Discrete Abelian Gauge Symmetries and Axions

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based on JHEP 1310(2013)146, PoS Corfu2012(2013)107, Fortsch.Phys. 62(2014)115-151 with **Wieland Staessens**
& JHEP1407(2014)124 with **Michael Blaszczyk, Isabel Koltermann**

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Motivation: Gauge Symmetries & Axions

- **Type II string theory**: a $U(1)$ per D-brane $\rightsquigarrow$ $\boxed{\sum_a U(1)_a}$
  - few **massless** in 4D: Y, $B - L$
  - most **massive** in 4D: $U(1)_{PQ}$ ...
- $U(1)_{\text{massive}}$ remains as *perturbative* global symmetry

$\swarrow$ $\searrow$

- non-pert: $\cancel{U(1)_{\text{massive}}}$
- $\mathbb{Z}_n \subset U(1)_{\text{massive}}$ survives

$\rightsquigarrow$ ultimate **selection rules** on matter **couplings** in 4D

- explicit breaking by $\langle \phi_{\text{matter}} \rangle$

$\rightsquigarrow$ $\cancel{U(1)_{PQ}}$ as **solution** to strong **CP problem**

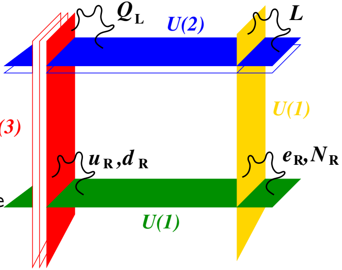
cf. also Mehta's talk

- Two kinds of **axions**:
  - Closed partner of (complex structure/Kähler) modulus & dilaton
  - Open: scalar matter with $U(1)_{\text{massive}}$ charge

$\rightsquigarrow$ **mixing** via Green-Schwarz coupling

Motivation: D-Brane Model Building

Spanish Quiver:



$$SU(3)_a \times SU(2)_b \times Y \times \begin{cases} U(1)^3_{\text{massive}} \\ (B-L) \times U(1)^2_{\text{massive}} \end{cases}$$

- 'Standard' realisation:

$$Y = \frac{Q_a}{6} + \frac{Q_c + Q_d}{2} \qquad B - L = \frac{Q_a}{3} + Q_d$$

- $\mathbb{Z}_3 \subset U(1)_a$ automatic, but selection rules agree with $SU(3)_a$
  - non-trivial $\mathbb{Z}_n \subset \sum_{x \in \{a,b,c,d\}} k_x U(1)_x$ possible
  - **generation dependent** $\mathbb{Z}_2$ found in extension: $U(4) \times U(2)^4$
- Natural candidate for $U(1)_{PQ}$ and **axion** σ:

- Q_L, L, (H_u, H_d), σ charged
- (u_R, d_R), (e_R, ν_R) neutral

$$\left.\right\} U(1)_{PQ} = U(1)_b \ \& \ \sigma = (\mathbf{Anti}_b)$$

Content

- **Massive & discrete gauge symmetries**
  - Reminder of the Green-Schwarz mechanism
  - $\mathbb{Z}_n$ symmetries in global D-brane models
- **Axions, strong CP problem & the dark sector**
  - Open & closed string sector
  - $\cancel{U(1)_{PQ}}$ & Higgs-axion potential in the DFSZ model
  - soft $\cancel{SUSY}$ terms in D-brane models
  - Lower bounds on M_{string} in global D-brane models
- **Intermezzo: $\cancel{SUSY}$ by deformations**
- **Conclusions**

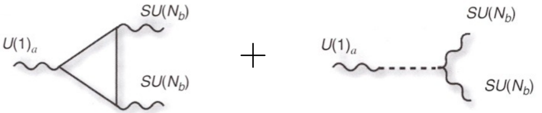
Massive & Discrete Gauge Symmetries

Argumentation for all string theories with multiple $U(1)$s analogous

- here: Type IIA language -

Massive & Discrete Gauge Symmetries - Type IIA Notation

- **Mixed anomalies** cancel by the **Green-Schwarz** mechanism:



$= 0$

- **Axions** ξ_i $(\star_4 d\xi_i \sim d\mathcal{B}_2^{(i)})$: longitudinal modes of $U(1)^k_{\text{massive}}$

$$\mathcal{S}_{CS} \supset \int_{\mathbb{R}^{1,3}} \sum_{i=0}^{h_{21}} \left(B_a^i \, \mathcal{B}_2^{(i)} \wedge \mathrm{tr} F_a + A_b^i \, \xi_i \, \mathrm{tr} F_b \wedge F_b \right)$$

with $\boxed{\mathcal{B}_2^{(i)} \propto \int_{\Pi_i^{\text{odd}}} C_5^{RR}}$; $\boxed{\xi_i \propto \int_{\Pi_i^{\text{even}}} C_3^{RR}}$

- $U(1)_X = \sum_a q_a U(1)_a$ **massless** if $\sum_a N_a q_a B_a^i = 0 \; \forall i$

- $\mathbb{Z}_n \subset U(1)^k_{\text{massive}}$ for suitable B_a^i ('mod n') due to **shift symmetry** of ξ_i

Axionic Shift Symmetry - Type IIA Notation

- **Closed string axions** within $\mathcal{N}=1$ chiral multiplets:
  - axion-dilaton: $S = \phi + \mathrm{i}\,\xi_0$
  - complex structure: $U_i = c_i + \mathrm{i}\,\xi_i$ $\qquad \xi_i \subset C_3^{RR}$
  - Kähler: $T_k = v_k + \mathrm{i}\,b_k$ $\qquad b_k \subset B_2^{NSNS}$
- $\mathcal{N}=1$ **SUGRA action** independent of $\xi_i \to \xi_i + 1$

$$\mathcal{K}_{\text{closed}} = -\ln \Re(S) - \sum_i \ln \Re(U_i) - \sum_k \ln \Re(T_k)$$

  - **perturbatively:** only couplings to $(\partial_\mu \xi_i)$
  - **non-perturbative** couplings via D-brane instantons: $e^{-\mathcal{S}_{\text{inst}}}$ with $\mathcal{S}_{\text{inst}} \supset 2\pi\mathrm{i}\,\xi_i$

in IIB: $U_i \leftrightarrow T_k$

- **Discrete** $\mathbb{Z}_n$ **symmetry** preserved if

$$A^\mu \to A^\mu + \partial^\mu \lambda \qquad \xi_i \to \xi_i + \underbrace{\overline{c}_i(B_a^i)}_{0 \bmod n \quad \forall i}\,\lambda$$

$\rightsquigarrow$ need to determine $\overline{c}_i(B_a^i)$!

