# Open Questions in CP Violation and Flavour

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# The Standard Model (SM)

has been **ruled out** by the discovery of neutrino oscilations, implying at least two non-vanishing neutrino masses.

In the SM, $\nu_i$ are strictly massless

Fortunately (or unfortunalely) it is easy to incorporate the observed neutrino masses

$$SM + 3\,\nu_R \longrightarrow \nu SM$$

Seesaw mechanism

- Present experimental results on Flavour and CP violation provide no clear evidence for New Physics beyond $\nu$SM

yet, there is plenty of motivation to have New Physics:

(i) Large number of free parameters in the $\nu$ SM. Just in the Flavour Sector:

10 (Quark Sector) + 12 (Lepton Sector) = 22 !

(ii) Most extensions of the SM introduce new sources of CP and Flavour Violation

(iii) Baryogenesis _ The SM cannot generate the observed BAU. In particular New sources of CP Violation are needed. Leptogenesis?

(iv) Strong CP problem.

Axions have not been found

Crucial Question: What is the Scale of New Physics?

From the non-observation (so far...) of clear signals of New Physics at low energies one concludes that: If there is New Physics at a scale $\lesssim 1\,\mathrm{TeV}$ and couples to fermions, then it should have a highly non-generic flavour structure (NGFS)

Question: Is it "Natural" to have New Physics with NGFS?

Answer : Yes, it is entirely natural because the flavour structure of the SM is also highly non-generic

What would be the "natural" expection for $V_{cb}$ before the measurement?

$$V_{cb} \approx V_{us}$$

Also from generalization of the Gatto–Sartori relation : $|V_{us}| \sim \sqrt{\frac{m_d}{m_s}} \longrightarrow \boxed{V_{cb} \sim \sqrt{\frac{m_s}{m_b}} - e^{i\varphi}\sqrt{\frac{m_c}{m_t}}}$

Fritzsch Ansatz ruled out by heavy top

Nature chose non-generic Flavour structure :

$$\boxed{V_{us} = \lambda \; ; \; V_{cb} \sim \lambda^2 \; ; \; V_{ub} \sim \lambda^3}$$

The non-observation of New Physics (so far!) in the Flavour sector motivated the Hypothesis of "Minimal Flavour Violation (MFV)

G. D'Ambrosio, G.F. Giudici, G. Isidori, A. Strumia
A. Buras et al

Crucial ingredient: The Flavour Structure of New Physics should only depend on $V^{CKM}$, in particular, no new sources of CP Violation

MFV is an Hypothesis, not a Model

Can one have a model where there is New Physics contributing to scalar-mediated FCNC at tree level, but Naturally Suppressed by small $V_{CKM}$ elements?

Yes!!

Consider a Two Higgs Doublet Model and introduce the following discrete symmetry:

$$\begin{pmatrix} u^0 \\ d^0 \end{pmatrix}_{L3} \to \exp(i\tau) \begin{bmatrix} u^0 \\ d^0 \end{bmatrix}_{L3} \; ; \; u^0_{R3} \to \exp(2i\tau) u^0_{Rj} \; ; \; \phi_2 \to e^{i\tau} \phi_2 \; ; \; \tau \neq 0, \pi$$

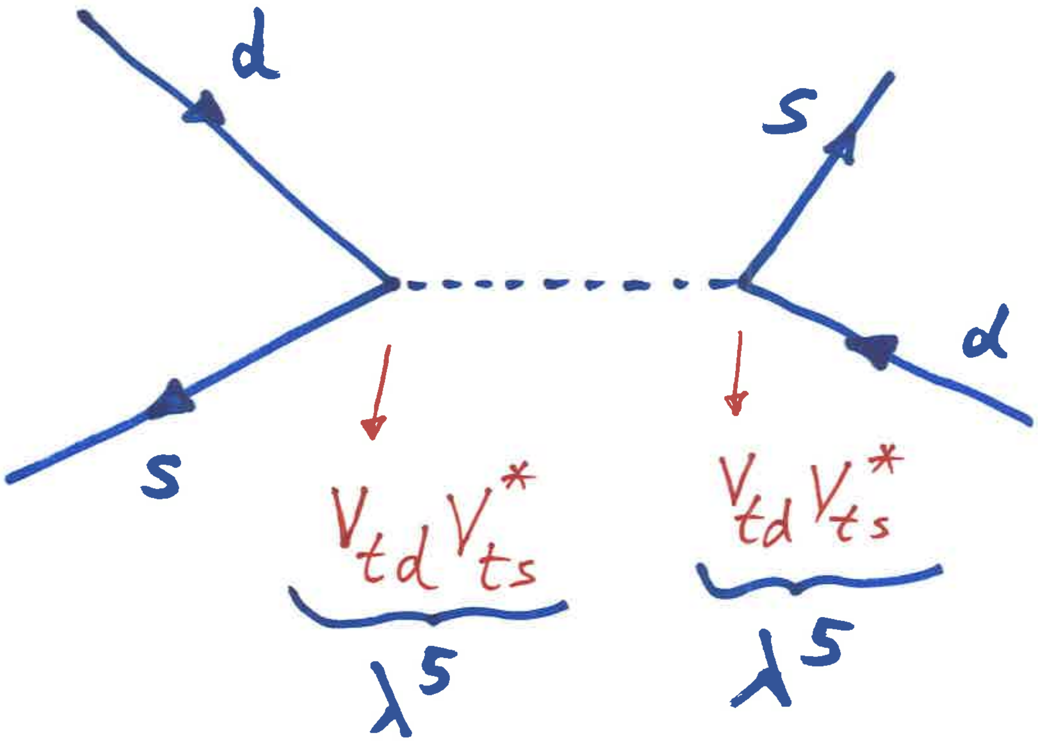
Yukawa matrices:

