S_3 and Q_6 as Flavour Symmetries in Multi-Higgs Models

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Some head breakers in HEP

- What happens as we approach the Planck scale? or just as we go up in energy...
- How are the gauge, Yukawa and Higgs sectors related at a more fundamental level?
- How do particles get their very different masses?
- Why is the mixing so strange?
- What is the nature of the Higgs?
- Is there one or many? How this affects all the above?
- **Where is the new physics??**

Facts

Some aspects of the flavour problem:

- Quark masses vastly different

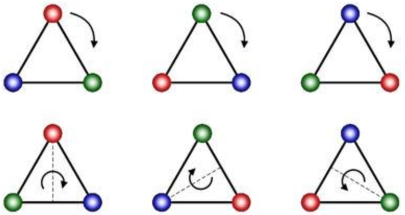
$$m_u : m_c : m_t \approx 10^{-6} : 10^{-3} : 1,$$
$$m_d : m_s : m_b \approx 10^{-4} : 10^{-2} : 1,$$
$$m_e : m_\mu : m_\tau \approx 10^{-5} : 10^{-2} : 1$$

- Quark weak mixing angles:
    - $\theta_{12} \approx 13.0^o$
    - $\theta_{23} \approx 2.4^o$
    - $\theta_{13} \approx 0.2^o$

- Lepton masses not known (only difference of squared masses), but extremely small
- Lepton weak mixing angles, best fit of recent experimental data
    - $\theta_{12} \approx 34^o$
    - $\theta_{23} \approx 46^o$
    - $\theta_{13} \approx 9^o$ Tórtola et al, 2014

    $\Rightarrow \theta_{13} \neq 0$
- CP-violation occurs in the weak sector

- The structure of quarks and leptons suggests some symmetry behind it
- What kind of symmetry?



- Abelian or non-Abelian?
  Abelian has irreps uni-dimensional, cannot reproduce mixing pattern with a single one
- Continuous or discrete?
  Continuous: broken symmetries lead to Goldstone bosons
  Discrete: no Goldstone bosons, small size and finite number of irreps
- Many different approaches

Discrete non-Abelian symmetries

- Studied extensively since it is possible to accommodate the three families of quarks and leptons
- From a bottom-up approach could be the last stage of breaking of a more fundamental theory
- Many groups studied: $A_4, S_4, S_3, Q_6, \Delta(27)$
- They lead to different patterns of masses and mixings $\Rightarrow$ different predictions

How do we choose a flavour symmetry?

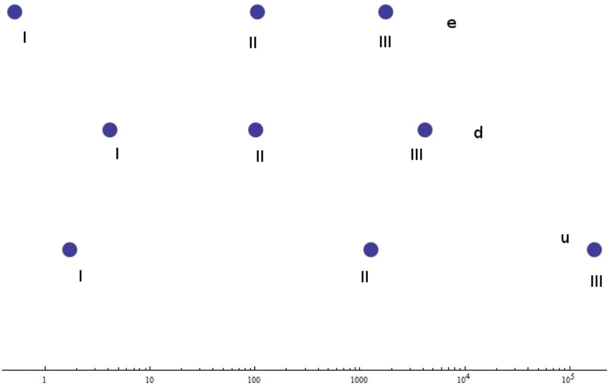
At low energies: **A postmodernist approach**

- Find the smallest possible flavour symmetry **suggested by the data**
- Explore how generally ("universally") it can be applied
- Follow it to the end
- Compare with the data, see if our guess works **(wild...)**

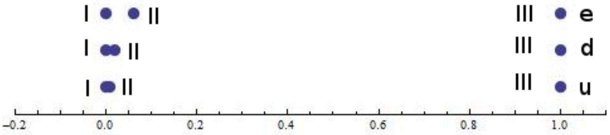
At high energies: **A conventional approach**

- Look for a bigger group that contains our symmetry
- Add more symmetry: SUSY? GUT? Flavour?
- Try to keep it minimal/simple, if possible... **(not conventional at all)**

Logarithmic plot of fermion masses



Plot of fermion mass ratios

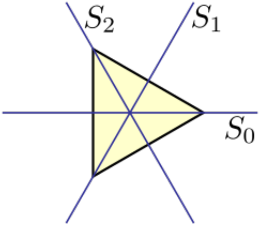


Fundamental fermions normalized by the heaviest of each type

suggests $\mathbf{2} \oplus \mathbf{1}$ structure

Also, prior to electroweak symmetry breaking all three families are interchangeable

The S_3 symmetry group: permutations of 3 objects.



