NOVEL T-VIOLATION OBSERVABLE OPEN TO ANY DECAY CHANNEL AT MESON FACTORIES

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Collaboration

- This work has been done with J. Bernabeu and M. Nebot: published in Phys.Lett. B728 (2014) 95-98 (arXiv:1309.0439) and work in progress.

Introduction I

- The BABAR Collaboration reported, in the $B^0 - \overline{B}^0$system, the first direct observation of T-violation [1207.5832], in the time evolution of any system with high statistical significance.
- The measurement is based in a method described in [JHEP 1208,064 (2012)] following the concepts originally proposed in [Phys. Lett. B 464,117(1999),4 and Nucl. Phys. B 590, 19 (2000)]. (Bernabeu, Bañuls, Martinez-Vidal and Villanueva).
- In the $\left\{ \left| P^0 \right\rangle, \left| \overline{P}^0 \right\rangle \right\}$ system , where P^0 stands for a neutral meson K^0, D^0, B_d^0 or B_s^0 (and $\overline{P}^0$ for the corresponding antimeson) , two arbitrary states are:

$$\begin{aligned} \left| P_1 \right\rangle &= p_1^0 \left| P^0 \right\rangle + \overline{p}_1^0 \left| \overline{P}^0 \right\rangle \\ \left| P_2 \right\rangle &= p_2^0 \left| P^0 \right\rangle + \overline{p}_2^0 \left| \overline{P}^0 \right\rangle \end{aligned}$$

Introduction II

- The probability that an initially prepared state P_1, evolving after time t to $P_1(t)$, behave like state P_2 is

$$\Pr\left[P_1 \rightarrow P_2(t)\right] = \left|\langle P_2 | P_1(t)\rangle\right|^2 = \left|\langle P_2 | U(t,0) | P_1\rangle\right|^2$$

- The T-VIOLATION observable is related to compare the reversed probabilities

$$\Pr\left[P_1 \rightarrow P_2(t)\right] - \Pr\left[P_2 \rightarrow P_1(t)\right]$$

- The BABAR asymmetry is build from $P_1 = B_d^0, \overline{B}_d^0$ and $P_2 = B_\pm$ where $B_\pm$ are the B-states tagged or filtered by the decays to CP-eigenstates with definite flavour content. These asymmetries are experimentally independent of CP violation. The Kabir asymmetry, with $P_1, P_2 = K^0, \overline{K}^0$ was measured by the CPLEAR Collaboration [Phys.Lett. B444 (1998) 43-51] with a non-vanishing value near 4 standard deviations.

Reference transition at C = - initial Entangled State I

- **We want to understand which is the Reference Transition** $P_1 \rightarrow P_2\,(t)$ **between** **<u>meson states</u>** **associated to a given pair of** **<u>decays</u>:** f_1 **at** t_1 **and** f_2 **at** $t_2 > t_1$.
- Two Quantum effects are relevant to answer this question:
  - **Entanglement** between the two neutral mesons produced at a meson factory is essential to prepare the initial sate.
  - The **filtering measurement** induced by the meson decay is used to connect to the final state.
- The entangled state of the two mesons is in an antisymmetric combination of individual orthogonal states,

$$\left|\Phi_{(C=-)}\right\rangle = \frac{1}{\sqrt{2}}\left\{\left|P^0\left(\overrightarrow{k}\right)\right\rangle\left|\overline{P}^0\left(\overrightarrow{-k}\right)\right\rangle - \left|\overline{P}^0\left(\overrightarrow{k}\right)\right\rangle\left|P^0\left(-\overrightarrow{k}\right)\right\rangle\right\}$$

Reference transition at C = - initial Entangled State II

- Therefore the (still living) meson at time t_1 is tagged as "**the state that does not decay into** f"

$$|P_{\nrightarrow f}\rangle = \frac{1}{\sqrt{|A_f|^2 + |\overline{A}_f|^2}} \left[\overline{A}_f \left|P^0\right\rangle - A_f \left|\overline{P}^0\right\rangle\right]$$

where A_f $(\overline{A}_f)$ is the decay amplitude from P^0 $\left(\overline{P}^0\right)$ to f:

$$A_f = \langle f|\, W \left|P^0\right\rangle \quad ; \quad \overline{A}_f = \langle f|\, W \left|\overline{P}^0\right\rangle$$

to first order in the weak Hamiltonian H_w and to all orders in strong interactions $W = U_S(\infty, 0)\, H_w$. $U_S(\infty, 0)$ is the strong evolution operator and is equal to the identity if we can neglect final sate interactions. (For hadronic decays we will assume transitions with one helicity amplitude: $0 \rightarrow 0 + j$)

