Contribution ID: **13** Type: **not specified**

Variationl Study of SU(3) Gauge Theory by Stationary Variance

Wednesday, 3 December 2014 17:10 (35 minutes)

The principle of stationary variance is advocated as a viable variational approach to gauge theories where the simple Gaussian Effective Potential (GEP) is known to be useless. The method can be regarded as a second-order extension of the GEP and seems to be suited for describing the strong coupling limit of non-Abelian gauge theories.
The single variational parameter of the GEP is replaced by trial unknown two-point functions, with infinite variational parameters to be optimized by the solution of a set of integral equations.
The stationary conditions can be easily derived by the self-energy, without having to write the effective
potential, making use of a general relation between self-energy and functional derivatives that has been proven to any order.
By that method, the low-energy limit of pure Yang-Mills $SU(3)$ gauge theory has been studied in Feynman gauge. In terms of standard irreducible graphs, the stationary equations are written as a set of coupled non-linear integral equations for the gluon and ghost propagators.
A physically sensible solution is found for any strength of the coupling.
The gluon propagator is finite in the infrared, with a dynamical mass that decreases as a power at high energies. At variance with some recent findings in Feynman gauge, the ghost dressing function does not vanish in the infrared limit and a decoupling scenario emerges as recently reported for the Landau gauge.

INTRODUCTION

There is a growing consensus on the utility of variational methods as analytical tools for a deeper understanding
of the infrared (IR) limit of non-Abelian gauge theories. The IR slavery of these theories makes the standard perturbation theory useless below some energy scale, and our theoretical knowledge of the IR limit relies on lattice simulation and on non-perturbative techniques like functional renormalization group and Dyson-Schwinger equations. Variational methods have been developed[1-7] as a complement to these analytical approaches and quite
recently the method of stationary variance[8, 9] has been advocated as a powerful second order extension of the Gaaussian Effective Potential (GEP)[10–13]. The GEP is a genuine variational method and has been successfully
applied to many physical problems in field theory, from scalar and electroweak theories[14–20] to superconductivity[21–23] and antiferromagnetism[24], but turns out to be useless for gauge interacting fermions[25-26]. Actually, since the GEP only contains first order terms, it is not suited for describing the minimal coupling of gauge theories that has no first-order effects.
Several methods have been explored for including fermions[20] and higher order corrections[25,26], sometimes spoiling the genuine variational character of the method.

By a formal higher order extension of the GEP[27] the method of stationary variance has been developed as a genuine variational method that keeps in due account second order effects and seems to be suited to deal with the minimal coupling of gauge theories. While the method has been shown to be viable for the simple
Abelian case of QED[28], its full potentialities have not been explored yet. As a non-perturbative tool that

can deal with fermions in gauge theories, the method seems to be very useful for exploring the IR limit of QCD,
and its natural application field is the non-Abelian SU(3) gauge theory[29].

YANG-MILLS $SU(3)$ BY STATIONARY VARIANCE

A full study of QCD by the method of stationary variance
is still far away. As a first step, in Ref.[29] we explored the solution of the stationary equations
for pure Yang-Mills SU(3) theory.
While the method is a genuine variational tool that does not require any small parameter, the technique is based
on standard Feynman rules of perturbation theory. The single variational parameter of the GEP is replaced by trial unknown two-point functions, with infinite variational parameters to be optimized by the solution of a
set of integral equations, the stationary equations. However, these equations can be easily derived by
the self-energy, without having to write the effective potential, making use of a general relation between self-energy and functional derivatives that has been proven to any order[27].

For pure Yang-Mills theory the method of stationary variance provides a set of non-linear coupled integral equations whose solutions are the propagators for gluons and ghosts. Therefore the work has a double motivation:
the technical aim of showing that the method is viable and a solution does exist
(which was not obvious nor proven in general), and the physical interest on the gluon propagator in the IR limit,
where its properties seem to be related to the important issue of confinement.
On the technical side, having shown that a sensible untrivial solution does exist is a major achievement
that opens the way to a broader study of QCD by the same method. Inclusion of quarks would be straightforward
as some fermions, the ghosts, are already present in the simple Yang-Mills theory, and they already seem to play
well their role of canceling the unphysical degrees of freedom.