Green-Schwarz Couplings & $\mathbb{Z}_n$ Symmetries - Type IIA Notation

$$\mathcal{S}_{CS} \supset \int_{\mathbb{R}^{1,3}} \sum_{i=0}^{h_{21}} \left(B_a^i \, \mathcal{B}_2^{(i)} \wedge \mathrm{tr} F_a + A_b^i \, \xi_i \, \mathrm{tr} F_b \wedge F_b \right)$$

$$\text{with } \boxed{\mathcal{B}_2^{(i)} \propto \int_{\Pi_i^{\text{odd}}} C_5^{RR}} \; ; \; \boxed{\xi_i \propto \int_{\Pi_i^{\text{even}}} C_3^{RR}}$$

- Expand 3-cycles and $\Omega\mathcal{R}$-images as:

$$\Pi_a = \sum_{i=0}^{h_{21}} \left(A_a^i \, \Pi_i^{\text{even}} + B_a^i \, \Pi_i^{\text{odd}} \right), \quad \Pi_a' = \sum_{i=0}^{h_{21}} \left(A_a^i \, \Pi_i^{\text{even}} - B_a^i \, \Pi_i^{\text{odd}} \right)$$

- If $\boxed{\Pi_i^{\text{even}} \circ \Pi_j^{\text{odd}} = m_i \, \delta_{ij}}$ with $m_i \in \mathbb{Z}$ $\quad \rightsquigarrow d\mathcal{B}_2^{(i)} = m_i \star_4 d\xi_i$
- $\overset{U(1)=\sum_a k_a U(1)_a \text{ shift}}{\rightsquigarrow}$ $\xi_i \to \xi_i + \left(m_i \sum_a N_a k_a B_a^i \right) \lambda$
  - $\{\Pi_i^{\text{even}}, \Pi_j^{\text{odd}}\}$ span **sublattice** of finite index: $\Lambda_3^{\text{even}} \oplus \Lambda_3^{\text{odd}} \subsetneq \Lambda_3$
  - **all** known global D-brane models of **this type**
  - $A_a^i, B_a^i \in \frac{1}{m_i} \mathbb{Z}$
- resolve ambiguities (A_a^i, B_a^i): rewrite '0 mod n' as intersection $\#$

$\mathbb{Z}_n$ Symmetries in Terms of Intersection Numbers - Type IIA

- ambiguities of normalisation factors m_i in B_a^i and Π_i^{odd} cancel

$U(1)_{\text{massless}} = \sum_a q_a U(1)_a$	$\mathbb{Z}_n \subset U(1)_{\text{massive}} = \sum_a k_a U(1)_a$
$\Pi_i^{\text{even}} \circ \sum_a N_a q_a \Pi_a = 0 \; \forall i$ $\Leftrightarrow \quad \sum_a N_a q_a B_a^i = 0 \; \forall i$	$\Pi_i^{\text{even}} \circ \sum_a N_a k_a \Pi_a = 0 \bmod n \; \forall i$ $\Leftrightarrow \quad m_i \sum_a N_a k_a B_a^i = 0 \bmod n \; \forall i$
$q_a \in \mathbb{Q}$	$k_a \in \mathbb{Z}$, $0 \leqslant k_a < n$, $\gcd(k_a, n) = 1$

- derivation of m_i, B_a^i for all **orbifolds** with particle physics models ✓
  - basis of $\{\Pi_i^{\text{even}}\}$ needed

$\rightsquigarrow$ $\mathbb{Z}_n$ symmetries in any *global* model can be derived ✓

- **Cross-check**: K-theory constraint can be written as $\mathbb{Z}_2$ ✓

GH, Staessens '13

Comments

- **Bottom-up** *(local)* **models**: $\{\Pi_i^{\text{even}}\}$ not known
  - use $\left(\Pi_x + \Pi'_x\right)_{x\in\{b,c,d\}} \circ \Pi_a = \Pi_x \circ \left(\Pi_a - \Pi'_a\right)$
  - **4 necessary conditions** from $U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d$

    $\Leftrightarrow (h_{21}+1) \overset{\text{typical}}{\sim} \mathcal{O}(10)$ nec.+suff. conditions in *global* models

- **Redundant $\mathbb{Z}_N$ symmetries**:
  - $\mathbb{Z}_N \subset U(1)_{\text{massive}} \subset U(N) \simeq SU(N)_{U(1)}$ automatic & trivial:

$$(\mathbf{Adj})_0 + (\mathbf{1})_0 \qquad (\mathbf{N})_1 \qquad (\mathbf{Sym})_2 + (\mathbf{Anti})_2$$

- **But:** non-trivial sums of $\mathbb{Z}_{N_a} \subset U(N_a)$ charges can arise

  $\rightsquigarrow$ **generation dependent** $\mathbb{Z}_n$ symmetries

example of generation dependent $\mathbb{Z}_2$ later

Related Works on Abelian Discrete Symmetries

SUSY field theory:

- *Discrete gauge symmetries and the origin of baryon and lepton number conservation in supersymmetric versions of the standard model* L.E.Ibáñez, G.G.Ross: Nucl.Phys.B368(1992)3-37
- *What is the discrete gauge symmetry of the MSSM?* H.K.Dreiner, C.Luhn, M.Thormeier: Phys.Rev.D73(2006)075007

see Luhn's talk

$\rightsquigarrow$ R-parity ($\mathbb{Z}_2$), baryon triality ($\mathbb{Z}_3$), proton hexality ($\mathbb{Z}_6$) for e.g. **proton stability**

D-brane models:

- *Discrete gauge symmetries in D-brane models* M.Berasaluce-Gonzalez, L.E.Ibáñez, P.Soler, A.M.Uranga: JHEP1112(2011)113
- *Discrete Gauge Symmetries in Discrete MSSM-like Orientifolds* L.E.Ibáñez, A.N.Schellekens, A.M.Uranga: Nucl.Phys.B865(2012)509-540
- *String Constraints on Discrete Symmetries in MSSM Type II Quivers* P.Anastasopoulos, M.Cvetič, R.Richter, P.K.S.Vaudrevange: JHEP1303(2013)011
- *Zp charged branes in flux compactifications* M.Berasaluce-Gonzalez, P.G.Camara, F.Marchesano, A.M.Uranga: JHEP1304(2013)138
- ...

GH, W. Staessens '13

$\rightsquigarrow$ **ultimate** coupling **selection rules** beyond massless gauge symmetries

M- & F-theory models: see Acharya's, Leontaris', Athanasopoulos' talks

F-theory also: Mayrhofer, Palti, Till, Weigand '14; García-Etxebarria, Grimm, Keitel '14

$\mathbb{Z}_n$ Symmetries in Global Models on Orbifolds of IIA/$\Omega\mathcal{R}$

- $\dim(\Lambda_3^{\text{even}}) = h_{21} + 1$ conditions
- **phenomenologically** interesting:
  - $\boxed{T^6/\mathbb{Z}_6}$: $h_{21} = 5$ — **GH**, Ott '04
  - $\boxed{T^6/\mathbb{Z}_6'}$: $h_{21} = 5$ $(+6)^*$ — Gmeiner, **GH** '07-08
  - $\boxed{T^6/\mathbb{Z}_2 \times \mathbb{Z}_6}$: $h_{21} = 15$ $(+4)^*$ — **GH**, Ripka, Staessens '12
  - $\boxed{T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'}$: $h_{21} = 15$ — Ecker, **GH**, Staessens '14 & in progress

* D-branes wrap only untwisted & $\mathbb{Z}_2$ twisted cycles

- shape of Λ_3^{even} depends on lattice orientations under $\Omega\mathcal{R}$

$$\mathcal{R} : z \to \bar{z}$$

- L-R symmetric & Pati-Salam models *'natural'* on D-branes

  $\rightsquigarrow$ $U(1)_Y$ (SM) or $U(1)_{B-L}$ (L-R) to rotate some charges to 0

  $\rightsquigarrow$ some additional $\mathbb{Z}_n$ charges trivial (in observable sector)

Example I: L-R Symmetric Model on $T^6/\mathbb{Z}_6$ - $h_{21}=5$

GH, Ott '04; see also Gmeiner, **GH** '09

- $\boxed{U(3)_a \times U(2)_b \times USp(2)_c \times U(1)_d \times USp(2)_e}$
- $U(1)_{B-L} = (\frac{Q_a}{3} + Q_d)_{\text{massless}}$ & $U(1)^2_{\text{massive}}$
- $USp(2)_{x\in\{c,e\}} \to U(1)_{x,\text{massless}}$ by brane displacement
- only $x \in \{a,b,d\}$ contribute to $\mathbb{Z}_n$ conditions
- after $B-L$ rotation:

GH, Staessens '13

Discrete sym.		**Charge assignment for the MSSM states**										
$\mathbb{Z}_n$	$\subset k_x U(1)_x$	Q_L	$\overline{U}_R$	$\overline{D}_R$	L	$\overline{E}_R$	$\overline{N}_R$	$H_u^{(1)}$	$H_u^{(2)}$	$H_d^{(1)}$	$H_d^{(2)}$	Σ_b
$\mathbb{Z}_2$	Q_a+Q_d	0	0	0	0	0	0	0	0	0	0	0
$\mathbb{Z}_2$	Q_b	0	1	1	0	1	1	1	1	1	1	0
$\mathbb{Z}_3$	Q_a	0	0	0	0	0	0	0	0	0	0	0

open string axion: $\Sigma_b \simeq (\mathbf{1}_{\overline{\mathbf{Anti}}_b})_{-2_b}$

not listed: mild amount of vector-like exotics

- $(k_a, k_b, k_d) = (1,1,1) \simeq \mathbb{Z}_2$ of K-theory constraint
- $\mathbb{Z}_2^{(b)}$ gives **no extra constraints** beyond $SU(2)_b$ charges
  $\rightsquigarrow$ **all $\mathbb{Z}_n$ appear trivial from 4D perspective**

Example II: L-R Symm. Model on $T^6/\mathbb{Z}_6'$ - $h_{21} = 15\ (+4)^*$

Gmeiner, **GH** '07-'08

- $\boxed{U(3)_a \times U(2)_b \times USp(2)_c \times U(1)_d\ (\times USp(6)_{\text{hidden}})}$
- $U(1)_{B-L} = (\frac{Q_a}{3} + Q_d)_{\text{massless}}$ & $U(1)^2_{\text{massive}}$
- $USp(2)_c \to U(1)_{c,\text{massless}}$ by brane displacement σ
- $USp(6)_{\text{hidden}}$ **cannot** be broken by σ or Wilson line τ (~~SUSY~~)
- after $B-L$ rotation:

GH, Staessens '13

Discrete sym.		**Charge assignment for the chiral states**									
$\mathbb{Z}_n$	$\subset \sum_x k_x\, U(1)_x$	Q_L	$\overline{U}_R$	$\overline{D}_R$	L	$\overline{L}$	$\overline{E}_R$	$\overline{N}_R$	H_u	H_d	Σ_b
$\mathbb{Z}_2$	$Q_a + Q_d$	0	0	0	0	0	0	0	0	0	0
$\mathbb{Z}_3$	Q_a	0	0	0	0	0	0	0	0	0	0
$\mathbb{Z}_6$	Q_b	0	1	1	4	4	3	3	5	5	4
	$\xrightarrow{U(1)_c}$	0	0	2	4	4	4	2	0	4	4

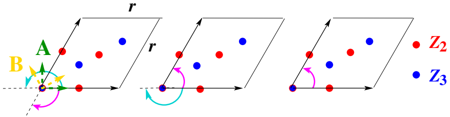
open string axion: $\Sigma_b \simeq (\mathbf{1}_{\overline{\mathbf{Anti}_b}})_{-2_b}$

not listed: mild amount of vector-like exotics

- **non-trivial**: $\boxed{\mathbb{Z}_3 \subset U(1)_b}$

Example III: A Pati-Salam Model on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$ - $h_{21} = 15$

- $\mathbb{Z}_2 \times \mathbb{Z}_6'$ shifts: $\vec{v} = (\frac{1}{2}, \frac{-1}{2}, 0)$, $\vec{w}' = (\frac{-1}{3}, \frac{1}{6}, \frac{1}{6})$ on $SU(3)^3$



- $\Pi_a^{\text{frac}} = \frac{1}{4}\left(X_a\,\rho_1 + Y_a\,\rho_2 + \sum_{k=1}^{3}\sum_{\alpha=1}^{5}\left[x_{a,\alpha}^{(k)}\,\varepsilon_\alpha^{(k)} + y_{a,\alpha}^{(k)}\,\tilde{\varepsilon}_\alpha^{(k)}\right]\right)$
  with $\rho_1 \circ \rho_2 = -\varepsilon_\alpha^{(k)} \circ \tilde{\varepsilon}_\alpha^{(k)} = 4$
- $h_{21} + 1 = 16$ $\Omega\mathcal{R}$-even & odd 3-cycles: **GH**, Staessens '13

$$\Pi_0^{\text{even},\mathbb{1}} = \rho_1, \qquad \Pi_0^{\text{odd},\mathbb{1}} = -\rho_1 + 2\,\rho_2,$$
$$\Pi_{\alpha\in\{1,2,3\}}^{\text{even},\mathbb{Z}_2^{(k)}} = \varepsilon_\alpha^{(k)}, \qquad \Pi_{\alpha\in\{1,2,3\}}^{\text{odd},\mathbb{Z}_2^{(k)}} = -\varepsilon_\alpha^{(k)} + 2\,\tilde{\varepsilon}_\alpha^{(k)},$$
$$\Pi_4^{\text{even},\mathbb{Z}_2^{(k)}} = \varepsilon_4^{(k)} + \varepsilon_5^{(k)}, \qquad \Pi_4^{\text{odd},\mathbb{Z}_2^{(k)}} = 2\,(\tilde{\varepsilon}_4^{(k)} + \tilde{\varepsilon}_5^{(k)}) - (\varepsilon_4^{(k)} + \varepsilon_5^{(k)}),$$
$$\Pi_5^{\text{even},\mathbb{Z}_2^{(k)}} = 2\,(\tilde{\varepsilon}_4^{(k)} - \tilde{\varepsilon}_5^{(k)}) - (\varepsilon_4^{(k)} - \varepsilon_5^{(k)}), \qquad \Pi_5^{\text{odd},\mathbb{Z}_2^{(k)}} = \varepsilon_4^{(k)} - \varepsilon_5^{(k)},$$

- Intersection numbers

$$\Pi_{\tilde{\alpha}}^{\text{even},\mathbb{Z}_2^{(k)}} \circ \Pi_{\tilde{\beta}}^{\text{odd},\mathbb{Z}_2^{(l)}} = \delta^{kl}\delta_{\tilde{\alpha}\tilde{\beta}} \times \begin{cases} 8 & \tilde{\alpha} = 0 \\ -8 & 1\ldots 3 \\ -16 & 4 \\ 16 & 5 \end{cases} \quad \text{with} \quad \mathbb{Z}_2^{(0)} \equiv \mathbb{1}$$

- wrapping numbers *a priori* $A_a^i, B_a^i \in \frac{1}{8}\,\mathbb{Z}$ $\qquad m_i \in \{\pm 8, \pm 16\}$

A Pati-Salam Model on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$: $\mathbb{Z}_n$ Conditions

