Mass Insertion Parameters from SU(5)xS$_4$xU(1) model of flavour

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Outline

Introduction

- ❖ The SM & SUSY Flavour Problem.
- ❖ Solving it by imposing a Family symmetry.

The SU(5)xS_4xU(1) Model

- ❖ Review of fermionic sector.

- ❖ Construction of SUSY breaking sector:
  - SCKM basis
  - **Mass Insertion (MI) parameters:**

- ❖Predictions for low energy MIs Vs experimental constraints.

Summary

The Flavour Problem

Why are there 3 families of quarks & leptons?

Why are their masses so hierarchical?

Why is lepton mixing so large compared to quark mixing?

Why are neutrino masses so small?

More than 1 generations → **Yukawa coupling terms become matrices**

Understanding pattern of fermion masses & mixings =
= Understanding structure of Yukawa matrices.

Here is an idea…

Family Symmetry

Extend symmetry group with a Family symmetry G_F.

- admits triplet reps

(*3 families in a triplet*)

Introduce heavy scalar fields: **Flavons**: Φ

- couple to usual matter fields

Write down operators allowed by all symmetries

- typically non-renormalisable

$$O_Y = f^i \frac{\Phi_i \Phi_j}{M^2} f^{cj} H$$

M: heavy mass scale; UV cut-off

Spontaneously **break** G_F, as Φs develop **≠0 vevs**

- effective Yukawa couplings generated:

$$Y_{ij} = \frac{\langle\Phi_i\rangle\langle\Phi_j\rangle}{M^2} = f\left(\boxed{\lambda = \frac{\langle\Phi\rangle}{M}}\right)$$

expansion parameter

build up desired hierarchical Yukawa textures

Explain form of Yukawa matrices ⟺ Find appropriate symmetry G_F, field content & vacuum alignment for flavons

Extend to **SUSY GUTs**

- Fields become superfields.
- Yukawa operators arise from the superpotential W:

$$W = fw\left(\frac{\Phi^n}{M^n}\right) f^c H$$

flavon vevs aligned via minimization of potential

- Kinetic terms & scalar masses arise from the Kähler potential K.
- **Spartner masses & mixings must also be explained.**
- **Control FC processes induced by loop diags involving sfermion masses** which are non-diagonal in the basis where Yukawa matrices are diagonal (SCKM basis).
- GUT models more constraining due to boundary conditions between hadronic & leptonic sectors.

$\theta^{\nu}_{13} \ll \theta^{\nu}_{12}, \theta^{\nu}_{23}$

- An interesting Family symmetry G_F would predict **TB-mixing** in the neutrino sector.

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \boxed{0} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

- Neutrino mass matrix:

✓ diagonalised by U_{TB}.

✓ invariant under Klein symmetry: $Z_2^S \otimes Z_2^U$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \quad U = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$\theta^{\nu}_{13} \approx 9^{o}$

- Need deviations from TB.

Neutrino flavour symmetry arising from G_F

- G_F would contain the S & U generators
- preserved in the neutrino sector (*m^{ν}_{eff} invariant under S & U*).

A specific model :

$SU(5) \times S_4 \times U(1)$

permutations of 4 objects

Minimal GUT with smallest discrete group that contains S&U generators.

The SU(5) x S_4 x U(1) Model

$T = \mathbf{10} = (Q, u^c, e^c)$ $F = \bar{\mathbf{5}} = (L, d^c)$

Field	T_3	T	F	N	H_5	$H_{\bar{5}}$	$H_{\overline{45}}$	Φ_2^u	$\widetilde{\Phi}_2^u$	Φ_3^d	$\widetilde{\Phi}_3^d$	Φ_2^d	$\Phi_{3'}^\nu$	Φ_2^ν	Φ_1^ν
$SU(5)$	$\mathbf{10}$	$\mathbf{10}$	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$	$\bar{\mathbf{5}}$	$\overline{\mathbf{45}}$	$\mathbf{1}^{\forall}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
S_4	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{2}$	$\mathbf{1}$
$U(1)$	0	x	y	$-y$	0	0	z	$-2x$	0	$-y$	$-x-y-2z$	z	$2y$	$2y$	$2y$

❖ **<u>U(1) symmetry</u>:** different flavons couple to distinct sectors at LO (according to their f label);

❖"Leading" operators: U(1) charges add up to zero $\forall$ x ,y, z ϵ Z.

❖Subleading operators allowed when values of x,y,z are fixed.
Forbid the unwanted ones by choosing the most appropriate values:

(x,y,z)=(5,4,1)

Part I

The fermion sector

C. Hagedorn, S.F. King, C. Luhn:

arXiv:1003.4249
arXiv:1205.3114

Constructing Y^u

$T = \mathbf{10} = (Q, u^c, e^c)$

Write down all **operators** that form a **singlet** under all symmetries

combine up to 8 flavons with TTH_5 for the first two families & $T_3T_3H_5$ for the 3rd family.

$$\frac{1}{M^2} y_1^u T T \Phi_2^u \tilde{\Phi}_2^u H_5 + \frac{1}{M} y_2^u T T \Phi_2^u H_5 + y_t T_3 T_3 H_5 +$$

$$\frac{1}{M^5} Z_1 T T (\Phi_2^d)^2 (\Phi_3^d)^3 H_5 + \frac{1}{M^5} Z_2 T T_3 (\Phi_2^d)^3 (\Phi_3^d)^2 H_5 + \frac{1}{M^3} Z_{3,4} T_3 T_3 (\Phi_3^d)^2 \Phi_{2,3'}^\nu H_5$$

Break family symmetry with non-zero flavon vevs. e.g.

$$\frac{\langle \Phi_2^u \rangle}{M} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \phi_2^u \lambda^4 + \begin{pmatrix} \delta_{2,1_8}^u \lambda^8 \\ \sum_{m=5}^{8} \delta_{2,2_m}^u \lambda^m \end{pmatrix}$$

$\lambda \approx 0.22$: Wolfstein parameter

$$Y^u_{|GUT} = \begin{pmatrix} y_u e^{i\theta_u^y} \lambda^8 & 0 & 0 \\ 0 & y_c e^{i\theta_u^y} \lambda^4 & z_2^u e^{i\theta_2^{zu}} \lambda^7 \\ 0 & z_2^u e^{i\theta_2^{zu}} \lambda^7 & y_t \end{pmatrix} + \dots$$

CP spontaneously broken also by flavons. All phases come from complex flavon vevs

- Similarly, write down operators consisting of T , F & Φ^d_ρ

$$Y^d_{|GUT} = Y^d_\alpha + Y^d_\beta \qquad Y^e_{|GUT} = (Y^d_\alpha - 3Y^d_\beta)^T$$

$$Y^d_\alpha = \begin{pmatrix} 0 & x_2 e^{i\theta^x_2}\lambda^5 & -x_2 e^{i\theta^x_2}\lambda^5 \\ -x_2 e^{i\theta^x_2}\lambda^5 & 0 & x_2 e^{i\theta^x_2}\lambda^5 \\ z^d_3 e^{i\theta^{zd}_3}\lambda^6 & z^d_2 e^{i\theta^{zd}_2}\lambda^6 & y_b e^{i\theta^y_b}\lambda^2 \end{pmatrix} + \dots$$

$$Y^d_\beta = \begin{pmatrix} z^d_1 e^{i\theta^{zd}_1}\lambda^8 & 0 & z^d_{11} e^{i\theta^{zd}_{11}}\lambda^8 \\ z^d_{10} e^{i\theta^{zd}_{10}}\lambda^8 & y_s e^{i\theta^y_s}\lambda^4 & -y_s e^{i\theta^y_s}\lambda^4 \\ 0 & z^d_9 e^{i\theta^{zd}_9}\lambda^7 & 0 \end{pmatrix} + \dots$$

$$T = \mathbf{10} = (Q, u^c, e^c)$$

$$F = \bar{\mathbf{5}} = (L, d^c)$$

$$m_d : m_s : m_b \approx \lambda^4 : \lambda^2 : 1$$
$$m_e : m_\mu : m_\tau \approx (1/3)\lambda^4 : 3\lambda^2 : 1$$

$$\theta^d_{12} \approx \lambda, \quad \theta^d_{13} \approx \lambda^3, \quad \theta^d_{23}, \approx \lambda^2$$
$$\theta^e_{12} \approx (1/3)\lambda, \quad \theta^e_{13} \approx 0, \quad \theta^e_{23}, \approx 0$$

- Y^u almost diagonal, quark mixing coming from Y^d.

