

# HIGGS-DILATON COSMOLOGY

## Universality vs. Criticality

Javier Rubio



DISCRETE 2014  
King's College  
London



# HIGGS INFLATION

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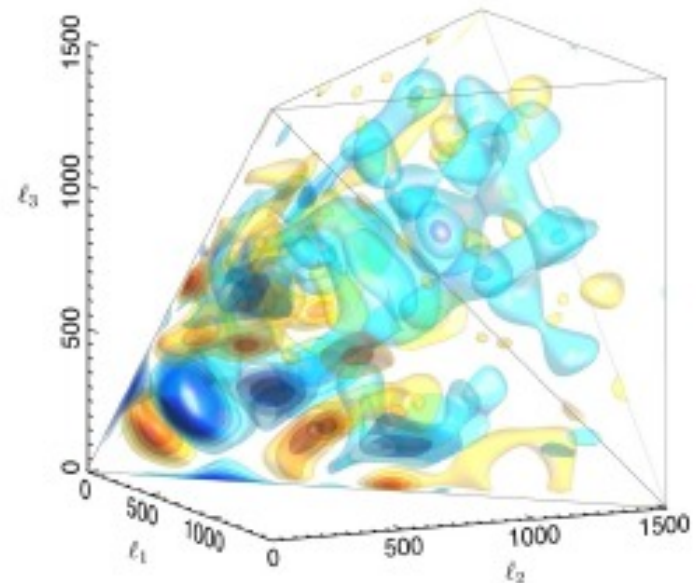
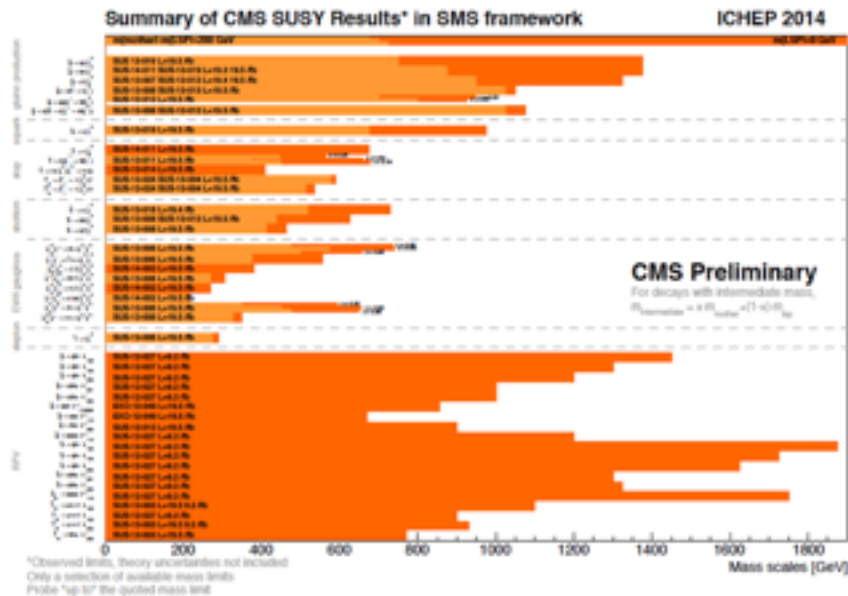
# The era of scalars has begun

## LHC

- “Clear evidence for a neutral boson with a mass of 126 GeV”
- “Extensive search without any significant deviations from the SM so far”

## PLANCK

- “Single field SR inflation has survived its most stringent
- “Small isocurvature contribution”
- “No evidence for primordial NG”



# Higgs inflation: Jordan frame

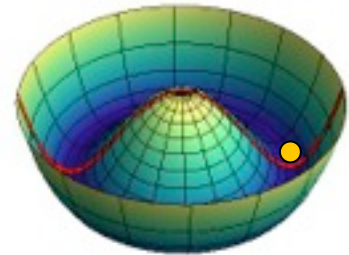
## 1. One field to rule them "all"

*Identify the Higgs with the inflaton*

"Erustra fit per plura quod potest fieri per pauciora". William of Ockham, Summa Totius Logicae

Scale invariance

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{M_P^2 + \xi_h h^2}{2} R - \frac{1}{2} (\partial h)^2 - V(h)$$



## 2. Physicists' Nightmare Scenario

*No new intermediate scales till "Planck" scale*

**Inflation easily studied in the Einstein frame**

$$\tilde{g}_{\mu\nu} = \Omega^2(h) g_{\mu\nu} \quad \Omega^2 = 1 + \frac{\xi_h h^2}{M_P^2}$$

# Higgs inflation: Einstein frame

$$\frac{\tilde{\mathcal{L}}}{\sqrt{-\tilde{g}}} = \frac{M_P^2}{2} \tilde{R} - \frac{1}{2} \left( \frac{\Omega^2 + 6\xi_h^2 h^2 / M_P^2}{\Omega^4} \right) \tilde{g}^{\mu\nu} \partial_\mu h \partial_\nu h - \frac{V(h)}{\Omega^4}$$

**Redefining the field to make it canonic. normalized**

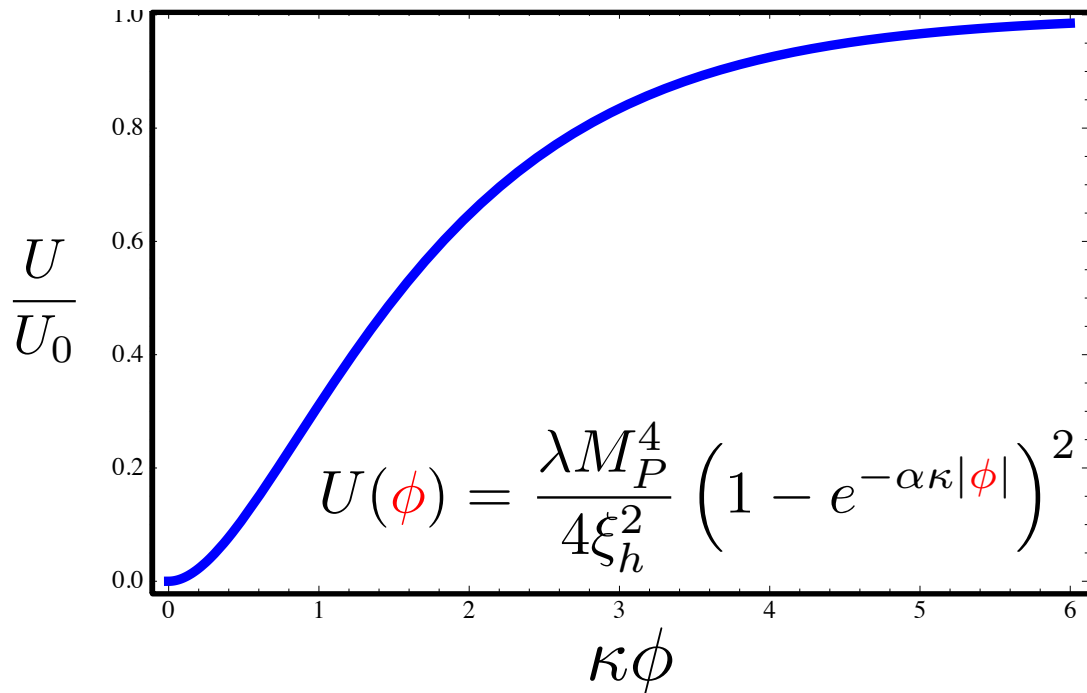
$$\frac{\tilde{\mathcal{L}}}{\sqrt{-\tilde{g}}} = \frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi)$$

$$U(\phi) = \frac{\lambda}{4} \phi^4 \quad \text{for } h < \frac{M_P}{\sqrt{\xi_h}}$$
$$U(\phi) = \frac{\lambda M_P^4}{4\xi_h^2} \left( 1 - e^{-\alpha\kappa|\phi|} \right)^2 \quad \text{for } h > \frac{M_P}{\sqrt{\xi_h}}$$