Reference transition at C = - initial Entangled State III

- The corresponding orthogonal state $\left\langle P^{\perp}_{\nrightarrow f} | P_{\nrightarrow f} \right\rangle = 0$ is given by

$$\left| P^{\perp}_{\nrightarrow f} \right\rangle = \frac{1}{\sqrt{\left|A_f\right|^2 + \left|\overline{A}_f\right|^2}} \left[A_f^* \left| P^0 \right\rangle + \overline{A}_f^* \left| \overline{P}^0 \right\rangle \right]$$

and **it is the one filtered by the decay**.

- What we call the "filtering identity" define the precise meaning of this statement:

$$\left| \left\langle P^{\perp}_{\nrightarrow f_2} | P_1(t) \right\rangle \right|^2 = \frac{\left| \langle f_2 | W | P_1(t) \rangle \right|^2}{\left(\left|A_{f_2}\right|^2 + \left|\overline{A}_{f_2}\right|^2 \right)}$$

- Experimentally**, the Reference Transition**

$$P_1 \rightarrow P_2(t)$$

is therefore **directly connected to** $P_1 = P_{\nrightarrow f_1}$, $P_2 = P^{\perp}_{\nrightarrow f_2}$, i.e.,

$$P_{\nrightarrow f_1}(t_1) \rightarrow P^{\perp}_{\nrightarrow f_2}(t_2)$$

Reference transition at C = - initial Entangled State IV

- And the T **transformed transition** ("reversed")

$$P^{\perp}_{\nrightarrow f_2}(t_1) \longrightarrow P_{\nrightarrow f_1}(t_2)$$

  does not correspond to the pair of decays f_2 **at** t_1 , f_1 **at** $t_2 > t_1$, **neither in the initial nor in the final decays**.
- This is the "orthogonality problem" that prevents taking an arbitrary pair of decay channels.
- To connect the T transformed transition with experiment, we need to find a pair of decay channels such that, for each of them,

$$\text{Given } f \longrightarrow \exists f' / \left|P_{\nrightarrow f'}\right\rangle = \left|P^{\perp}_{\nrightarrow f}\right\rangle$$

Reference transition at C = - initial Entangled State V

- This orthogonality condition is satisfied by either CP conjugate decay channels $\left(P^0, \overline{P}^0\right)$ or CP-eigenstates of opposite sign with the same flavour content (P_+, P_-) and no direct CP violation $\left(\lambda_{f'}\lambda_f^* = -\left|q/p\right|^2 \sim -1\right)$. Hence the exceptionality of the transitions between semileptonic and CP eigenstate decays. As a consequence, the orthogonality condition limits the pair of decay channels suitable for T-symmetry tests **if we start, as a Reference, from the "experimental" transition** $\left(P_{\nrightarrow f_1}(t_1) \longrightarrow P^{\perp}_{\nrightarrow f_2}(t_2)\right)$.

Reference transition at C = - initial Entangled State VI

- Having identified the orthogonality problem, we give a bypass consisting in having an alternative reference transition once we fix the two channels f_1, f_2.

We make the following replacements

$$\left\{ \begin{array}{ccccc} & \text{BABAR} & & \text{NEW} \\ \text{REFERENCE} \\ \text{TRANSITION} & P_{\nrightarrow f_1}(t_1) \rightarrow P^{\perp}_{\nrightarrow f_2}(t_2) & \Longrightarrow & P_{\nrightarrow f_1}(t_1) \rightarrow P_{\nrightarrow f_2}(t_2) \\ \text{T TRANSFORMED} \\ \text{TRANSITION} & P^{\perp}_{\nrightarrow f_2}(t_1) \rightarrow P_{\nrightarrow f_1}(t_2) & \Longrightarrow & P_{\nrightarrow f_2}(t_1) \rightarrow P_{\nrightarrow f_1}(t_2) \end{array} \right\}$$

- Whereas now the two initial P-states are directly connected to experiment, the price to be paid now is that the two final state are not. One has to work out what is the connection of the novel genuine theoretical T-asymmetry observable to experimental measurements.