- Georgi-Jarlskog (GJ) relations: $m_b \approx m_\tau$, $m_\mu \approx 3m_s$, $m_d \approx 3m_e$ and GST relation: $\theta_{12} \approx \sqrt{(m_d/m_s)}$ incorporated at LO.

Neutrino sector

LO operators:
$NFH_5 \to M_D$, $NN\Phi^\nu_\rho \to M_R$

Type I see-saw formula:
$\mathbf{m^\nu_{eff}} = M_D\, \mathbf{M_R^{-1}}\, M_D^{\,T}\, \boldsymbol{\upsilon_u^2}$

$\langle\Phi_\rho^\nu\rangle$: eigenvectors of **S&U**

$Z_2^S x Z_2^U$ Klein subgroup of S_4 preserved

TB-mixing in the neutrino sector at LO

$$\mathbf{U_{PMNS} = U^{e\dagger}_L\, U^\nu_L = U^{e\dagger}_L U_{TB}}$$

$\boldsymbol{\theta^l_{12}}$, $\boldsymbol{\theta^l_{23}}$ of the correct order & $\boldsymbol{\theta^l_{13} \sim 3^o}$

Deviation from TB due to charged-lepton sector not enough as $\boldsymbol{\theta^l_{13}{}^{exp} \approx 9^o}$

Further deviation from **TB:** new flavon **η** (S_4 singlet).

$$\frac{1}{M}\eta\Phi_2^d NN$$

breaks Z_2^U as $\langle\Phi_2^d\rangle$
<u>Not</u> eigenvector of U

θ^ν_{13} , θ^ν_{23} receive corrections $O(\lambda)$ $\to$ agreement with exp.

Part II

Canonical normalisation effects

The soft SUSY breaking sector

A-trilinear terms:

$$\mathcal{L}_{soft} = \boxed{\epsilon_{\alpha\beta}\left(-H_u^\alpha \tilde{Q}^{\beta i} A^u_{ij} \tilde{u}^{cj} - H_d^\alpha \tilde{Q}^{\beta i} A^d_{ij} \tilde{d}^{cj} - H_u^\alpha \tilde{L}^{\beta i} A^\nu_{ij} \tilde{\nu}^{cj} - H_d^\alpha \tilde{L}^{\beta j} A^e_{ij} \tilde{e}^{cj} + h.c\right)}$$

$$+ \tilde{Q}^*_{i\alpha}(m_Q^2)^i_j \tilde{Q}^{\alpha j} + \tilde{u}^{c*}_i (m^2_{u^c})^i_j \tilde{u}^{cj} + \tilde{d}^{c*}_i (m^2_{d^c})^i_j \tilde{d}^{cj} + \tilde{L}^*_{i\alpha}(m_L^2)^i_j \tilde{L}^{\alpha j}$$

$$+ \tilde{e}^{c*}_i (m^2_{e^c})^i_j \tilde{e}^{cj} + \tilde{\nu}^{c*}(m^2_{\nu^c})^i_j \tilde{\nu}^{cj}$$

Scalar mass terms

Superpotential W

Gives rise to Yukawa & A-trilinear terms through $<\int d^2\theta\, W>$

$$\frac{X}{M_X} H f \sum_{\Phi\Phi'} a^{f f^c}_{\Phi\Phi'} \frac{\Phi \otimes \Phi'}{M^2} f^c$$

picks up **F-terms** from hidden sector fields X & from flavons.

$$\langle F_{\Phi_A} \rangle_i = m_0\, x_A \langle \Phi_A \rangle_i$$

$$H f \sum_{\Phi\Phi'} y^{f f^c}_{\Phi\Phi'} \frac{\Phi \otimes \Phi'}{M^2} f^c$$

trilinears have same structure as Yukawas but different O(1) coefs.

trilinears & Yukawas can not be simultaneously diagonalised.

Origin of **off-diagonalities** in the SCKM basis

Kähler potentials K_F, K_T, K_N

$\langle\int d^4\theta\, K\rangle$ give rise to kinetic and soft scalar masses

$$\mathcal{L}_K \supset \tilde{K}_{ij}\left(\partial_\mu \varphi_i^* \partial^\mu \varphi_j + i\eta_i^* \partial_\mu \bar{\sigma}^\mu \eta_j\right)$$

$$\frac{\langle |F_X|^2\rangle}{M_X^2} = m_0^2$$

Kähler metric: $\tilde{K}_{ij} = \left.\frac{\partial^2 K}{\partial f_i^\dagger f_j}\right|_{f=\varphi}$ *generic sfields*

Flavon expansion :

$$K_F = F^\dagger\left[\left(c_0^{K_F} + c_0^{M_F}\frac{X^\dagger X}{M_X^2}\right)\mathbb{1}_3 + \sum_{\Phi\Phi'}\left(c_{\Phi\Phi'}^{K_F} + c_{\Phi\Phi'}^{M_F}\frac{X^\dagger X}{M_X^2}\right)\frac{\Phi\otimes\Phi'}{M^2}\right]F$$

❖ Kähler metrics & soft masses: same structure, different O(1) coefs.

❖ Generation of **off-diagonalities** is inevitable.

❖ Work in a basis where: $\tilde{\mathbf{K}}^{ij}=\mathbf{1}$. $\longrightarrow$ **Canonical Normalisation**

Canonical Normalisation: change of basis such that: $(\mathbf{P}^{\dagger})^{-1}\,\tilde{\mathbf{K}}\,\mathbf{P}^{-1} = \mathbf{1}$

- Bring all quantities into that basis.
- Y^u_c: zero entries are populated; (23) & (32) entries reduced by two orders of λ.
- Y^ν_c: (12), (21) & (33) entries also reduced by two orders of λ.
- Rest of the effects just consist of changing the O(1) coefs.

Successful fermionic masses & mixings survive.

Now the off-diagonalities in the soft sector have to be controlled in order to lead to predictions that agree with the FCNC bounds.

The SUSY Flavour Problem

The SUSY Flavour Problem

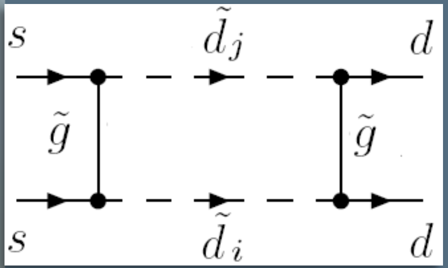
FC

generation mixing…

NC

$$\bar{d}^i_L\,(U_L^{d\dagger})^{ik}\gamma^\mu (U_L^d)^{kj}\,d^j_L \underset{U_L^{d\dagger}U_L^d=\mathbb{1}}{\Longrightarrow} \bar{d}^i_L\,\gamma^\mu d^i_L$$

SM *No generation mixing at tree level*

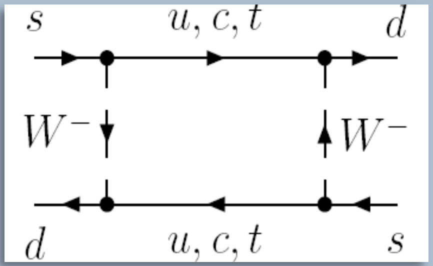


$$\bar{d}^i_L\,(U_L^{d\dagger})^{ik}\lambda_G\,(\Gamma_L^d)^{kj}\,\tilde{d}^j_L \Longrightarrow \bar{d}^i_L\,\lambda_G K^{ij}\,\tilde{d}^j_L$$

$$U_L^{d\dagger}\Gamma_L^d = K \neq \mathbb{1}$$

SUSY *tree level FCNC mediated by gluino*

CC



$$\bar{u}^i_L\,(U_L^{u\dagger})^{ik}\gamma^\mu (U_L^d)^{kj}\,d^j_L \Longrightarrow \bar{u}^i_L\,\gamma^\mu V^{ij}_{CKM} d^j_L$$

$$U_L^{u\dagger}U_L^d = V_{CKM} \neq \mathbb{1}$$

SM *Only through loops with charged particles*

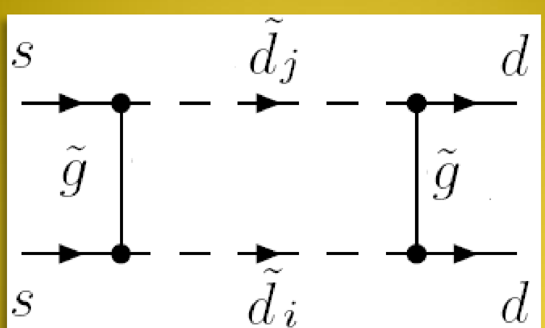
The SUSY Flavour Problem

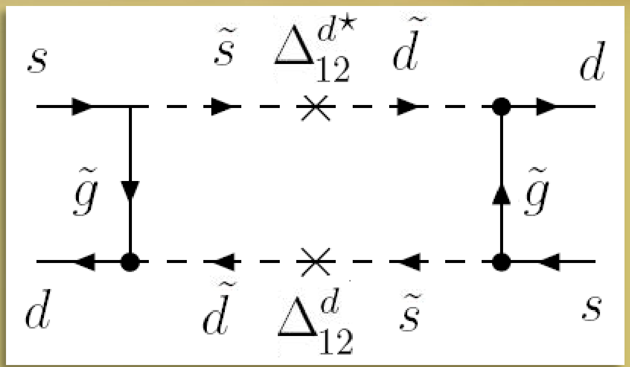
MI — *Mass Insertion approximation*

- Work in Super-CKM basis (diagonal m_d)
- gluino vertex diagonal in flavour but non-diagonal $\tilde{m}^2_d$.
- Approximate squark propagator.