$$\underbrace{\Gamma_1 = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{bmatrix} ; \; \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{bmatrix}}_{\text{Yukawa Couplings giving mass to down quarks}} ; \; \underbrace{\Delta_1 = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{bmatrix} ; \; \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{bmatrix}}_{\text{Yukawa couplings giving mass to up quarks}}$$

- FCNC at tree level in the down sector
- No FCNC in the up-sector

Violates the Dogma of NFC!!

# FCNC at tree-level, but naturally suppressed




See M.N. Rebelo

Parallel Session

G.C.B, W. Grimus, L. Lavoura (BGL)

F. Botella, G.C.B., A. Carmona, M. Nebot, L. Pedro (JHEP 1407 (2014))

BGL is quite unique !!

# What is the Origin of CP Violation?

- Is CP violated at the Lagrangian level, through Complex Yukawa coupling like in the SM?

  Kobayashi - Maskawa (1973)

or

- CP is a good symmetry of the Lagrangian, only broken by the vacuum? T. D. Lee (1973)

Independently of **Origin of CP Violation** by now it has been established experimentally, that $V_{CKM}$ is <u>Complex</u>

Crucial Point : Measurement of $\gamma$ !

Models with Spontaneous CP violation have two important challenges:

(i) They have to generate a complex $V^{CKM}$

(ii) They have to avoid/suppress in a natural way scalar mediated FCNC

Actually, the original Lee Model generates a complex $V_{CKM}$

$$M_d M_d^\dagger = \frac{1}{2}\left\{ v_1^2 \left(Y_1 Y_1^\dagger\right) + v_2^2 \left(Y_2 Y_2^\dagger\right) + 2 v_1 v_2 \left(Y_1 Y_2^\dagger + Y_2 Y_1^\dagger\right)\cos\theta + \underbrace{2 i\, v_1 v_2 \sin\theta \left(Y_2 Y_1^\dagger - Y_1 Y_2^\dagger\right)} \right\}$$

Similar for $M_u M_u^\dagger$

In general this leads to complex $V^{CKM}$. But too large FCNC !!

An important, but very difficult Question:
How to conceive an experiment that could distinguish between spontaneous and explicit CP Violation.

In my opinion, this question can only be answered within a realistic framework/model where both spontaneous and explicit CP violation can be achieved. Some interesting work was done using CP-odd invariants relevant for the Scalar Sector

G.C.B., M.N. Rebelo, J. Silva Marcos; Gunion and Haber; B. Grzadkowski, O.M. Ogreid, P. Osland
M. Krawczyk, D. Sokolowska

# Minimal realistic model with Spontaneous CP Violation

$SM + 3\,\nu_R + \underbrace{D_L, D_R}$ , $S$ complex scalar

vector-like, singlet under $SU(2)$

$\langle S \rangle = V e^{i\varphi} \rightarrow$ The phase $\varphi$ is the only source of CP Violation and generates a complex $V_{CKM}$

Various Collaborations
GCB, P. Parada, L. Bento, F. Botella, M.N. Rebelo, M. Nebot ....
see
M. Nebot, Parallel Section

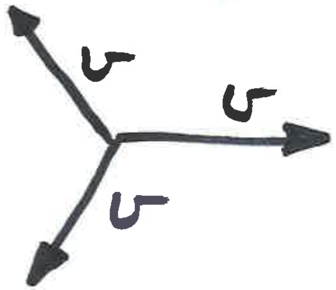
Some features of the Model:

- Possible solution to the Strong CP problem
- Naturally Small violations of $3\times3$ unitarity
  $\Downarrow$
- Naturally suppressed $Z$-mediated FCNC
- Provides a Framework for the Common Origin of all CP violations

# Geometrical CP Violation

The presence of an extra-symmetry in the Lagrangian may lead to a situation where the minimum of a multi-Higgs potential contains complex vevs, where the phases are fixed by the symmetry and do not depend on the specific values of the parameters of the potential.

