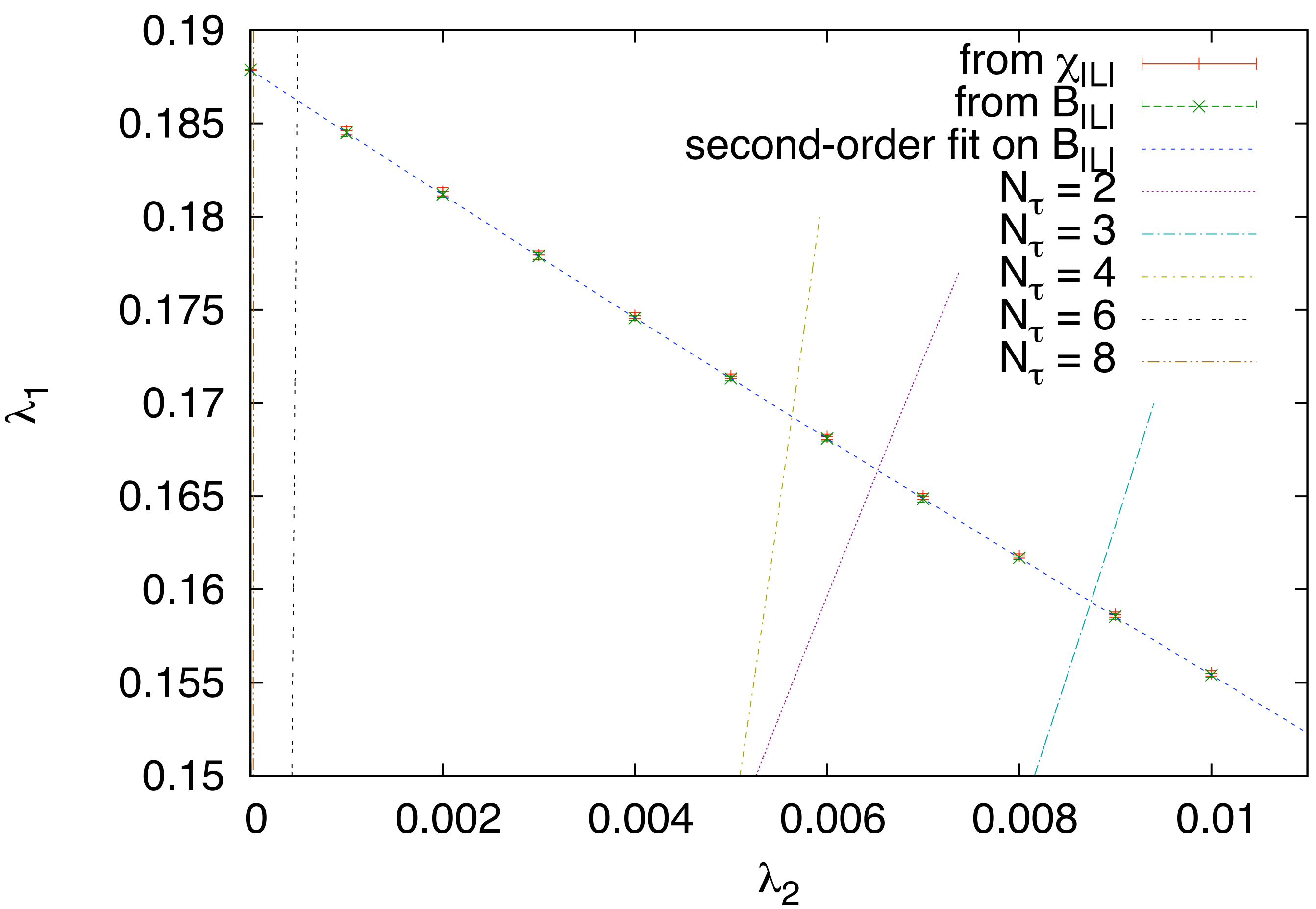


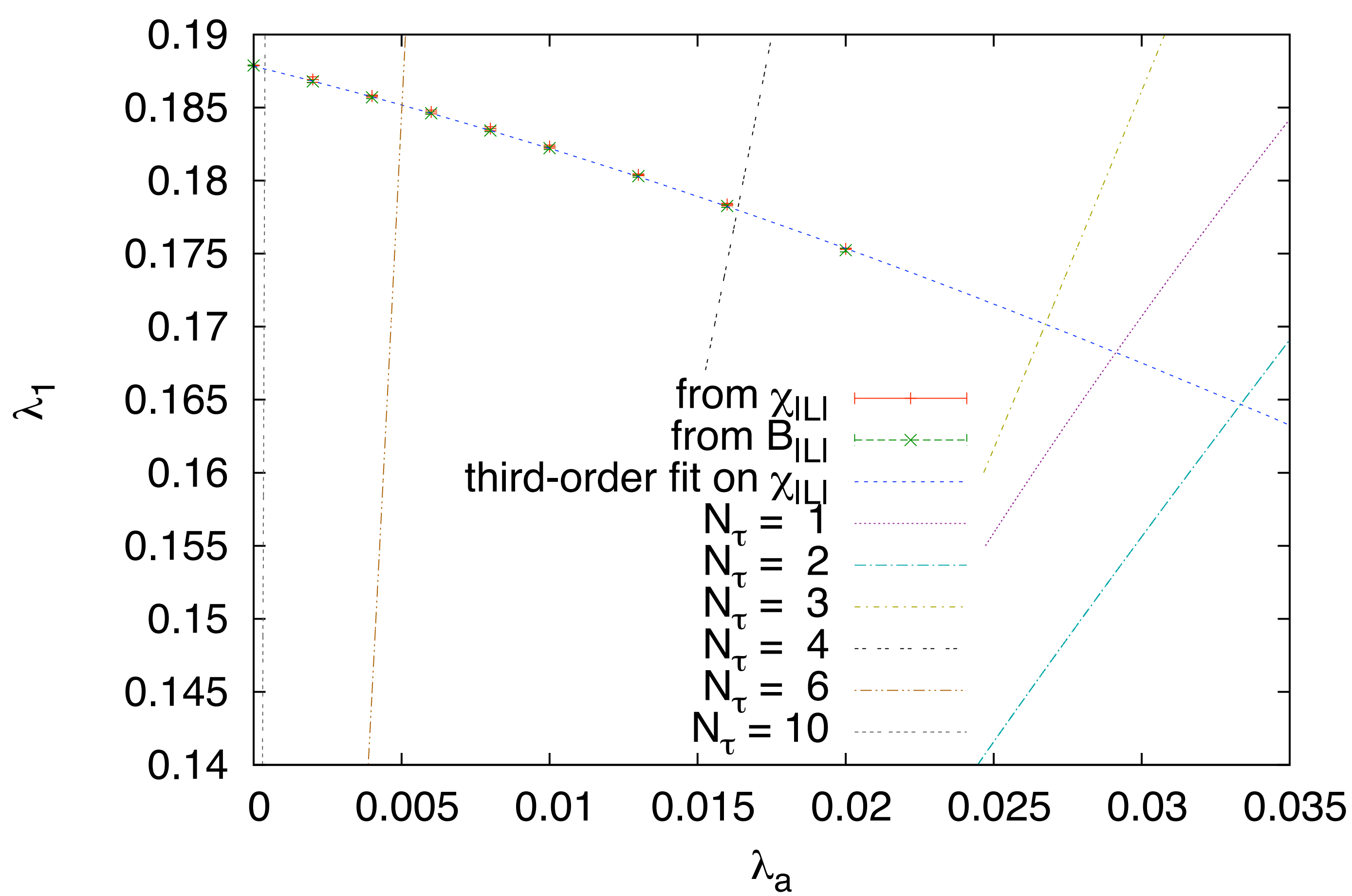
Second order (3d Ising) phase transition for SU(2):