The New T-violating Asymmetry I

- The probability associated to **the new reference transition is**

$$P_{12}(t) = \left|\left\langle P_{\nrightarrow f_2} | P_{\nrightarrow f_1}(t)\right\rangle\right|^2$$

- **It is not directly connected to the double decay rate as measured at meson factories**

$$I_{12}(t) = \frac{\left|\langle f_2|\, W \,|P_{\nrightarrow f_1}(t)\rangle\right|^2}{\left(\left|A_{f_2}\right|^2 + \left|\bar{A}_{f_2}\right|^2\right)} = \left|\left\langle P^{\perp}_{\nrightarrow f_2} | P_{\nrightarrow f_1}(t)\right\rangle\right|^2$$

but using closure it easy to relate both quantities

$$\begin{aligned} P_{12}(t) &= \left\langle P_{\nrightarrow f_1}(t) | P_{\nrightarrow f_2}\right\rangle \left\langle P_{\nrightarrow f_2} | P_{\nrightarrow f_1}(t)\right\rangle = \\ &= \left\langle P_{\nrightarrow f_1}(t)\right| \left[I - \left|P^{\perp}_{\nrightarrow f_2}\right\rangle \left\langle P^{\perp}_{\nrightarrow f_2}\right| \right] \left|P_{\nrightarrow f_1}(t)\right\rangle \end{aligned}$$

- **The key result is**

$$P_{12}(t) = N_1(t) - I_{12}(t)$$

The New T-violating Asymmetry II

- $N_1(t)$ being the **total survival probability of the $P_{\nrightarrow f_1}$ state after time** t

$$N_1(t) = \langle P_{\nrightarrow f_1}(t) | P_{\nrightarrow f_1}(t) \rangle \quad (1)$$

  a well defined and measurable quantity: the ratio between the number of mesons that have not decayed after a time t and the initial number tagged at t_1 as $|P_{\nrightarrow f_1}\rangle$ by the observation of the first decay f_1. We may then call this term the "total survival probability" of the state $|P_{\nrightarrow f_1}\rangle$.

- With the probability of the reference transition and its time reversed theoretically and experimentally well-defined we present **the motion reversal asymmetry**

$$A_R(f_1, f_{2;}t) = P_{12}(t) - P_{21}(t)$$

  this new observable becomes entirely measurable in a 1^{--} meson factory for any pair of decay channels f_1, f_2. We have not imposed any particular condition to the pair of decay channels f_1, f_2, so one is not forced to use flavour specific or CP eigenstate decay channels. We will discuss later whether the measurable $A_R(f_1, f_2; t)$ becomes a genuine Time-Reversal-Violating Asymmetry for any pair of decays .

The New T-violating Asymmetry III

- This asymmetry can be written explicitly as

$$A_R(f_1, f_2; t) = P_{12}(t) - P_{21}(t) = [N_1(t) - N_2(t)] + [I_{21}(t) - I_{12}(t)]$$

The second piece in the r.h.s. corresponds to the typical $t \leftrightarrow -t$ asymmetry of the double decay rate in the meson factory: comparing the rate of a process decaying first one side to the final state f_1 and after a time t the other side decays to f_2 with the one with f_i reversed. The first piece is the difference of the total survival probability of the two states $\left|P_{\nrightarrow f_i}\right\rangle$: this is the new piece to be measured that allows to work with the here proposed new reference transition