$$\left(\tilde{m}^d\right)^2_{ij} = \left(\tilde{m}^d\right)^2 \mathbb{1} + \Delta^d_{ij}$$

$$\frac{i}{\left(\left(k^2 - (\tilde{m}^d)^2\right)\mathbb{1} - \Delta^d\right)_{ij}} \approx i\frac{\delta_{ij}}{k^2 - (\tilde{m}^d)^2} + i\frac{\Delta^d_{ij}}{\left(k^2 - (\tilde{m}^d)^2\right)^2} + \dots$$





The SUSY Flavour Problem

MI

Mass Insertion approximation

$$(\delta^d_{AB})_{ij} = \frac{(\Delta^d_{AB})_{ij}}{(\tilde{m}^d)^2}\ , \quad \{A,B\} = \{L,R\}$$

$$\mathcal{H}_{\rm eff}^{\Delta F=2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \tilde{C}_i \tilde{Q}_i + \text{h.c.},$$

$$C_1^{\tilde{g}} \simeq -\frac{\alpha_s^2}{\tilde{m}^2}[(\delta_d^{LL})_{21}]^2 g_1^{(1)}(x_g),$$

$$\tilde{C}_1^{\tilde{g}} \simeq -\frac{\alpha_s^2}{\tilde{m}^2}[(\delta_d^{RR})_{21}]^2 g_1^{(1)}(x_g),$$

$$C_4^{\tilde{g}} \simeq -\frac{\alpha_s^2}{\tilde{m}^2}[(\delta_d^{LL})_{21}(\delta_d^{RR})_{21}] g_4^{(1)}(x_g),$$

$$C_5^{\tilde{g}} \simeq -\frac{\alpha_s^2}{\tilde{m}^2}[(\delta_d^{LL})_{21}(\delta_d^{RR})_{21}] g_5^{(1)}(x_g),$$

$$x_g = M_{\tilde{g}}^2/\tilde{m}^2$$

- ❖ Since the observed FCNCs are strongly suppressed, experiment sets strong bounds on these parameters.

- ❖ In our particular example, the relevant observable is:

$$M_{12}^{(K^0)} \equiv \langle K^0 | \mathcal{H}_{\rm eff}^{\Delta F=2} | \bar{K}^0 \rangle$$

- ❖ Need to check whether our model predicts MIs that agree with the current bounds!

Mass Insertion (MI) Parameters

- Change to the basis where $(U^f_L)^\dagger Y^f_C U^f_R = Y^f$ **: diag** SCKM basis
- Full fermion & sfermion 3x3 mass matrices defined as:

$$(\tilde{m}^2_u)_{LL} = (U^u_L)^\dagger m^2_Q U^u_L$$

$$(\tilde{m}^2_u)_{RR} = (U^u_R)^\dagger m^2_{u^c} U^u_R$$

$$m^2_{\tilde{f}_{LL}} = (\tilde{m}^2_f)_{LL} + \tilde{Y}_f \tilde{Y}_f^\dagger v^2_{u,d} \quad , \quad m^2_{\tilde{f}_{RR}} = (\tilde{m}^2_f)_{RR} + \tilde{Y}_f^\dagger \tilde{Y}_f v^2_{u,d}$$

$$m^2_{\tilde{f}_{LR}} = \tilde{A}_f v_{u,d} - \mu \tilde{Y}_f v_{d,u} \quad , \quad m^2_{\tilde{f}_{RL}} = \tilde{A}_f^\dagger v_{u,d} - \mu \tilde{Y}_f^\dagger v_{d,u}$$

- Theoretical predictions in terms of the dim/less parameters:

$$(\delta^f_{LL})_{ij} = \frac{(m^2_{\tilde{f}_{LL}})_{ij}}{\langle m_{\tilde{f}} \rangle^2_{LL}}, \quad (\delta^f_{RR})_{ij} = \frac{(m^2_{\tilde{f}_{RR}})_{ij}}{\langle m_{\tilde{f}} \rangle^2_{RR}},$$

$$(\delta^f_{LR})_{ij} = \frac{(m^2_{\tilde{f}_{LR}})_{ij}}{\langle m_{\tilde{f}} \rangle^2_{LR}}, \quad (\delta^f_{RL})_{ij} = \frac{(m^2_{\tilde{f}_{RL}})_{ij}}{\langle m_{\tilde{f}} \rangle^2_{RL}}$$

$$\langle m_{\tilde{f}} \rangle^2_{AB} = \sqrt{(m^2_{\tilde{f}_{AA}})_{ii} (m^2_{\tilde{f}_{BB}})_{jj}}$$

Mass Insertion (MI) Parameters

GUT scale orders of magnitude…

dropping O(1) coefs…

$$(\delta^u)_{LL} = \begin{pmatrix} 1 & \lambda^4 & \lambda^6 \\ \cdot & 1 & \lambda^5 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad (\delta^u)_{RR} = \begin{pmatrix} 1 & \lambda^4 & \lambda^6 \\ \cdot & 1 & \lambda^5 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad (\delta^u)_{LR} = \begin{pmatrix} \lambda^8 & 0 & 0 \\ 0 & \lambda^4 & \lambda^7 \\ 0 & \lambda^7 & 1 \end{pmatrix}$$

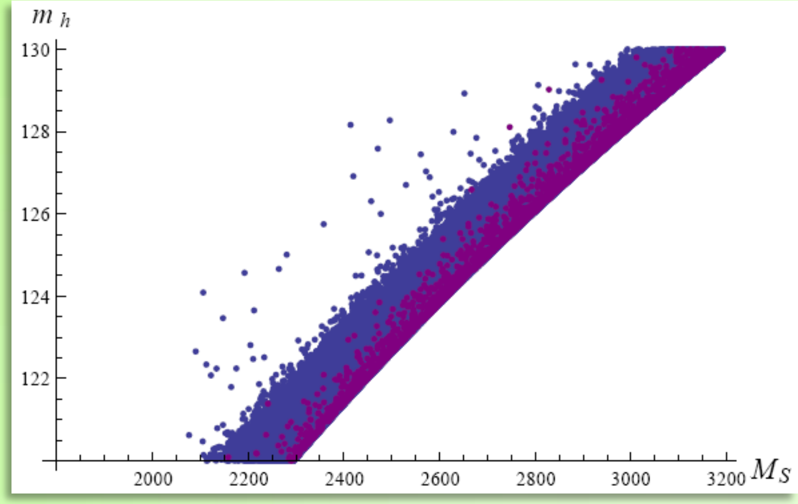
$$(\delta^d)_{LL} = \begin{pmatrix} 1 & \lambda^3 & \lambda^4 \\ \cdot & 1 & \lambda^2 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad (\delta^d)_{RR} = \begin{pmatrix} 1 & \lambda^4 & \lambda^4 \\ \cdot & 1 & \lambda^4 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad (\delta^d)_{LR} = \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^6 & \lambda^6 & \lambda^2 \end{pmatrix}$$

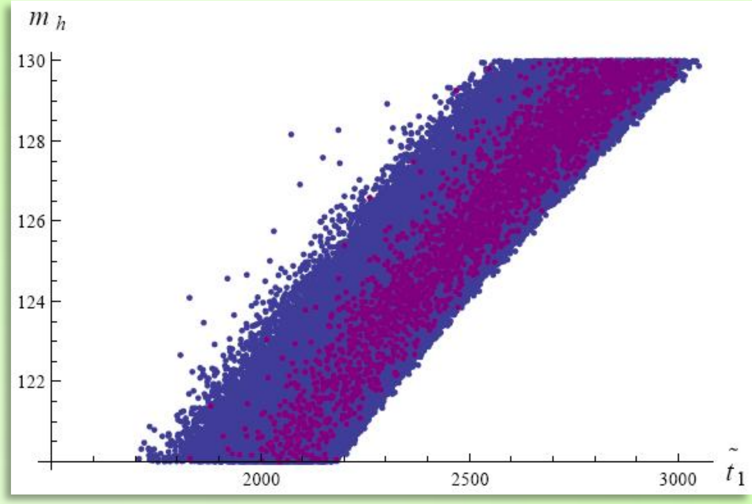
$$(\delta^e)_{LL} = \begin{pmatrix} 1 & \lambda^4 & \lambda^4 \\ \cdot & 1 & \lambda^4 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad (\delta^e)_{RR} = \begin{pmatrix} 1 & \lambda^3 & \lambda^4 \\ \cdot & 1 & \lambda^2 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad (\delta^e)_{LR} = \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^6 \\ \lambda^5 & \lambda^4 & \lambda^6 \\ \lambda^5 & \lambda^4 & \lambda^2 \end{pmatrix}.$$

- Small off-diagonalities but…small enough?
- RG run down to the low energy scale where experiments are performed and compare with given bounds.