Example : $S_3 \longrightarrow$




In general, these vacua do not lead to CP Violation, but an exception was found.

GCB, J. M. Gérard, W. Grimus (1984)

Recently very interesting papers on the subject See I. Medeiros → Parallel

Incomplete List : I. De Medeiros, D. E. Costa, F. Feruglio, C. Hagedorn, M. Holthausen, M. Linder, M. A. Schmidt
Mu-Chun-Chen, M. Rätz, M. Fallbacher, K. Mahanthappa

# Degeneracy with Three Majorana Neutrinos

## Work based on:

GCB, M. N. Rebelo, J. I. Silva-Marcos and D. Wegman,
arXiv:1405.5120 [hep-ph] to appear in Phys. Rev. D

GCB, M. N. Rebelo and J. I. Silva-Marcos,
Phys. Rev. Lett. 82 (1999) 683 [hep-ph/9810328].

# The Limit of Exact Degeneracy

**Without loss of generality, one can choose to work in a Weak Basis where the charged lepton mass matrix is diagonal, real.**

Assume three left-handed neutrinos and consider a Majorana mass term with the form:

$$L_{mass} = -(\nu_{L\alpha})^T C^{-1} (M_o)_{\alpha\beta} \nu_{L\beta} + h.c.$$

**where $\nu_{L\alpha}$ stand for the left-handed weak eigenstates and $M_o$ is a 3×3 symmetric complex mass matrix.**

**In general, M is diagonalised by a unitary matrix U through:**

$$U_0^T M_0 U_0 = \mathrm{diag}\,(m_{\nu 1}, m_{\nu 2}, m_{\nu 3})$$

**it follows that in the limit of exact neutrino mass degeneracy, $M_0$ can be written:**

$$M_0 = \mu S_0$$

**where $\mu$ is the common neutrino mass and $S_0 = U_0^* U_0^\dagger$. In the limit of exact degeneracy, a novel feature arises, namely $M_0$ is proportional to the symmetric unitary matrix $S_0$.**

In the case of neutrinos with different CP parities the most general matrix $S_0$ can be parametrized in the form:

$$S_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & s\phi \\ 0 & s\phi & -c\phi \end{bmatrix} \begin{bmatrix} c_\theta & s_\theta & 0 \\ s_\theta & -c_\theta & 0 \\ 0 & 0 & e^{i\alpha} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & s\phi \\ 0 & s\phi & -c\phi \end{bmatrix} , \text{ or}$$

$$S_0 = O_{23}(\phi)\, O_{12}(\theta) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\alpha} \end{bmatrix} O_{23}(\phi)$$

Using the fact that:

$$\boxed{S_0 = U_0^* U_0^\dagger}$$

One concludes that the leptonic mixing matrix $U_0$ is given by:

$$U_0 = O_{23}(\phi)\, O_{12}\left(\frac{\theta}{2}\right) \begin{bmatrix} 1 & & \\ & i & \\ & & e^{-i\alpha/2} \end{bmatrix}$$

# An important point:

$U_0$ always has one zero entry which in the above parametrization appears in the (1, 3) position.

This may be a hint that the limit of exact degeneracy is a good starting point to perform a small perturbation around it, leading to the lifting of the degeneracy and the generation of a non-zero $U_{e3}$

Since $S_0$ is a unitary matrix one can consider $S_0$ unitarity triangles which are analogous to the ones encountered in the $U^{PMNS}$, but with a <u>different physical meaning</u>!