Follow the symmetry...

Assignment between fermion fields and irreps:

$$\Phi \to F \quad = \quad F(\Phi_1, \Phi_2, \Phi_3)$$

F is a S_3 reducible representation $\mathbf{1_S} \oplus \mathbf{2}$

$$F_s \quad = \quad \frac{1}{\sqrt{3}}\Big(\Phi_1 + \Phi_2 + \Phi_3\Big); \qquad F_D = \begin{pmatrix} \frac{1}{\sqrt{2}}(\Phi_1 - \Phi_2) \\ \\ \frac{1}{\sqrt{6}}(\Phi_1 + \Phi_2 - 2\Phi_3) \end{pmatrix}$$

Some references to work with an S_3 symmetry

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- S. Dev et al, Phys.Lett. B708 (2012) 284-289
- S. Zhou, Phys.Lett. B704 (2011) 291-295
- E. Barradas et al, 2014
- P. Das et al, 2014
- A.E. Cárcamo et. al, 2014
- There are many more, I apologize for those not included.

- In the SM (one Higgs eW doublet) with S_3, to give masses to the particles the flavor symmetry has to be broken

A. Mondragón, E. Rodríguez-Jáuregui, 1999, 2000

- The breaking of the symmetry can be parametrized with Z, satisfies cubic equation
- Possible to classify texture zeroes in equivalence classes, simplify analysis A. Mondragón and F. González, 2011
- If S_3 it's **not explicitly broken** then... $\Rightarrow$

we need to introduce additionally two Higgs weak-doublets more to the SM to preserve the permutational symmetry (at least before eW breaking)

J. Kubo, A. Mondragón, M. Mondragón, E. Rodríguez, 2003

S_3 symmetry with 3 Higgs doublets in the Lagrangian

- To build an S_3 invariant Lagrangian:
  assign the first two families to the doublet irrep $\mathbf{2}$
  the third one either to the singlet symmetric $\mathbf{1}_S$
  or the singlet antisymmetric $\mathbf{1}_A$.
- Add three right handed neutrinos to implement the seesaw mechanism
- All sectors follow this assignment: quarks, leptons (left and right) and Higgs
- Different assignments lead to different models
  Also important in symmetries left in the Higgs potential after eW breaking

González-Felipe, Ivanov, Nishi, Serodio and Silva, 2014

After electroweak symmetry breaking, the Higgs $SU(2)_L$ doublets acquire real vacuum expectation values (vev's),

$$w_1 \equiv \langle 0|H_1|0\rangle, \;\; w_2 \equiv \langle 0|H_2|0\rangle,$$

$$v_S \equiv \langle 0|H_S|0\rangle, \;\; \text{and} \;\; v_A \equiv \langle 0|H_A|0\rangle,$$

J. Kubo, A. Mondragón, M. M., E. Rodríguez-Jáuregui, 2003

We get for every Dirac fermion the generic mass matrix:

$$\mathcal{M}_f = \begin{pmatrix} \mu_1 + \mu_2 & \mu_4 & \mu_5 \\ \mu_4 & \mu_1 - \mu_2 & \mu_6 \\ \mu_7 & \mu_8 & \mu_3 \end{pmatrix}$$

Furthermore, the concept of flavour is extended to the Higgs sector

Quarks

Numerical study of quarks and three Higgses showed compatibility with data

Kubo, Mondragón, M, Rodríguez-Jáuregui, 2003

FCNC's in quark sector are suppressed

Teshima, 2012

Quark lepton complementarity studied

Barranco, A. Mondragón, González Canales, 2009

Data on quarks has improved considerably $\Rightarrow$ important to go back to data and check compatibility with symmetry

Possible to classify different S_3 models in equivalence classes and obtain known textures

Comparison with recent data gives very good agreement between theoretical and experimental V_{CKM}

González-Canales, A& M Mondragón, Saldaña-Salazar, Velasco-Sevilla, 2013

- Depending on whether the singlet Higgs in the symmetric or anti-symmetric singlet irrep:
- $H_S \Rightarrow$ Viable models (quarks) only when left and right parts of the third family share the same assignement
- $H_A \Rightarrow$ Viable models (quarks) only when left and right part of the third family are in different irreps
- They lead to two zero textures (Fritzsch type) or NNI form
- Both known to give good phenomenology
- Case with four Higgs (H_A and H_S) reduces to the case with three Higgses, but Higgs potential may differ