**All the non-linearities moved to the scalar sector**

# A sufficiently flat potential

*Scale invariance becomes shift symmetry*



$$\frac{\lambda}{\xi_h^2} \sim 10^{-11}$$

COBE

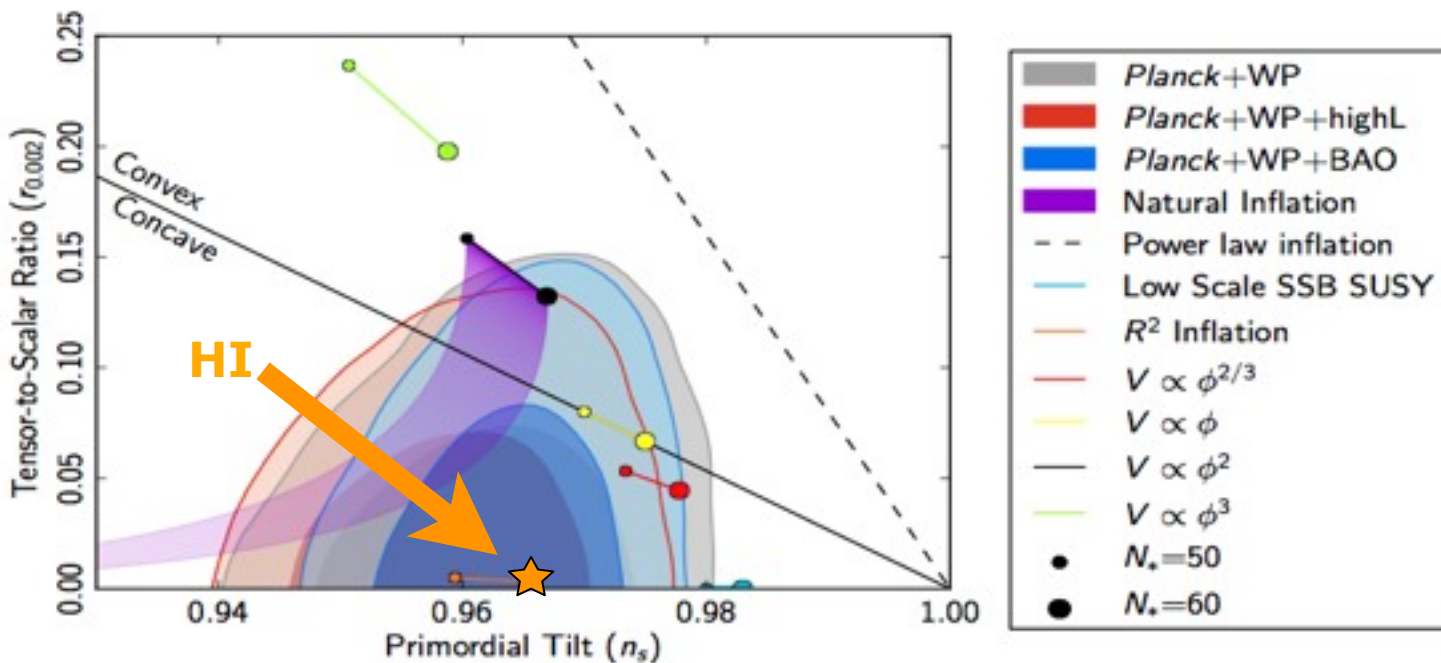
Only  $\lambda/\xi_h^2$  is important

# The primordial spectra

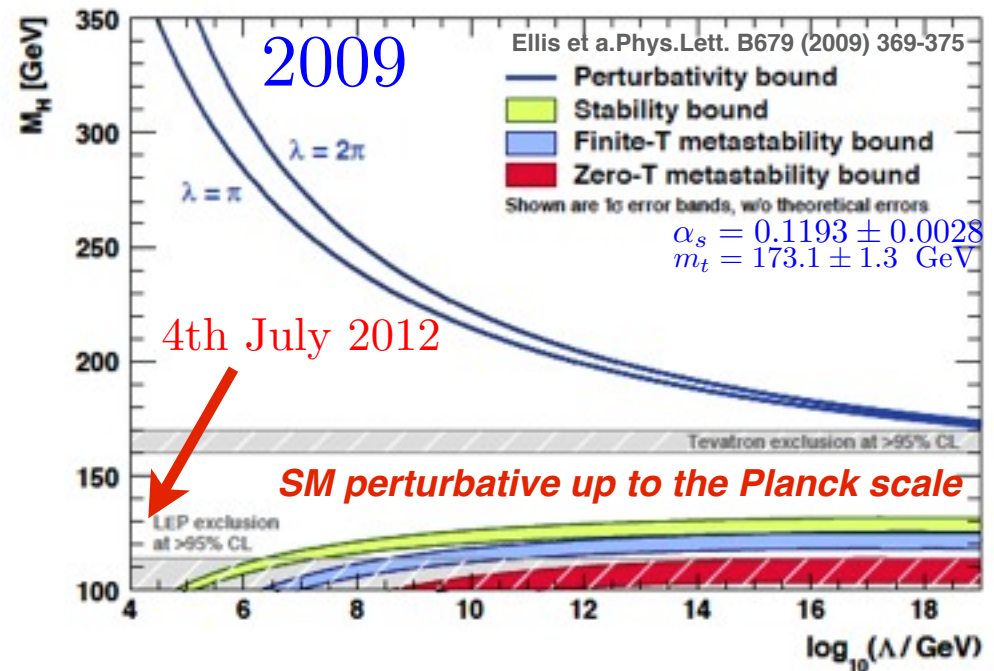
**Scalar pert.**  $\mathcal{P}_S(k) = \mathcal{A}_s \left( \frac{k}{k^*} \right)^{n_s - 1 + \frac{1}{2}\alpha_s \ln(k/k^*) + \frac{1}{6}\beta_s (\ln(k/k^*))^2 + \dots}$

**Tensor pert.**  $\mathcal{P}_T(k) = \mathcal{A}_t \left( \frac{k}{k^*} \right)^{n_t + \frac{1}{2}\alpha_t \ln(k/k^*) + \dots}$

$$n_s \equiv 1 + \frac{d \ln \mathcal{P}_s}{d \ln k} \quad r(k) \equiv \frac{\mathcal{P}_T(k)}{\mathcal{P}_S(k)} \quad \alpha_s \equiv \frac{d n_s}{d \ln k} \quad \beta_s \equiv \frac{d^2 n_s}{d \ln k^2}$$



# The Higgs is like Odysseus between Scylla and Charybdis

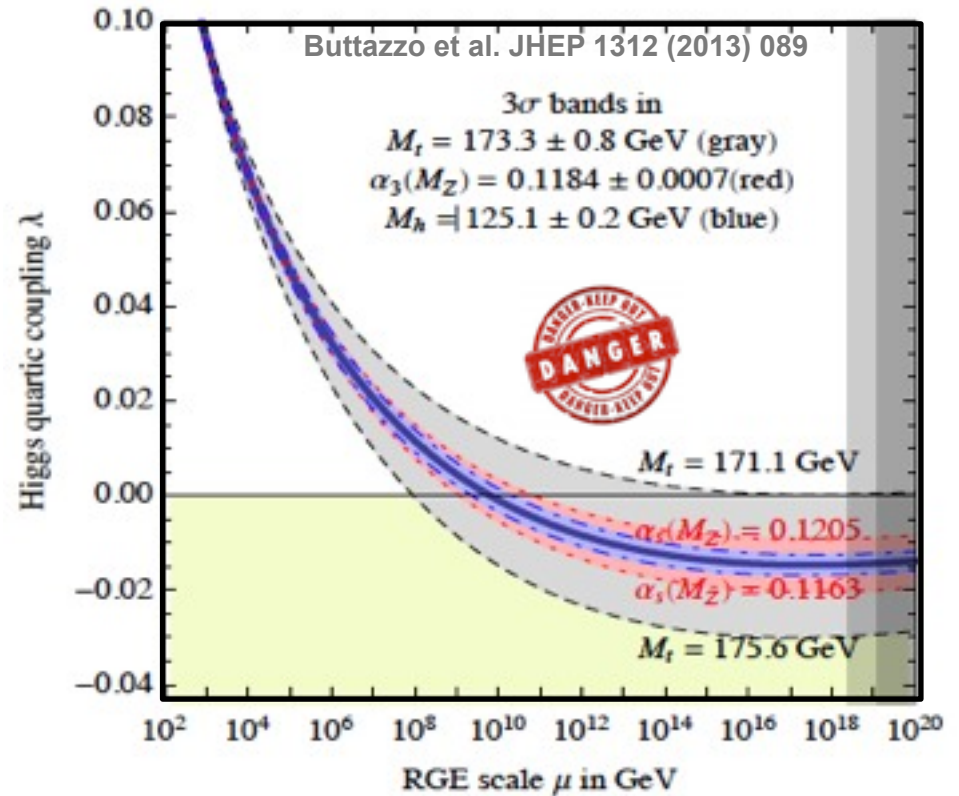
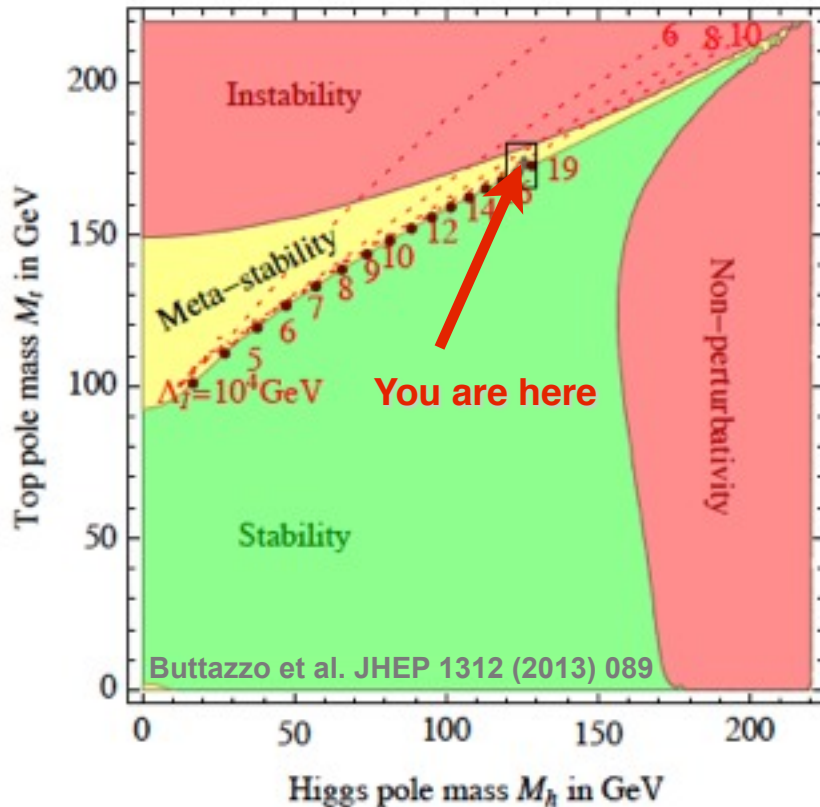