The theoretical expressions I

- Using the time evolution imposed by Quantum Mechanics we know the time dependent structure of the needed measurable quantities in terms of Δm, Γ and $\Delta\Gamma$, determined by the eigenvalues of the entire Hamiltonian for the $\left(P^0, \overline{P}^0\right)$.
- The motion reversal asymmetry is given by:

$$A_R\left(f_1, f_2; t\right) = e^{-\Gamma t} \left\{ \begin{array}{c} \mathcal{C}^N\left[f_1, f_2\right]\left(\cosh\left(\frac{\Delta\Gamma t}{2}\right) - \cos\left(\Delta m t\right)\right) + \\ +\mathcal{S}_h^N\left[f_1, f_2\right]\sinh\left(\frac{\Delta\Gamma t}{2}\right) + \mathcal{S}_c^N\left[f_1, f_2\right]\sin\left(\Delta m t\right) \end{array} \right\}$$

and depends on three "asymmetry parameters": the non-vanishing value of any of these asymmetry parameters would be a signal of Time-Reversal-Violation.

The theoretical expressions II

- The theoretical connection of these measurable parameters to the matrix element of the meson evolution Hamiltonian H_{ij} involved in the Weisskoppf-Wigner Approach (WWA) for the $\left(P^0, \overline{P}^0\right)$ system is

$$
\begin{aligned}
\mathcal{C}^N\left[f_1, f_2\right] &= \delta\left(C_{f_1} - C_{f_2}\right) - \mathrm{Im}\left(\theta\right)\left(S_{f_1} - S_{f_2}\right) \\
\mathcal{S}_h^N\left[f_1, f_2\right] &= \delta\left(C_{f_1} R_{f_2} - C_{f_2} R_{f_1}\right) + \mathrm{Im}\left(\theta\right)\left(R_{f_1} S_{f_2} - R_{f_2} S_{f_1}\right) \\
\mathcal{S}_c^N\left[f_1, f_2\right] &= \left(C_{f_1} S_{f_2} - C_{f_2} S_{f_1}\right)\left\{1 + \delta\left(C_{f_1} + C_{f_2}\right)\right\} + \\
&\quad + \delta\left(S_{f_1} - S_{f_2}\right) + \mathrm{Re}\left(\theta\right)\left(R_{f_1} S_{f_2} - R_{f_2} S_{f_1}\right)
\end{aligned}
$$

- We present the results at leading order in terms of the usual mixing parameters -see the book of G. Branco et al. - $\delta = \left(1 - |q/p|^2\right) / \left(1 + |q/p|^2\right)$, that is real, T and CP violating, where $(q/p)^2 = H_{21}/H_{12}$, and the complex parameter θ that is CPT and CP violating $\theta = \left(H_{22} - H_{11}\right) / \left(\Delta m - i\Delta\Gamma/2\right)$.

The theoretical expressions III

- We also use the parameter R_i, S_i and C_i defined in terms of the well-known $\lambda_i = q\bar{A}_{f_i}/pA_{f_i}$

$$C_i = \frac{1-|\lambda_i|^2}{1+|\lambda_i|^2} \quad : \quad S_i = \frac{2\,\mathrm{Im}(\lambda_i)}{1+|\lambda_i|^2} \quad ; \quad R_i = \frac{2\,\mathrm{Re}(\lambda_i)}{1+|\lambda_i|^2}$$

  Note that this parameters are not all independent $C_i^2 + S_i^2 + R_i^2 = 1$.

- It is worthwhile to point out that all these pieces should be small, except **the term** $\left(C_{f_1} S_{f_2} - C_{f_2} S_{f_1}\right)$ that **could be of order 1**. In fact this term is the reminiscent of the BaBar T-violating measurement translated to the present context. **If the asymmetry $A_R\left(f_1, f_2; t\right)$ were always a time reversal asymmetry this observable would represent the generalization of the T violating program to "any pair of decay channels"**.

Condition to have a true T violating asymmetry I

- The principle microreversibility strictly means that T-invariance implies the cancellation of the asymmetry

$$A_T\left(f_1, f_{2;}t\right) = P_{12}\left(t\right) - P_{2_T 1_T}\left(t\right)$$

where

$$P_{2_T 1_T}\left(t\right) = \left|\left\langle U_T P_{\nrightarrow f_1} \middle| U_T P_{\nrightarrow f_2}\left(t\right)\right\rangle\right|^2$$

This asymmetry is not exactly $A_R\left(f_1, f_{2;}t\right)$.