2 zeroes mass matrices

We can bring the quark mass matrices from a symmetric basis to a hierarchical basis via a rotation and a shift

$$\mathcal{M}^f_{S_3} \longrightarrow \mathcal{M}^f_{Hier} \equiv \mathcal{R}(\theta)_{12}\mathcal{M}^f_{S_3}\mathcal{R}(\theta)^T_{12} = \begin{pmatrix} \mu_0^f & a^f & 0 \\ a^{f*} & b^f & c^f \\ 0 & c^{f*} & d^f \end{pmatrix}$$
$$= \mu_0^f \mathbf{1}_{3\times 3} + \widehat{\mathcal{M}}^f_{Hier} \ ,$$

The matrix $\widehat{\mathcal{M}}^f_{Hier}$ has two texture zeroes

$$\widehat{\mathcal{M}}^f_{Hier} = \begin{pmatrix} 0 & a^f & 0 \\ a^{f*} & b'^f & c^f \\ 0 & c^{f*} & d'^f \end{pmatrix} = \begin{pmatrix} 0 & a^f & 0 \\ a^{f*} & b^f - \mu_0^f & c^f \\ 0 & c^{f*} & d^f - \mu_0^f \end{pmatrix},$$

and eigenvalues denoted as σ_i^f, $i = 1, 2, 3$.

To achieve these textures we

- Perform a rotation
- May choose a particular value for the rotation angle θ, i.e. a particular change of basis or
- If mass matrices hermitian or symmetric
- Shift the matrix and use the known two zeroes textures reparameterization

Rodríguez-Jáuregui, Mondragón, 2000;Barranco, González-Canales, Mondragón, 2008

- Reparameterize the mass matrices in terms of its invariants and one constrained parameter

Then, the physical masses m_i^f are related to the shifted masses σ_i^f simply by $m_i^f = \mu_0^f + \sigma_i^f$.

Then, we can express the entries in the CKM mixing matrix as analytical relations between the parameters, determined by the symmetry

The CKM matrix

The V_{CKM} matrix is defined as

$$V_{CKM}^{th} = \mathbf{U}_{u_L}^{\dagger}\mathbf{U}_{d_L} = \mathbf{O}_u^T P^{(u-d)}\mathbf{O}_d,$$

where $P^{(u-d)} = \text{diag}[1, e^{i\phi_1}, e^{i(\phi_1+\phi_2)}]$ with $\phi_i \equiv \phi_{iu} - \phi_{id}$, and $\mathbf{O}_{u,d}$ are the real orthogonal matrices, that diagonalize the mass matrix.

Thus, we can express the V_{CKM} as function of the quark masses:

exact, analytical expressions

V_{CKM} entries are written as functions of the quark mass ratios, two free parameters, and a CP violating phase. For example:

$$
\begin{aligned}
\mathbf{V}_{us}^{th} &= -\left(\frac{\tilde{m}_c(1-\tilde{m}_u-\delta_u)\tilde{m}_d(1+\tilde{m}_s-\delta_q)}{(1-\delta_u)(1-\tilde{m}_u)(\tilde{m}_c+\tilde{m}_u)(1-\delta_q)(1+\tilde{m}_s)(m_s+m_d)}\right)^{1/2} \\
&+ \left(\frac{\tilde{m}_u\tilde{m}_s}{(1-\tilde{m}_u)(\tilde{m}_c+\tilde{m}_u)(\tilde{m}_d+\tilde{m}_s)}\right)^{1/2}\left\{\left(\frac{(1-\tilde{m}_u-\delta_u)(1+\tilde{m}_s-\delta_d)}{(1+\tilde{m}_s)}\right)^{1/2}\right. \\
&+ \left.\left(\frac{(1+\tilde{m}_c-\delta_u)\delta_u(1-\tilde{m}_d-\delta_d)\delta_d}{(1-\delta_u)(1-\delta_d)(1+\tilde{m}_s)}\right)^{1/2}\right\}e^{i\phi}
\end{aligned}
$$

We find an excellent agreement between our theoretical V_{CKM}^{th} and V_{CKM}^{PDG}

$\chi^2 = 3.10 \times 10^{-1}/4$ for the fit using the central values
$\chi^2 = 4/4$ for the fit using the values with restricted precision

for appropriate values of the constrained parameters δ_f and a CP violating phase $\sim 80 \sim 100^o$, i.e. maximal

F. González-Canales, A. Mondragón, M. Mondragón, L. Velasco-Sevilla, U. Saldaña-Salazar, arXiv:13046644, Phys.Rev. D88 (2013) 096004.