The influence of a second coupling

NLO-couplings: next-to-nearest neighbour, adjoint rep. loops



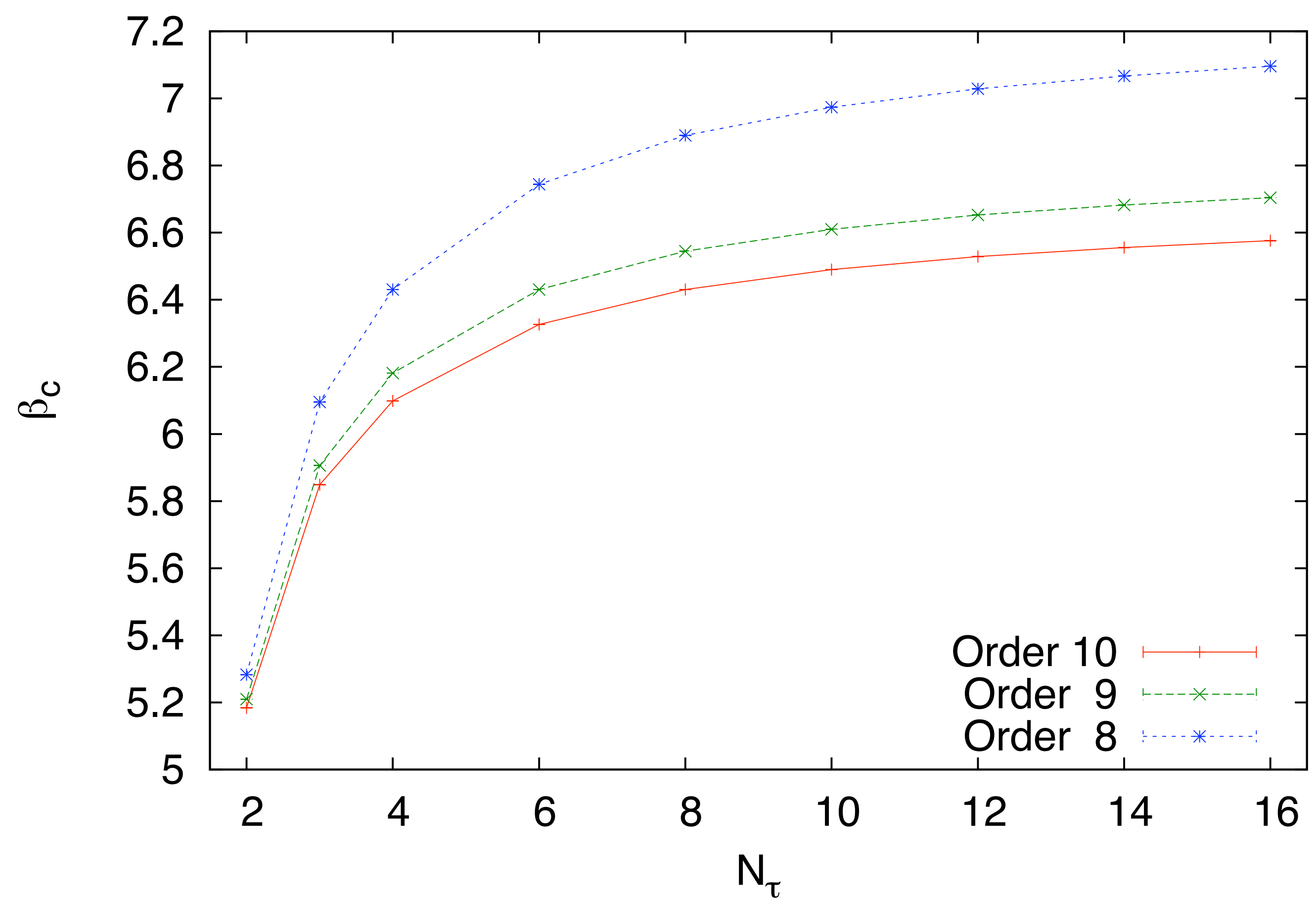


...gets very small for large N_τ !

Mapping back to 4d finite T Yang-Mills

Inverting

$\lambda_1(N_\tau, \beta) \rightarrow \beta_c(\lambda_{1,c}, N_\tau)$ ...points at reasonable convergence

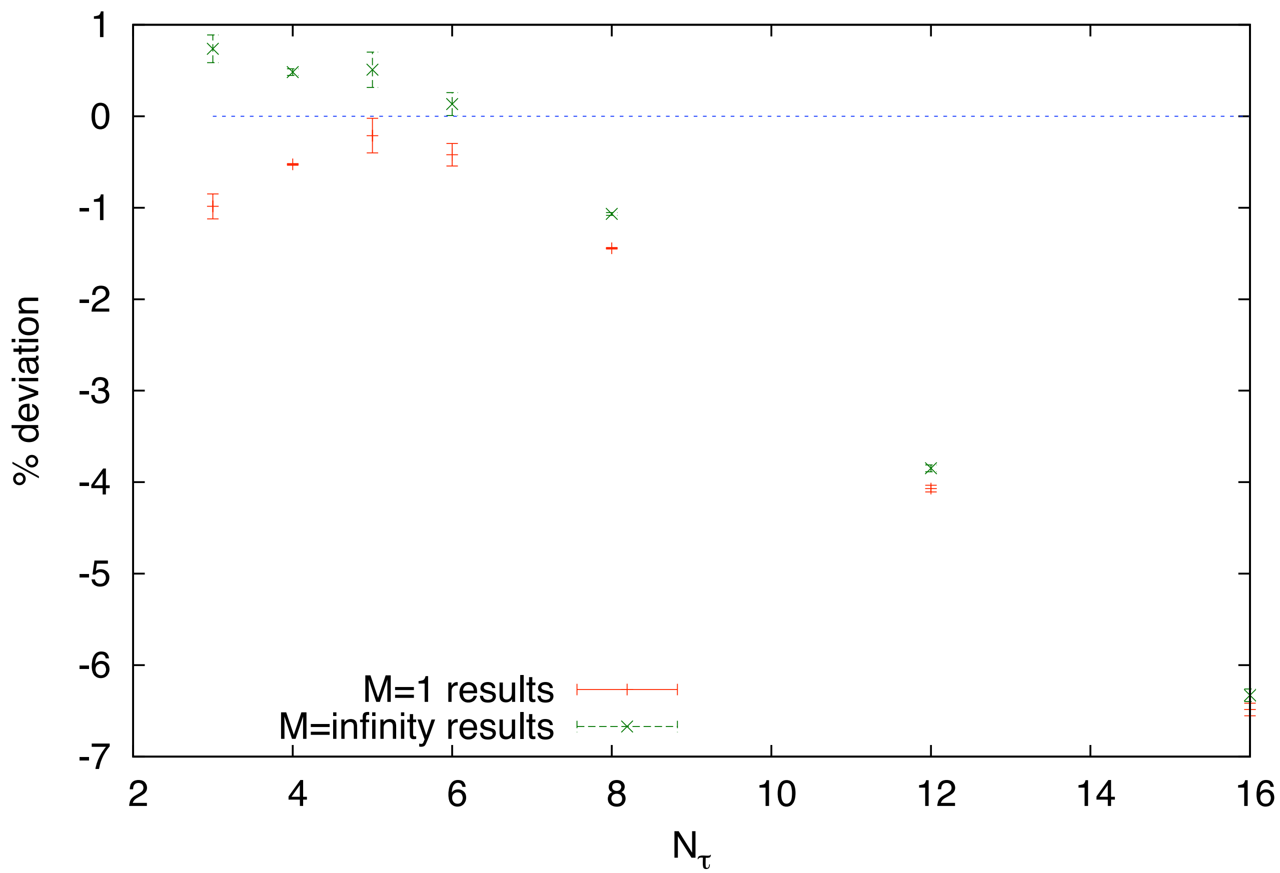


SU(3)

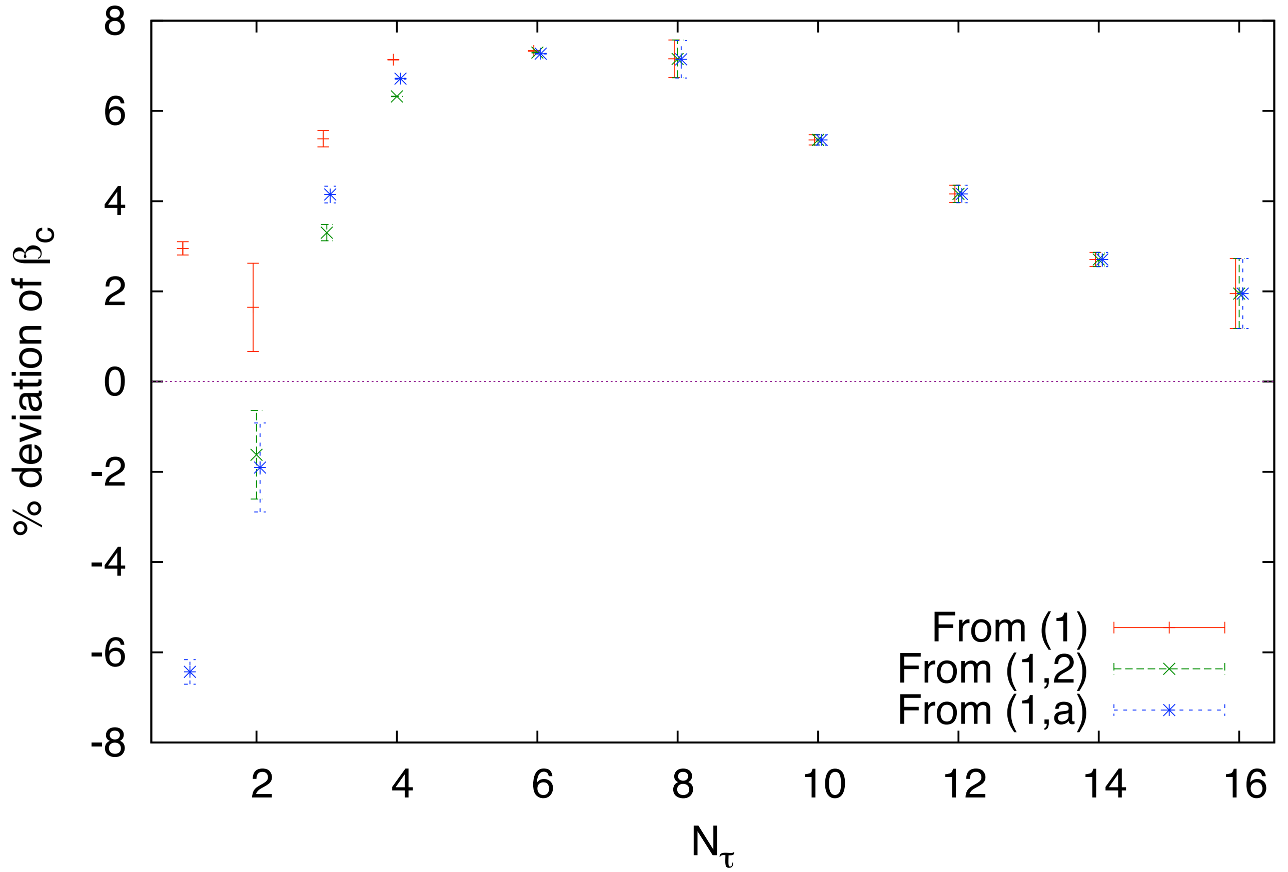
Comparison with 4d Monte Carlo

Relative accuracy for β_c compared to the full theory

SU(2)