$$
\sum_a k_a N_a \left(\begin{array}{c}
Y_a \\ \hline
-y^{(1)}_{a,1} \\
-y^{(1)}_{a,2} \\
-y^{(1)}_{a,3} \\
-(y^{(1)}_{a,4} + y^{(1)}_{a,5}) \\
2\,(x^{(1)}_{a,4} - x^{(1)}_{a,5}) + (y^{(1)}_{a,4} - y^{(1)}_{a,5}) \\ \hline
-y^{(2)}_{a,1} \\
-y^{(2)}_{a,2} \\
-y^{(2)}_{a,3} \\
-(y^{(2)}_{a,4} + y^{(2)}_{a,5}) \\
2\,(x^{(2)}_{a,4} - x^{(2)}_{a,5}) + (y^{(2)}_{a,4} - y^{(2)}_{a,5}) \\ \hline
-y^{(3)}_{a,1} \\
-y^{(3)}_{a,2} \\
-y^{(3)}_{a,3} \\
-(y^{(3)}_{a,4} + y^{(3)}_{a,5}) \\
2\,(x^{(3)}_{a,4} - x^{(3)}_{a,5}) + (y^{(3)}_{a,4} - y^{(3)}_{a,5})
\end{array} \right)
\stackrel{!}{=} 0 \bmod n \stackrel{!}{=} \sum_a k_a N_a \left(\begin{array}{c}
\frac{Y_a - \sum_{i=1}^3 [y^{(i)}_{a,1} + y^{(i)}_{a,2} + y^{(i)}_{a,3}]}{4} \\
\frac{Y_a - [y^{(1)}_{a,1} + y^{(1)}_{a,2} + y^{(1)}_{a,3}]}{2} \\
\frac{Y_a - [y^{(2)}_{a,1} + y^{(2)}_{a,2} + y^{(2)}_{a,3}]}{2} \\
-\frac{y^{(2)}_{a,1} + y^{(2)}_{a,3} + y^{(3)}_{a,1} + y^{(3)}_{a,3}}{2} \\
-\frac{y^{(1)}_{a,1} + y^{(1)}_{a,3} + y^{(3)}_{a,2} + y^{(3)}_{a,3}}{2} \\
-\frac{y^{(1)}_{a,2} + y^{(1)}_{a,3} + y^{(2)}_{a,2} + y^{(2)}_{a,3}}{2} \\ \hline
\frac{Y_a + [y^{(1)}_{a,3} + x^{(1)}_{a,4} + y^{(1)}_{a,4} - x^{(1)}_{a,5}] + \sum_{j=2}^3 [y^{(j)}_{a,2} - (y^{(j)}_{a,4} + y^{(j)}_{a,5})]}{4} \\
\frac{Y_a + \sum_{j=1,2} [y^{(j)}_{a,1} - x^{(j)}_{a,4} + x^{(j)}_{a,5} + y^{(j)}_{a,5}] + [y^{(3)}_{a,3} + x^{(3)}_{a,4} + y^{(3)}_{a,4} - x^{(3)}_{a,5}]}{4} \\
\frac{Y_a + [y^{(2)}_{a,2} - (y^{(2)}_{a,4} + y^{(2)}_{a,5})]}{2} \\
\frac{Y_a + [y^{(1)}_{a,1} - x^{(1)}_{a,4} + x^{(1)}_{a,5} + y^{(1)}_{a,5}]}{2} \\
-\frac{y^{(2)}_{a,4} + y^{(2)}_{a,5} + y^{(3)}_{a,4} + y^{(3)}_{a,5}}{2} \\
-\frac{x^{(1)}_{a,4} - x^{(1)}_{a,5} + y^{(1)}_{a,5} + x^{(2)}_{a,4} - x^{(2)}_{a,5} + y^{(2)}_{a,5}}{2} \\ \hline
\frac{Y_a + \sum_{i=1}^3 [y^{(i)}_{a,3} + x^{(i)}_{a,4} + y^{(i)}_{a,4} - x^{(i)}_{a,5}]}{4} \\
\frac{Y_a + y^{(1)}_{a,3} + x^{(1)}_{a,4} + y^{(1)}_{a,4} - x^{(1)}_{a,5}}{2} \\
\frac{Y_a + y^{(2)}_{a,3} + x^{(2)}_{a,4} + y^{(2)}_{a,4} - x^{(2)}_{a,5}}{2} \\
\frac{Y_a + \sum_{i=1}^3 y^{(i)}_{a,3}}{2}
\end{array} \right)
$$

A Pati-Salam Model on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$: Spectrum

GH, Ripka, Staessens '12

$$SU(4)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d \times SU(2)_e \times U(1)^5_{\text{massive}}$$

- Standard Model particles plus **one Higgs**

$$(\mathbf{4},\overline{\mathbf{2}},\mathbf{1};\mathbf{1},\mathbf{1})+2\,(\mathbf{4},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1})+(\overline{\mathbf{4}},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})+2\,(\overline{\mathbf{4}},\mathbf{1},\overline{\mathbf{2}};\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{2},\overline{\mathbf{2}};\mathbf{1},\mathbf{1})$$

$\rightsquigarrow$ **one massive generation** at leading order by charge selection rules

- chiral w.r.t. anomalous $U(1)^5_{\text{massive}}$

$$(\mathbf{1},\mathbf{2},\mathbf{1};\overline{\mathbf{2}},\mathbf{1})+3(\mathbf{1},\overline{\mathbf{2}},\mathbf{1};\overline{\mathbf{2}},\mathbf{1})+(\mathbf{1},\overline{\mathbf{2}},\mathbf{1};\mathbf{1},\overline{\mathbf{2}})+(\mathbf{1},\mathbf{1},\overline{\mathbf{2}};\mathbf{2},\mathbf{1})+3(\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{2})$$

but non-chiral w.r.t. $SU(4)_a \times SU(2)_b \times SU(2)_c$

- non-chiral w.r.t. to full $U(4)_a \times U(2)^4$ with **GUT Higgses**

$$2\left[(\mathbf{4},\mathbf{1},\mathbf{1};\overline{\mathbf{2}},\mathbf{1})+h.c.\right]+[(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{2},\mathbf{2})+h.c.]+(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{4}_{\mathbf{Adj}},\mathbf{1})$$
$$+\,2\,[(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{3}_{\mathbf{S}},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1}_{\mathbf{A}},\mathbf{1})+h.c.]+[(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{3}_{\mathbf{S}})+(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1}_{\mathbf{A}})+h.c.]$$

Pati-Salam model cont'd: $\mathbb{Z}_n$ Symmetries in $U(1)^5_{\text{massive}}$

- 5 independent $\mathbb{Z}_n$ symmetries ($h_{21} = 15$) G.H., Staessens '13
- 4 family-independent & trivial: $\mathbb{Z}_N \subset U(N)$
- **family-dependent**:
  - $\mathbb{Z}_4 \subset \frac{1}{2}\sum_{x \in \{b,c,d,e\}} U(1)_x \rightsquigarrow$ **selection** rule on **Yukawas**