$$M_0 = \mu S_0$$

Under a **Weak basis transformation** corresponding to a rephasing of both $\nu_L$ and the charged lepton fields, $S_0$ transforms as:

$$S_0 = L\, S_0\, L$$

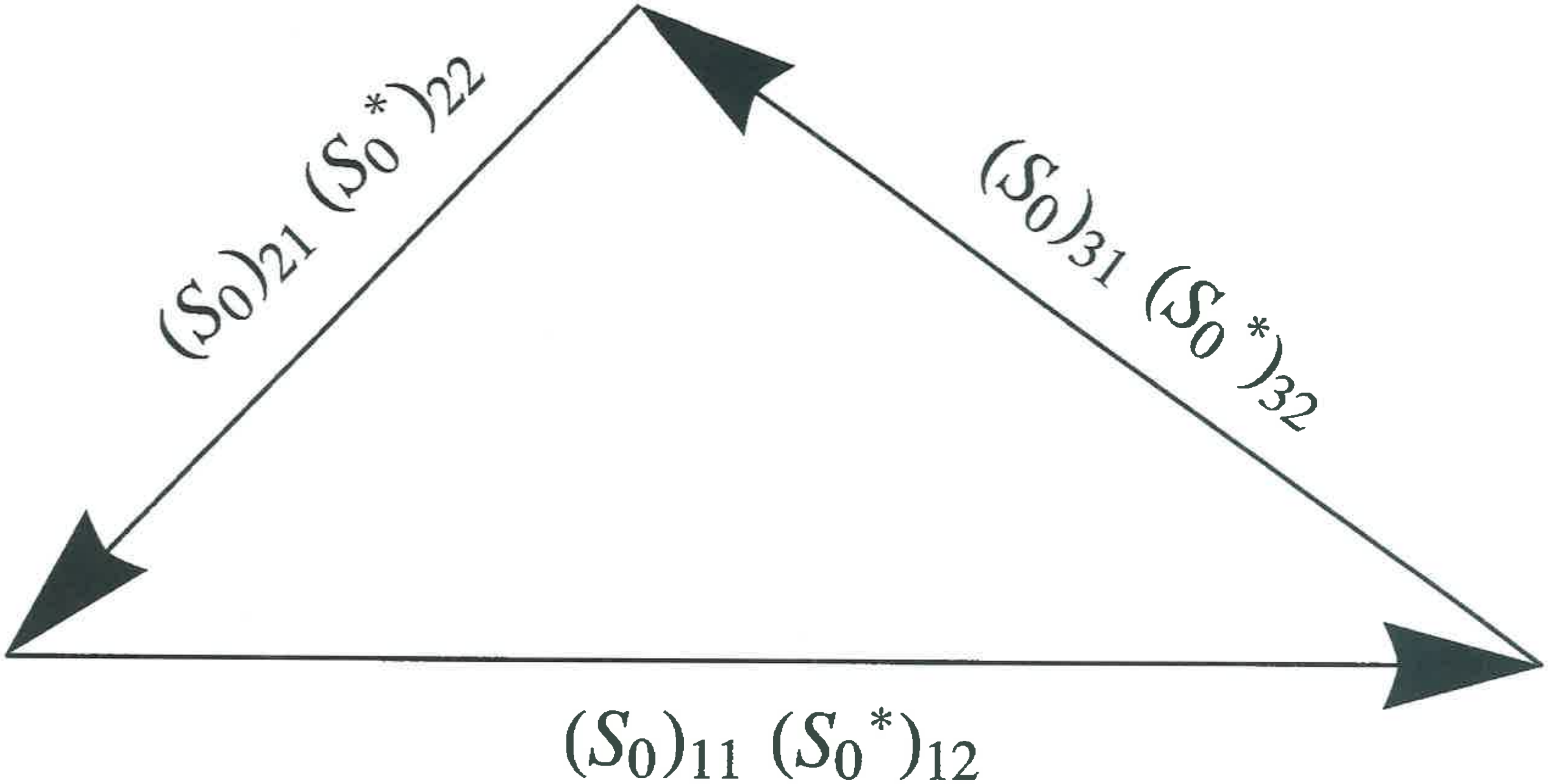
where $L = \mathrm{diag}\,(e^{i\varphi_1}\ e^{i\varphi_2}\ e^{i\varphi_3})$. As a result the individual phases of $S_0$ have <u>no physical meaning</u>

# On the physical meaning of

## $S_0$ triangles

$$\begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix}$$

$$\boxed{S_{12} S_{12}^{*} + S_{21} S_{22}^{*} + S_{31} S_{32}^{*} = 0}$$




Under rephasing, the triangle rotates.
However, its area remains invariant.
Question: What is the physical meaning of this area?

# General Approach to the Study of CP Violation

J. Bernabéu, GCB, M. Gronau
(1986)

The best way of studying the CP properties of a model is to write the Lagrangian as:

$$\mathcal{L} = \mathcal{L}_{CP} + \mathcal{L}_{remaining}$$

, where $\mathcal{L}_{CP}$ is the part of the Lagrangian which one knows that conserves CP. Typically, $\mathcal{L}_{CP}$ contains the gauge interactions since they necessarily conserve CP (W. Grimus, M.N. Rebelo)

- Construct the most general CP transformation which leaves $\mathcal{L}_{CP}$ invariant
- Study CP invariance restrictions on $\mathcal{L}_{remaining}$

What are the CP properties of the Leptonic sector with Majorana Neutrinos?