Leptons

$-$	$+$
$H_S,\ \nu_{3R}$	$H_I,\ L_3,\ L_I,\ e_{3R},\ e_{IR},\ \nu_{IR}$

- In the leptonic sector we add a Z_2 symmetry
- FCNC's are strongly suppressed by the $S_3 \times Z_2$ symmetry and the mass hierarchy of the charged leptons
- Possible to write the mixing angles in terms of the lepton masses
- Predictions for neutrino masses and mixings
- S_3 gives $\theta_{13} \neq 0$
- If $M_{1R} = M_{2R}$, θ_{13} too small: lower bound, and θ_{12}, θ_{23} within experimental limits

A. Mondragón, M. M., E. Peinado, 2007,2008

- If $M_{1R} \neq M_{2R} \quad \Rightarrow \quad \theta_{12}, \theta_{23}, \theta_{13}$ compatible with recent data

A. Mondragón, M. M., F. González, 2012

Charged leptons

The mass matrix of the charged leptons takes the form

$$\mathbf{M}_e = m_\tau \begin{pmatrix} \tilde{\mu}_2 & \tilde{\mu}_2 & \tilde{\mu}_5 \\ \tilde{\mu}_2 & -\tilde{\mu}_2 & \tilde{\mu}_5 \\ \tilde{\mu}_4 & \tilde{\mu}_4 & 0 \end{pmatrix}. \tag{1}$$

Reparametrized in terms of its eigenvalues and written to order $\left(m_\mu m_e/m_\tau^2\right)^2$ and $x^4 = (m_e/m_\mu)^4$, is

$$\mathbf{M}_e \approx m_\tau \begin{pmatrix} \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & -\frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & 0 \end{pmatrix}.$$

Only free parameter is the Dirac phase δ_e

Neutrinos

The $S_3 \times Z_2$ gives the following matrix for Dirac neutrinos

$$\mathbf{M}_{\nu_D} = \begin{pmatrix} \mu_2^\nu & \mu_2^\nu & 0 \\ \mu_2^\nu & -\mu_2^\nu & 0 \\ \mu_4^\nu & \mu_4^\nu & \mu_3^\nu \end{pmatrix}, \qquad (2)$$

Kubo et al 2003, Felix, Mondragón 2006, Mondragon 2007

and considering the following form to the mass matrix of right-handed neutrinos $\mathbf{M}_{\nu_R} = \mathrm{diag}\{M_1, M_2, M_3\}$ Then, the mass matrix $\mathbf{M}_{\nu_L}$ takes the form

$$\mathbf{M}_{\nu_L} = \begin{pmatrix} \frac{2(\mu_2^\nu)^2}{\overline{M}} & \frac{2\lambda(\mu_2^\nu)^2}{\overline{M}} & \frac{2\mu_2^\nu\mu_4^\nu}{\overline{M}} \\ \frac{2\lambda(\mu_2^\nu)^2}{\overline{M}} & \frac{2(\mu_2^\nu)^2}{\overline{M}} & \frac{2\mu_2^\nu\mu_4^\nu\lambda}{\overline{M}} \\ \frac{2\mu_2^\nu\mu_4^\nu}{\overline{M}} & \frac{2\mu_2^\nu\mu_4^\nu\lambda}{\overline{M}} & \frac{2(\mu_4^\nu)^2}{\overline{M}} + \frac{(\mu_3^\nu)^2}{M_3} \end{pmatrix}, \quad \lambda = \frac{1}{2}\left(\frac{M_2 - M_1}{M_1 + M_2}\right), \text{ and } \overline{M} = 2\frac{M_1 M_2}{M_2 + M_1}.$$

Neutrino mixing angles I

The solar angle θ_{12}: strongly dependent on the neutrino masses, weakly dependent on the charged lepton masses

$$\tan\theta_{12}^2 = \frac{(\Delta m_{12}^2 + \Delta m_{13}^2 + |m_{\nu_3}|^2)^{1/2} - |m_{\nu_3}|}{(\Delta m_{13}^2 + |m_{\nu_3}|^2 \cos^2\phi_\nu)^{1/2} + |m_{\nu_3}|}$$

The numerical value of $\tan^2\theta_{12}$ fixes the origen and scale of the neutrino masses.