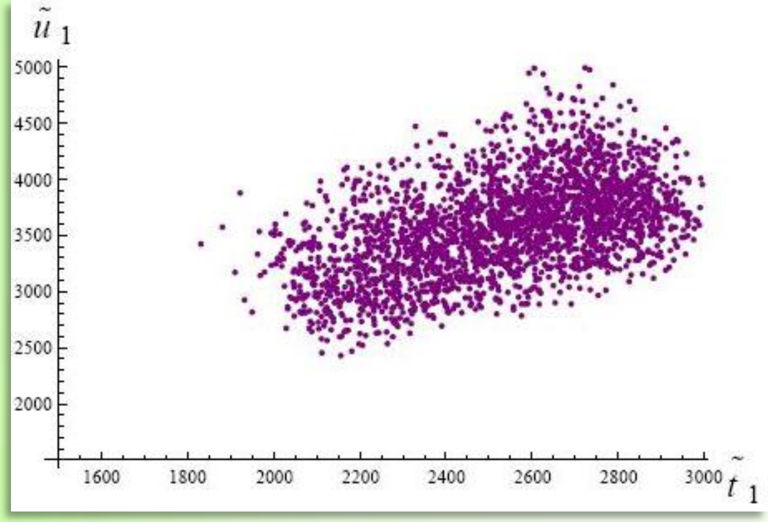
Low energy predictions

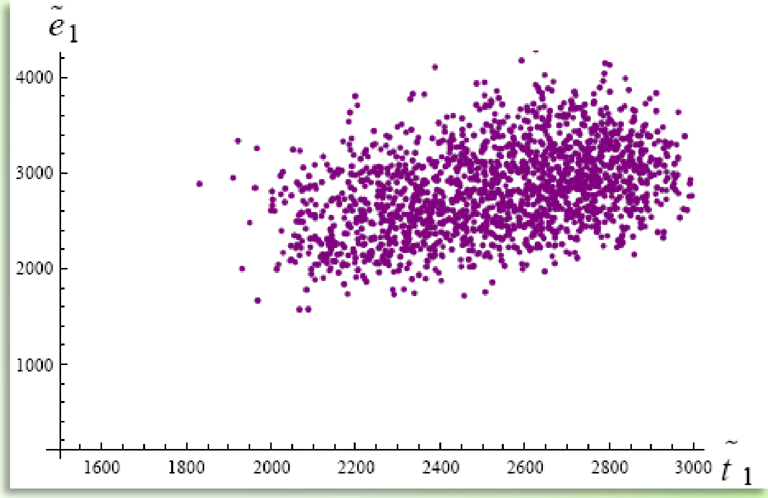
- SUSY scale already pushed high by the Higgs mass





- Strongest constraint from **μ→eγ** pushes even further:

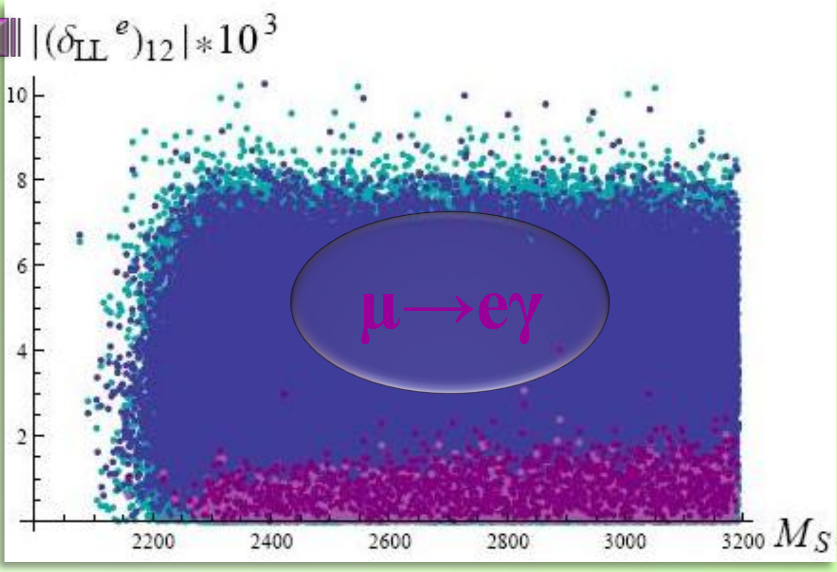




Low energy predictions

Around its max allowed value:

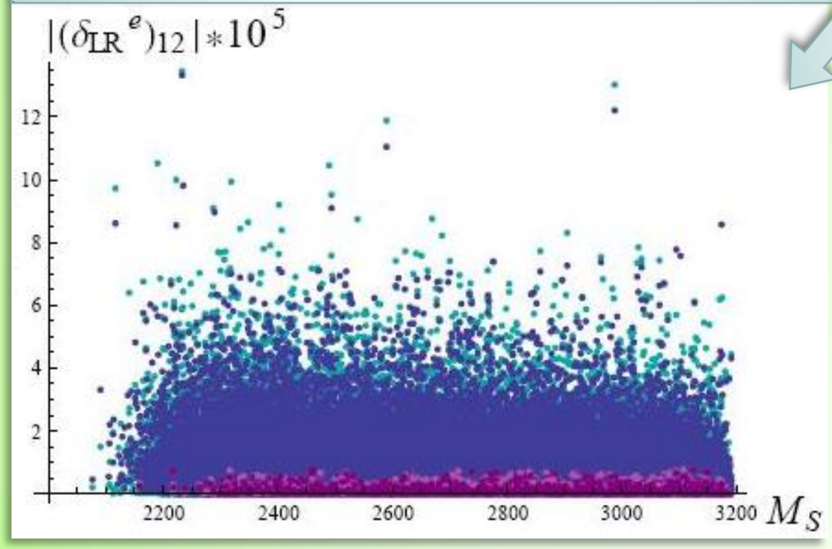
$|(\delta^e_{LL})_{12}|_{max} \sim O(10^{-5},-10^{-4})$

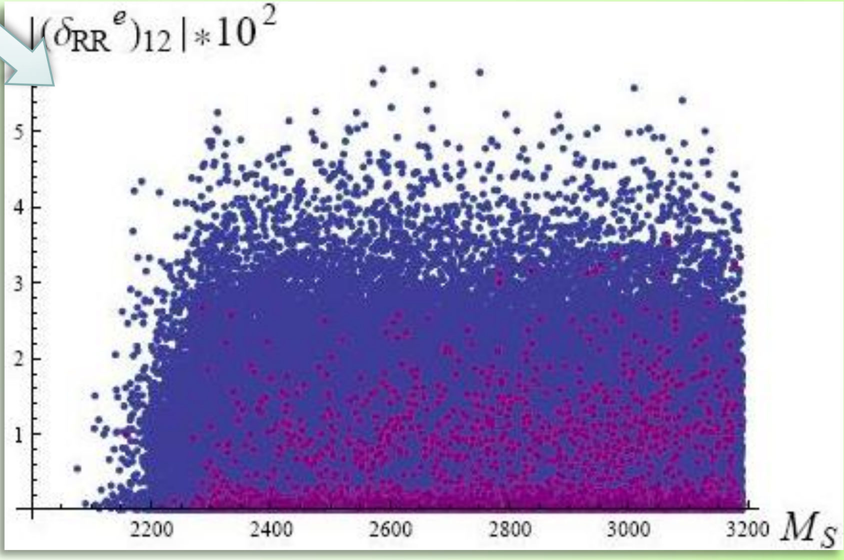


Also close to their limits:

$|(\delta^e_{LR})_{12}|_{max} \sim O(10^{-6},-10^{-5})$

$|(\delta^e_{RR})_{12}|_{max} \sim O(10^{-3},-10^{-2})$





Low energy predictions

Parameter	Prediction	Bound
• $\lvert(\delta^e_{LL})_{13,23}\rvert$	$O(10^{-4},10^{-3})$	$O(10^{-2},10^{-1})$
• $\lvert(\delta^e_{LR})_{13,31,23}\rvert$	$O(10^{-6},10^{-5})$	$O(10^{-2},10^{-1})$
• $\lvert(\delta^e_{LR})_{32}\rvert$	$O(10^{-5},10^{-4})$	$O(10^{-2},10^{-1})$
• $\lvert(\delta^e_{RR})_{13}\rvert$	$O(10^{-5},10^{-4})$	$O(10^{-1}, 1)$
• $\lvert(\delta^e_{RR})_{23}\rvert$	$O(10^{-3},10^{-2})$	$O(10^{-1}, 1)$

Bounds from:
arXiv: 1405.6960
arXiv: 1304.2783
arXiv: 1207.3016

Low energy predictions

Parameter	Prediction	Bound
$\lvert(\boldsymbol{\delta}^{\mathbf{d}}_{\mathbf{LL}})_{\mathbf{23}}\rvert$	$O(10^{-3},\mathbf{10^{-2}})$	$O(\mathbf{10^{-2}},10^{-1})$
$\lvert\mathbf{Im}(\boldsymbol{\delta}^{\mathbf{d}}_{\mathbf{LL}})_{\mathbf{12}}\rvert$	$\mathbf{O(10^{-3},10^{-2})}$	$\mathbf{O(10^{-3},10^{-2})}$
$\lvert\mathrm{Im}(\delta^{d}_{LR})_{12}\rvert$	$O(10^{-7},10^{-6})$	$O(10^{-4},10^{-3})$
$\lvert(\delta^{d}_{RR})_{23}\rvert$	$O(10^{-4},10^{-3})$	$O(10^{-1}, 1)$
$\lvert(\delta^{d}_{RL})_{23}\rvert$	$O(10^{-7},10^{-6})$	$O(10^{-2})$
$\lvert(\delta^{d}_{LR})_{23}\rvert$	$O(10^{-6},10^{-5})$	$O(10^{-3},10^{-2})$
$\lvert(\delta^{u}_{LR})_{23}\rvert$	$O(10^{-6})$	0.3 (MS~1TeV) 0.1 (MS~3TeV)

Summary

- SU(5) x S_4xU(1) Flavour model successfully predicts the fermionic masses and mixing angles.

- Considering canonical normalisation effects does not spoil the original features of the fermionic sector.

- Predicted off-diagonalities of soft terms (and MIs) small even at M_{GUT}. Easy to overcome most of experimental bounds.

- Strongest constraint from **$\mu\rightarrow e\gamma$.** **$(\delta^{e}_{LL})_{12}$** around its upper limit.

- Comparison with the rest of the bounds suggests study of phenomenology related to edms, ϵ_K and $b\rightarrow s$ transitions.

Thank you for your attention

On the vacuum alignment…

- Introduce **driving fields** that couple to the flavons.

- Require their F-terms to vanish: $(F^i = \partial W/\partial \phi^i = 0)$

e.g. couple the driving field $\mathbf{X_1^d}$ (S_4 singlet) Φ_2^d (S_4 doublet):

$$X_1^d(\Phi_2^d)^2 = 2X_1^d\Phi_{2,1}^d\Phi_{2,2}^d$$

require: $$\frac{\partial}{\partial X_1^d} 2X_1^d\Phi_{2,1}^d\Phi_{2,2}^d = 0 \Rightarrow \langle \Phi_2^d \rangle \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{or} \quad \langle \Phi_2^d \rangle \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

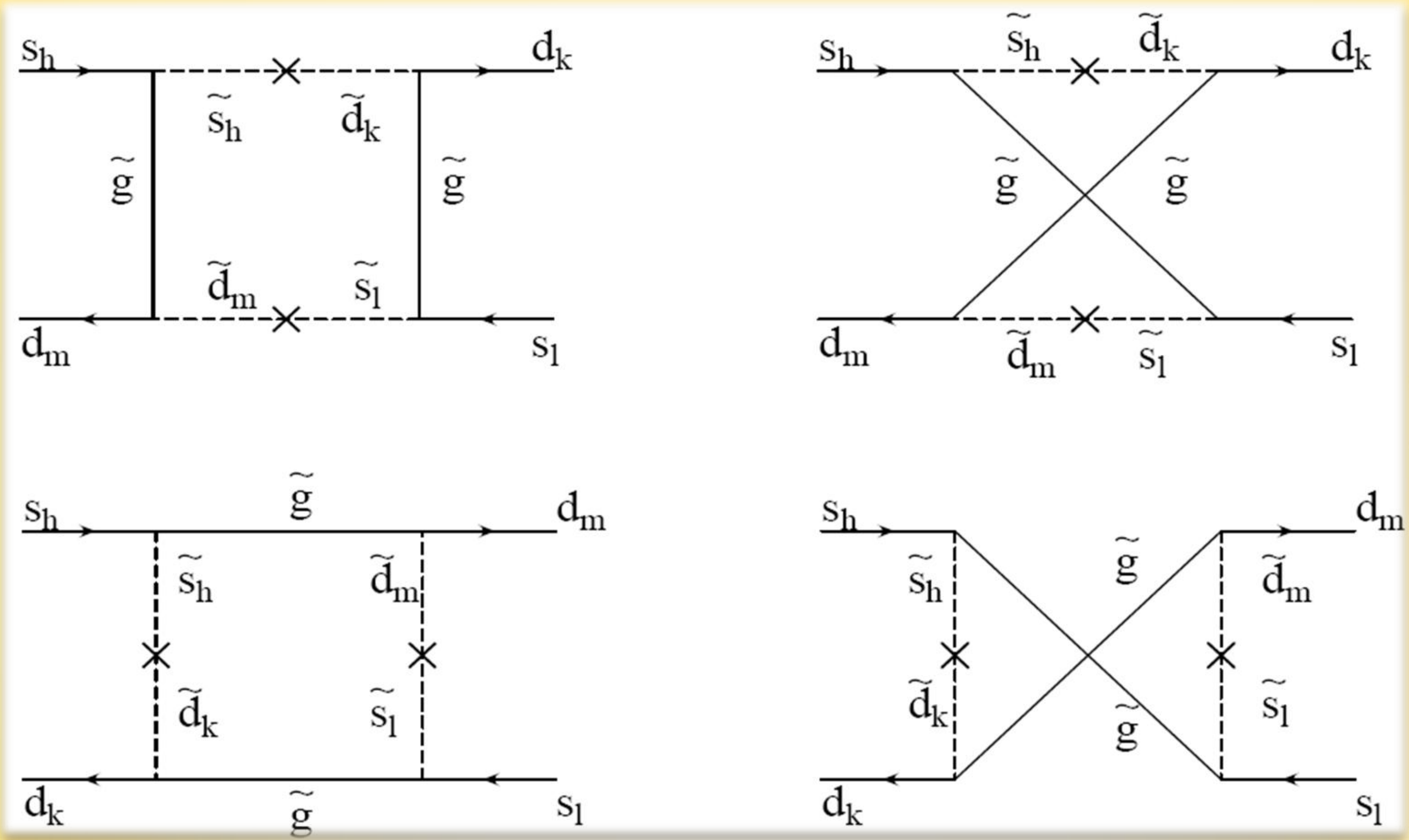
Without loss of generality, pick $\Phi_{2,1}^d \neq 0$.

- In a similar way, all flavons are aligned through vanishing F-terms of driving fields. For the neutrino sector in particular, this process not only fixes $<\Phi_i^\nu>$ but also requires that: $\varphi_1^\nu \sim \varphi_1^\nu \sim \varphi_3^{\prime\nu}$

Experimental Constraints

e.g.

SUSY contributions to K^0-$\overline{K}^0$ mixing & δ^d_{12}



Low Energy MIs

- Calculations so far hold just below M_{GUT}.
- Need RGE evolution to M_W where experiments are performed.

- Recast our parameters in a form:

$$(\delta^f)^{ij}_{AB}|_{M_W} = S^{f(ij)}_{AB} \times (\delta^f)^{ij}_{AB}|_{M_{GUT}}, \quad i,j = 1,2,3, \quad A,B \in L,R$$

- where S^f <1 accounts for the effects of RG-running.