*“incidit in scyllam cupiens vitare charybdim”*

*“he runs on Scylla, wishing to avoid Charybdis”*



# On the edge of stability



Non trivial interplay between top Yukawa and Higgs self-coupling....evil *Scylla* hidden in error bars  
 Stable or metaestable? An experimental rather than theoretical issue ...

Is there a reason for that?

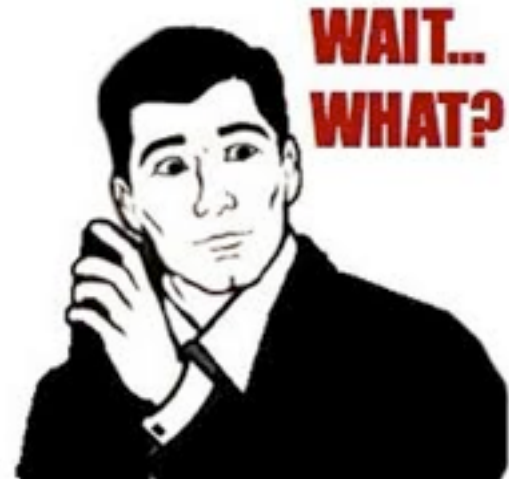
M. Lindner, M. Sher, and H. W. Zaglauer, Phys.Lett., B228, 139 (1989), C. Froggatt and H. B. Nielsen, Phys.Lett., B368,96 (1996), M. Shaposhnikov and C. Wetterich, Phys.Lett., B683, 196 (2010), M. Veltman, Acta Phys.Polon., B12, 437 (1981), B683, 196 (2010), M. Holthausen, K.S.Lim, M.Lindner and references therein....

Gravitational corrections?

Lalak,Lewicki, Olszewki arXiv 14.02.3826, Branchina, Massina Phys.Rev.Lett. 111 (2013) 241801 etc...

# Vacuum stability vs inflation ?

“If  $m_h$  and  $m_t$  are close to the measured central value, Higgs inflation is not possible and  $V_{eff}$  becomes negative much before  $M_P$ ”



# SM + gravity is non-renormalizable

1. Which is the sensitivity to higher dimensional operators?
2. Which are the renormalization group equations?  
Which is the relation between high and low energy parameters?

In the **lack of UV completion** the answer to these questions can be only based on the

**SELF-CONSISTENCY OF PROCEDURE**

# 1. Sensitivity to high dim. operators

**Defining the cutoff from the theory itself by considering all possible reactions**

- ◎ **The energy at which perturbative unitarity in high-energy scattering processes is violated**

## 1. Compute the quadratic lagrangian

$$\Phi(\mathbf{x}, t) = \bar{\Phi}(t) + \delta\Phi(\mathbf{x}, t) \longrightarrow c_n \frac{\mathcal{O}_n(\delta\Phi)}{[\Lambda(\bar{\Phi})]^{n-4}} \quad \triangle ! \quad \text{Background dependent!}$$

## 2. Get rid of the mixings in the quadratic action

## 3. Read out the cutoff from higher order oper.

## Example

$$\phi = \bar{\phi} + \delta\phi \quad g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

### 1. Compute the quadratic lagrangian (Jordan F.)

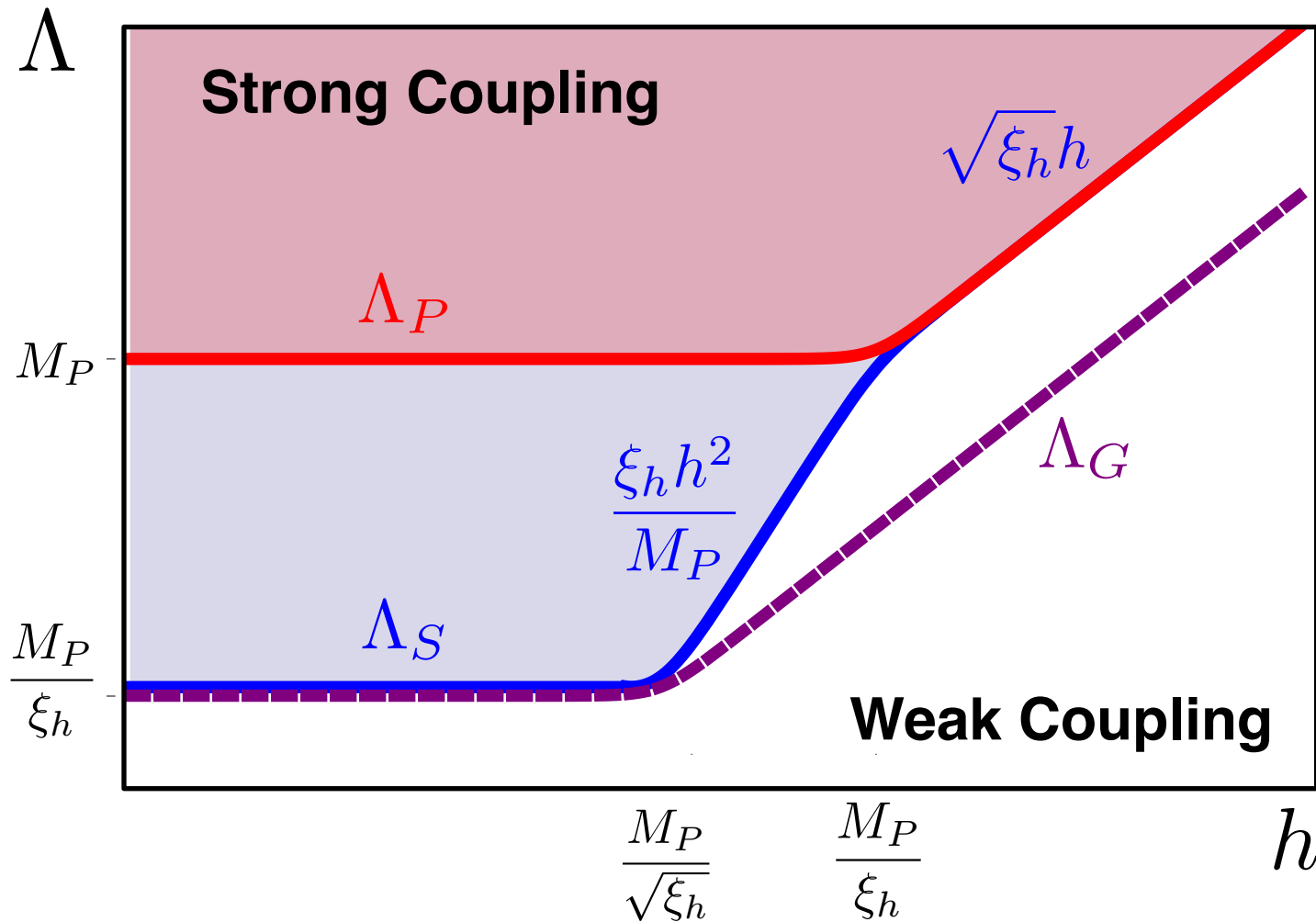
$$\mathcal{L}^{(2)} = -\frac{M_P^2 + \xi\bar{\phi}^2}{8} (h^{\mu\nu}\square h_{\mu\nu} + 2\partial_\nu h^{\mu\nu}\partial^\rho h_{\mu\rho} - 2\partial_\nu h^{\mu\nu}\partial_\mu h - h\square h) \\ + \frac{1}{2}(\partial_\mu\delta\phi)^2 + \xi\bar{\phi}(\square h - \partial_\lambda\partial_\rho h^{\lambda\rho})\delta\phi,$$