The atmospheric mixing angle θ_{23}: depends mostly on the charged lepton masses

$$\sin\theta_{23} \approx \frac{1}{\sqrt{2}} \frac{1 - 2\tilde{m}_\mu^2 + \tilde{m}_\mu^4}{\sqrt{1 - 4\tilde{m}_\mu^2 + x^2 + 6\tilde{m}_\mu^4}} = 0.7071$$

$$x = m_e/m_\mu = 4.84 \times 10^{-3}, \qquad \tilde{m}_\mu = m_\mu/m_\tau = 5.95 \times 10^{-2}$$

Neutrino mixing angles II

The reactor mixing angle θ_{13}: mostly determined by the interplay of S_3 and the mass spliting of the right-handed neutrinos in the seesaw mechanism plus a very small contribution from the charged leptons

$$\sin\theta_{13} \approx \frac{2(\lambda\mu)m_{\nu_3}}{m_{\nu_1}-m_{\nu_2}}\left(1-\sqrt{\frac{(m_{\nu_2}-m_{\nu_3})}{m_{\nu_3}-m_{\nu_1}}}\right)\left(\cos\eta-\sqrt{(1-\frac{m_{\nu_1}}{m_{\nu_3}})(\frac{m_{\nu_2}}{m_{\nu_3}}-\frac{m_{\nu_1}}{m_{\nu_3}})}\sin\eta\right)$$

$$+\quad \frac{1}{\sqrt{2}}\frac{m_e}{m_\mu}\frac{(1+4(\frac{m_e}{m_\mu})^2-(\frac{m_\mu}{m_\tau})^4)}{\sqrt{1+(\frac{m_\mu}{m_\tau})^2+5(\frac{m_e}{m_\mu})^2-(\frac{m_\mu}{m_\tau})^4}}$$

where

$$\cos\eta=\sqrt{\frac{m_{\nu_2}-m_{\nu_3}}{m_{\nu_2}-m_{\nu_1}}}, \qquad \sin\eta=\sqrt{\frac{m_{\nu_3}-m_{\nu_1}}{m_{\nu_2}-m_{\nu_1}}}$$

We get

$$\sin\theta_{13}^{th}\approx 0.17 \qquad \text{with} \qquad (\lambda\mu)\approx 0.02$$

Neutrino mixing angles: Theory vs Experiment

The experimental values of the neutrino mixing angles θ_{13} and θ_{23}

(D.V. Forero, M. Tórtola and J.W.F. Valle, arXiv: : arXiv:1405.7540 [hep-ph] May 2014)

$$\bar{\theta}^{o\ exp}_{13} = 8.8(8.9) \pm 0.4 \rightarrow \quad \sin^2 \bar{\theta}^{o\ exp}_{13} = 0.0234 \pm 0.002 \quad (.0240 \pm .0019);$$
$$\bar{\theta}^{o\ exp}_{23} = 48.9^{+1.8}_{-7.4} \quad (49.2^{+1.4}_{-2.5}) \rightarrow \quad \sin^2 \bar{\theta}^{o\ exp}_{23} = 0.567^{+.032}_{-.028} \quad (0.573^{+.025}_{-.043})$$

Our theoretical predictions:

$$\theta^{o\ th}_{13} = 9.6 \pm 0.07 \qquad \sin^2 \theta^{o\ th}_{13} = 0.028 \pm 0.002$$

$$\theta^{o\ th}_{23} = 44.97 \pm 1.2 \qquad \sin^2 \theta^{o\ th}_{23} = 0.50 \pm 0.02$$

in very good agreement with the latest experimental values!

A.Mondragón, M. Mondragón and E. Peinado Phys. Rev. D **76**, 076003 (2007), F. González Canales and A. Mondragón, J. Phys: Conference Series **387** (2012) 012008 and F. González Canales, A. Mondragón and M. Mondragón, Fortsch.Phys. 61 (2013)

Neutrino mass spectrum

- We wrote the neutrino mass differences, $m_{\nu_i} - m_{\nu_j}$, in terms of the differences of the squared masses $\Delta_{ij}^2 = m_{\nu_i}^2 - m_{\nu_j}^2$ and one of the neutrino masses, say m_{ν_3}.
- The mass m_{ν_2} was taken as a free parameter in the fitting of our formula for $\tan\theta_{12}$ to the experimental value

$$\Delta m_{21}^2 = 7.6 \times 10^{-5} eV^2 \qquad \Delta m_{13}^2 = 2.4 \times 10^{-3} eV^2$$

and

$$\tan\theta_{12} = 0.696$$

we get

$$|m_{\nu_3}| \approx 0.019 \ eV \Longrightarrow |m_{\nu_2}| \approx 0.053 \ eV \ , \quad |m_{\nu_1}| \approx 0.052 \ eV$$

And now? What about the Higgs sector?