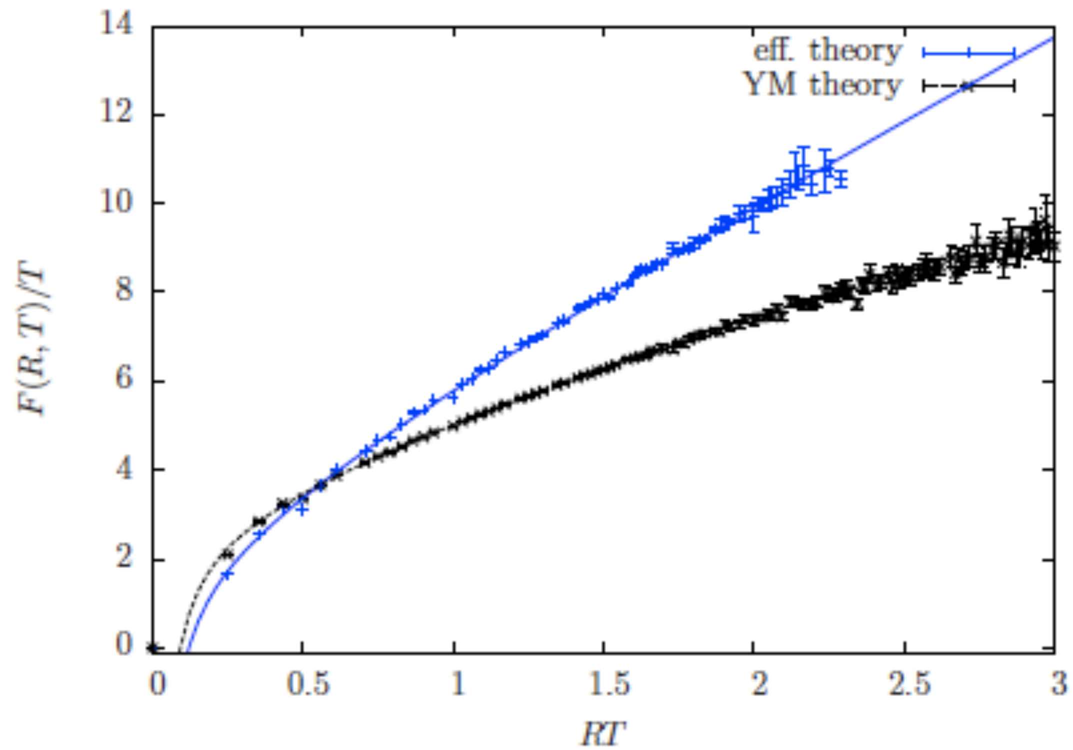


SU(3)



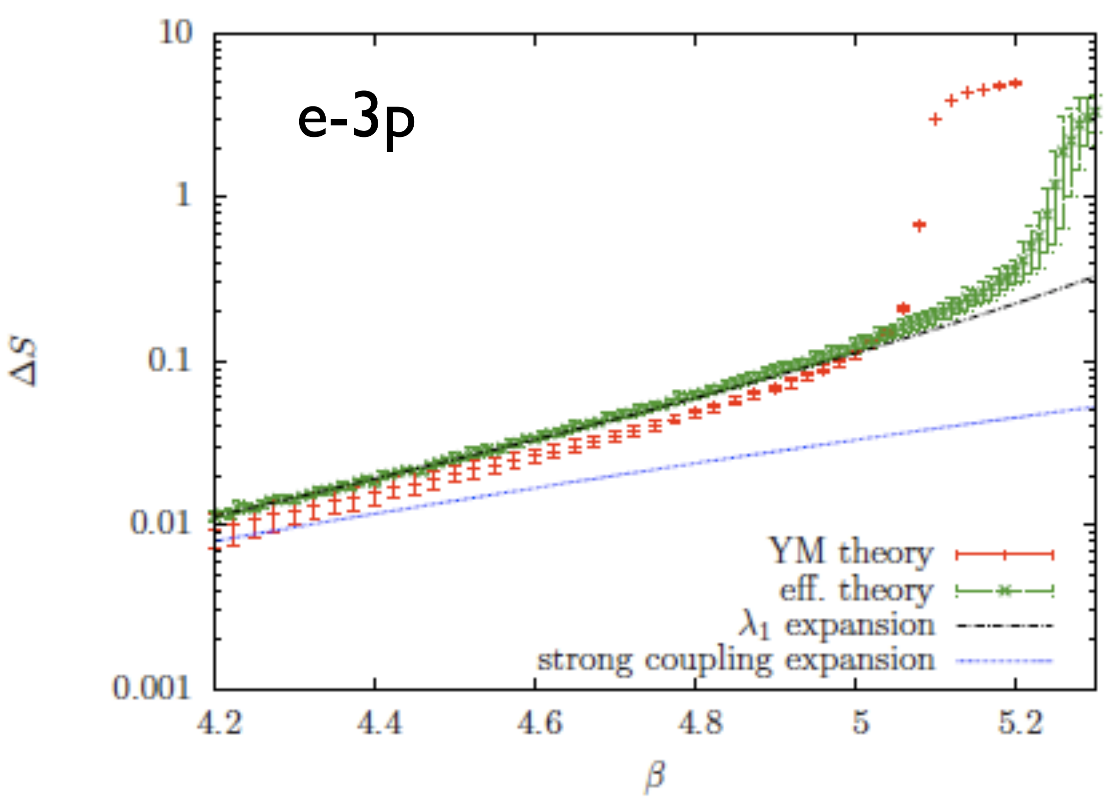
Note: influence of additional couplings checked explicitly!

What does and does not work?



Correlation functions and spectrum:
NO

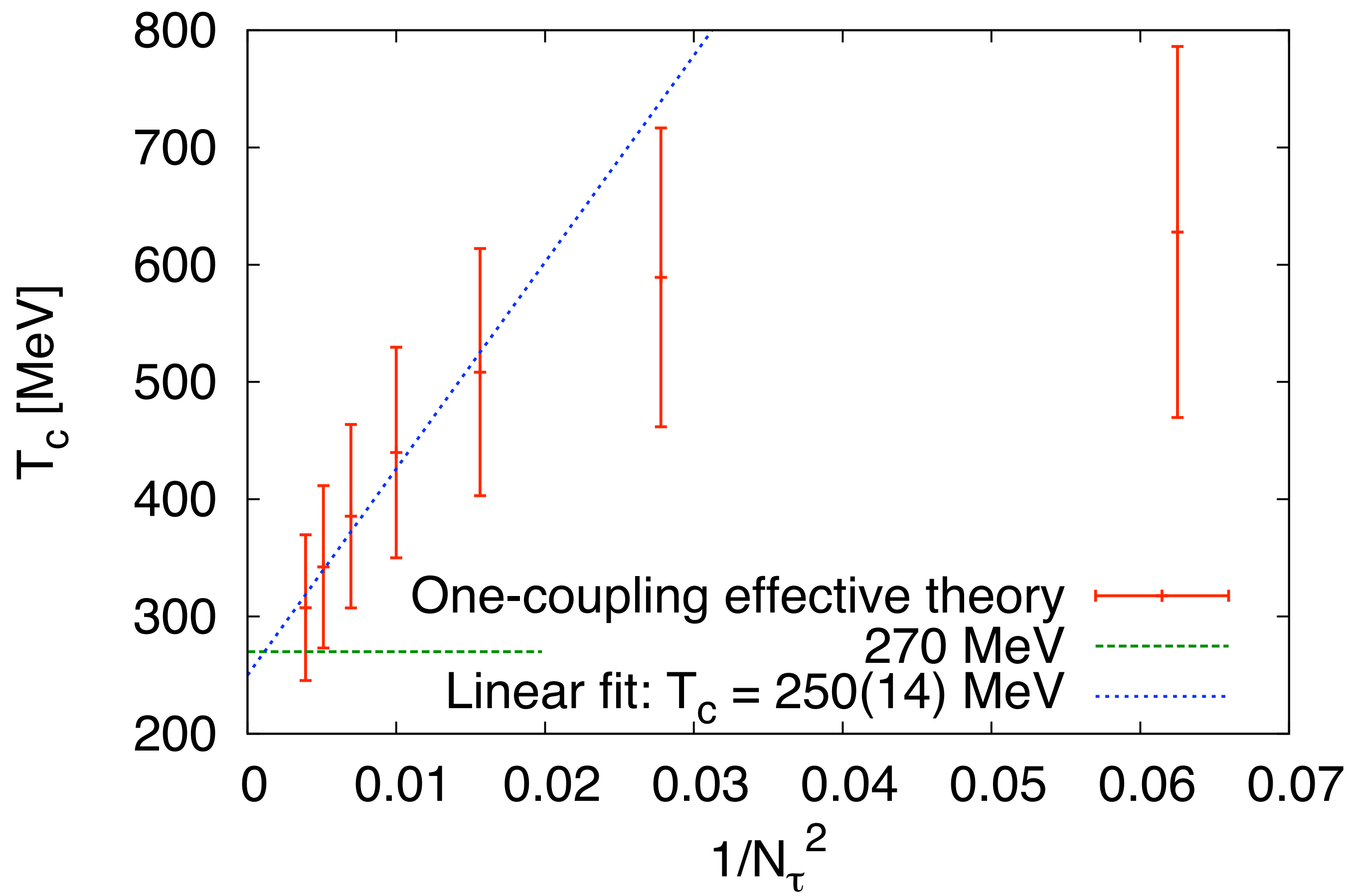
couplings over large distances needed



Thermodynamics and critical coupling:
YES

partition function needed, ultra-local!