- U_T is the antiunitary time reversal operator $U_T = \widehat{U}_T K$, where $\widehat{U}_T$ is unitary and K is the complex conjugation operator. For pseudoscalars, the effect of $\widehat{U}_T$ is safe with the only change $\overrightarrow{p} \rightarrow -\overrightarrow{p}$ and using rotational invariance. The possible difficulty in assigning a genuine character of T-violation to the asymmetry $A_R\left(f_1, f_{2;}t\right)$ is concentrated on K.

Condition to have a true T violating asymmetry II

- It is therefore clear that a sufficient condition that must satisfy "both" channels in order to be $A_R\left(f_1, f_{2;}t\right)$ a true T-violating asymmetry is that **in the T invariant limit**

$$U_T\left|P_{\nrightarrow f_i}\right\rangle = e^{i\varphi_i}\left|P_{\nrightarrow f_i}\right\rangle$$

- In the $\left\{P^0, \overline{P^0}\right\}$meson space one can take

$$\widehat{U}_T = \begin{pmatrix} e^{i(\nu-\xi)} & 0 \\ 0 & e^{i(\nu+\xi)} \end{pmatrix}$$

where ν is the phase of the CPT operator $U_{CPT}\left|P^0\right\rangle = e^{i\nu}\left|\overline{P}^0\right\rangle$ and $U_{CP}\left|P^0\right\rangle = e^{i\xi}\left|\overline{P}^0\right\rangle$ $\left(U_{CP}\left|\overline{P}^0\right\rangle = e^{-i\xi}\left|P^0\right\rangle\right)$ defines de phase of the CP

operator. Therefore we can write

$$
\begin{aligned}
\left|P_{\not\to f_i}\right\rangle &= N_{f_i}^{*} A_{f_i} \left(\frac{\overline{A}_{f_i}}{A_{f_i}} \left|P^0\right\rangle - \left|\overline{P}^0\right\rangle \right) \\
U_T \left|P_{\not\to f_i}\right\rangle &= N_{f_i}^{*} A_{f_i}^{*} e^{i(\nu+\xi)} \left(\frac{\overline{A}_{f_i}^{*}}{A_{f_i}^{*}} e^{-2i\xi} \left|P^0\right\rangle - \left|\overline{P}^0\right\rangle \right)
\end{aligned}
$$

- **In order the motion reversal asymmetry $A_R\left(f_1, f_{2;}t\right)$ be a true T-violating asymmetry both channels f_i should satisfy, in the T invariant limit, the relation:**

$$
A_f^{*}\overline{A}_f = \overline{A}_f^{*} A_f e^{-2i\xi}
$$

- In the limit of T invariance it is well-known that $q/p = e^{2i\xi}$ therefore we have to choose decay channels f_i in $A_R\left(f_1, f_{2;}t\right)$ such that they **verify in the T invariant limit**

$$
\lambda_{f_i} = \lambda_{f_i}^{*}
$$

Condition to have a true T violating asymmetry IV

- Or equivalently: **channels that in the T invariant limit verifies**

$$S_{f_i} = 0$$

- This condition seems quite reasonable but revels the difficulties associated to T violating observables. Namely in the absence of final state interactions there only remains weak phases that should vanish in the T invariant limit and therefore making $S_{f_i} = 0$. But by the same token we discovered that also in these observables final state interactions can give fake T violation.
- If we look at the expression of $A_R(f_1, f_2; t)$ we discovered that $A_R(f_1, f_2; t) = 0$ provided the quantity δ -violating T- is equal to zero, together with the condition $S_{f_1} = S_{f_2} = 0$. Therefore if the chosen channels f_i are such that S_{f_i} are T violating quantities, then $A_R(f_1, f_2; t)$ is a T violating quantity.