Discrete charges for the five-stack Pati-Salam model on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_6' \times \Omega\mathcal{R})$												
Discrete symmetries		Charge assignment for the 'chiral' states										
$\mathbb{Z}_n$	$U(1) = \sum_x k_x\, U(1)_x$	(Q_L, L) ab	(Q_L, L) ab'	(Q_R, R) ac	(Q_R, R) ac'	(H_d, H_u)	X_{bd}	$X_{bd'}$	$X_{be'}$	X_{cd}	$X_{cd'}$	$X_{ce'}$
$\mathbb{Z}_2$	$U(1)_e$	0	0	0	0	0	0	0	1	0	0	1
	$U(1)_d$	0	0	0	0	0	1	1	0	1	1	0
	$U(1)_c$	0	0	1	1	1	0	0	0	1	1	1
	$U(1)_b$	1	1	0	0	1	1	1	1	0	0	0
$\mathbb{Z}_4$	$U(1)_a$	1	1	3	3	0	0	0	0	0	0	0
	$U(1)_b + U(1)_c + U(1)_d + U(1)_e$	3	1	1	3	0	0	2	2	0	2	2

Reduction of the *Family Dependent* Symmetry: $\mathbb{Z}_4 \to \mathbb{Z}_2$

- unwritten lore: **mod out centers** of $SU(N)$:
  $((\mathbb{Z}_4)^2 \times (\mathbb{Z}_2)^3)/(\mathbb{Z}_4 \times (\mathbb{Z}_2)^4) \simeq \boxed{\mathbb{Z}_2}$
- search consistent charge assignment by hand:
  - $(\mathbf{4}, \overline{\mathbf{2}}, \mathbf{1}, \mathbf{1}, \mathbf{1}).(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1}).(\mathbf{1}, \mathbf{2}, \overline{\mathbf{2}}, \mathbf{1}, \mathbf{1})$ *perturbatively* allowed
  - $(\mathbf{4}, \overline{\mathbf{2}}, \mathbf{1}, \mathbf{1}, \mathbf{1}).(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{1}).(\mathbf{1}, \overline{\mathbf{2}}, \mathbf{1}, \overline{\mathbf{2}}, \mathbf{1})$ *pert.* forbidden by $U(1)_b$
    - $\mathbb{Z}_4$ charge: 2 mod 4
  - $(\mathbf{4}, \overline{\mathbf{2}}, \mathbf{1}, \mathbf{1}, \mathbf{1}).(\overline{\mathbf{4}}, \mathbf{1}, \overline{\mathbf{2}}, \mathbf{1}, \mathbf{1}).(\mathbf{1}, \mathbf{2}, \overline{\mathbf{2}}, \mathbf{1}, \mathbf{1})$ *pert.* forbidden by $U(1)_c$
  - ...

	(Q_L, L) ab	ab'	(Q_R, R) ac	ac'	(H_d, H_u)	X_{bd}	$X_{bd'}$	$X_{be'}$	X_{cd}	$X_{cd'}$	$X_{ce'}$
$\mathbb{Z}_2$	0	1	0	1	0	0	1	1	0	1	1

- $\mathbb{Z}_2$ remains **family-dependent**

... very special D-brane configuration

interesting for flavour physics cf. King's talk

- **cannot** be obtained from 'mod 2' on $\mathbb{Z}_4$ charges
  $\rightsquigarrow$ unwritten lore doesn't really help

Axions, Strong CP Problem, Dark Sector

Axions and the Strong CP Problem

- **Axions** originally invoked to solve **strong CP-problem**

$$\mathcal{L}_\alpha \supset \frac{1}{2}\left(\partial_\mu \alpha\right)\left(\partial^\mu \alpha\right) - \frac{1}{32\pi^2}\frac{\alpha(x)}{f_\alpha}\,\mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu})$$

  - *global* Pecci-Quinn symmetry $U(1)_{PQ}$ — Pecci, Quinn '77
  - **axion** α arises from rewriting two Higgs doublets
  - ~~electro-weak~~ & ~~PQ~~ scales identical
  - axions $\leftrightarrow$ photon conversion assumed *(Primakoff effect)*
    $\rightsquigarrow$ astrophysical & lab searches (e.g. ALPs@DESY)
    pure QCD axion experimentally excluded

    see ongoing parallel session

- modified models contain **SM singlet field** σ
  - σ couples to Higgs doublets $\rightsquigarrow$ new terms in V_{Higgs}
  - ~~PQ~~ by $\langle\sigma\rangle$ at higher energy than $\cancel{SU(2)_L \times Y}$

Zhitnitsky '80; Dine, Fischler, Srednicki '81; ...

Axions in Type II String Theory

- realisation in **D-brane models**
  - **open** string **axions** Kiritsis '05; Svrcek, Witten '06; Berenstein, Perkins '12 ...
    - can mix with **closed** string axions
  - $\boxed{U(1)_{PQ} \to U(1)_{\text{massive}}}$
    - *perturbative* symmetry
  - 'exotic' scalars abundant - adjustments to **SUSY** required
  - suitable ~~SUSY~~ minimum of V_{Higgs}? **GH**, Staessens '13

Open String Axions & DFSZ Model

- $U(1)_{PQ}$ must allow:

$$\mathcal{L}_{\text{Yukawa}} = f_u\, Q_L \cdot H_u\, u_R + f_d\, Q_L \cdot H_d\, d_R + f_e\, L \cdot H_d\, e_R + f_\nu\, L \cdot H_u\, \nu_R$$

- introduce **SM singlet** σ with $U(1)_{PQ} \simeq U(1)_{\text{massive}}$ charge
- (H_u, H_d) charged under $U(1)_{PQ}$