In order to answer this question, one has to recall the conditions for CP invariance when one has $n$ generations of left-handed Majorana neutrinos. Under CP the lepton fields transform as:

$$(CP)\, l_L\, (CP)^{\dagger} = U_L\, \gamma^0 C\, \overline{l}_L^{\,T}$$

$$(CP)\, \nu_L\, (CP)^{\dagger} = U_L\, \gamma^0 C\, \overline{\nu}_L^{\,T}$$

$$(CP)\, l_R\, (CP)^{\dagger} = U_R\, \gamma^0 C\, \overline{l}_R^{\,T}$$

where $U_L$, $U_R$ are unitary matrices acting in flavour space.

In order to have CP invariance in the leptonic sector, the mass matrices $m_\nu$, $m_\ell$ have to satisfy the conditions:

G.C.B, L.Lavoura,
M.N.Rebelo

$$U_L^T (m_\nu) U_L = - m_\nu^*$$

$$U_L^\dagger (m_\ell) U_R = m_\ell^*$$

$$\Downarrow$$

$$U_L^\dagger h_\ell U_L = h_\ell^*$$

These conditions are weak-basis invariant!!

From the previous Eqs., one can derive that a necessary condition for CP invariance, for an arbitrary number of generations is:

$$\mathrm{tr}\left[m_\nu m_\nu^\dagger , h_\ell\right]^3 = 0$$

C. Jarlskog

For 3 generations one has: $\mathrm{tr}[h_\nu, h_\ell]^3 \propto \det[h_\nu, h_\ell]$

$$\mathrm{tr}\left[m_\nu m_\nu^\dagger , h_\ell\right]^3 \propto i\,(m_\mu^2 - m_e^2)(m_\tau^2 - m_\mu^2)(m_\tau^2 - m_e^2)\times$$

Dirac type CP violation

$$\times (m_2^2 - m_1^2)(m_3^2 - m_2^2)(m_3^2 - m_1^2)\left[\mathrm{Im}\, Q_\ell\right]$$

$Q_\ell$ invariant quartet of $U_{PMNS}$

Can one have a WB invariant sensitive to Majorana-type CP Violation?

The best procedure to find such an invariant is to consider the case of two Majorana neutrinos, where one does know that there is no Dirac-type CP violation, but there is Majorana type CP-Violation

The simplest invariant of this type is:

$$I_{\text{Majorana}} \equiv \operatorname{Im} \operatorname{tr}\left(h_\ell\, m^* m m^* h^* m\right)$$

For two generations:

$$\boxed{I_{\text{Maj}} = \frac{1}{2} m_1 m_2 \left(m_2^2 - m_1^2\right)\left(M_\mu^2 - M_e^2\right)^2 \times \sin^2(2\theta) \sin 2\gamma}$$

Invariants are very clever!

The invariant $I_{Maj.}$ vanishes in the limit of degenerate neutrinos even for 3 generations.

Question : Can one have a CP-odd WB invariant which does not vanish even in the limit of 3 exactly mass degenerate Majorana neutrinos?

Answer : Yes!

$$I_{deg} \equiv tr\left[(m\,h\,m^*), h^*\right]^3$$

One can write $I_{deg.}$ in terms of physical quantities:

Area of $S_0$ triangle!!

$$I_{deg} = 6i\, \Delta m\, \mathrm{Im}\left[(S_0)_{11}(S_0)_{22}(S_0)^*_{12}(S_0)^*_{21}\right]$$

$$= \frac{3i}{2}\, \Delta m \cos\theta \sin^2\theta \sin^2(2\phi)\sin\alpha$$

$$\Delta m = \mu^6 (m_\tau^2 - m_\mu^2)^2 (m_\tau^2 - m_e^2)^2 (m_\mu^2 - m_e^2)^2$$

$\mu$ — degenerate neutrino mass

# Experimental Data

Table 1: Neutrino oscillation parameter summary. For $\Delta m^2_{31}$, $\sin^2\theta_{23}$ , $\sin^2\theta_{13}$, and $\delta$ the upper (lower) row corresponds to normal (inverted) neutrino mass hierarchy.