- In general, $$S^{f(ij)}_{AB} \sim \frac{\langle \tilde{m}_f \rangle^2_{AB\,(ij)}|_{M_{GUT}}}{\langle \tilde{m}_f \rangle^2_{AB(ij)}|_{M_W}}$$

$$\langle m_{\tilde{f}} \rangle^2_{AB} = \sqrt{(m^2_{\tilde{f}_{AA}})_{ii}(m^2_{\tilde{f}_{BB}})_{jj}}$$

Experimental Constraints

$$\Delta m_K = 2\,\mathrm{Re}\langle K^0 | \mathcal{H}_{eff}\, | \bar{K}^0 \rangle$$

$$\begin{aligned}
\Delta m_K = -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\frac{2}{3} m_K f_K^2 \Bigg\{ & \left(\delta_{12}^d\right)_{LL}^2 \left(24\, x\, f_6(x) + 66\, \tilde{f}_6(x)\right) + \left(\delta_{12}^d\right)_{RR}^2 \left(24\, x\, f_6(x) + 66\, \tilde{f}_6(x)\right) \\
& + \left(\delta_{12}^d\right)_{LL} \left(\delta_{12}^d\right)_{RR} \left[\left(384 \left(\frac{m_K}{m_s + m_d} \right)^2 + 72 \right) x\, f_6(x) + \left(-24 \left(\frac{m_K}{m_s + m_d} \right)^2 + 36 \right) \tilde{f}_6(x) \right] \\
& + \left(\delta_{12}^d\right)_{LR}^2 \left[-132 \left(\frac{m_K}{m_s + m_d} \right)^2 x\, f_6(x) \right] + \left(\delta_{12}^d\right)_{RL}^2 \left[-132 \left(\frac{m_K}{m_s + m_d} \right)^2 x\, f_6(x) \right] \\
& + \left(\delta_{12}^d\right)_{LR} \left(\delta_{12}^d\right)_{RL} \left[-144 \left(\frac{m_K}{m_s + m_d} \right)^2 - 84 \right] \tilde{f}_6(x) \Bigg\}
\end{aligned}$$

$$x = m_{\tilde{g}}^2 / m_{\tilde{q}}^2$$

- ❖ Require that the abs. value of each term does not exceed the measured Δ_{mK}.

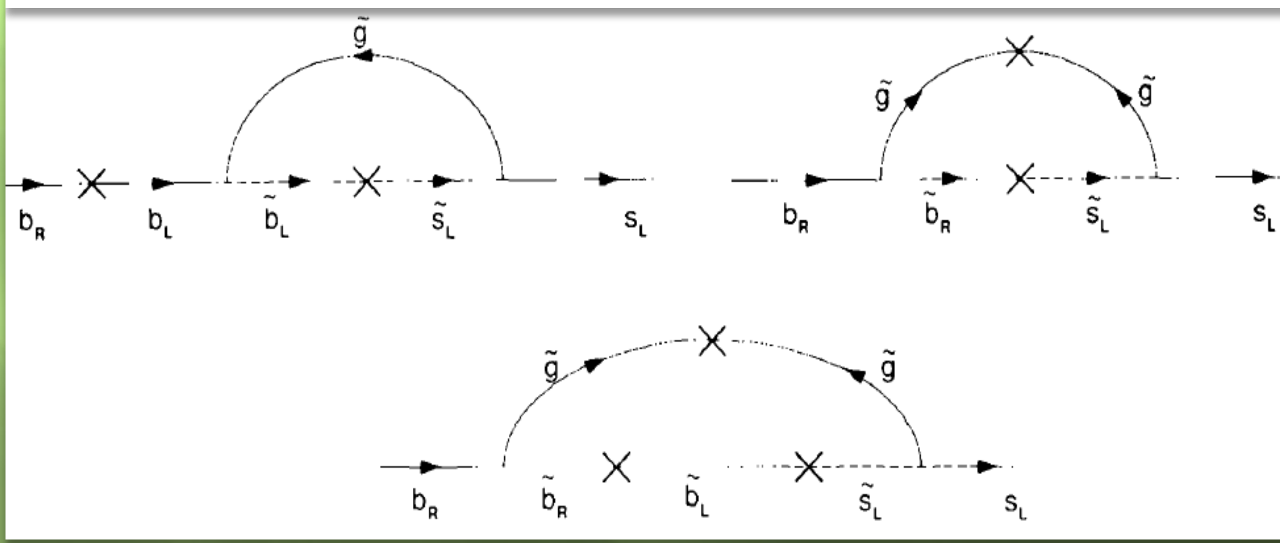
- ❖ Constraint $|(\delta^d_{12})_{AB}|$

Experimental Constraints

- Similar expressions for Δ_{mD}, Δ_{mB} and Δ_{mBs} constraint $|(\delta^u_{12})_{AB}|$, $|(\delta^d_{13})_{AB}|$ and $|(\delta^d_{23})_{AB}|$ respectively.

- $|(\delta^d_{23})_{AB}|$ is also constrained through:

$$\mathrm{BR}(b \to s\gamma) = \frac{\alpha_s^2 \alpha}{81\pi^2 m_{\tilde q}^4} m_b^3 \tau_B \left\{ \left| m_b M_3(x) \left(\delta^d_{23}\right)_{LL} + m_{\tilde g} M_1(x) \left(\delta^d_{23}\right)_{LR} \right|^2 + L \leftrightarrow R \right\}$$



- Similar constraints for lepton sector…

Neutrino sector

$$y_D F N H_5 + \alpha N N \Phi_1^\nu + \beta N N \Phi_2^\nu + \gamma N N \Phi_3^\nu$$

$$Y_D^\nu = y_D \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Choose appropriate vacuum alignment for the flavons to get **TB-mixing**.

M_R

- **Type I see-saw** formula: $m^\nu_{eff} = Y^\nu M_R^{-1} Y^{\nu T} \upsilon_u^2$
- m^ν_{eff} diagonalised by U_{TB}, to give the light neutrino masses

flavon vevs have been chosen as: $\varphi_1^\nu \approx \lambda^4 M, \quad \varphi_2^\nu \approx \lambda^4 M, \quad \varphi_{3'}^\nu \approx \lambda^4 M$

such that: $m_1, m_2, m_3 \sim 0.1$ eV $\sim y_D^2 \upsilon_u^2/(\lambda^4 M)$

Resulting predictions for leptonic mixing angles….

$U_{PMNS} = U^e_L\, U^{\nu\dagger}_L = U^e_L U_{TB}$ contains:

$$\sin\theta^l_{13} \approx \lambda/(3\sqrt{2}), \quad \sin^2\theta^l_{23} \approx 1/2, \quad \sin^2\theta^l_{12} \approx 1/3 + 2/9\lambda\cos\delta^l$$

2012 global fits for exp. data: $\sin^2\theta^l_{13} \approx 0.024$

$\sin^2\theta^l_{23} \approx 0.4$

$\sin^2\theta^l_{12} \approx 0.3$

For $\lambda \approx 0.22$

- predicted θ^l_{13} , θ^l_{23} of the correct order
- predicted $\theta^l_{13} \sim 3^o$
- $\theta^{l\,\exp}_{13} \approx 9^o$

Deviation from TB due to charged-lepton sector not enough

- **partly break** Klein symmetry

TB-mixing in the neutrino sector

alignments of flavon vevs **preserve** the $Z_2^S x Z_2^U$ subgroup of S_4.

$<\Phi_i^\nu>$: eigenvectors of **S&U**

Deviation from **TB-mixing:** new flavon **η** (S_4 singlet).

$$\frac{1}{M}\eta\Phi_2^d NN$$

break Z_2^U as $<\Phi_2^d>$: Not eigenvector of U

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}\varphi_2^d = \begin{pmatrix} 0 \\ 1 \end{pmatrix}\varphi_2^d$$

with $<\eta> \approx \lambda^4 M$, $\varphi_2^d \approx \lambda M$

$$m_\nu^{eff'} = \frac{v_u^2}{\lambda^4 M}\begin{pmatrix} -A_\nu + B_\nu + C_\nu & A_\nu & A_\nu \\ A_\nu & B_\nu & C_\nu \\ A_\nu & C_\nu & B_\nu \end{pmatrix} + \boxed{\frac{v_u^2}{\lambda^4 M}\begin{pmatrix} 0 & 0 & \lambda D_\nu \\ 0 & \lambda D_\nu & 0 \\ \lambda D_\nu & 0 & 0 \end{pmatrix}}$$

θ^ν_{12} remains tri-maximal, **θ^ν_{13} & θ^ν_{23} receive corrections O(λ).**

- **SUSY** : approximate symmetry at low energies.
- **Broken** in a different, ***hidden*** **sector** of the theory by some sfields **X**.
- SUSY breaking communicated to MSSM sfields through interactions between hidden & visible sectors.
- **Integrate out X** $\longrightarrow$ effective L_{MSSM}
- Coupling constants become functions of X.

e.g. $Y \to Y\left(\frac{X}{M^X}\right)$

M: characteristic scale of interactions.