Other important technical issues are gauge invariance, renormalization and the choice of a physical scale. The method is not gauge invariant, and no effort has been made to restore gauge invariance at this first stage. While there are several ways to attempt it[1,30], in this first step a fixed gauge has been used,
namely the Feynman gauge where the calculation is easier. In this gauge, the properties of the solution are explored in order to see if any unphysical feature emerges for the propagator and the polarization function. Actually the polarization function is found approximately transverse up to a constant mass shift due to the dynamical mass generation. As far as the solution satisfies, even approximately, the constraints imposed by gauge invariance, the method is acceptable on the physical ground.
On the other hand the gluon propagator is not a physical observable and is known to be a gauge-dependent quantity.
Of course, since the solution depends on the gauge, the choice of working in Feynman gauge could be non-optimal,
and the method could be improved by exploring other gauge choices, like Landau gauge.
Besides being easier, working in Feynman gauge is also interesting from the physical point of view,
as there are very few data available on the gluon propagator in this gauge.

Since lattice simulations are the most natural benchmark for any variational calculation in the IR limit, the regulating scheme is borrowed from lattice simulation, with an energy cutoff and a bare coupling
that depends on it. Renormalization Group (RG) invariance requires that the physical observables are left invariant by a change of the cutoff that is followed by the corresponding change of the bare coupling.
Then, renormalized physical quantities can be defined that do not depend on the cutoff.
The only free parameter of the theory is the energy scale, that must be fixed by a comparison with the experimental data or lattice simulations. No other fit parameter has been introduced in the method, especially mass counterterms that are forbidden by the gauge invariance of the Lagrangian.

On the physical side, the properties of the gluon propagator in Feynman gauge are basically unexplored.
In Coulomb gauge[2–5, 7] and in Landau gauge[6, 31–38] there has been an intense theoretical work in the last years. In Landau gauge theoretical and lattice data are generally explained in terms of a decoupling regime,
with a finite ghost dressing function and a finite massive gluon propagator.
The more recent findings confirm the original prediction[39] of a dynamical mass generation for the gluon.
In Feynman gauge we do not expect a very different scenario.
A finite ghost propagator has been recently proposed[40], but there are no lattice data available that
could confirm it. That makes the study of the Feynman gauge more interesting.

As discussed in Ref.[29], in the present work no important differences are found with respect to the Landau gauge.
A decoupling scenario emerges, with very flat ghost dressing functions, flatter than expected, and a ghost propagator that diverges in the IR limit where the ghost behaves like a free zero-mass particle. A finite gluon propagator is found in the IR limit, with a dynamical mass that saturates at about 0.5-0.8 GeV and decreases as a power in the high energy limit.
Unfortunately the quantitative predictions are biased by an approximate estimate of the energy scale due to the lack of lattice data in Feynman gauge.
Any quantitative estimate of the gluon mass requires that an accurate energy scale should be fixed first. We can only say that a qualitative agreement is found with other predictions in Feynman gauge.

CONCLUDING REMARKS

In summary, one of the major achievements of the present work is the proof that a physically consistent solution does exist for the coupled set of non-linear integral equations that arise from the condition of stationary variance. Since pure Yang-Mills theory already contains fermions (the ghosts), inclusion of quarks in the formalism is straightforward, and would open the way to a broader study of QCD by the same technique.

The method can be improved in many way. We did not bother about gauge invariance in this first approach, but the properties of the polarization function, namely the correct cancellations of the unphysical degrees of freedom by the ghosts, show that the constraints of gauge invariance can be satisfied, at least approximately, by the variational solution.

While some attempts could be made for enforcing gauge invariance[1,30], a physically motivated choice for the gauge would probably improve the approximation. Landau gauge would be a good candidate, as it would enforce the transversality in the polarization function from the beginning.

An other interesting further development would come from the extension of the formalism to the general case of a finite external background field. For a scalar theory that kind of approach allows a consistent definition of approximate vertex functions by the functional derivative of the effective action. For the GEP these functions can be shown to be the sum of an infinite set of bubble graphs[16]. A similar approach would give a more consistent approximation for the gluon propagator in the present variational framework.

Eventually, the inclusion of quarks would lead to a direct comparison with the low energy phenomenology of QCD.

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Session Classification: Parallel 5: Strongly coupled gauge theories