### 2. Get rid of the mixings in the quadratic action

$$\delta\phi = \sqrt{\frac{M_P^2 + \xi\bar{\phi}^2}{M_P^2 + \xi\bar{\phi}^2 + 6\xi^2\bar{\phi}^2}} \delta\hat{\phi}, \\ h_{\mu\nu} = \frac{1}{\sqrt{M_P^2 + \xi\bar{\phi}^2}} \hat{h}_{\mu\nu} - \frac{2\xi\bar{\phi}}{\sqrt{(M_P^2 + \xi\bar{\phi}^2)(M_P^2 + \xi\bar{\phi}^2 + 6\xi^2\bar{\phi}^2)}} \bar{g}_{\mu\nu} \delta\hat{\phi}$$

### 3. Read out the cutoff from higher order operat.

$$\frac{\xi\sqrt{M_P^2 + \xi\bar{\phi}^2}}{M_P^2 + \xi\bar{\phi}^2 + 6\xi^2\bar{\phi}^2} (\delta\hat{\phi})^2 \square \hat{h}$$



**A consistent EFT** : Cutoffs are parametrically larger than all the energy scales involved in the history of the Universe

# 2. Running of the couplings

Gauge bosons/fermions action invariant under conformal transformations except for the mass terms

$$\mathcal{L}_F = \frac{y_f}{\sqrt{2}} h \bar{\psi} \psi \quad \longrightarrow \quad \tilde{\mathcal{L}}_F = \frac{y_f}{\sqrt{2}} \frac{h}{\Omega} \bar{\psi} \psi \equiv \frac{y_f}{\sqrt{2}} F(\phi) \bar{\psi} \psi$$

## Low energies

$$F = \phi$$

$$F'(0) = 1$$

$$\tilde{m}_{A,f} = m_{A,f}$$

**USUAL SM**

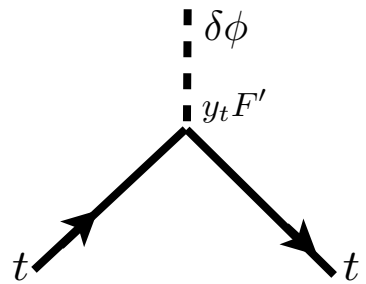
## During inflation

$$F = \frac{M_P}{\sqrt{\xi_h}} \left( 1 - e^{-\alpha\kappa|\phi|} \right)^{1/2}$$

$$F'(\phi_0) = 0$$

$$\tilde{m}_{A,f} = \text{const.}$$

**CHIRAL SM**

$$\begin{aligned}\tilde{\mathcal{L}}_t(\phi + \delta\phi) &= \frac{y_t}{\sqrt{2}} F(\phi + \delta\phi) \bar{\psi}_t \psi_t \\ &= \frac{y_t}{\sqrt{2}} F(\phi) \bar{\psi}_t \psi_t + \frac{y_t}{\sqrt{2}} \frac{dF(\phi)}{d\phi} \delta\phi \bar{\psi}_t \psi_t + \dots\end{aligned}$$


In a background field the coupling of the top to the Higgs pert. is proportional to  $F'$ .

**At low energies**

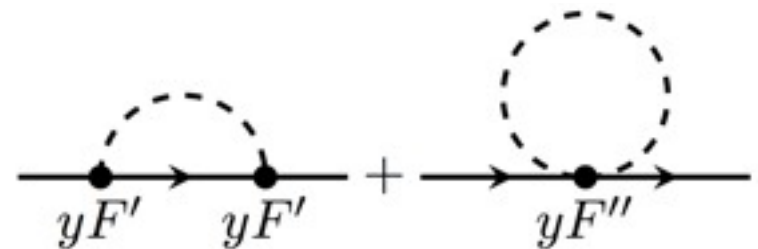
$$F = \phi \quad F'(0) = 1$$

**At high energies**

$$F = \text{const.} \quad F'(\phi_0) = 0$$

Consider the propagation of the top quark

Add counterterms to cancel divergencies



$$\delta\mathcal{L}_{\text{ct}} \sim \left( \# \frac{y_t^3}{\bar{\epsilon}} + \delta y_{t1} \right) F'^2 F \bar{\psi} \psi + \left( \# \frac{y_t \lambda}{\bar{\epsilon}} + \delta y_{t2} \right) F'' (F^4)'' \bar{\psi} \psi,$$