Channels useful for the asymmetry I

1. Flavour specific channels verify $S_f = 0$ always. Therefore **flavour specific channels can be used in** $A_R(f_1, f_2; t)$.
2. If $U_T|f\rangle = e^{i\upsilon_f}|f'\rangle$, and we take rotationally invariant states $f = f'$ (for example a two body state with one spin zero particle and total spin zero) and neglecting final state interactions (FSI) $W = H_{wk}$ one can show that in the T invariant limit

$$\overline{A}_f = \langle f| U_T^\dagger U_T H_{wk} U_T^\dagger U_T \left|\overline{P}^0\right\rangle = e^{-i\upsilon_f}\overline{A}_f^* e^{-i(\nu+\xi)}$$
$$A_f = \langle f| U_T^\dagger U_T H_{wk} U_T^\dagger U_T \left|P^0\right\rangle = e^{-i\upsilon_f} A_f^* e^{-i(\nu-\xi)}$$

that reproduces

$$A_f^*\overline{A}_f = \overline{A}_f^* A_f e^{-2i\xi}$$

This confirms our previous comment in a precise way: **Any channel f_i where we can neglect FSI can be used in** $A_R(f_1, f_2; t)$.

Channels useful for the asymmetry II

3. As in the precedent case but including final state interactions one get under T invariance

$$\begin{aligned} A_f^* &= \langle U_T f | \, S^\dagger U(\infty, 0) \, H_{wk} \left| U_T P^0 \right\rangle = \sum_\beta \langle U_T f | \, S^\dagger \, |\beta\rangle \, e^{i(\nu - \xi)} A_\beta \\ \overline{A}_f^* &= \langle U_T f | \, S^\dagger U(\infty, 0) \, H_{wk} \left| U_T \overline{P}^0 \right\rangle = \sum_\beta \langle U_T f | \, S^\dagger \, |\beta\rangle \, e^{i(\nu + \xi)} \overline{A}_\beta \end{aligned}$$

where we have used the strong scattering matrix $S = U(+\infty, -\infty)$. If f is an eigenstate of the S matrix $\langle \beta | \, S \, |f\rangle = e^{-i2\delta_f} \delta_{f,\beta}$ then it is straightforward to get for rotationally invariant f states:

$$A_f^* \overline{A}_f = A_f \overline{A}_f^* e^{-2i\xi}$$

We conclude that **one can also use in** $A_R(f_1, f_2; t)$ f_i **eigenstates of the strong scattering matrix**. This can be relevant at Φ factories where Kaons can decay to a two pion state with well-defined isospin: $(\pi\pi)_I$.

Channels useful for the asymmetry III

4. If the CP operator acting on the state f is $CP\left|f\right\rangle = e^{-i\xi_f}\left|\overline{f}\right\rangle$ it is easy to prove that in the CP invariant limit in the decay amplitudes we get

$$\overline{A}_{\overline{f}} = e^{i(\xi_f - \xi)} A_f \quad ; \quad A_{\overline{f}} = e^{i(\xi_f + \xi)} \overline{A}_f$$

therefore in the case of CP eigenstates $f = \overline{f}$, and $e^{-i\xi_f} = \eta_f = \pm 1$ and we reproduce again the sufficient condition. Therefore **we can use** in our T violating asymmetry all the CP **eigenstates channels** f **where we can neglect** CP **violation in the corresponding decay amplitudes**. This argument can be transformed in a more general one related to CPT.

5. If we consider just decays to stays f mediated by CPT invariant decay amplitudes, then in the T invariant limit we also have CP invariance and consequently we get again the desired condition . An important difference with the previous condition is that assuming CP invariance in the decay we just can use the decay $B^0 \to (\pi\pi)_{I=2}$ in the two pion mode. Under CPT invariance in the decay we can use both $B^0 \to \pi^+\pi^-$ and $B^0 \to \pi^0\pi^0$. **Assuming** CPT **invariance in the decay amplitude we can use any** CP **eigenstate in our T violating asymmetry** $A_R(f_1, f_2; t)$.