  $\rightsquigarrow Q_L$ **or** (u_R, d_R) have $U(1)_{PQ}$ charge

- **Higgs potential** of the DFSZ model

$$\begin{aligned} V_{\text{DFSZ}}(H_u, H_d, \sigma) \quad = \quad & \lambda_u (H_u^\dagger H_u - v_u^2)^2 + \lambda_d (H_d^\dagger H_d - v_d^2)^2 + \lambda_\sigma (\sigma^* \sigma - v_\sigma^2)^2 \\ & + (a\, H_u^\dagger H_u + b\, H_d^\dagger H_d)\, \sigma^* \sigma + c\, (H_u \cdot H_d\, \sigma^2 + h.c.) \\ & + d\, |H_u \cdot H_d|^2 + e\, |H_u^\dagger H_d|^2 \end{aligned}$$

- **SUSY** version: $V = V_F + V_D + V_{\text{soft}}$
- modify $c\, (H_u \cdot H_d\, \sigma^2 + h.c.) \longrightarrow c\, (H_u \cdot H_d\, \sigma + h.c.)$; $\sigma \sim e^{ia}$

Matter	Q_L	$\overline{u}_R$	$\overline{d}_R$	H_u	H_d	L	$\overline{e}_R$	$\overline{\nu}_R$	Σ
$U(1)_{PQ}$	∓ 1	0	0	± 1	± 1	∓ 1	0	0	∓ 2

- identify $\Sigma = (\textbf{Anti})_{U(2)_b}$ in global D-brane models

e.g. SM on $T^6/\mathbb{Z}_6$: **GH**, Ott '04 & $T^6/\mathbb{Z}_6'$: Gmeiner, **GH** '08

Mixing of Open and Closed String Axions

GH, Staessens '13

- open string axion a from $\sigma = \frac{v+s(x)}{\sqrt{2}} e^{i\frac{a(x)}{v}}$
- **open** axion a mixes with **closed** axion ξ ($\leftarrow U(1)_{\text{massive}}$)

$$\zeta = \frac{M_{\text{string}}\,\xi + qv\,a}{\sqrt{M_{\text{string}}^2 + q^2v^2}}, \qquad \alpha = \frac{M_{\text{string}}\,a - qv\,\xi}{\sqrt{M_{\text{string}}^2 + q^2v^2}}$$

$$\Rightarrow \qquad \mathcal{L}_{\text{CP-odd}} = \frac{1}{2}\left(\partial_\mu\zeta + m_B B_\mu\right)^2 + \frac{1}{2}(\partial_\mu\alpha)^2$$

- axion **decay constant** f_α from dim. reduction:
$\mathcal{L}_{\text{anom}} = \frac{1}{16\pi^2}\frac{\zeta(x)}{f_\zeta}\mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu}) + \frac{1}{32\pi^2}\frac{\alpha(x)}{f_\alpha}\mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu})$

$$\text{with} \qquad f_\zeta = \frac{\sqrt{M_{\text{string}}^2 + (qv)^2}}{2}, \qquad f_\alpha = \frac{M_{\text{string}}\,qv\sqrt{M_{\text{string}}^2 + (qv)^2}}{(M_{\text{string}}^2 - (qv)^2)}$$

- For $\boxed{M_{\text{string}} \gg v}$: $\zeta \simeq \xi_{\text{closed}}$, $\alpha \simeq a_{\text{open}}$

Soft $\cancel{\text{SUSY}}$ Terms

Origin of $\boxed{V = V_F + V_D + V_{\text{soft}}}$

$$
\begin{aligned}
V_{\text{DFSZ}}(H_u, H_d, \sigma) \quad = \quad & \lambda_u(H_u^\dagger H_u - v_u^2)^2 + \lambda_d(H_d^\dagger H_d - v_d^2)^2 + \lambda_\sigma(\sigma^*\sigma - v_\sigma^2)^2 \\
& +(a\, H_u^\dagger H_u + b\, H_d^\dagger H_d)\,\sigma^*\sigma + c\,(H_u \cdot H_d\,\sigma + h.c.) \\
& +d\,|H_u \cdot H_d|^2 + e\,|H_u^\dagger H_d|^2
\end{aligned}
$$

in **SUSY field theory**

- $\mathcal{W} = \mu \Sigma H_d \cdot H_u$
- $K^{\text{SUSY}}(\Phi^\dagger e^{2gV}\Phi) = \Phi^\dagger e^{2gV}\Phi$
- $\mathcal{W}_{\text{soft}} = \eta\, c H_u \cdot H_d\, \Sigma \rightsquigarrow \mathcal{A}$-terms
- $K_{\text{soft}} = \eta\bar{\eta}\; m_\Phi^2\, \Phi^\dagger e^{2gV}\Phi \rightsquigarrow m_{\text{soft}}$

in **Type II string models**

- strongly coupled hidden group e.g. $USp(6)$ in $T^6/\mathbb{Z}_6'$ model
- gaugino condensate: $\langle \lambda\lambda \rangle = \Lambda_c^3 \rightsquigarrow M^2_{\cancel{SUSY}} = \langle F^{\mathcal{H}} \rangle \sim \frac{\Lambda_c^3}{M_{\text{Planck}}}$
- gravity (+ gauge) mediation to SM sector

Lower Bounds on M_{string}

- typical **phenomenological** constraints from $f_\zeta \sim M_{\text{string}}$, $f_\alpha \sim qv$: $M_{\text{string}} \geq 10^9$ GeV
- supplemented by constraints on gauge couplings
  - $\frac{M^2_{\text{Planck}}}{M^2_{\text{string}}} \stackrel{\text{examples}}{=} \frac{4\pi \; v_1 v_2 v_3}{g^2_{\text{string}}}$
  - @ tree-level: $\frac{4\pi}{g^2_{SU(N_a)}} = \frac{\sqrt{v_1 v_2 v_3}}{8\pi^3 3^{1/4} \, g_{\text{string}}} \times \mathcal{O}(1)_{\text{model}}$
  - @ 1-loop: linear dep. on v_i, $\ln \frac{v_1 v_3}{v_2^2}$ $\Leftarrow$ **cancellations** possible