$\phi\psi\psi$  at small  $\phi$

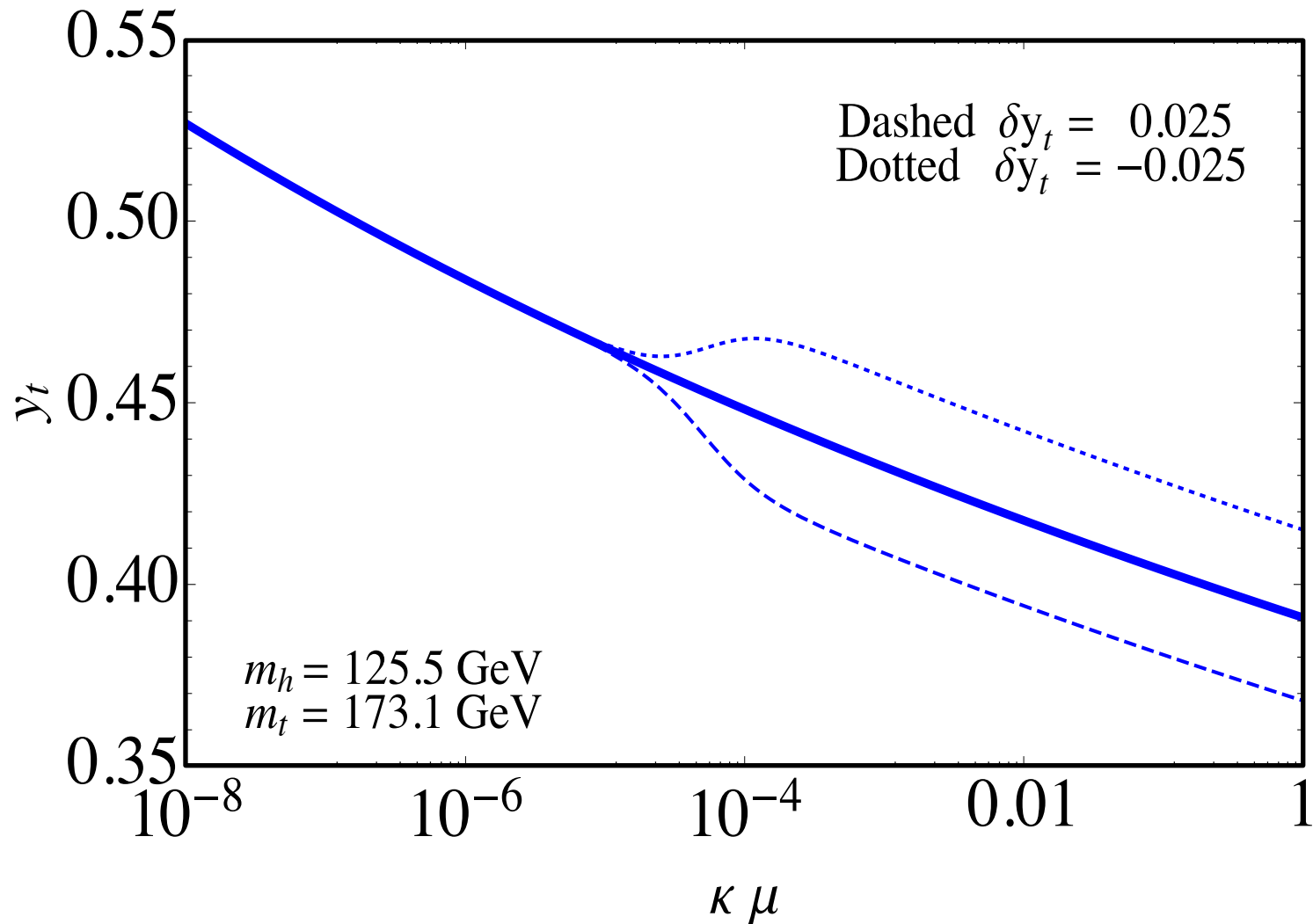


**At low energies**

$$F = \phi \quad F'(0) = 1$$

**At high energies**

$$F = \text{const.} \quad F'(\phi_0) = 0$$

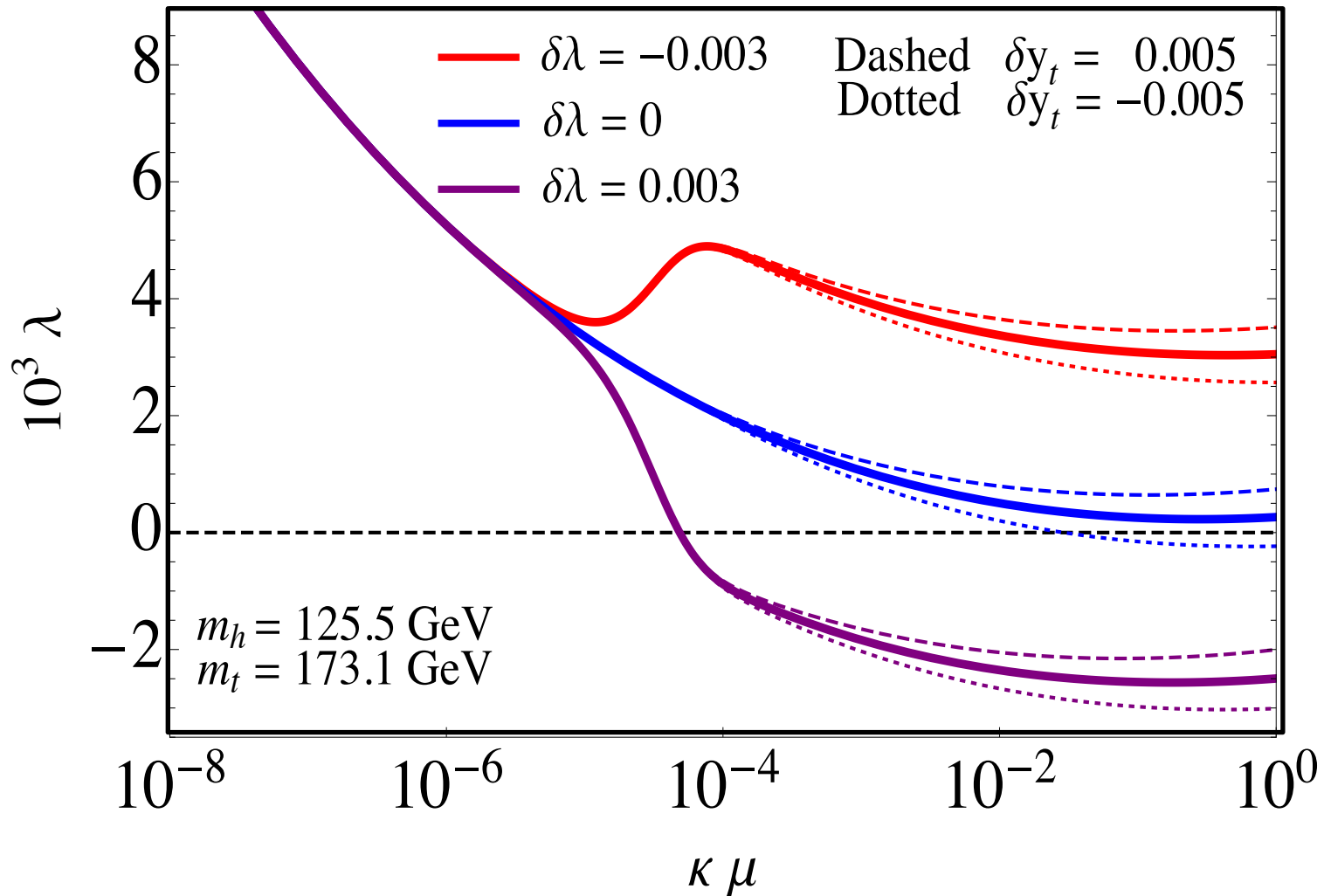


**At low energies**

$$F = \phi \quad F'(0) = 1$$

**At high energies**

$$F = \text{const.} \quad F'(\phi_0) = 0$$



**1. Run SM RGE until the chiral SM ---> Jumps**

**2. The obtained values are used as the input of the chiral phase, whose RG equations are run until a given scale.**

**3. This scale is chosen to minimize higher order corrections**

**4. RGE effective potential at inflation is computed**

$$U_{RGE}(\phi) = \frac{\lambda(\mu(\phi)) M_P^4}{4\xi_h^2(\mu(\phi))} (1 - e^{-\alpha\kappa\phi})^2$$

**7. Inflationary observables are computed**

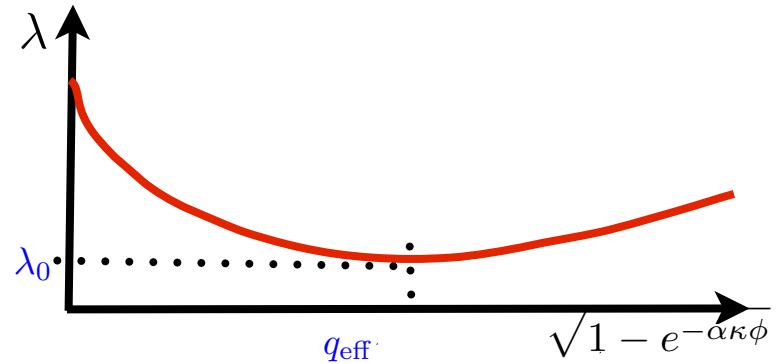
# Lets look at the asymptotics

$$\lambda(\mu(\phi)) = \lambda_0 + b \log^2 \left( \frac{\sqrt{1 - e^{-\alpha\kappa\phi}}}{q_{\text{eff}}} \right)$$

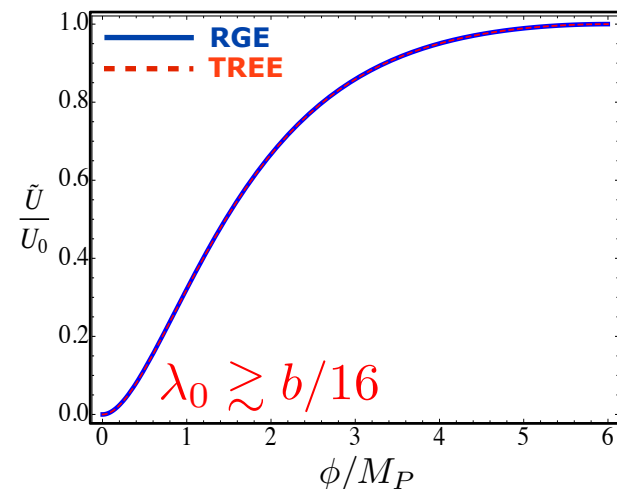
$$b \simeq 2.3 \times 10^{-5}$$

$$\lambda_0 = \lambda_0(m_h^*, m_t^*) \ll 1$$

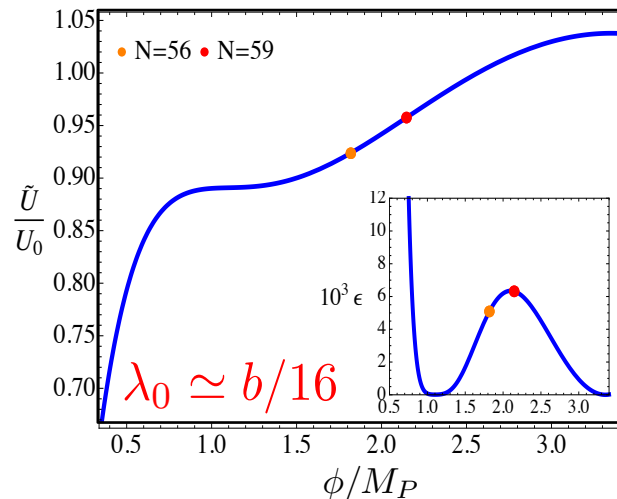
$$q_{\text{eff}} = q_{\text{eff}}(m_h^*, m_t^*) \sim \mathcal{O}(1)$$



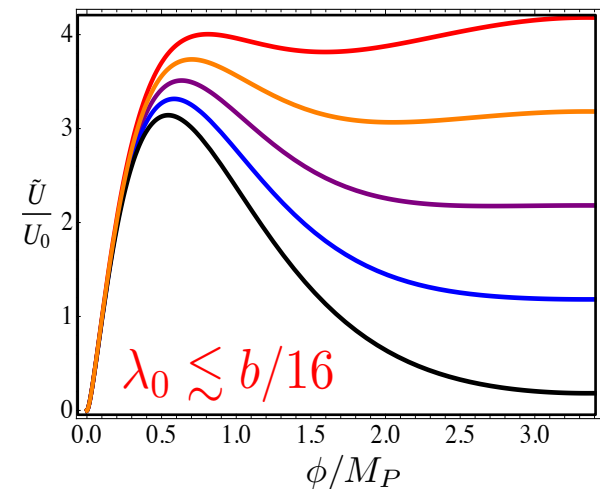
## UNIVERSALITY



## CRITICALITY



## NO INFLATION



# Universal regime

$$\lambda_0 \gtrsim b/16$$

$$\lambda(\mu(\phi)) = \lambda_0 + b \log^2 \left( \frac{\sqrt{1 - e^{-\alpha \phi}}}{q_{\text{eff}}} \right)$$