Continuum limit feasible!



-error bars: difference between last two orders in strong coupling exp.

-using non-perturbative beta-function (4d T=0 lattice)

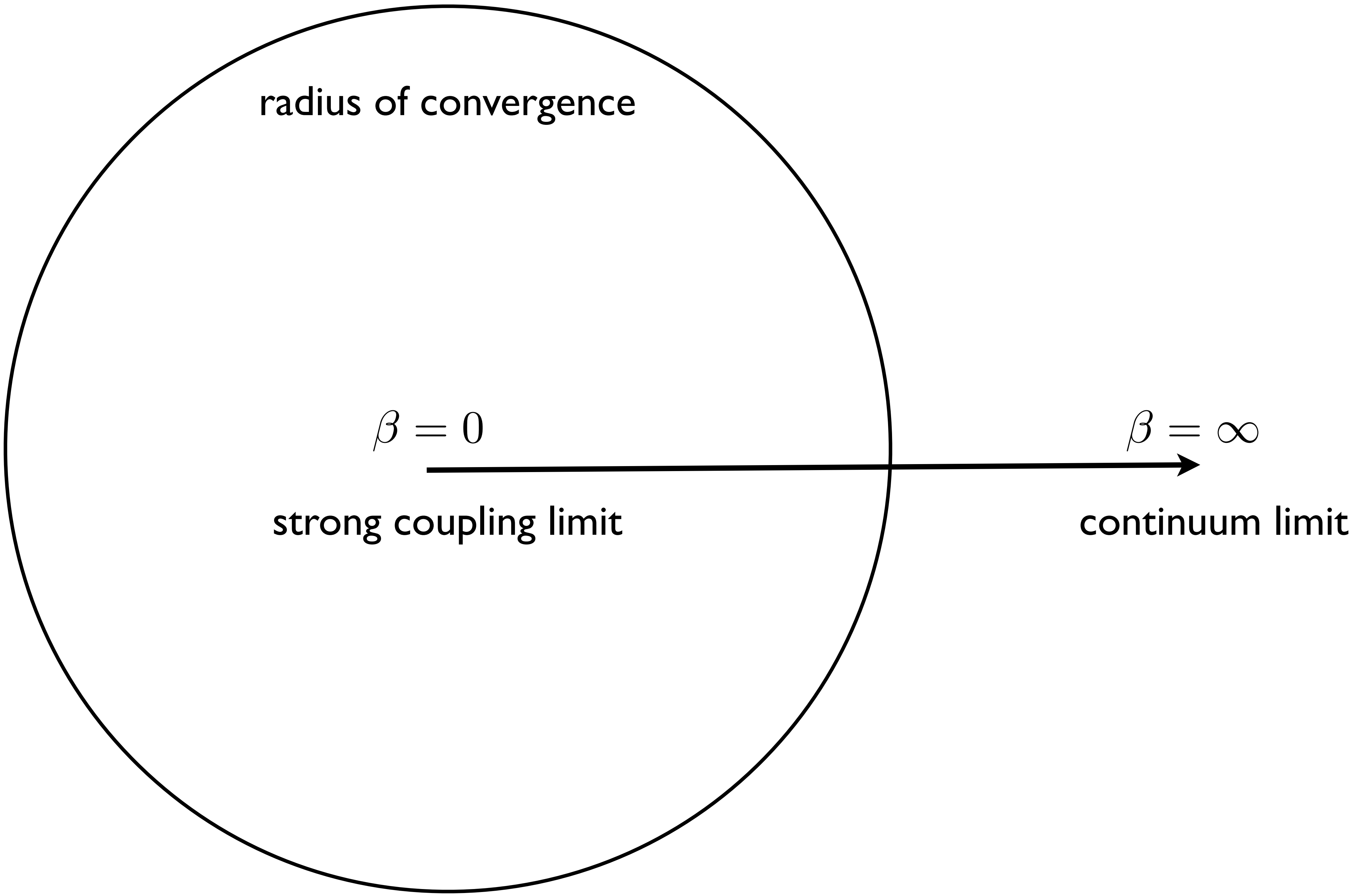
-all data points from one single 3d MC simulation!

How is this possible?

$\beta = 0$ → $\beta = \infty$

strong coupling limit → continuum limit

How is this possible?



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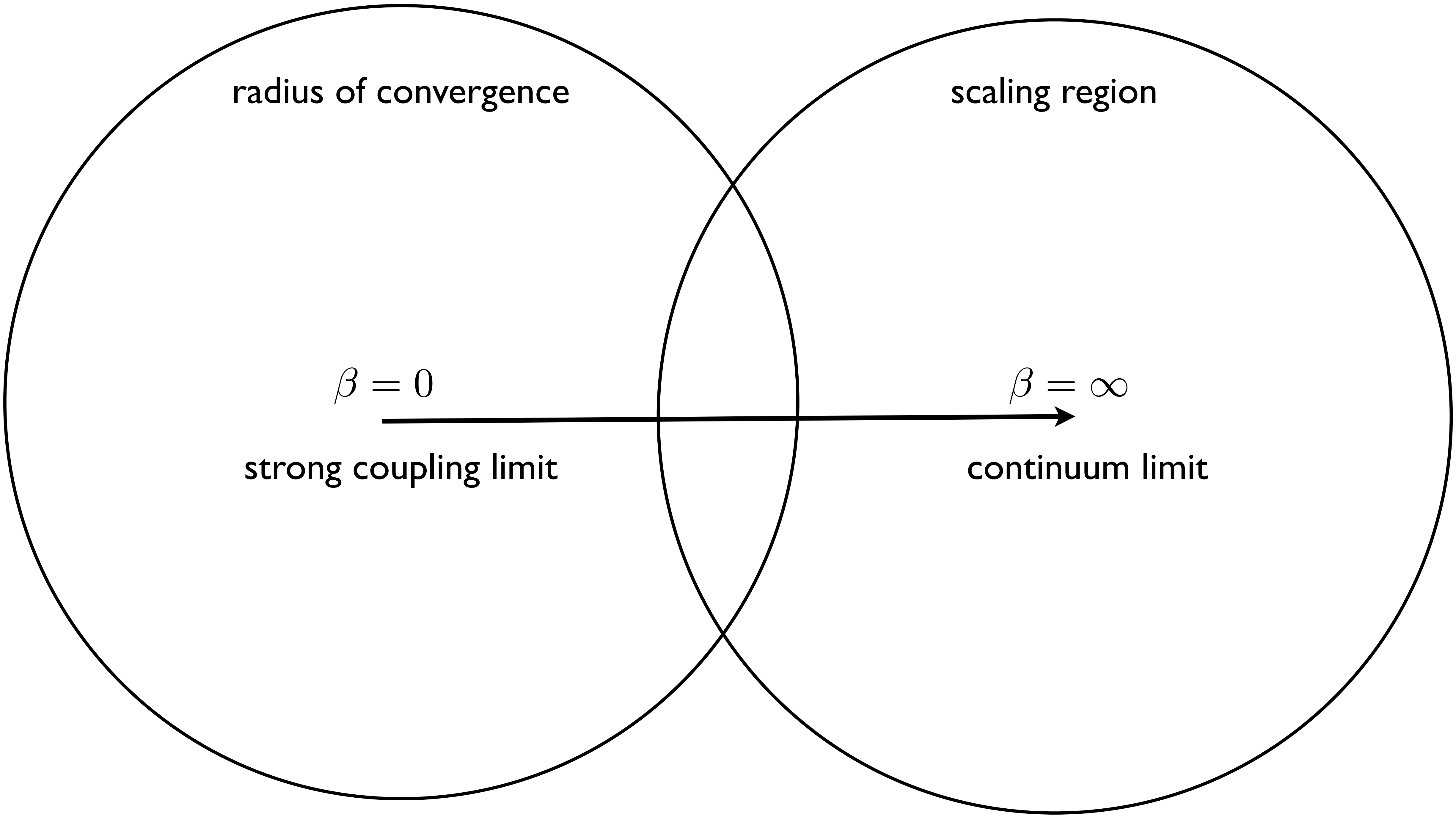
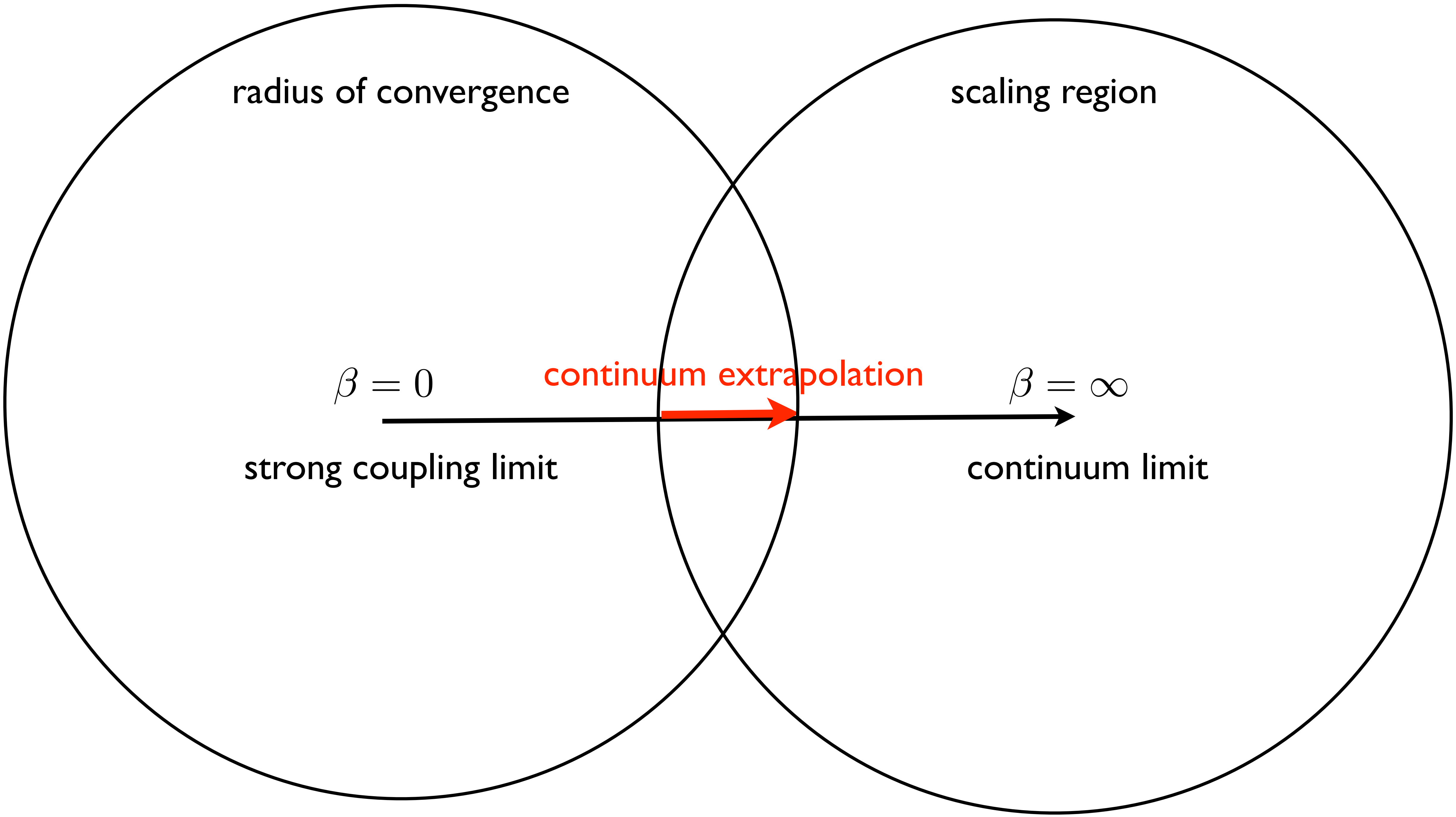