| Parameter | Best fit | $1\sigma$ range |
|---|---|---|
| $\Delta m^2_{21}$ $[10^{-5} eV^2]$ | 7.62 | 7.43 – 7.81 |
| $\Delta m^2_{31}$ $[10^{-3} eV^2]$ | 2.55 | 2.46 – 2.61 |
| $\Delta m^2_{31}$ $[10^{-3} eV^2]$ | 2.43 | 2.37 – 2.50 |
| $\sin^2\theta_{12}$ | 0.320 | 0.303 – 0.336 |
| $\sin^2\theta_{23}$ | 0.613 (0.427) | 0.400 –0. 461 and 0.573 – 0.635 |
| $\sin^2\theta_{23}$ | 0.600 | 0.569 – 0.626 |
| $\sin^2\theta_{13}$ | 0.0246 | 0.0218 –0.0275 |
| $\sin^2\theta_{13}$ | 0.0250 | 0.0223 – 0.0276 |
| $\delta$ | 0.80 $\pi$ | 0 –2 $\pi$ |
| $\delta$ | -0.03 $\pi$ | 0 –2 $\pi$ |

# Lifting the Degeneracy

- For definiteness and without loss of generality, we work in the weak Basis where the charged lepton mass matrix is diagonal and real.

- Several textures for leptonic mixing have been proposed. In most schemes, the pattern of leptonic mixing is predicted, but the spectrum of masses is not constrained by the simetries.

- Therefore, it is consistent to consider these schemes, together with the hypothesis of quasi-degeneracy of Majorana neutrinos.

- Until recently, one of the most favoured Ansätze, from the experimental point of view, was the tribimaximal Ansatz.

- The discovery of $\theta_{13} \neq 0$ motivated a series of studies of how to generate $\theta_{13} \neq 0$ through a small perturbation of the tribimaximal Ansatz.

The distinctive feature of our analysis:
we start from a non-trivial limit of 3 exactly degenerate Majorana neutrinos, where the mixing can be written:

$$U_0(\theta, \psi) K \quad ; \quad K = \mathrm{diag.}(1, i, e^{-i\alpha/2})$$

Lifting the degeneracy corresponds to adding a small perturbation to $S_0$

$$M = \mu\left(S_0 + \varepsilon^2 Q_0\right)$$

$$\varepsilon^2 \equiv \frac{\Delta m_{31}^2}{2\mu^2}$$

- We assume that the physics responsible to the lifting of the degeneracy, does not introduce new sources of CP violation, beyond the phase $\alpha$, already present in the degeneracy limit:

$$U_{PMNS} = U_0 \, O$$

where $O$ is an orthogonal matrix parametrized by small angles:

$$O = O_{12}(\phi_1)\, O_{13}(\phi_3)\, O_{23}(\phi_2)$$

The matrix $Q_0$ is determined by:

$$\varepsilon^2 Q_0 = U_0^* \, O \left( \frac{1}{\mu} D_\nu - \mathbb{1} \right) O^T U_0^\dagger$$

$$D_\nu = \text{diag.}\left( m_{\nu_1}, m_{\nu_2}, m_{\nu_3} \right)$$

Let us consider:

$$U_0 = U_{TBM} \cdot K \quad ; \quad K = \text{diag.}\left(1, i\, e^{-i\alpha/2}\right)$$

$$U_{TBM} = \begin{bmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \end{bmatrix} \rightarrow$$

corresponds to $\phi = 45^\circ$ and $\cos\theta/2 = 2/\sqrt{6}$, $\theta/2 = 35.26^\circ$

In this scenario, one may find a particularly simple solution : one can reach agreement with experiment, by choosing a matrix O with only one parameter different from 0, namely $\phi_2$. In this case

$$\boxed{\sin^2(\theta_{13}) = \frac{\sin^2(\phi_2)}{3}}$$

$$\boxed{\sin^2\theta_{12} = \frac{1-\sin^2\phi_2}{3-\sin^2\phi_2} = \frac{1/3-|U_{13}|^2}{1-|U_{13}|^2}}$$

$$\boxed{\sin^2(\theta_{23}) = \frac{1}{2} - \frac{\sqrt{6}\,\sin\alpha/2\,\sin\phi_2\cos\phi_2}{3-\sin^2\phi_2}}$$

With the Standard Parametrization one has:

$$I_{CP} \equiv \mathrm{Im}\, Q_{\text{leptonic}} = \frac{1}{8}\Big| \sin(2\theta_{12}) \sin(2\theta_{13}) \cdot$$

$$\cdot \sin(2\theta_{23}) \cos\theta_{13} \cdot \sin\delta \Big|$$

In our framework

$$I_{CP} = \left| \frac{\cos\alpha/2 \, \sin\phi/2 \, \cos\phi/2}{3\sqrt{6}} \right|$$

and $I_{CP}$ is predicted to be of order $10^{-2}$.