Only  $\lambda_0/\xi_h^2$  is important

**For masses slightly above the critical Higgs mass \***

$$m_h^* > m_{\text{crit}} - 0.1 \log \frac{\xi}{1000} \text{ GeV}$$

$$m_{\text{crit}} = \left[ 129.1 + \frac{y_t(173.2 \text{ GeV}) - 0.9361}{0.0058} \times 2.0 \right] \text{ GeV}$$

Bezrukov et al. JHEP 1210 (2012) 140, Buttazzo et al. JHEP 1312 (2013) 089,

**the predictions of the model are universal**

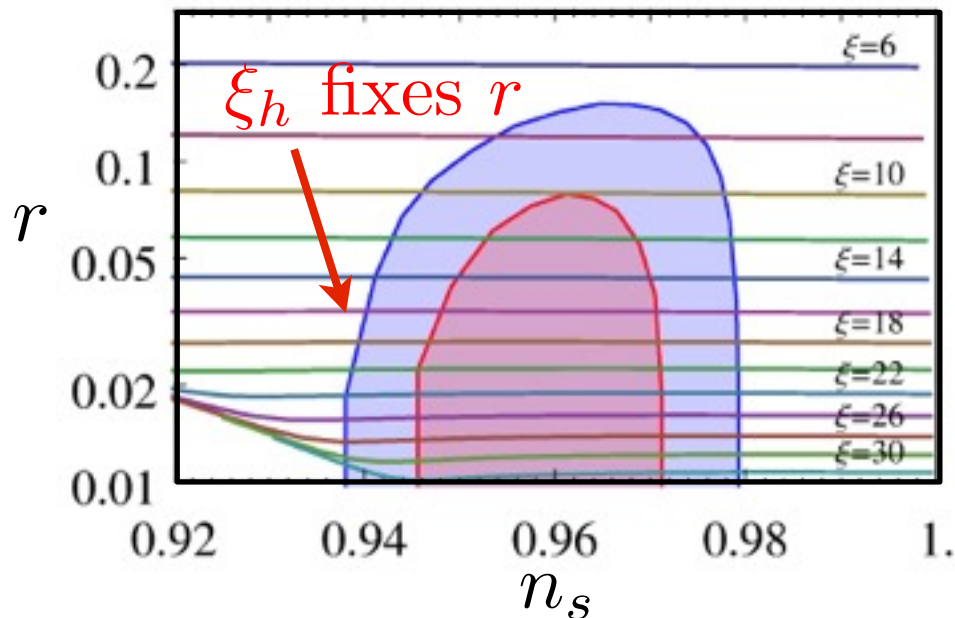
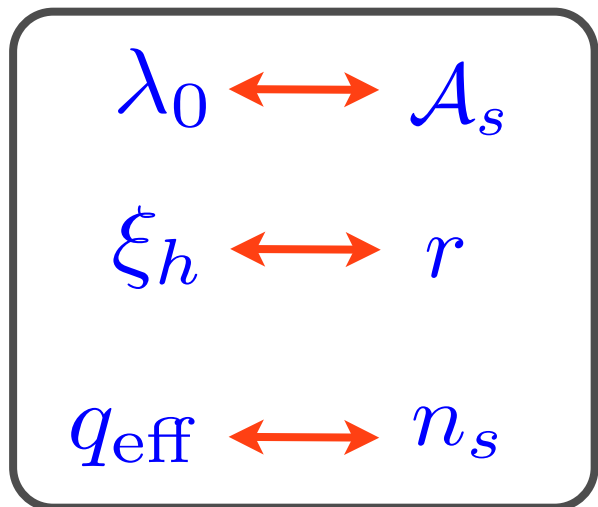
$$n_s \simeq 0.97$$

$$r \simeq 0.003$$

# Critical Regime

$$\lambda_0 \simeq b/16 \quad \lambda(\mu(\phi)) = \lambda_0 + b \log^2 \left( \frac{\sqrt{1 - e^{-\alpha\kappa\phi}}}{q_{\text{eff}}} \right)$$