- The details of the Higgs potential are crucial to distinguish between scenarios
- Flavour symmetry breaking related directly to the electroweak breaking
- In S_3 models in the quark sector at least one of the global minima is compatible with good phenomenology $\theta = \pi/6$
- Other minima have to be analysed
- Full analysis of leptonic and Higgs sectors underway
  The predictions come from these two sectors

Calculate you must...

Q_6 as flavour symmetry

S_3 works well at low energies, how do we go to higher energies?

S_3 might be the last stage in a chain of breakings, each one of those obscures the underlying symmetry, and we don't have much experimental information...

- Keep the $2 + 1$ structure
- Embed it in a SUSY GUT

Try with a SUSY $SU(5)$ GUT, with Q_6 as flavour group

- Q_6 binary dihedral group, double covering of S_3
- Q_6 suppresses some proton violating operators
- One $S_3 \times Z_2$ gives same matrices as $SU(5) \times Q_6$
  $\Rightarrow$ NNI form with interesting neutrino sector

J.C. Gómez Izquierdo, F. González Canales, M.M.

- It works well with $\not{R}$

also extensively studied: Kubo et al, 2005,2007, 2009, 2011, 2013 ; Kajiyama, 2007; Araki et al, 2012; Gómez Izquierdo, González Canales, M.M., 2014

The matter content $SU(5)\times Q_6$

	$SU(5)$	Q_6
(H_1^d, H_2^d)	$\bar{\mathbf{5}}$	$\mathbf{2}_1$
H_3^d	$\bar{\mathbf{5}}$	$\mathbf{1}_{+,2}$
(H_1^u, H_2^u)	$\mathbf{5}$	$\mathbf{2}_1$
H_3^u	$\mathbf{5}$	$\mathbf{1}_{+,2}$
(F_1, F_2)	$\bar{\mathbf{5}}$	$\mathbf{2}_2$
F_3	$\bar{\mathbf{5}}$	$\mathbf{1}_{-,3}$
(T_1, T_2)	$\mathbf{10}$	$\mathbf{2}_2$
T_3	$\mathbf{10}$	$\mathbf{1}_{-,3}$
(N_1^c, N_2^c)	$\mathbf{1}$	$\mathbf{2}_2$
N_3^c	$\mathbf{1}$	$\mathbf{1}_{-,1}$
Y_B	$\mathbf{1}$	$\mathbf{1}_{+,2}$
$H_{\bar{45}}$	$\bar{\mathbf{45}}$	$\mathbf{1}_{+,2}$
H_{45}	$\mathbf{45}$	$\mathbf{1}_{+,2}$
Φ	$\mathbf{24}$	$\mathbf{1}_{+,0}$

$SU(5) \times Q_6$

We have an $SU(5)$ SUSY GUT, with extended Higgs sector and Q_6 as flavour group

Quark and charged lepton masses follow very similar analyses as S_3

To avoid the $M_d = M_e^T$ relation we added a **45** irrep (can be done with non-renormalizable operators)

e.g.Berezhiani, Z. Tavartkiladze, and M. Vysotsky, 1998; Bajc, Fileviez, Senjanovic 2002

Neutrino masses can be achieved:

- Via a seesaw adding right handed neutrinos
- Via non-renormalizable operators — e.g. Machado and Pleitez, 2007
- Adding an extra $U(1)$ — e.g. like a breaking of $SO(10)$
- Via R parity violation — e.g. Romao and Valle, 1992

Look at the most straightforward one for the moment

All the others currently under investigation

Quark and lepton mass matrices

The quark and charged lepton mass matrices have the form In this particular model

$$\mathbf{M}_u = \begin{pmatrix} 0 & -2\tilde{Y}_1 v_{45} & \bar{y}^u h_2^{0u} \\ 2\tilde{Y}_1 v_{45} & 0 & -\bar{y}^u h_1^{0u} \\ \bar{y}^u h_2^{0u} & -\bar{y}^u h_1^{0u} & y_4^u h_3^{0u} \end{pmatrix}, \ \mathbf{M}_d = \begin{pmatrix} 0 & y_1^d h_3^{0d} + 2Y_1 v_{\bar{45}} & y_2^d h_2^{0d} \\ -y_1^d h_3^{0d} - 2Y_1 v_{\bar{45}} & 0 & -y_2^d h_1^{0d} \\ y_3^d h_2^{0d} & -y_3^d h_1^{0d} & y_4^d h_3^{0d} + 2Y_2 v_{\bar{45}} \end{pmatrix};$$

$$\mathbf{M}_\ell = \begin{pmatrix} 0 & -\left(y_1^d h_3^{0d} - 6Y_1 v_{\bar{45}}\right) & y_3^d h_2^{0d} \\ y_1^d h_3^{0d} - 6Y_1 v_{\bar{45}} & 0 & -y_3^d h_1^{0d} \\ y_2^d h_2^{0d} & -y_2^d h_1^{0d} & y_4^d h_3^{0d} - 6Y_2 v_{\bar{45}} \end{pmatrix}.$$

where $\bar{y}^u \equiv \left(y_2^u + y_3^u\right)/2$.