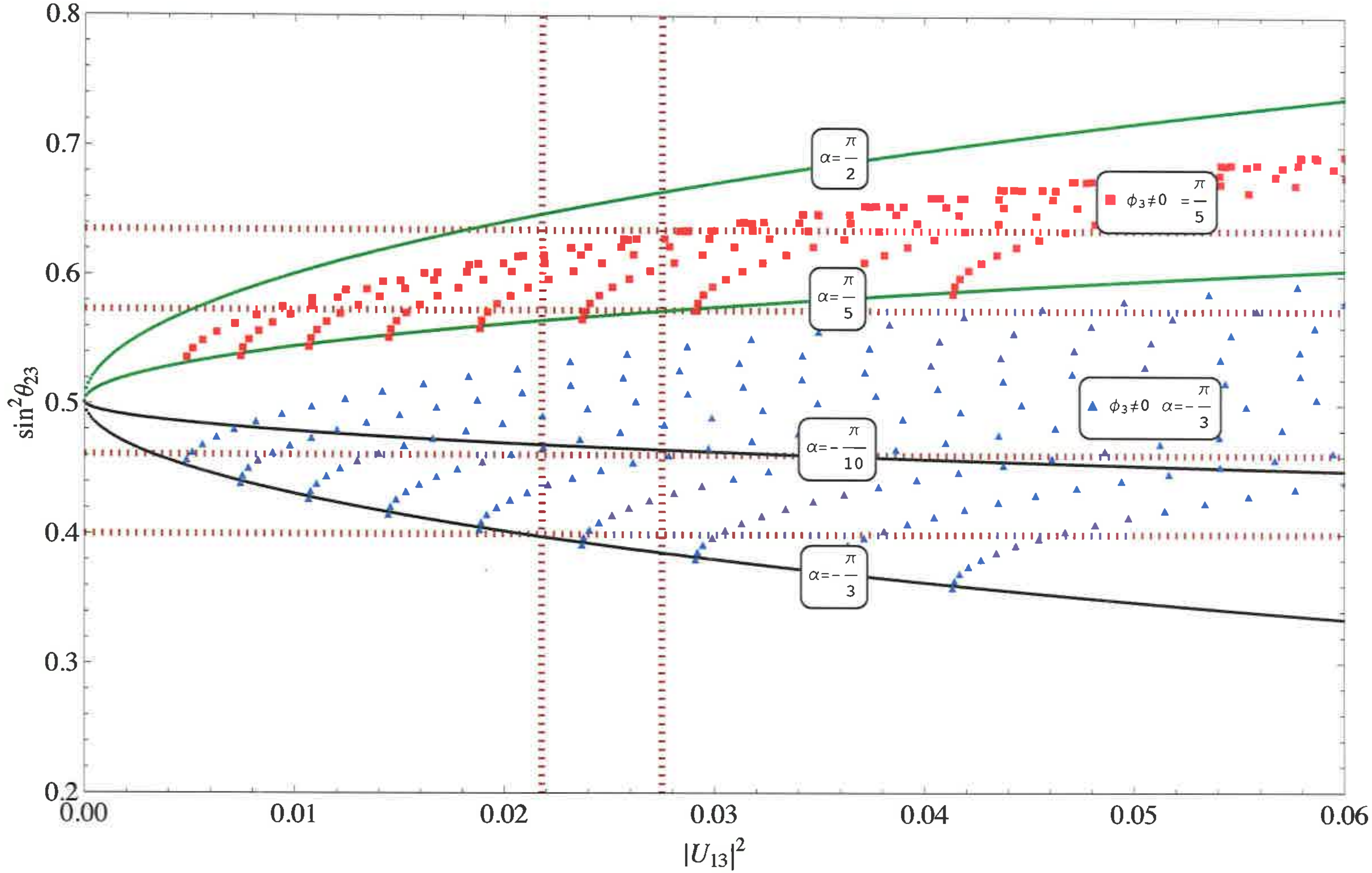



Figure 2: $\sin^2\theta_{23}$ versus $|U_{13}|^2$ obtained by perturbing tribimaximal mixing with $\phi_3 = 0$. Each curve corresponds to a fixed $\alpha$ and to $\phi_1 = 0$ , therefore $\phi_2$ is the only variable. The points drifting away from each curve were obtained by varying also $\phi_3$.