As an example, let's see the contractions of a simple term..

$$y_3^u T_3 T_3 H_5 + y_2^u \frac{1}{M} T T \Phi_2^u H_5 + y_1^u \frac{1}{M^2} T T \Phi_2^u \tilde{\Phi}_2^u H_5$$

Remember: T ~ **2**, Φ_2^u ~ **2**, H_5 ~ **1**

We need:

- *Kronecker product:* $\boxed{\mathbf{2} \times \mathbf{2} = \mathbf{1} + \mathbf{1}' + \mathbf{2}}$

- *Clebsh-Gordan coefs:*

$$\mathbf{2} \times \mathbf{2}: \quad b_1\tilde{b}_2 + b_2\tilde{b}_1 \sim \mathbf{1}\,, \quad b_1\tilde{b}_2 - b_2\tilde{b}_1 \sim \mathbf{1}'\,, \quad (b_2\tilde{b}_2, b_1\tilde{b}_1)^t \sim \mathbf{2}$$

$$(b_1, b_2)^t, (\tilde{b}_1, \tilde{b}_2)^t \sim \mathbf{2}$$

Constructing Y^d *&* Y^e

LO operators responsible for down-quark & charged -lepton masses:

$$T = \mathbf{10} = (Q, u^c, e^c)$$
$$F = \bar{\mathbf{5}} = (L, d^c)$$

$$y_2^d \frac{1}{M} F T_3 \Phi_3^d H_{\bar{5}} + y_1^d \frac{1}{M^2} (F \tilde{\Phi}_3^d)_{\mathbf{1}} (T \Phi_2^d)_{\mathbf{1}} H_{\overline{45}} + y_3^d \frac{1}{M^3} (F \Phi_2^d \Phi_2^d)_{\mathbf{3}} (T \tilde{\Phi}_3^d)_{\mathbf{3}} H_{\bar{5}}$$

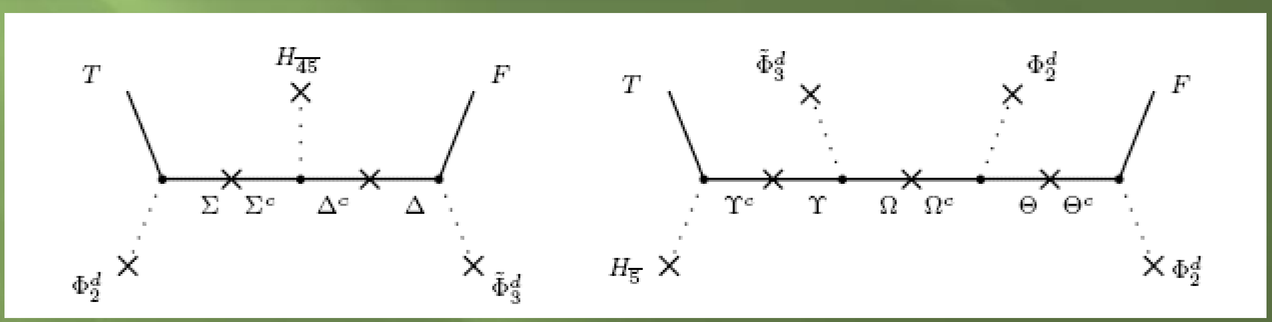
using the following vevs…

$$\langle \Phi_2^d \rangle = \varphi_2^d \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \langle \Phi_3^d \rangle = \varphi_3^d \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \langle \tilde{\Phi}_3^d \rangle = \tilde{\varphi}_3^d \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$$v_d \frac{\phi_3^d}{M} y_2^d T_3 F_3$$

$$v_d \frac{\tilde{\phi}_3^d \phi_2^d}{M^2} y_1^d \left(T_2 F_2 - T_2 F_3 \right)$$

$$v_d \frac{\tilde{\phi}_3^d (\phi_2^d)^2}{M^3} y_3^d \left(-T_1 F_3 + T_2 F_3 - T_2 F_1 + T_1 F_2 \right)$$



Comments…

- Y^u is diagonal.

- All quark mixing is coming from Y^d.

- The specific contractions of the down-quark/charged-lepton operators have been assumed to be dominant, such that Y^d & Y^e have the desired form, after the insertion of the appropriate flavon vevs.

$$y_2^d \frac{1}{M} F T_3 \Phi_3^d H_{\bar{5}} + \boxed{y_1^d \frac{1}{M^2} (F \tilde{\Phi}_3^d)_{\mathbf{1}} (T \Phi_2^d)_{\mathbf{1}} H_{\overline{45}}} + \boxed{y_3^d \frac{1}{M^3} (F \Phi_2^d \Phi_2^d)_{\mathbf{3}} (T \tilde{\Phi}_3^d)_{\mathbf{3}} H_{\bar{5}}}$$

$$\upsilon_d y_1^d \left(T_2 F_2 - T_2 F_3\right) \lambda^4$$

"-3"

$m_b \approx m_\tau$, $m_\mu \approx 3m_s$, $m_d \approx 3m_e$

$$\upsilon_d y_3^d \left(-T_1 F_3 + T_2 F_3 - T_2 F_1 + T_1 F_2\right) \lambda^5$$

(12)=(21) , (11)=0

$\theta_{12} \approx \sqrt{(m_d/m_s)}$

- Applying the **Type I see-saw** formula: $\mathbf{m^{\nu}_{eff} = Y^{\nu} M_R^{-1} Y^{\nu T} \upsilon_u^2}$ we get the effective neutrino mass matrix,
- diagonalised by the TB-mixing matrix $\mathbf{U_{TB}}$, to give the light neutrino masses (*we have reparametrised*...):

$$U_{TB}^T m_{eff}^{\nu} U_{TB} = \begin{pmatrix} -2A_\nu + B_\nu + C_\nu & 0 & 0 \\ 0 & A_\nu + B_\nu + C_\nu & 0 \\ 0 & 0 & B_\nu - C_\nu \end{pmatrix} \frac{v_u^2}{\lambda^4 M}$$

- The flavon vevs have been chosen as:

$$\varphi_1^\nu \approx \lambda^4 M, \quad \varphi_2^\nu \approx \lambda^4 M, \quad \varphi_{3'}^\nu \approx \lambda^4 M$$

such that: $\mathbf{m_1, m_2, m_3 \approx 0.1\ eV.}$

$$m_\nu^{eff'} = \frac{v_u^2}{\lambda^4 M}\begin{pmatrix} -A_\nu + B_\nu + C_\nu & A_\nu & A_\nu \\ A_\nu & B_\nu & C_\nu \\ A_\nu & C_\nu & B_\nu \end{pmatrix} + \frac{v_u^2}{\lambda^4 M}\begin{pmatrix} 0 & 0 & \lambda D_\nu \\ 0 & \lambda D_\nu & 0 \\ \lambda D_\nu & 0 & 0 \end{pmatrix}$$

Introducing the parameter:

$$n = \frac{(-2A_\nu^* + B_\nu^* + C_\nu^*)D_\nu + (B_\nu - C_\nu)D_\nu^*}{4(\mathbb{R}e[B_\nu C_\nu^* - A_\nu(B_\nu^* + C_\nu^*)] + |A_\nu|^2])}$$

we can express the angles of the matrix that diagonalises $m_\nu^{eff'}$ as:

$$\sin\theta_{13}^\nu \approx \frac{|n|}{\sqrt{2}}\lambda, \quad \sin\theta_{23}^\nu \approx \frac{1}{\sqrt{2}}\left(1 - \frac{\mathrm{Re}(n)}{2}\lambda\right)$$

Multiplying with U^e_L, we get the PMNS matrix with the corrected mixing angles,

$$\sin\theta_{13}^l \approx \frac{\lambda}{\sqrt{2}}\left(\frac{1}{3} + |n|\right), \quad \sin\theta_{23}^l \approx \frac{1}{\sqrt{2}}\left(1 - \frac{\mathrm{Re}(n)}{2}\lambda\right)$$

that can account for a θ^l_{13} as large as measured by experiment.

Forbidding “unwanted ” operators using the U(1) symmetry…

- Want to forbid any additional operators that would disturb the forms of the fermion masses & mixings that have been successfully found.