Channels useful for the asymmetry IV

6. It is clear that if f is not a CP eigenstates, strong phases can make $S_f \neq 0$ even in the absence of weak phases (in the T invariant limit). This means that our asymmetry $A_R(f_1, f_2; t)$ cannot be used alone with this non CP eigenstate f. But let us assume CP invariance of the decay amplitudes, then it can be shown that

$$\lambda_{\overline{f}} = \frac{q\overline{A}_{\overline{f}}}{pA_{\overline{f}}} = \frac{q}{p}\frac{e^{i(\xi_f-\xi)}A_f}{e^{i(\xi_f+\xi)}\overline{A}_f} = e^{-2i\xi}\frac{q}{p}\frac{A_f}{\overline{A}_f} = e^{-2i\xi}\left(\frac{q}{p}\right)^2\frac{1}{\lambda_f}$$

and **in the T invariant limit we get**

$$\lambda_f\lambda_{\overline{f}} = 1$$

7. From here it is straightforward to prove that assuming CP invariance in the decay amplitudes A_f and $\overline{A}_f$ the violation of the following identities will be a clear signal of T violation.

$$C_f + C_{\overline{f}} = 0 \quad ; \quad S_f + S_{\overline{f}} = 0 \quad ; \quad R_f - R_{\overline{f}} = 0$$

Channels useful for the asymmetry V

8. As in the case of CP eigenstates we can change the role of CP by CPT. The fact that for example $S_f + S_{\overline{f}} \neq 0$ is a signal of T violation for non CP eigenstates suggest us to combine two asymmetries

$$\langle A_R(f,g;t)\rangle_f = A_R(f,g;t) + A_R\left(\overline{f},g;t\right)$$

in such a way that **if g fulfill the conditions to participate then the new "averaged" asymmetry is a T violating one provided the decay amplitudes A_f and $A_{\overline{f}}$ are invariant under CPT.**

Avoiding the measurement of total survival probabilities I

- The T violating program here presented relays on the measurement of the asymmetry

$$A_R(f_1, f_2; t) = P_{12}(t) - P_{21}(t) = [N_1(t) - N_2(t)] + [I_{21}(t) - I_{12}(t)]$$

The second piece in the r.h.s. corresponds to the typical $t \leftrightarrow -t$ asymmetry of the double decay rate in the meson factory. The first piece is the difference of the total survival probability of the two states $\left|P_{\nrightarrow f_i}\right\rangle$. In order to measured $N_i(t)$ one has to have a good control of all decay channels f_i to be sure that our $\left|P_{\nrightarrow f_i}\right\rangle$ has not decay at time t. In some cases a good approximation is $N_i(t) \sim e^{-\Gamma t}$ and therefore the contribution of the total survival probability almost cancels out.

Avoiding the measurement of total survival probabilities II

- Nevertheless we have presented a method to eliminate these $N_i\left(t\right)$ contributions. The idea is to sum three T violating asymmetries to cancel out theses unwanted pieces in the following way:

$$\begin{aligned} A_3\left(f_1, f_2, f_3; t\right) &= A_R\left(f_1, f_2; t\right) + A_R\left(f_2, f_3; t\right) + A_R\left(f_3, f_1; t\right) = \\ &= I_{21}\left(t\right) - I_{12}\left(t\right) + I_{32}\left(t\right) - I_{23}\left(t\right) + I_{13}\left(t\right) - I_{31}\left(t\right) \end{aligned}$$

  As long as the three asymmetry are T violating, we have constructed a T violating observable by comparing three time inverted and normalized doubled decay rate processes at a meson factory.

- The result we obtain do not contain terms whose time dependence goes with $\cosh\left(\frac{\Delta\Gamma t}{2}\right)$ and $\cos\left(\Delta m t\right)$. These pieces coming from the total survival probabilities cancels out. We get therefore

$$A_3\left(f_1, f_2, f_3; t\right) = e^{-\Gamma t}\left\{\mathcal{S}_h^N\left[f_1, f_2, f_3\right]\sinh\left(\frac{\Delta\Gamma t}{2}\right) + \mathcal{S}_c^N\left[f_1, f_2, f_3\right]\sin\left(\Delta m t\right)\right\}$$