**3 observational inputs for 3 unknown parameters**



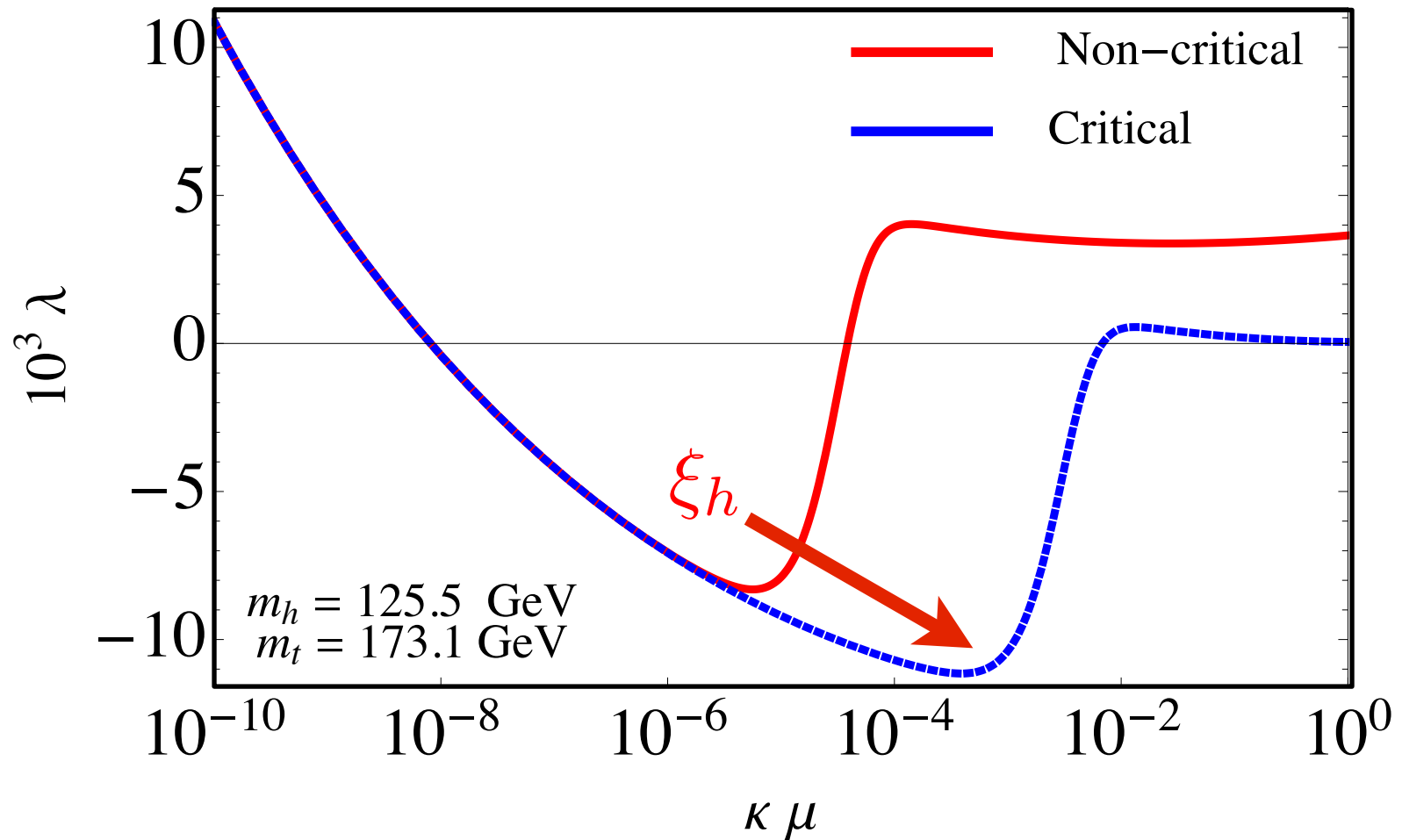
**No additional scales**

$$\frac{M_P}{\xi} \sim \frac{M_P}{\sqrt{\xi}} \sim M_P$$

# Living beyond the edge?



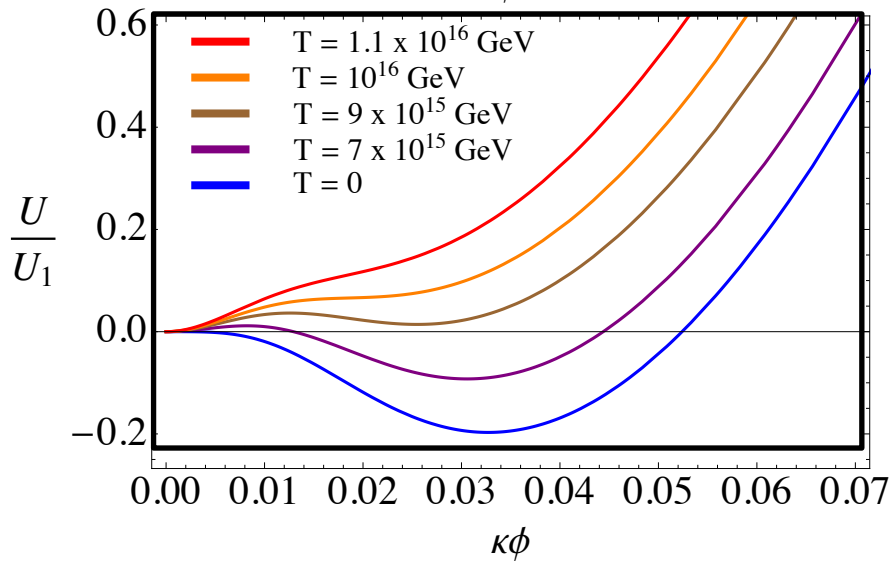
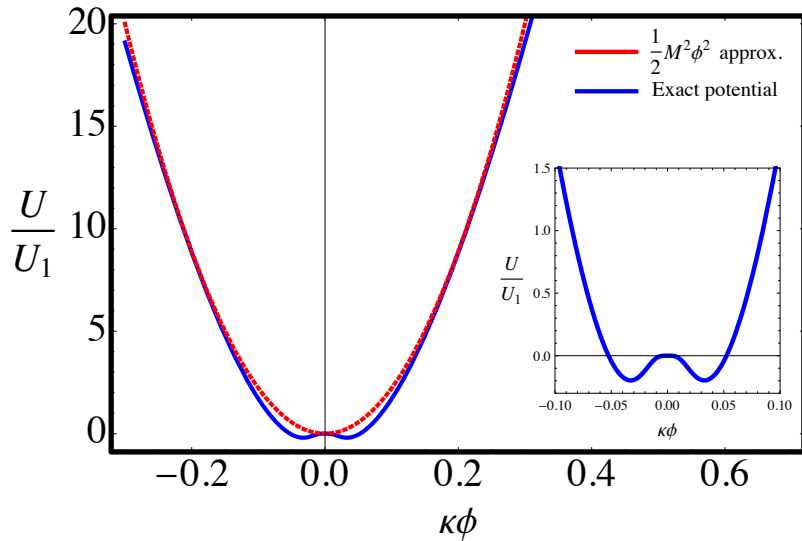
# Restoring asymptotics



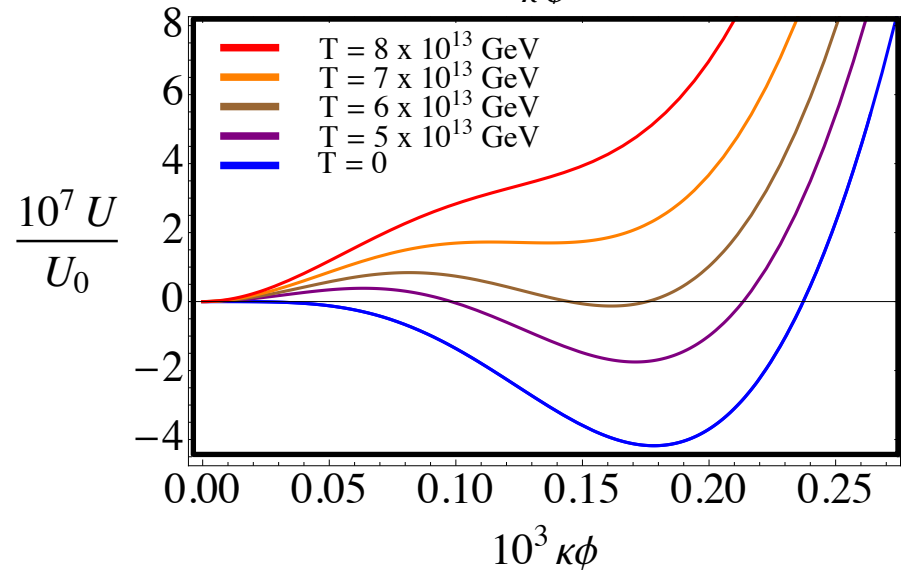
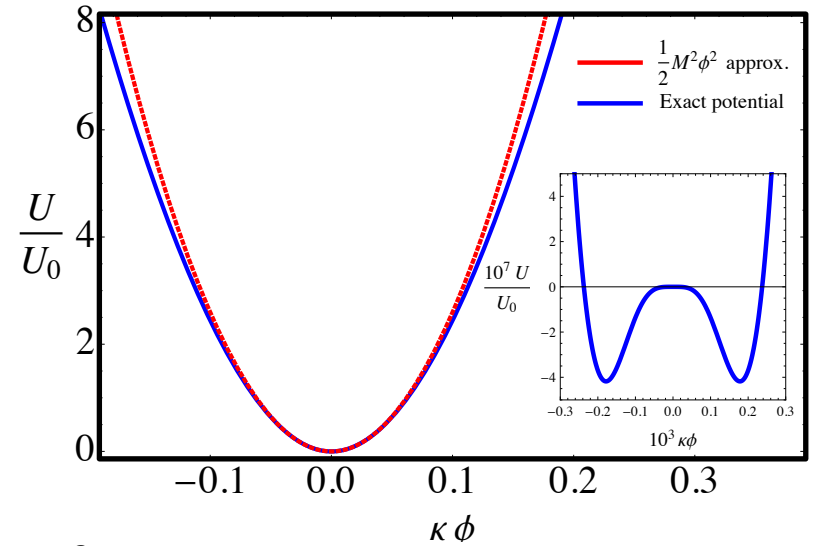


# Thermal corrections

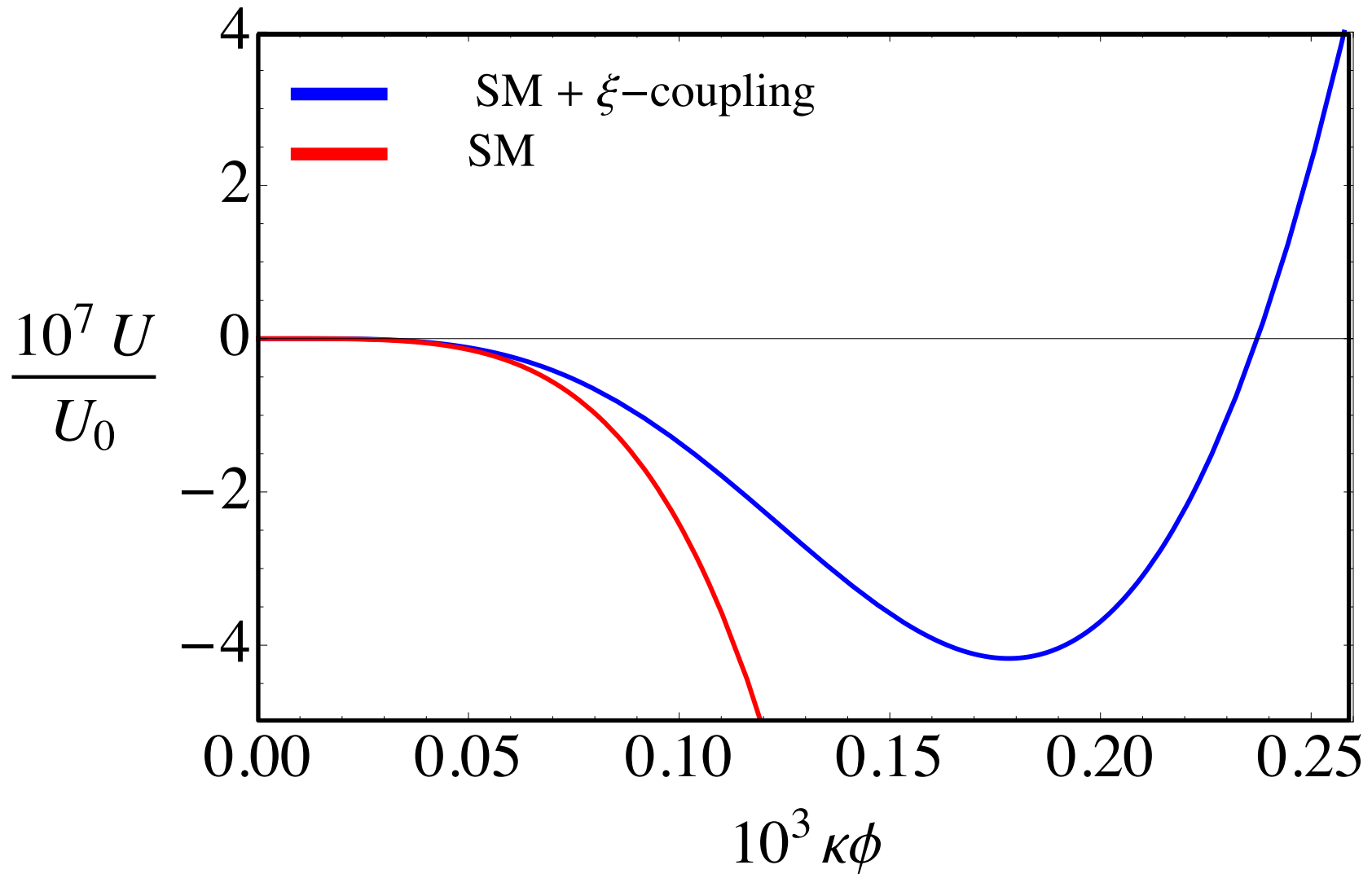
## Critical case



## Non-critical case



# At low temperatures...



**Life beyond the edge**



# Life beyond the edge

- ✓ The Higgs can make the Universe flat, homogeneous and isotropic
- ✓ A consistent EFT if the UV completion respects scale invariance