How is this possible?



Including heavy, dynamical Wilson fermions

N_f (degenerate) fermions $\Longrightarrow \quad S = S_{\text{gauge}} + S_q[U, \psi, \overline{\psi}]$

$$S_q = \sum_{x,y;f} \overline{\psi}_{f,y} \Big(1\!\!1 - \kappa H[U] \Big)_{yx} \psi_{f,x} \quad , \quad H[U]_{yx} = \sum_{\pm\mu} \delta_{y,x+\hat{\mu}} (1\!\!1 + \gamma_\mu) U_{x,\mu}$$

Integrate the Grassmann variables $\psi, \overline{\psi}$:

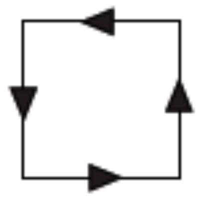
$$S = S_{\text{gauge}} - N_f \operatorname{Tr} \log(1\!\!1 - \kappa H)$$

Expand in the *hopping parameter* $\kappa = 1/(2aM + 8)$: [*]

$$S = S_{\text{gauge}} + N_f \sum_{\ell=1}^{\infty} \frac{\kappa^\ell}{\ell} \operatorname{Tr} H[U]^\ell$$

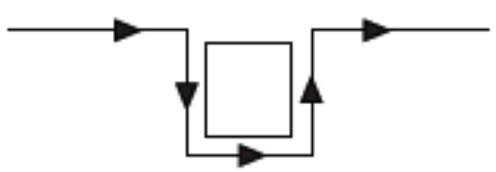
Similar to de Pietri, Feo, Seiler, Stamatescu 07, Aarts, Stamatescu 08 ...

Links along imaginary time gain $\exp(\pm\mu a)$



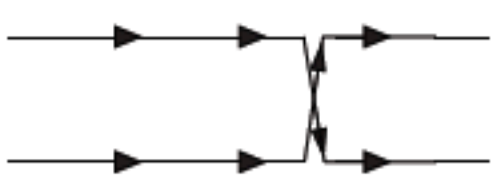
reabsorbed in gauge part: $\begin{cases} \beta \to \beta + \mathcal{O}(\kappa^4) \\ u(\beta) \to u(\beta,\kappa) \end{cases}$

LO Polyakov "magnetic" term $\sim \begin{cases} \underbrace{(2\kappa e^{+a\mu})^{N_\tau}}_{h_1} L \\ \underbrace{(2\kappa e^{-a\mu})^{N_\tau}}_{\bar{h}_1} L^* \end{cases}$



higher corrections to the above:

$$h_1 = (2\kappa e^{a\mu})^{N_\tau}\left[1 + \mathcal{O}(k^2) f(u) + \ldots\right]$$



other (suppressed) terms, such as $h_2(L_x L_{x+\hat{i}})$,

$$h_2 \sim (2\kappa e^{a\mu})^{2N_\tau}\kappa^2 \ldots$$

In general the model becomes (with $\overline{h}_i(\mu) = h_i(-\mu)$)

$$-S_{\rm eff} = \sum_i \lambda_i(u,\kappa,N_\tau) S_i^{\rm S} - 2N_f \sum_i \Big[h_i(u,\kappa,\mu,N_\tau) S_i^{\rm A} + \overline{h}_i(u,\kappa,\mu,N_\tau) S_i^{\dagger\rm A} \Big]$$

Now, keep only $\lambda_1 S_1^{\rm S}$ and $h_1 S_1^{\rm A} + \overline{h}_1 S_1^{\dagger\rm A}$ (now called just λ, h)

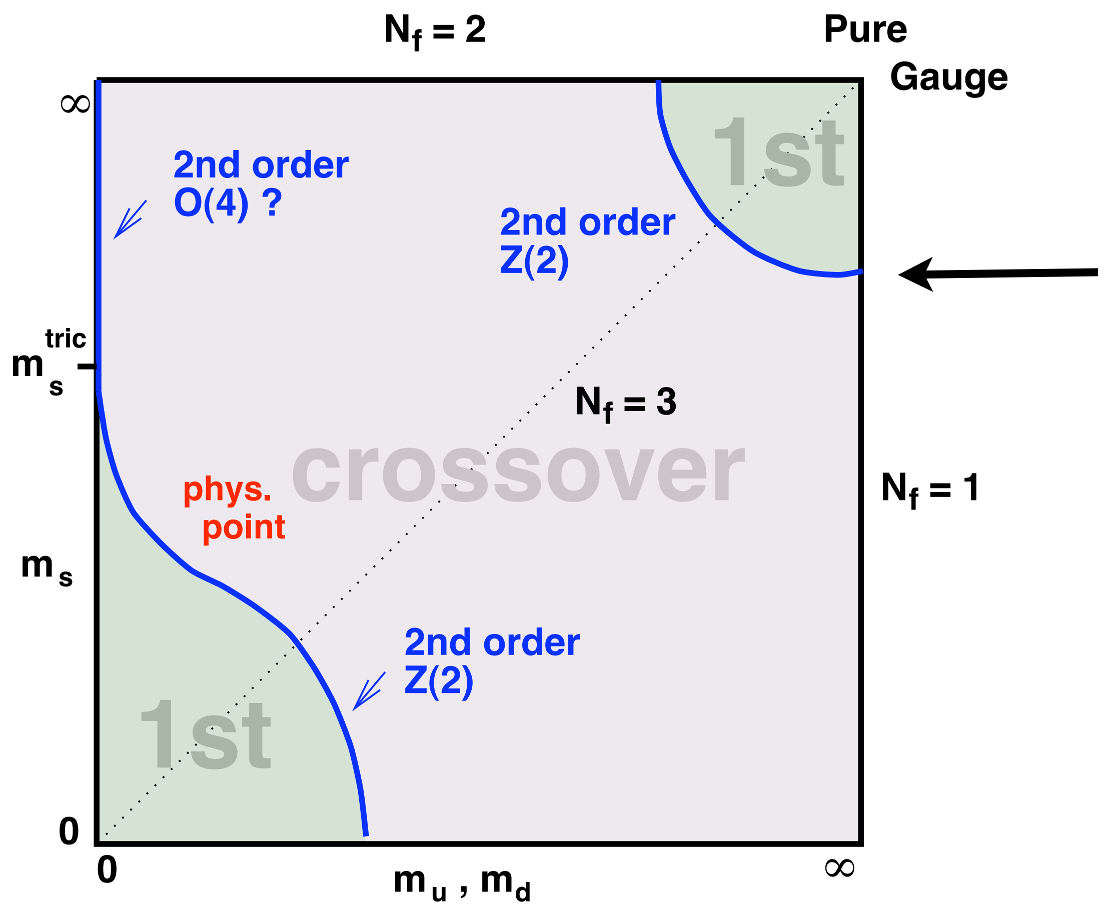
Higher powers of loops are resummed into a determinant:

$$\begin{aligned} Z_{\rm eff}(\lambda_1, h_1, \overline{h}_1; N_\tau) \;=\; & \int [{\rm d}L] \Big(\prod_{<ij>} [1 + 2\lambda_1 {\rm Re} L_i L_j^*] \Big) \\ & \Big(\prod_x \underbrace{\det[(1 + h_1 W_x)(1 + \overline{h}_1 W_x^\dagger)]^{2N_f}}_{\equiv Q(L_x, L_x^*)^{N_f}} \Big) \end{aligned}$$