As can be seen, the $\mathbf{M}_u$ mass matrix turns out almost symmetric due to the flavour structure.

Quark sector

Quark mixing matrix can be fitted fixing the quark mass ratios and two free parameters, similar as in S_3 (assume that the form is preserved from GUT to eW scale, not quite true):
We take

$$\chi^2 = \frac{(|V_{ud}^{th}| - |V_{ud}^{ex}|)^2}{\sigma_{V_{ud}}^2} + \frac{(|V_{us}^{th}| - |V_{us}^{ex}|)^2}{\sigma_{V_{us}}^2} + \frac{(|V_{ub}^{th}| - |V_{ub}^{ex}|)^2}{\sigma_{V_{ub}}^2} + \frac{(\mathcal{J}_q^{th} - \mathcal{J}_q^{ex})^2}{\sigma_{\mathcal{J}_q}^2}$$

and using $\mathcal{J}$ implies unitarity as a constraint and $\chi^2 = 0.0515$ as the minimal value.
These correspond to the following values for the V_{CKM} elements

$$\begin{aligned} |V_{ud}^{th}| &= 0.97428^{+0.00016}_{-0.00014}, \quad |V_{us}^{th}| = 0.2252^{+0.00062}_{-0.00067} \\ |V_{ub}^{th}| &= 0.00351^{+0.00017}_{-0.00016}, \quad \mathcal{J}_q = 2.95^{+0.20}_{-0.19} \times 10^{-5}. \end{aligned}$$

A more detailed numerical analysis without assuming unitarity under way.

Neutrino sector I

Two mass degenerate right-handed neutrinos (RHNs)

- Again, similar to S_3, $M_{1R} = M_{2R}$ gives us a lower bound for the reactor angle θ_{13} and the mass scale of the neutrinos.
- In this case we obtain the following sum rule

$$m_{\nu_3} \leq (\sqrt{m_{\nu_2}} - \sqrt{m_{\nu_1}})^2,$$

where the equality in the above expression means an upper bound for the m_{ν_3} lightest mass.
- Normal hierarchy ruled out, only inverted hierarchy possible
- Reactor and atmospheric angles depend only on the charged lepton masses and one free parameter
- Solar mixing angle depends also on the neutrino masses

Neutrino sector II

All RHNs have different masses

- PMNS matrix depends on the charged lepton and neutrino masses
- Through a χ^2 fit we can determine the best values for the free parameters: $m_{\nu_{3[2]}}$, $\widetilde{\mu}_0$ and δ_ν
- $\Longrightarrow$ three mixing angles are within experimental bounds
- Range for neutrino masses

The neutrino masses and mixing

The neutrino masses are written as:

$$\widetilde{m}_{\nu_1} = \frac{m_{\nu_1}}{m_{\nu_3}} = \sqrt{1 - \frac{\Delta m^2_{\rm ATM}}{m^2_{\nu_3}}}, \quad \widetilde{m}_{\nu_2} = \frac{m_{\nu_2}}{m_{\nu_3}} = \sqrt{1 - \frac{\Delta m^2_{32}}{m^2_{\nu_3}}} \quad ({\rm NH}),$$
$$\widetilde{m}_{\nu_3} = \frac{m_{\nu_3}}{m_{\nu_2}} = \sqrt{1 - \frac{\Delta m^2_{23}}{m_{\nu_2}}}, \quad \widetilde{m}_{\nu_1} = \frac{m_{\nu_1}}{m_{\nu_2}} = \sqrt{1 - \frac{\Delta m^2_{\odot}}{m^2_{\nu_2}}} \quad ({\rm IH}), \tag{3}$$

where $\Delta m^2_{32} = \Delta m^2_{\rm ATM} - \Delta m^2_{\odot}$ and $\Delta m^2_{23} = \Delta m^2_{\rm ATM} + \Delta m^2_{\odot}$

Thus $m_{\nu_{3[2]}} < \Delta m^2_{32[\odot]}$ and $\sum_i m_{\nu_i} < 0.23$ eV constrain $m_{\nu_{3[2]}}$