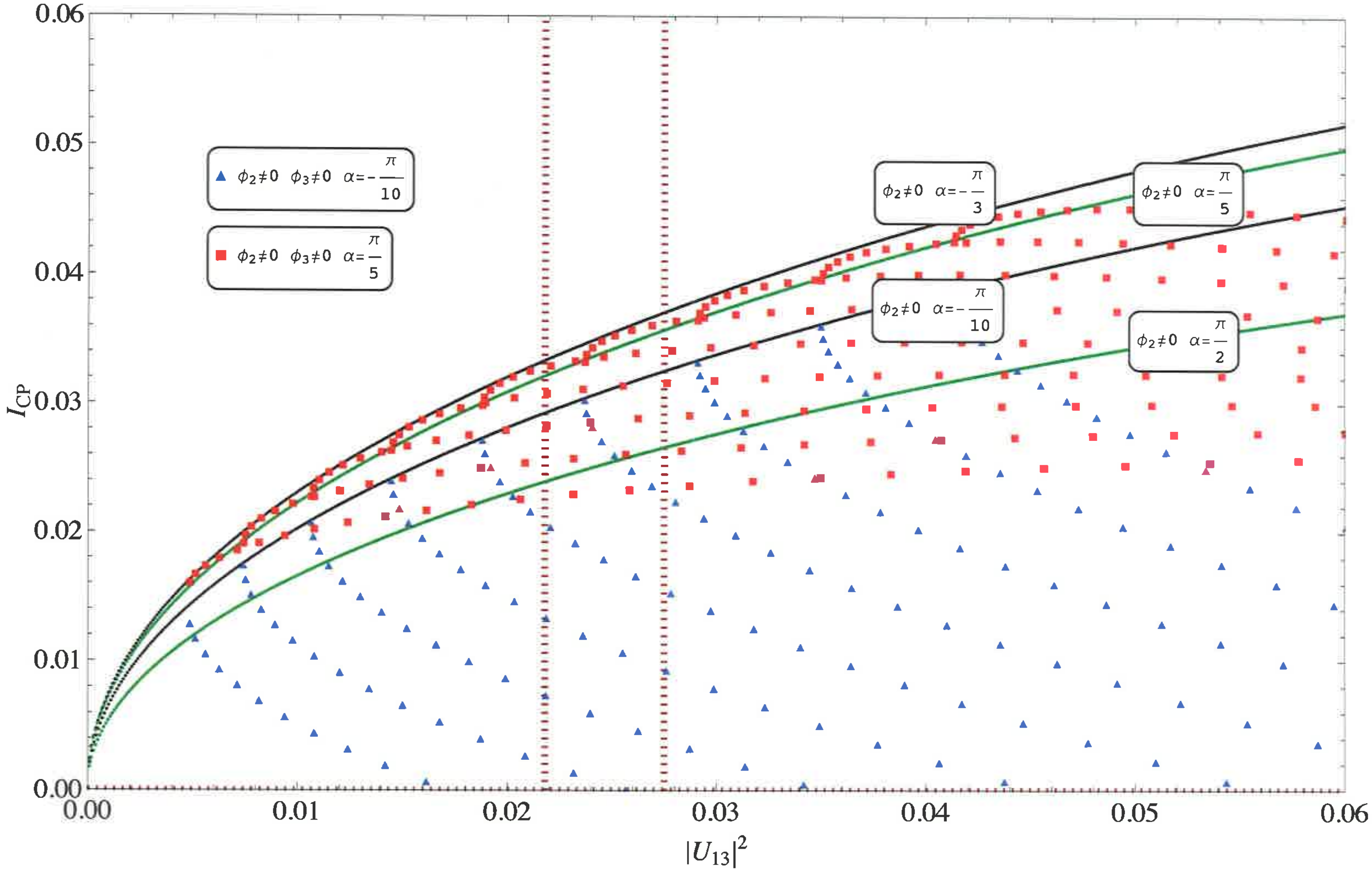



Figure 3: $I_{CP}$ versus $|U_{13}|^2$ obtained by perturbing tribimaximal mixing with $\phi_3 = 0$. Each curve corresponds to a fixed $\alpha$ and to $\phi_1 = 0$ , therefore $\phi_2$ is the only variable. The points drifting away from each curve were obtained by varying also $\phi_3$.

# Conclusions

- There are various plausible, realistic scenarios where there may be New Physics at the TeV scale, without violating naturalness criteria:
    - Multi-Higgs Models
    - Models with vector-like quarks
- Origin of CP Violation (explicit, spontaneous?) is still an open question, hopefully to be answered by experiment.
- The limit of exact degeneracy of 3 Majorana neutrinos is very subtle. It may provide a key to the understanding of the observed pattern of leptonic mixing.