No chemical potential ($h = \overline{h}$): the full (λ, h)-model has then

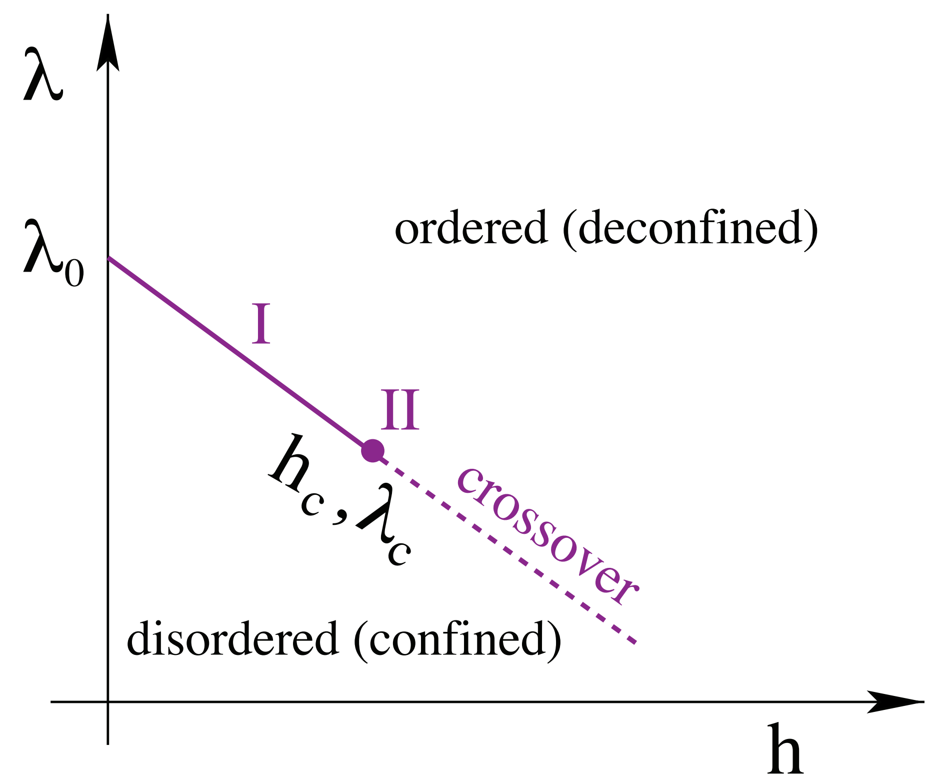
- a "spin-spin" interaction between neighbour Polyakov loops
- a "magnetic-field" term acting on sites

QCD: first order deconfinement transition region



deconfinement p.t.:
breaking of global $Z(3)$ symmetry;
explicitly broken by quark masses
transition weakens

Phase diagram in eff. theory:



The critical point

$$\lambda_c = 0.18672(7), h_c = 0.000731(40)$$

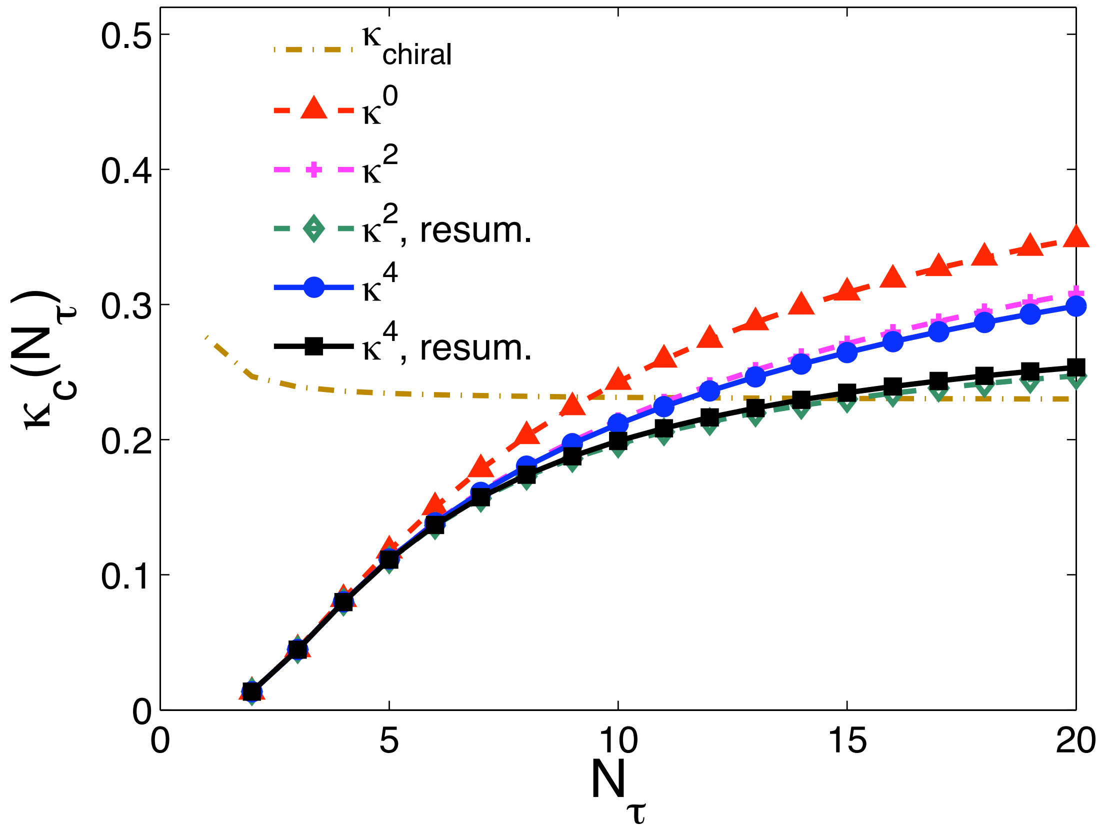
Mapping back to QCD:

		eff. theory	4d MC, WHOT	4d MC, de Forcrand et al
N_f	M_c/T	$\kappa_c(N_\tau = 4)$	$\kappa_c(4)$, Ref. [23]	$\kappa_c(4)$, Ref. [22]
1	7.22(5)	0.0822(11)	0.0783(4)	~ 0.08
2	7.91(5)	0.0691(9)	0.0658(3)	–
3	8.32(5)	0.0625(9)	0.0595(3)	–

$e^{-M/T} \simeq h/N_f$ [linear approximation in $h \ll 1 \ldots$]

