#### Phenomenology of scenarios with flavor and CP symmetries

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# Outline

- lepton mixing: parametrization and data
- combination of flavor and CP symmetries: general idea
- highlights of survey of  $G_f = \Delta(3 n^2)$  and  $G_f = \Delta(6 n^2)$  and CP
- examples for predictions of  $0\nu\beta\beta$  decay and leptogenesis
- conclusions



# Lepton mixing: parametrization



with

$$\tilde{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

and  $s_{ij} = \sin \theta_{ij}$ ,  $c_{ij} = \cos \theta_{ij}$ 



**Global fits** (Gonzalez-Garcia et al. ('14) [after Neutrino'14]) best fit and  $1\sigma$  error  $3\sigma$  range  $\sin^2 \theta_{13} = 0.0219^{+0.0010}_{-0.0011}$  $0.0188 < \sin^2 \theta_{13} \le 0.0251$  $\sin^2 \theta_{12} = 0.304^{+0.012}_{-0.012}$  $0.270 < \sin^2 \theta_{12} < 0.344$  $\sin^2 \theta_{23} = \begin{cases} [0.451^{+0.06}_{-0.03}] \\ 0.577^{+0.027}_{-0.035} \end{cases}$  $0.385 \le \sin^2 \theta_{23} \le 0.644$  $\delta = (1.39^{+0.37}_{-0.33}) \ \pi \qquad 0 \le \delta \le 2 \ \pi$  $lpha \;,\;\; eta$ unconstrained

Global fits NH [IH] (Gonzalez-Garcia et al. ('14) [after Neutrino'14])

$$||U_{PMNS}|| \approx \begin{pmatrix} 0.83 & 0.55 & 0.15 \\ 0.39[4] & 0.64[57] & 0.66[75] \\ 0.41[5] & 0.54[62] & 0.73[64] \end{pmatrix}$$

and no information on Majorana phases



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 $\Downarrow$  Mismatch in lepton flavor space is large!



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CP phases have not been measured up to now!

 $\Downarrow$ 



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Use flavor and CP symmetry to explain/predict these features

 $\Downarrow$ 



- interpret this mismatch in lepton flavor space as mismatch of residual symmetries  $G_{\nu}$  and  $G_{e}$
- if we want to predict lepton mixing, we have to derive this mismatch
- let us assume that there is a symmetry, broken to  $G_{\nu}$  and  $G_{e}$
- this symmetry is in the following a combination of a

finite, discrete, non-abelian symmetry  $G_f$  and CP (Grimus/Rebelo ('95), Feruglio et al. ('12,'13), Holthausen et al. ('12), Chen et al. ('14))

[Masses do not play a role in this approach.]

• three generations of LH leptons are in 3 of group  $G_f$ 



- three generations of LH leptons are in 3 of group  $G_f$
- effect of residual symmetry  $G_e$  in charged lepton sector: constraints on mass matrix combination  $m_l^{\dagger}m_l$  for  $e^c m_l l$

$$Q^{\dagger}m_l^{\dagger}m_lQ = m_l^{\dagger}m_l$$

for Q generating  $G_e$ 

• matrix  $U_e$  diagonalizing  $m_l^{\dagger}m_l$  is determined by choice of Q



- three generations of LH leptons are in 3 of group  $G_f$
- effect of residual symmetry  $G_{\nu} = Z_2 \times CP$  in neutrino sector: constraints on Majorana mass matrix  $m_{\nu}$

$$Z^T m_{\nu} Z = m_{\nu}$$
 and  $X m_{\nu} X = m_{\nu}^{\star}$ 

for Z generating  $Z_2$  and CP transformation X

• matrix  $U_{\nu}$  diagonalizing  $m_{\nu}$  is constrained by choice of Z and X



- three generations of LH leptons are in 3 of group  $G_f$
- residual symmetries  $G_e$  and  $G_{\nu} = Z_2 \times CP$
- matrix  $U_e$  diagonalizing  $m_l^{\dagger}m_l$  is determined by choice of Q
- matrix  $U_{\nu}$  diagonalizing  $m_{\nu}$  is constrained by choice of Z and X
- PMNS mixing matrix

$$U_{PMNS} = U_e^{\dagger} U_{\nu}$$

is constrained by choice of  $G_e$  and  $G_{\nu}$ , i.e. (Q, Z, X)

[Masses are free parameters in this approach.]