Avoiding the measurement of total survival probabilities III

- To simplify we choose Flavour Specific Channels (FS): $f_3 = FS$ and $f_2 = \overline{FS}$. Then if we chose f a CP eigenstates and assume CPT invariance in the decay to this channel, **for any channel $f = f_{CP}$ it turns out that $A_3\left(f_{CP}, \overline{FS}, FS; t\right)$ is a genuine T violating asymmetry. This procedure allows to extend to any CP eigenstate the T violanting program -initiated by Babar- in any meson factory like Belle II, Daphne II etc...**

$$A_3\left(f, \overline{FS}, FS; t\right) = e^{-\Gamma t}\left\{2\delta R_f \sinh\left(\frac{\Delta\Gamma t}{2}\right) + 2S_f\left(1 + \delta C_f\right)\sin\left(\Delta m t\right)\right\}$$

- The program can be extended to non CP eigenstates by combining

$$\begin{aligned} & A_3\left(f, \overline{FS}, FS; t\right) + A_3\left(\overline{f}, \overline{FS}, FS; t\right) \\ = \quad & e^{-\Gamma t}\left\{\begin{array}{c} 2\delta\left(R_f + R_{\overline{f}}\right)\sinh\left(\frac{\Delta\Gamma t}{2}\right) + \\ + \left[2\left(S_f + S_{\overline{f}}\right) + 2\delta\left(S_f C_f + S_{\overline{f}} C_{\overline{f}}\right)\right]\sin\left(\Delta m t\right) \end{array}\right\} \end{aligned}$$

Avoiding the measurement of total survival probabilities IV

where the T violating pieces are proportional to δ or $\left(S_f + S_{\overline{f}}\right)$. This second formula represents the extension of the T violating program to any non CP eigenstate.

Conclusions I

- We have modified the reference transition of BABAR T violating measurement $P_{\nrightarrow f_1}(t_1) \rightarrow P^{\perp}_{\nrightarrow f_2}(t_2)$ - directly connected to decay $\Upsilon(4S) \rightarrow \underset{\rightarrow f_1 \text{ at } t_1}{P} \ \underset{\rightarrow f_2 \text{ at } t_2}{P}$ - by $P_{\nrightarrow f_1}(t_1) \rightarrow P_{\nrightarrow f_2}(t_2)$.
- With the New Reference we proposed in addition to the Babar asymmetry

$$\Pr\left[P_{\nrightarrow f_1} \rightarrow P^{\perp}_{\nrightarrow f_2}(t)\right] - \Pr\left[P^{\perp}_{\nrightarrow f_2} \rightarrow P_{\nrightarrow f_1}(t)\right]$$

the new one

$$A_R(f_1, f_{2;} t) = P_{12}(t) - P_{21}(t)$$

$$P_{ij}(t) \equiv \Pr\left[P_{\nrightarrow f_i} \rightarrow P_{\nrightarrow f_j}(t)\right] = \left|\left\langle P_{\nrightarrow f_j} | P_{\nrightarrow f_i}(t)\right\rangle\right|^2$$

Conclusions II

- The connection to the experimental observables is given by the total survival probability and the normalized double decay rate to f_1 at t_1 and f_2 at a later time $t_2 = t + t_1$

$$\begin{aligned} P_{12}(t) &= N_1(t) - I_{12}(t) \\ N_1(t) &= \left\langle P_{\nrightarrow f_1}(t) \middle| P_{\nrightarrow f_1}(t) \right\rangle \\ I_{12}(t) &= \frac{\left| \langle f_2 | W \left| P_{\nrightarrow f_1}(t) \right\rangle \right|^2}{\left(\left| A_{f_2} \right|^2 + \left| \overline{A}_{f_2} \right|^2 \right)} \end{aligned}$$

- The proposed asymmetry can be used for decay products that include flavour specific, CP eigenstates and non CP eigenstates. In the last case one has to combine the reference transitions $P^0 \to g$ and $P^0 \to \overline{g}$. In this way we eliminate possible sources of fake T violation. Some channels needs additionally the assumption of CPT invariance (for example) in the weak decay Hamiltonian.

Conclusions III

- By combining different asymmetries in the following way

$$A_3\left(f_1, f_2, f_3; t\right) = A_R\left(f_1, f_2; t\right) + A_R\left(f_2, f_3; t\right) + A_R\left(f_3, f_1; t\right)$$

  one can avoid the measurements of the total survival probabilities.
- With the proposal here discussed, the way is more open to a full experimental program of studies of T violation observables at meson factories.