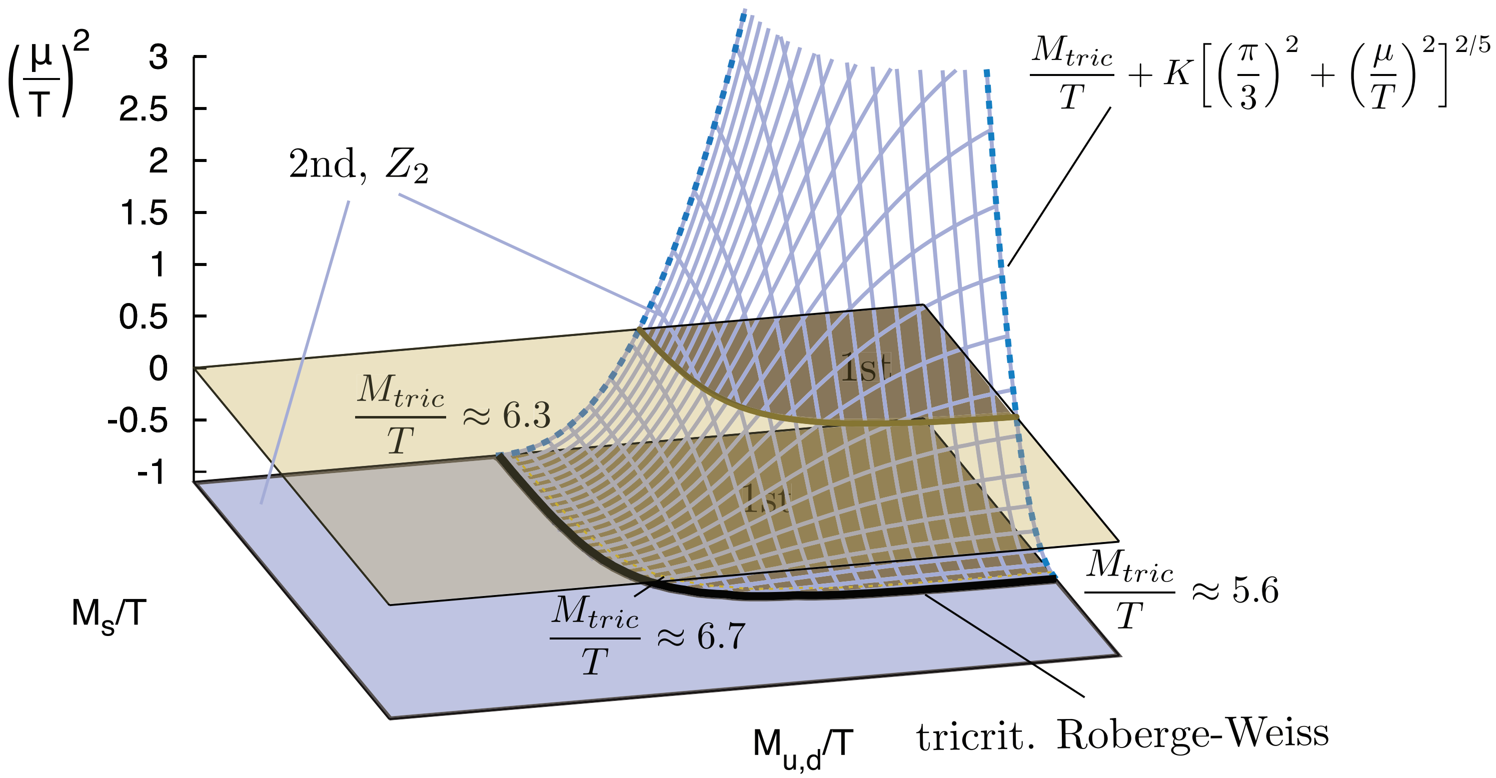
Accuracy ~5%, predictions for Nt=6,8,... available!

Convergence properties:



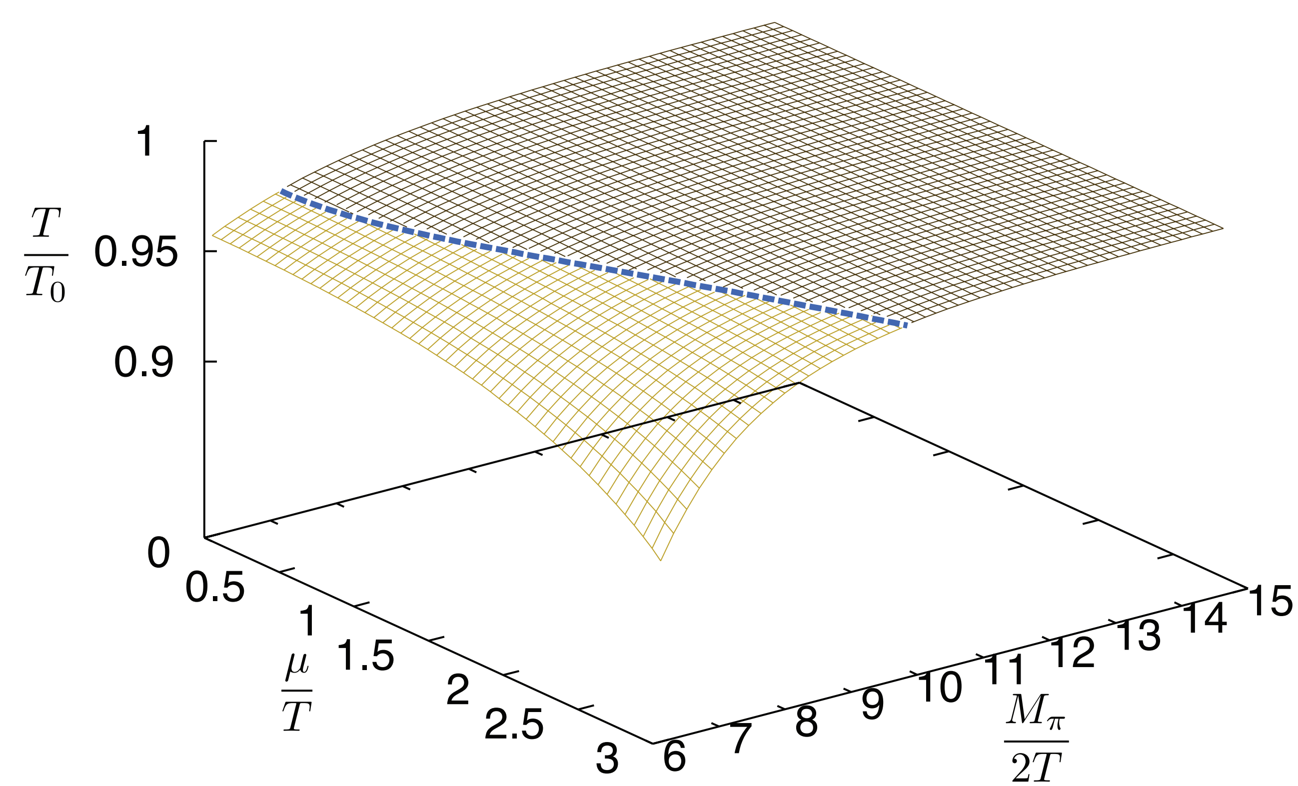
$\kappa_c(\text{chiral})$

The fully calculated deconfinement transition

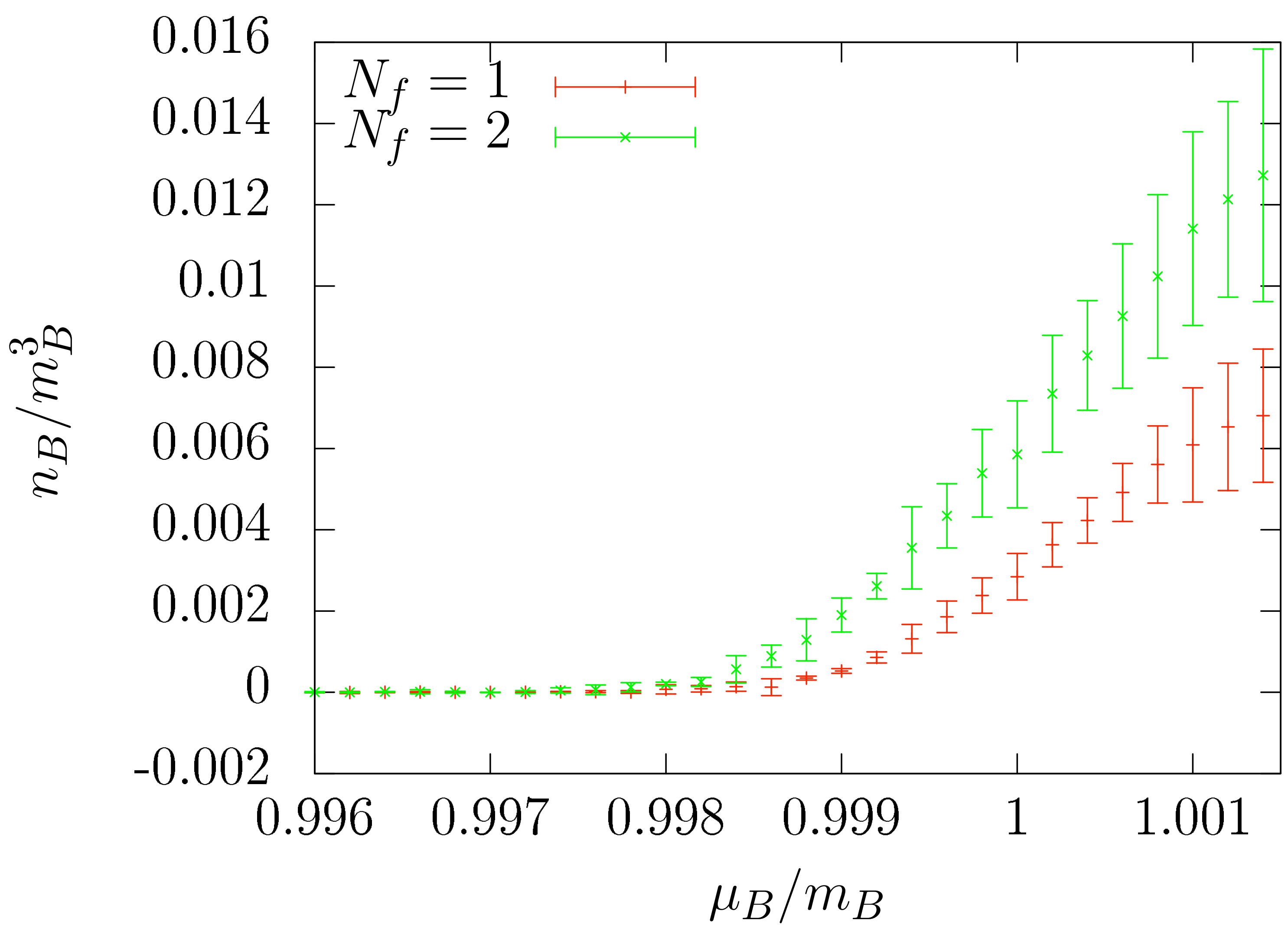


deconfinement critical surface

phase diagram for Nf=2, Nt=6



The equation of state for nuclear matter



$$S_{eff} \sim \kappa^n u^m, \quad n+m=4$$

$$m_\pi = 20 \text{ GeV}, T = 10 \text{ MeV}$$

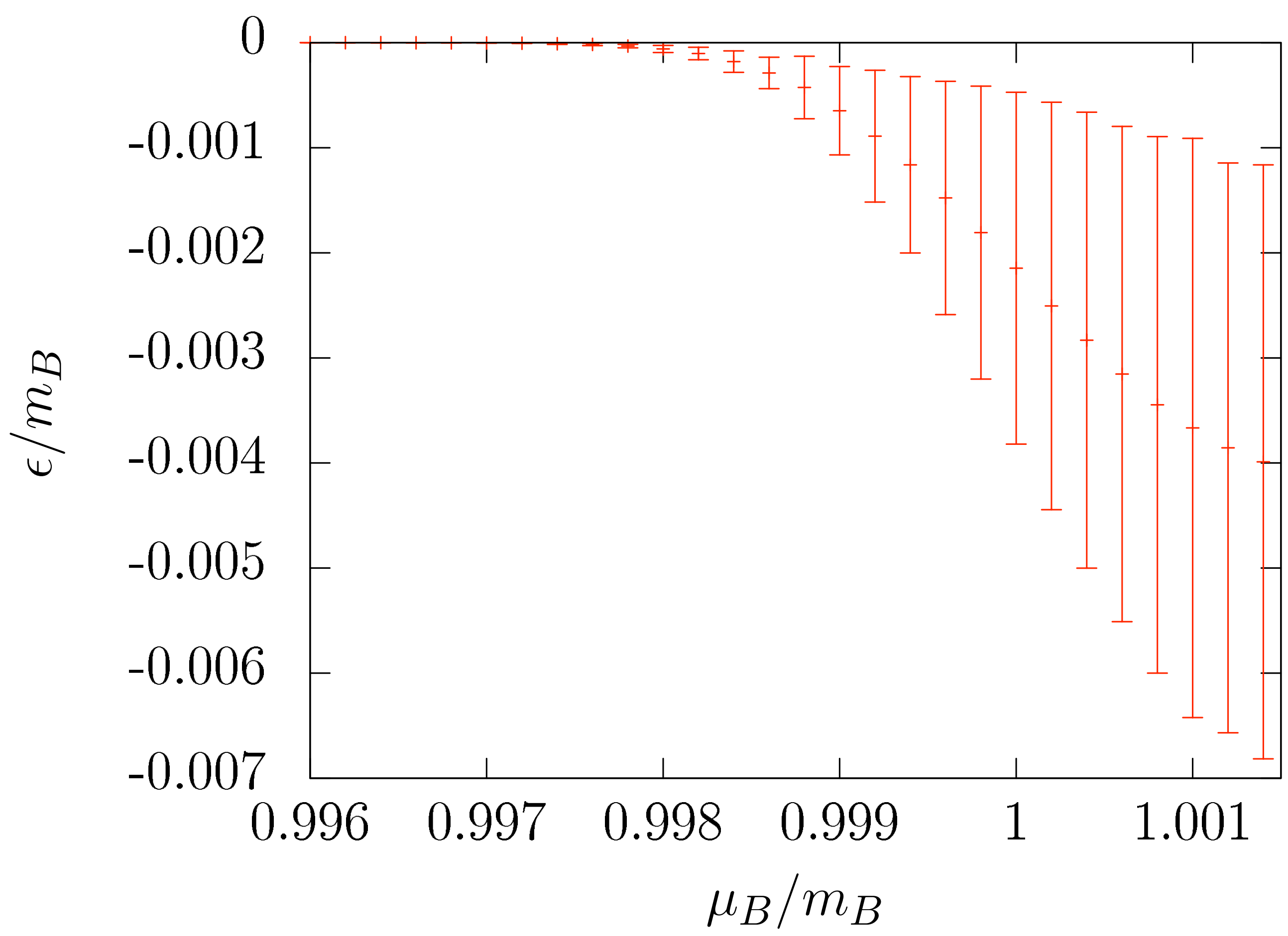
Effect of binding between baryons: $\mu_c < m_B$

Binding energy per nucleon: $\epsilon = \dfrac{\mu_c - m_B}{m_B} \sim 10^{-3}$

Transition is smooth crossover: $T > T_c \sim \epsilon\, m_B$

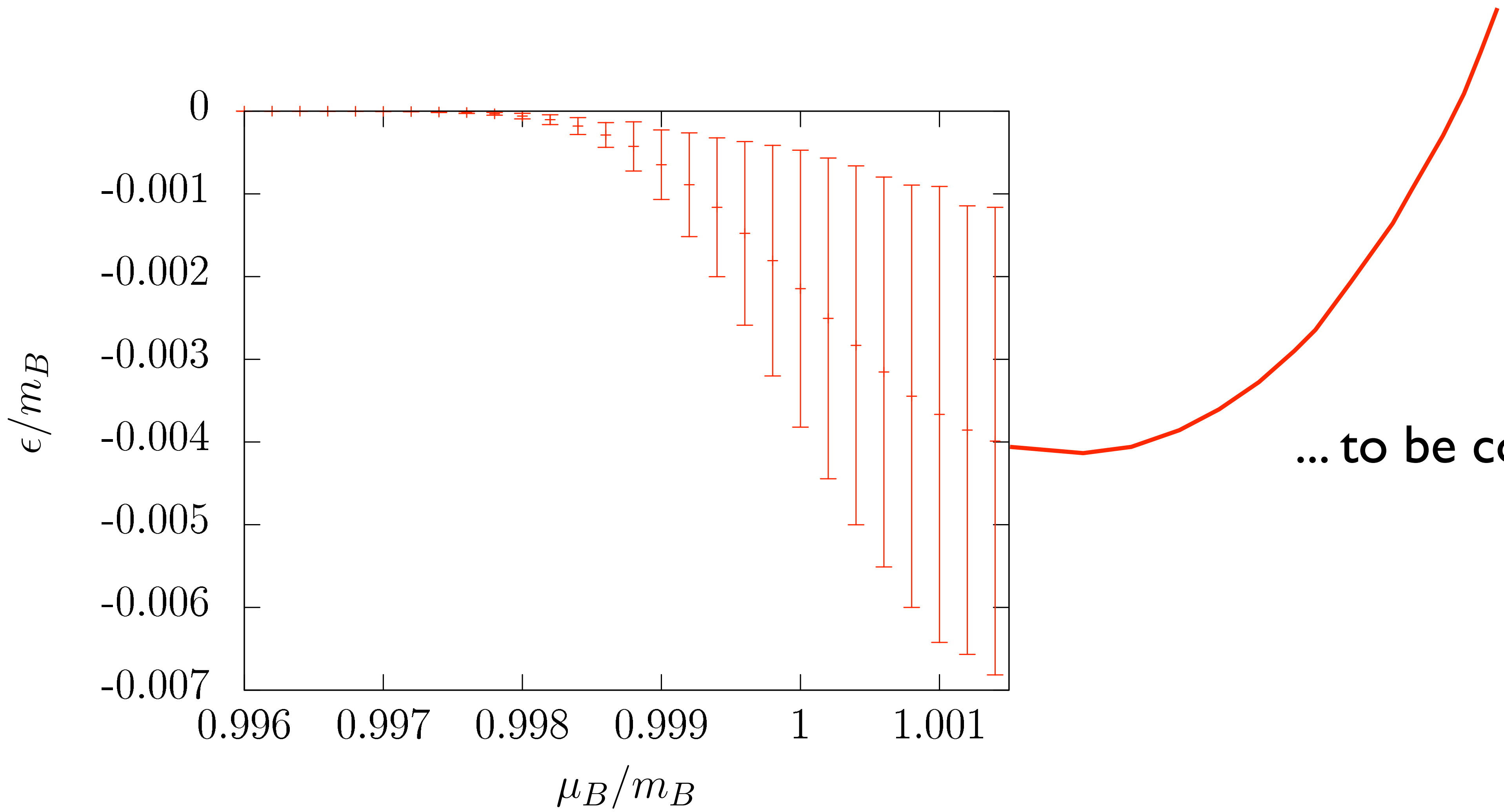
Binding energy per nucleon

$$\epsilon = \frac{e - n_B m_B}{n_B m_B} = \frac{e}{n_B m_B} - 1$$



Binding energy per nucleon

$$\epsilon = \frac{e - n_B m_B}{n_B m_B} = \frac{e}{n_B m_B} - 1$$



... to be continued...

Minimum: access to nucl. binding energy, nucl. saturation density!

$\epsilon \sim 10^{-3}$ consistent with the location of the onset transition

Conclusions

- Two-step treatment of QCD phase transitions:

  I. Derivation of effective action by strong coupling expansion
  II. Simulation of effective theory

- Z(N)-invariant effective theory for Yang-Mills, correct order of phase trans. Tc with 10% accuracy in the continuum limit!

- Finite T deconf. transition for heavy fermions and all chemical potentials

- Silver blaze property + phase transition to nuclear matter at T=0