 $U_{PMNS} = U_e^{\dagger} \Omega_{\nu} R(\theta) K_{\nu}$ 

- $U_{PMNS}$  contains one parameter  $\theta$
- permutations of rows and columns of  $U_{PMNS}$  possible
- 3 unphysical phases are removed by  $U_e \rightarrow U_e K_e$

#### Predictions:

 $\Downarrow$ 

Mixing angles and CP phases are predicted in terms of one parameter  $\theta$  only, up to permutations of rows/columns



To remember

- CP transformation X also acts on flavor space:  $\phi_i \xrightarrow{\text{CP}} X_{ij} \phi_j^*$ (X unitary)
- only "useful" choice in this context: X symmetric
- combination of flavor and CP symmetry requires

 $(X^*AX)^* = A'$  with in general  $A \neq A'$  and  $A, A' \in G_f$ 

- in particular  $G_{\nu} = Z_2 \times CP$ :  $XZ^* ZX = 0$
- LH leptons in irred rep 3 to be mapped into c.c. under CP (Chen et al. ('14))



•  $\Delta(3 n^2)$  can be characterized with three generators a, c and d (Luhn et al. ('07))

$$a^{3} = 1$$
 ,  $c^{n} = 1$  ,  $d^{n} = 1$  ,  
 $cd = dc$  ,  $aca^{-1} = c^{-1}d^{-1}$  ,  $ada^{-1} = c$ 

• all elements of the group can be written as

$$g = a^{lpha} c^{\gamma} d^{\delta}$$
 with  $lpha = 0, 1, 2$  ,  $0 \leq \gamma, \delta \leq n-1$ 

• for  $n \ge 2$ : group is non-abelian and has irred threedimensional reps **3** 



•  $\Delta(6 n^2)$  can be characterized with the generators a, c, dand b (Escobar/Luhn ('08))

$$b^2 = 1$$
 ,  $(ab)^2 = 1$  ,  
 $bcb^{-1} = d^{-1}$  ,  $bdb^{-1} = c^{-1}$ 

• all elements of the group can be written as

$$g = a^{\alpha}b^{\beta}c^{\gamma}d^{\delta}$$
 with  $\alpha = 0, 1, 2$  ,  $\beta = 0, 1$  ,  $0 \leq \gamma, \delta \leq n-1$ 

• for  $n \ge 2$ : group is non-abelian and has irred threedimensional reps **3** 



• we can choose as representation matrices for 3

$$a = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \quad \text{with} \quad \omega = e^{2\pi i/3}$$

and

$$c = \frac{1}{3} \begin{pmatrix} 1+2\cos\phi_n & 1-\cos\phi_n - \sqrt{3}\sin\phi_n & 1-\cos\phi_n + \sqrt{3}\sin\phi_n \\ 1-\cos\phi_n + \sqrt{3}\sin\phi_n & 1+2\cos\phi_n & 1-\cos\phi_n - \sqrt{3}\sin\phi_n \\ 1-\cos\phi_n - \sqrt{3}\sin\phi_n & 1-\cos\phi_n + \sqrt{3}\sin\phi_n & 1+2\cos\phi_n \end{pmatrix}$$
with  $\phi_n = \frac{2\pi}{n}$ 



• for 3 of  $\Delta(6 n^2)$  we also choose

$$b = \pm \left( \begin{array}{rrrr} 1 & 0 & 0 \\ 0 & 0 & \omega^2 \\ 0 & \omega & 0 \end{array} \right)$$

• Nota bene: we always take  $3 \nmid n$  and, if necessary, n even



## Choice of CP transformation

• we consider in the following X in 3 to be of the form

$$X = g P_{23} \quad \text{with} \quad P_{23} = \left(\begin{array}{rrrr} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{array}\right)$$

with g representing an element of the flavor group

fulfills relevant conditions



$$G_e = Z_3$$
  $Z$   $X$ 



-

•

 $G_e = Z_3$  Z X

#### can be reduced to

 $\begin{array}{lll} Q=a & Z=c^{n/2} & X=a\,b\,c^s\,d^{2s}\,P_{23} & ({\rm Case~1}) \\ & & & \\ Z=b\,c^m\,d^m & & X=c^s\,d^t\,P_{23} & ({\rm Case~2}) \\ & & & \\ & & X=b\,c^s\,d^{n-s}\,P_{23} & ({\rm Case~3a,} \end{array} \end{array}$ 