## UNIVERSAL HI

$$r \simeq 0.003$$

$$\xi_h \sim \mathcal{O}(10^3)$$

## CRITICAL HI

$$r \sim \mathcal{O}(0.1)$$

$$\xi_h \sim \mathcal{O}(10)$$

- ✓ The non-critical Higgs inflation scenario can be possible *even if our vacuum is metastable*
- ✓ The critical Higgs-inflation scenario *does require the stability of the vacuum all the way up till the inflationary scale*

# 2. Reliability of radiative corrections

Non-renormalizable -> Infinite number of counterterms

Respect scale invariance -> Dimensional regularization

## The choice of $\mu$

In renormalizable theories, it is arbitrary and field-independent

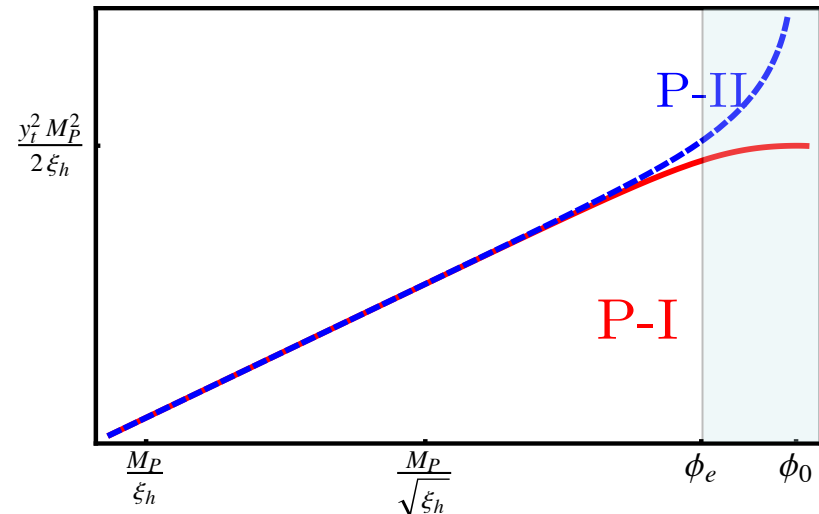
We modify this prescription in our non-renormalizable theory

Jordan frame def.

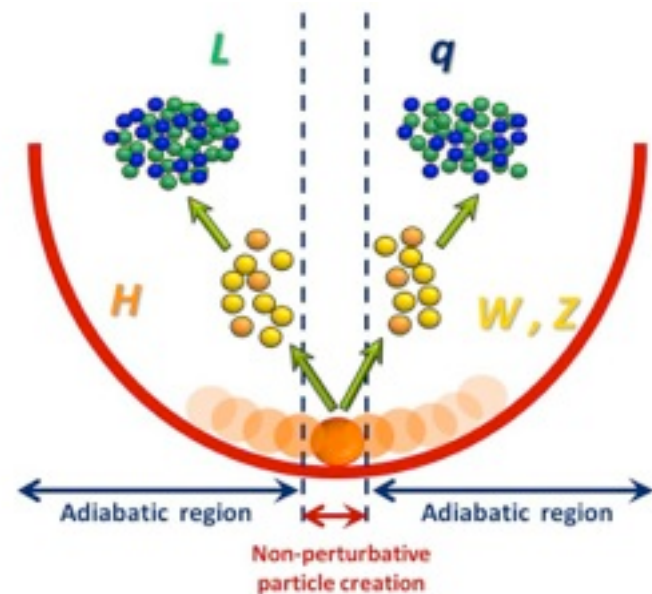
**P-I**  $M_P^2 + \xi_h h^2$

**P-II**  $M_P^2$

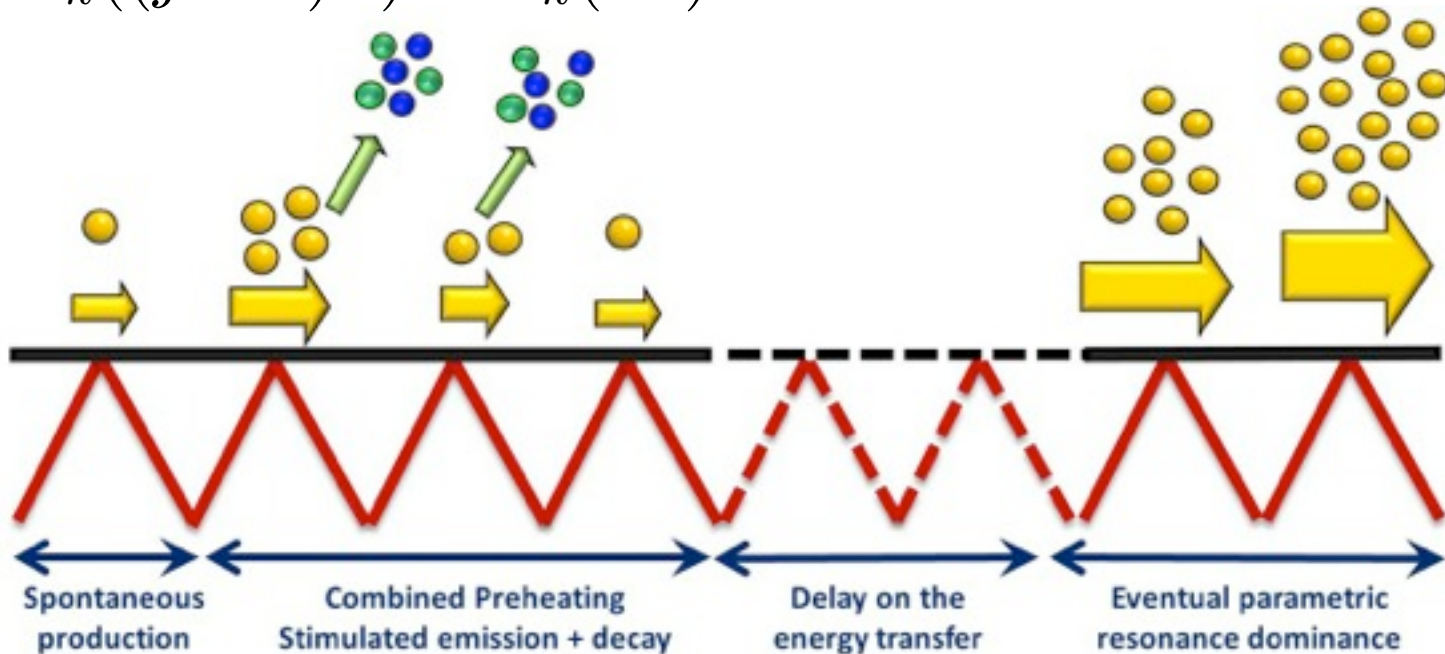
$$\lambda = \mu^{2\epsilon} \left( \lambda_R + \sum_n \frac{a_n}{\epsilon_n} \right)$$



# Combined Preheating



$$n_k((j+1)^+) = n_k(1^+) e^{-\gamma F_\Sigma(j)} e^{2\pi \sum_{i=1}^j \mu_k(i+1)}$$



# References

## Non-minimal coupling

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- ★ J. L. F. Barbon and J. R. Espinosa Phys. Rev. D79 (2009) 081302,
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- ★ D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, et al.

## Preheating

- ★ J. Garcia-Bellido, D.G. Figueroa, J. Rubio Phys.Rev. D79 (2009) 063531
- ★ F. Bezrukov, D. Gorbunov and M. Shaposhnikov JCAP 0906 (2009) 029

## Critical case

- ★ Y. Hamada, H. Kawai, K.-y. Oda, and S. C. Park arXiv:1403.5043.
- ★ F. Bezrukov and M. Shaposhnikov arXiv:1403.6078.
- ★ J. Rubio, M. Shaposhnikov Phys.Rev. D90 (2014) 027307