Our results

$\chi^2 = 0.020[0.014]$ for a normal [inverted] hierarchy. The neutrino masses at 1σ are:

$$m_{\nu_3} = \begin{cases} \left(5.35^{+4.32}_{-1.73}\right) \times 10^{-2}\mathrm{eV} \\ \left(4.44^{+4.21}_{-3.87}\right) \times 10^{-2}\mathrm{eV} \end{cases}, \; m_{\nu_2} = \begin{cases} \left(2.01^{+6.42}_{-0.98}\right) \times 10^{-2}\mathrm{eV} \\ \left(6.71^{+3.25}_{-1.81}\right) \times 10^{-2}\mathrm{eV} \end{cases}, \; m_{\nu_1} = \begin{cases} \left(1.08^{+6.59}_{-1.30}\right) \times 10^{-2}\mathrm{eV} \\ \left(6.65^{+3.27}_{-1.83}\right) \times 10^{-2}\mathrm{eV} \end{cases}$$

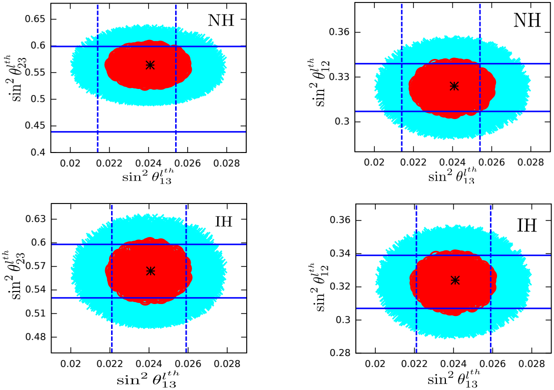
The free parameters $\widetilde{\mu}_0$ and δ_ν at 1 σ are:

$$\widetilde{\mu}_0 = \begin{cases} 0.22^{+0.63}_{-0.20} \\ 0.56^{+0.29}_{-0.56} \end{cases} \quad \text{and} \quad \delta_\nu = \begin{cases} 0.75^{+0.24}_{-0.15} \\ 0.73^{+0.25}_{-0.09} \end{cases} .$$

We obtain the following numerical values for the leptonic mixing angles, at 1 σ:

$$\theta_{12}^{\ell th} = \begin{cases} \left(34.71^{+0.91}_{-0.98}\right)^\circ \\ \left(34.73^{+0.89}_{-1.11}\right)^\circ , \end{cases} \; \theta_{23} = \begin{cases} \left(45.83^{+4.49}_{-3.98}\right)^\circ \\ \left(48.57^{+2.07}_{-2.76}\right)^\circ , \end{cases} \; \theta_{13} = \begin{cases} \left(8.77^{+0.40}_{-0.32}\right)^\circ \\ \left(8.93^{+0.33}_{-0.39}\right)^\circ , \end{cases}$$

A picture better than words...



Allowed regions for the lepton mixing angles for a normal and inverted hierarchy (NH and IH) in the mass spectrum of neutrinos. Turquoise: 90% C.L., red: 69% C.L.. The blue solid lines delimit the experimental data of $\sin^2\theta^{\ell}_{12(23)}$ at 1σ. The blue dashed lines delimit the experimental data of $\sin^2\theta^{\ell}_{13}$ at 1σ

Conclusions

- The permutational symmetry S_3 is the smallest non-Abelian discrete symmetry suggested by data
- Used as a flavour symmetry in the quark, lepton and Higgs sectors allows for a "unified" treatment of fermion masses
- Possible to find analytical expressions for mixing matrices of both quarks and leptons in terms of mass ratios
- Good phenomenology **both** in the quark and lepton sectors
- In the quark sector $\Rightarrow$ Fritzsch mass textures and the NNI form, both of which give good phenomenology $\Rightarrow$ fitting V_{CKM} with only three free parameters (and the mass ratios)
- **One parameter less than in SM**, despite the number of Higgses

Conclusions II

- In the leptonic sector it is possible to find analytical expressions for the mixing angles in terms of lepton masses $\Rightarrow$ fixing one of the mixing angles and using the experimental data $\Rightarrow$ predict the other two mixing angles correctly
- Allows to find a range for the neutrino masses
- Possible to extend it to an $SU(5) \times Q6$ SUSY GUT **with similarly good predictions**
- Full analysis of the quark and leptonic sectors underway
- The predictions come from these two sectors
- Full analysis of the Higgs sector underway
- Include SUSY breaking and RGE properly