Case 3b.1)

•





#### special cases have been discussed

(Feruglio et al. ('12,'13), Ding/Zhou ('13,'14), Ding/King ('14), King/Neder ('14))

recently, also  $G_e \neq Z_3$  has been studied

(Ding et al. ('14))



• Q = a with a being diagonal in our basis tells us

$$U_e = 1$$

• the form of  $Z = c^{n/2}$  does not depend on n

$$Z = \frac{1}{3} \left( \begin{array}{rrrr} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{array} \right)$$



• Q = a with a being diagonal in our basis tells us

$$U_e = \mathbb{1}$$

non-degenerate eigenvalue of Z = c<sup>n/2</sup> has trimaximal eigenvector
 thus one column of PMNS mixing matrix is trimaximal consequently, we find in Case 1 and Case 2

$$\sin^2 \theta_{12} = \frac{1}{3 \, \cos^2 \theta_{13}} \gtrsim \frac{1}{3}$$



# Case 2: $(Q, Z, X) = (a, c^{n/2}, c^s d^t P_{23})$

- consider all permutations of rows and columns: either pattern is excluded or results of mixing angles and CP phases can be formally written in unique way
- opportune choice of parameters

$$u = 2s - t \quad , \quad v = 3t$$



• results for mixing angles

$$\sin^2 \theta_{13} = \frac{1}{3} \left( 1 - \cos\left(\frac{\pi u}{n}\right) \cos 2\theta \right) \quad , \quad \sin^2 \theta_{12} = \frac{1}{2 + \cos\left(\frac{\pi u}{n}\right) \cos 2\theta} \quad ,$$
$$\sin^2 \theta_{23} = \frac{1}{2} \left( 1 + \frac{\sqrt{3} \sin\left(\frac{\pi u}{n}\right) \cos 2\theta}{2 + \cos\left(\frac{\pi u}{n}\right) \cos 2\theta} \right)$$



#### First consider only the reactor mixing angle



- p. 28/70

#### For the particular choice n = 8 you find



- p 29/70

#### Results for mixing angles put together



- p. 30/70

Numerical example: n = 8

u	$\sin^2 \theta_{13}$	$\sin^2 \theta_{12}$	$\sin^2 \theta_{23}$
u = 0	0.0218	0.341	0.5
u = -1	0.0254	0.342	0.387
u = 1	0.0254	0.342	0.613



- results for CP phases
  - in general all are non-trivial
  - however, for particular values, e.g.  $\theta = 0$ , some can vanish
  - most importantly:

 $\sin \delta$  and  $\sin \beta$  depend only on  $\theta$  and u/n, whereas  $\sin \alpha$  is the only quantity also depending on v

Here and in the following we set  $K_{\nu} = 1$ .



Numerical example: n = 8

u	$\sin^2 \theta_{13}$	$\sin^2 \theta_{12}$	$\sin^2 \theta_{23}$	$\sin \delta$	$\sin\beta$
u = 0	0.0218	0.341	0.5	1	0
u = -1	0.0254	0.342	0.387	0	0
u = 1	0.0254	0.342	0.613	0	0

values of  $\sin \alpha$  for u = 0

 $\sin \alpha = 0$  ,  $\sin \alpha = 1$  and  $\sin \alpha = -1/\sqrt{2}$ values of  $\sin \alpha$  for  $u = \pm 1$ 

 $\sin \alpha \approx -0.924$  and  $\sin \alpha \approx 0.383$ 



• Q = a with a being diagonal in our basis tells us

$$U_e = 1$$

• non-degenerate eigenvalue of  $Z = b c^m d^m$  has eigenvector of form

$$\frac{1}{\sqrt{6}} \begin{pmatrix} -1 + e^{2\pi i m/n} \\ -\omega^2 + e^{2\pi i m/n} \\ -\omega + e^{2\pi i m/n} \end{pmatrix}$$