$\rightsquigarrow$ M_{string} can be lowered to intermediary scale by **exponentially large volumes**:

GH, Staessens '13

M_{string} as a function of v_i and g_{string}								
$g_{\text{string}} = 0.1$			$g_{\text{string}} = 0.01$			$g_{\text{string}} = 0.001$		
$v_1 v_3$	$v^2_{2,\max}$	M_{string}	$v_1 v_3$	$v^2_{2,\max}$	M_{string}	$v_1 v_3$	$v^2_{2,\max}$	M_{string}
10^8	9.7×10^9	1.6×10^{10} GeV	10^6	1.5×10^{10}	1.6×10^{10} GeV	10^2	1.5×10^6	1.6×10^{12} GeV
10^{10}	1.5×10^{14}	2.8×10^9 GeV	10^8	1.6×10^{14}	1.5×10^8 GeV	10^4	1.6×10^{10}	1.5×10^{10} GeV
10^{12}	1.5×10^{18}	2.8×10^8 GeV	10^{10}	1.6×10^{18}	1.5×10^6 GeV	10^6	1.6×10^{14}	1.5×10^8 GeV

Low (TeV) string scale scenario: Antoniadis, Kiritsis, Rizos, Tomaras '03; Anchordoqui, Goldberg, Lüst, Nawata, Stieberger, Taylor'08; . . . Berenstein '14; . . .

~~SUSY~~ by Deformations

Intermezzo: ~~SUSY~~ by Deformations

Blaszczyk, **GH**, Koltermann '14

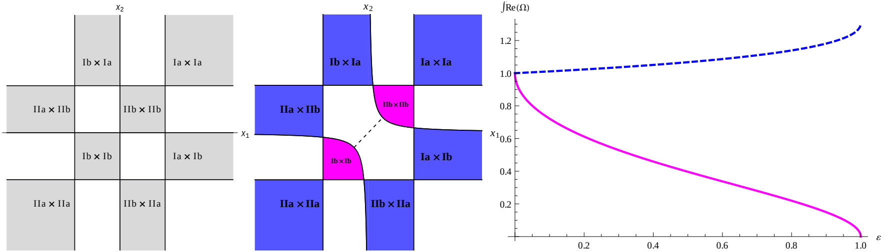
- What happens to $\mathrm{Vol}_{\mathrm{brane}}(\Pi)$ if $\mathbb{Z}_2$ **singularities** are **deformed**?
- use product of $\mathbb{P}^2_{112}$ with coord. (x_i, v_i, y_i) and for square tori $F_i = x_i v_i (x_i^2 - v_i^2)$

$$(T^2)^3/\mathbb{Z}_2 \times \mathbb{Z}_2 \simeq \{f = -y^2 + F_1 F_2 F_3 = 0\} \qquad \text{with } y \equiv y_1 y_2 y_3$$

- a **single** deformed fixed point: $f = -y^2 + F_1 F_2 F_3 + \varepsilon \delta F_1 \delta F_2 \cdot F_3 = 0 \rightsquigarrow y = y(x_1, x_2, x_3, \varepsilon)|_{v_i=1}$
- use $\Omega_3 = dz_1 dz_2 dz_3$ on $(T^2)^3$ with relation $dz_i = \frac{dx_i}{y_i}$
- compute $\boxed{\int_\Pi \Omega_3 = \int_\Pi \frac{dx_1 dx_2 dx_3}{y}}$ for deformed geometry:
  - decrease with $\sqrt{\varepsilon}$ if Π contains singularity
  - change linear in ε otherwise

Visualisation of Deformation of Singularity along $T_1^2 \times T_2^2$

- deformation of singularity at $x_1 = x_2 = 0$ along $T_1^2 \times T_2^2$



- volume of 3-cycle passing through singularity changes $\sim -\sqrt{\varepsilon}$

$$y = \sqrt{(x_1^2 - 1)(x_2^2 - 1)(x_1 x_2 + \varepsilon) x_3 (x_3^2 - 1)}$$

- $\boxed{\int_\Pi \Omega_3 = \int_\Pi \frac{dx_1 dx_2 dx_3}{y} \overset{\text{SUSY}}{=} \int_\Pi \Re(\Omega_3)}$
  - $\boxed{\varepsilon > 0}$ $\Pi_a^{\mathbb{Z}_2} = \Pi_{a'}^{\mathbb{Z}_2}$, $SO(2N)$& $USp(2N)$ branes stay SUSY
  - $\boxed{\varepsilon < 0}$ $\Pi_a^{\mathbb{Z}_2} = -\Pi_{a'}^{\mathbb{Z}_2}$, ~~SUSY~~ on $U(N)$ branes

i.e. **orbifold point** is **only SUSY** point of SM branes

Conclusions:

- $\boxed{\mathbb{Z}_n}$ expressed via **intersection numbers** in Type IIA:
  - $(h_{21}+1) \sim \mathcal{O}(10)$ nec. + suf. conditions per orbifold
  - many $\mathbb{Z}_n$ trivial in 4D field theory (e.g. $\mathbb{Z}_N \subset U(N)$)
  - **family-dependent** $\mathbb{Z}_4$ ($\mathbb{Z}_2$) constrains Yukawas

$\rightsquigarrow$ ultimate **selection rules**

. . . details in **GH**, Staessens '13

- $\boxed{U(1)_{PQ} \simeq U(1)_{\text{massive}}}$ and **axions** as **(Anti)**$_{U(2)}$
  - Mixing of **axions** from **open/closed** string sector
  - $\cancel{U(1)_{PQ}}$ and $\cancel{SU(2) \times Y}$ scales decouple
  - **intermediary** M_{string} and exponentially large volumes

. . . to be explored in greater detail

. . . details in **GH**, Staessens '13

- ~~**SUSY**~~ by deformation of 3-cycles

. . . how are SM fields affected?

. . . details in Blaszczyk, **GH**, Koltermann '14 & work in progress