• Q = a with a being diagonal in our basis tells us

$$U_e = \mathbb{1}$$

• this eigenvector can be identified with the third column of the PMNS mixing matrix, then we find  $\sin^2 \theta_{13}$  and  $\sin^2 \theta_{23}$  as functions of m/n(Case 3a)



• Q = a with a being diagonal in our basis tells us

$$U_e = \mathbb{1}$$

• this eigenvector can be identified with the first column of the PMNS mixing matrix, for m = n/2 the vector reads

$$\frac{1}{\sqrt{6}} \left( \begin{array}{c} -2 \\ \omega \\ \omega^2 \end{array} \right)$$



• Q = a with a being diagonal in our basis tells us

$$U_e = 1$$

 this eigenvector can be identified with the first column of the PMNS mixing matrix,

for m = n/2 we then know that

$$\sin^2\theta_{12} \lesssim \frac{1}{3}$$

(special choice for Case 3b.1)



# $(Q,Z,X)=(a,b\,c^m\,d^m,b\,c^s\,d^{n-s}\,P_{23})$

- $\sin^2 \theta_{13}$  and  $\sin^2 \theta_{23}$  only depend on m/n
- m/n = 1/16 (m/n = 15/16) leads to good fit of data:

$$\sin^2 \theta_{13} \approx 0.0254$$
 and  $\sin^2 \theta_{23} \approx \begin{cases} 0.613 \\ 0.387 \end{cases}$ 

• solar mixing angle depends on additional parameters s and heta



Results for solar mixing angle for m/n = 1/16



- p. 39/70

- CP phases depend on all parameters: m, n, s and  $\theta$
- CP phases are non-trivial in general
- particular choices of parameters lead to no CP violation



Results for Dirac phase  $\delta$  for m/n=1/16



- p. 41/70

Results for Majorana phases  $\alpha$  and  $\beta$  for m/n=1/16



- p. 42/70

Numerical example: n = 16, m = 1 $\sin^2 \theta_{13} \approx 0.0254$  and  $\sin^2 \theta_{23} \approx 0.613$ some viable choices of s

S	$\sin^2 \theta_{12}$	$\sin\delta$	$\sin lpha$	$\sin eta$
s = 0	0.304	0	0	0
s = 1	0.304	0.458	0.939	0.662
	0.304	0.0594	-0.939	0.0383
s = 3	0.317	-0.533	0	-0.357



- neutrinos can be their own antiparticles
- if true, a process called  $0\nu\beta\beta$  decay is allowed





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$$m_{ee} = \left| \cos^2 \theta_{12} \, \cos^2 \theta_{13} \, m_1 + \sin^2 \theta_{12} \, \cos^2 \theta_{13} \, e^{i\alpha} \, m_2 + \sin^2 \theta_{13} \, e^{i\beta} \, m_3 \right|$$

using the experimentally preferred 3  $\sigma$  ranges of  $\sin^2 \theta_{13}$ ,  $\sin^2 \theta_{12}$  and of the mass splittings and varying the unknown Majorana phases  $\alpha$  and  $\beta$  and the lightest neutrino mass we get ...



- neutrinos can be their own antiparticles
- if true, a process called  $0\nu\beta\beta$  decay is allowed



Case 2 with n = 8, u = 0 and normal ordering



– p. 47/70

Case 2 with n = 8, u = 0 and inverted ordering



Case 2 with n = 8, u = 1 and normal ordering



 $-n \frac{49}{70}$ 

Case 2 with n = 8, u = 1 and inverted ordering



Case 3a with n = 16, m = 1, s = 0 and normal ordering



Case 3a with n = 16, m = 1, s = 0 and inverted ordering



Case 3a with n = 16, m = 1, s = 1 and normal ordering



- p. 53/70

Case 3a with n = 16, m = 1, s = 1 and inverted ordering



Case 3a with n = 16, m = 1, s = 3 and normal ordering



– p. 55/70

Case 3a with n = 16, m = 1, s = 3 and inverted ordering



# Leptogenesis

baryon asymmetry of the Universe is measured well

$$Y_B = \left. \frac{n_B - n_{\bar{B}}}{s} \right|_0 = (8.77 \pm 0.24) \times 10^{-11} \qquad \text{(WMAP ('08), Planck ('13))}$$

- this asymmetry can be explained by decay of heavy right-handed neutrinos (Fukugita/Yanagida ('86))
- the three Sakharov conditions are fulfilled (Sakharov ('67))



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- this asymmetry can be explained by decay of heavy right-handed neutrinos (Fukugita/Yanagida ('86))
- the three Sakharov conditions are fulfilled (Sakharov ('67))
- simplest scenario:

 $Y_B \sim 10^{-3} \epsilon \eta$  with  $\epsilon$  CP asymmetry ,  $\eta$  washout factor



# Leptogenesis

• CP asymmetry  $\epsilon_i$  for right-handed neutrino  $N_i$ 

$$\epsilon_i = -\frac{\Gamma(N_i \to Hl) - \Gamma(N_i \to H^*l)}{\Gamma(N_i \to Hl) + \Gamma(N_i \to H^*\overline{l})}$$

• computation of  $\epsilon_i$  in case of unflavored leptogenesis

$$\epsilon_i = -\frac{1}{8\pi} \sum_{j \neq i} \frac{\operatorname{Im}\left( (\hat{Y}_D^{\dagger} \hat{Y}_D)_{ij}^2 \right)}{(\hat{Y}_D^{\dagger} \hat{Y}_D)_{ii}} f(x_{ji})$$

with  $\hat{Y}_D = Y_D U_R$  and  $U_R^T M_R U_R = \text{diag}(M_1, M_2, M_3)$ 

[ $Y_D$ : Dirac neutrino coupling,

 $M_R$ : Majorana mass matrix of right-handed neutrinos]

Case 2 with n = 8, u = 0, v = 0: Prediction for  $\epsilon_1$  and  $\epsilon_3$  vs lightest neutrino mass for normal ordering,  $\kappa = 1.6 \times 10^{-3}$ 



Case 2 with n = 8, u = 0, v = 0: Prediction for  $\epsilon_1$  and  $\epsilon_3$  vs lightest neutrino mass for inverted ordering,  $\kappa = 1.6 \times 10^{-3}$ 



– p. 61/70

Case 2 with n = 8, u = 0, v = 6: Prediction for  $\epsilon_1$  and  $\epsilon_3$  vs lightest neutrino mass for normal ordering,  $\kappa = 1.6 \times 10^{-3}$ 



Case 2 with n = 8, u = 0, v = 6: Prediction for  $\epsilon_1$  and  $\epsilon_3$  vs lightest neutrino mass for inverted ordering,  $\kappa = 1.6 \times 10^{-3}$ 



Case 2 with n = 8, u = 1, v = 3: Prediction for  $\epsilon_1$  and  $\epsilon_3$  vs lightest neutrino mass for normal ordering,  $\kappa = 1.6 \times 10^{-3}$ 



Case 2 with n = 8, u = 1, v = 3: Prediction for  $\epsilon_1$  and  $\epsilon_3$  vs lightest neutrino mass for inverted ordering,  $\kappa = 1.6 \times 10^{-3}$ 



Case 3a with n = 16, m = 1, s = 1: Prediction for  $\epsilon_1$  and  $\epsilon_3$  vs lightest neutrino mass for normal ordering,  $\kappa = 1.6 \times 10^{-3}$ 



Case 3a with n = 16, m = 1, s = 1: Prediction for  $\epsilon_1$  and  $\epsilon_3$  vs lightest neutrino mass for inverted ordering,  $\kappa = 1.6 \times 10^{-3}$ 



Case 3a with n = 16, m = 1, s = 3: Prediction for  $\epsilon_1$  and  $\epsilon_3$  vs lightest neutrino mass for normal ordering,  $\kappa = 1.6 \times 10^{-3}$ 



Case 3a with n = 16, m = 1, s = 3: Prediction for  $\epsilon_1$  and  $\epsilon_3$  vs lightest neutrino mass for inverted ordering,  $\kappa = 1.6 \times 10^{-3}$ 



# Conclusions

- approach with flavor and CP symmetry very interesting: allows to predict CP phases and free parameter θ helps to accommodate mixing angles
- very rich structure of results for  $G_f = \Delta(3 n^2)$  and  $\Delta(6 n^2)$
- comprehensive study and analytical understanding of results
- study of  $0\nu\beta\beta$  decay and leptogenesis in progress: constraints and correlations possible

Thank you for your attention.