- We refer to the operators that have been used so far as LO operators. They are the ones in which the U(1) charges of the participating superfields add up to zero for every x,y,z. e.g. $\mathbf{TT\Phi_2^u\tilde{\Phi}_2^u H_5}$

- If we start fixing the values of x,y,z, the segregation of different flavons associated with a particular type of fermion breaks down.

- The resulting operators are "unwanted" if their contribution to the mass matrices at an order of λ that spoils their structure.

e.g. An operator like $\mathbf{TT\Phi_2^d H_5}$ would contribute to $\mathbf{Y_u^{(11)}}$ a term of order λ, IF **2x+z=0**, spoiling the desired structure!

Require: $\boxed{2x+z \neq 0}$

Canonical Normalisation

- Kähler metrics $\mathcal{L}_K \supset K^{ij}(\partial_\mu \phi_i^* \partial^\mu \phi_j + i\eta_{i*}\partial_\mu \bar{\sigma}^\mu \eta_j + F_i^* F_j)$

  result from an **expansion in terms of flavons**
  ($K_{T,F}$ same as $M^2_{T,F}/m_0^2$ with different O(1) coefs.)

- Leads to non-canonical metrics
- Redefine matter fields $F \to P_F^{-1} F, \qquad T \to P_T^{-1} T$

  such that Kähler metrics are identified with the identity matrix.

$$(P_F^\dagger)^{-1} K_F P_F^{-1} = \mathbb{I} \quad \Longrightarrow \quad K_F = P_F^\dagger P_F$$
$$(P_T^\dagger)^{-1} K_T P_T^{-1} = \mathbb{I} \quad \Longrightarrow \quad K_T = P_T^\dagger P_T$$

- All quantities are rotated into that basis.
- **Pattern of fermion masses & mixings survives.**

Canonical Normalisation

❖ Canonically normalise the soft sector as well.

$$\frac{A^u_C}{A_0} = (P_T^{-1})^T \frac{A^u}{A_0} P_T^{-1} \qquad \frac{A^d_C}{A_0} = (P_T^{-1})^T \frac{A^d}{A_0} P_F^{-1}$$

$$\frac{M^2_{T_C}}{m_0^2} = (P_T^{-1})^\dagger \frac{M^2_T}{m_0^2} P_T^{-1} \qquad \frac{M^2_{FC}}{m_0^2} = (P_F^{-1})^\dagger \frac{M^2_F}{m_0^2} P_F^{-1}$$

❖ In the special case where the $K_{T,F}$ & $M^2{}_{T,F}/m_0{}^2$ coefs are proportional to each other, the off-diagonal entries would vanish.

Effects of RG-running

- Calculations so far hold just below M_{GUT}.
- Need RGE evolution to M_W where experiments are performed.

e.g.

$$\frac{d(\tilde{m}_u^2)^{ij}_{RR}}{dt} = -\frac{1}{4\pi^2}\boxed{G_u}\delta^{ij} + \frac{1}{4\pi^2}\boxed{(\tilde{F}_u)^{ij}}$$

Yukawa & soft driven matrix

gaugino driven scalar

$$\tilde{F}_u \sim \begin{pmatrix} \lambda^{16} & \lambda^{12} & \lambda^{7} \\ \lambda^{12} & \lambda^{8} & \lambda^{5} \\ \lambda^{7} & \lambda^{5} & 1 \end{pmatrix}$$

Diagonal Elements

$$(\tilde{m}_u^2)^{ii}_{RR}|_{M_W} \approx (\tilde{m}_u^2)^{ii}_{RR}|_{M_{GUT}} + 6.15 m_{1/2}^2\,, \quad i = 1, 2$$

$$(\tilde{m}_u^2)^{33}_{RR}|_{M_W} \approx (\tilde{m}_u^2)^{33}_{RR}|_{M_{GUT}} + 6.15 m_{1/2}^2 - \\ -\frac{1}{4\pi^2}\left(|\tilde{Y}_u^{33}|^2\left((\tilde{m}_u^2)^{33}_{RR} + (\tilde{m}_u^2)^{33}_{LL} + (\tilde{m}^2_{Hu})\right) + |\tilde{A}_u^{33}|^2\right)\log\left(\frac{M_{GUT}}{M_W}\right)$$

Effects of RG-running

Off-diagonal Elements

$$t = \log\left(\frac{\mu}{M}\right)$$

$$\frac{d(\tilde{m}_u^2)^{ij}_{RR}}{dt} = -\frac{1}{4\pi^2} G_u \delta^{ij} + \frac{1}{4\pi^2} \boxed{(\tilde{F}_u)^{ij}}$$

Yukawa & soft driven matrix

$$\tilde{F}_u \sim \begin{pmatrix} \lambda^{16} & \lambda^{12} & \lambda^{7} \\ \lambda^{12} & \lambda^{8} & \lambda^{5} \\ \lambda^{7} & \lambda^{5} & 1 \end{pmatrix}$$

- ❖ Stem purely from the $\tilde{F}_u$-term.
- ❖ Small compared to the O(1) terms that appear in the diagonal elements.

$$\boxed{(\tilde{m}_u^2)^{ij}_{RR}|_{M_{GUT}} \approx (\tilde{m}_u^2)^{ij}_{RR}|_{M_W} \,, i \neq j}$$

- ❖ Similarly, we run down $(\tilde{m}^2_d)_{RR}$, $(\tilde{m}^2_e)_{RR}$ *(with different G_f & F_f).*
- ❖ Worth commenting on the LL parameters though…

Effects of RG-running

arXiv:0103324v2

- Numerical estimates for the low energy masses in the literature do NOT take into account the effects of going to the SCKM basis.

- No big difference for RR-parameters because:
- G_f is independent of basis. So are $(\tilde{F}_f)^{ii}$ but only to LO in λ, while $(\tilde{F}_f)^{ij}$ are usually small and ignored anyway.

- However, the effects of the SCKM matrix $V=(U^u{}_L)^\dagger\, U^d{}_L$ can potentially be important for the LL-parameters.

$$\tilde{F}^u_Q = V\,\tilde{F}^d_Q V^\dagger$$

$$\tilde{F}^u_Q \sim \begin{pmatrix} \lambda^{10} & \lambda^9 & \lambda^7 \\ \lambda^9 & \lambda^8 & \lambda^5 \\ \lambda^7 & \lambda^5 & 1 \end{pmatrix} \quad , \qquad \tilde{F}^d_Q \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

Effects of RG-running

❖ Finally, we remark on the dominance of the **right-handed neutrinos** in the RGE for **$(\tilde{m}^2_e)_{LL}$**.

$$\tilde{F}^\nu_L = \frac{1}{2}\left(\frac{1}{2}\tilde{Y}_\nu \tilde{Y}_\nu^\dagger (\tilde{m}^2_e)_{LL} + \frac{1}{2}(\tilde{m}^2_e)_{LL}\tilde{Y}_\nu \tilde{Y}_\nu^\dagger + \tilde{Y}_\nu (\tilde{m}^2_N)_{RR}\tilde{Y}_\nu^\dagger + (m^2_{H_u})\tilde{Y}_\nu \tilde{Y}_\nu^\dagger + \tilde{A}_\nu \tilde{A}_\nu^\dagger\right)$$

$$Y_\nu = y_D \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad A_\nu = \alpha y_D \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad M_N = \begin{pmatrix} M_{N_1} & 0 & 0 \\ 0 & M_{N_2} & 0 \\ 0 & 0 & M_{N_3} \end{pmatrix}$$

$$\boxed{M_N \sim 10^{14}\mathrm{GeV}}$$

$$\boxed{\tilde{F}^\nu_L \sim \begin{pmatrix} 1 & \lambda & \lambda \\ \lambda & 1 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}}$$

$$(\tilde{m}^2_e)^{ii}_{LL}|_{M_W} \approx (\tilde{m}^2_e)^{ii}_{LL}|_{M_{GUT}} + 0.5\ m^2_{1/2} - \frac{1}{4\pi^2}\tilde{F}^\nu_L \log\left(\frac{M_{GUT}}{M_N}\right), \quad i = 1, 2$$

Low Energy MIs

$$(\delta^f)^{ij}_{AB}|_{M_W} = S^{f(ij)}_{AB} \times (\delta^f)^{ij}_{AB}|_{M_{GUT}}, \;\; i,j = 1,2,3, \quad A,B \in L,R,$$

e.g.

$$S^{u(12)}_{RR} = \frac{\langle \tilde{m}_u \rangle^2_{RR(12)}|_{M_{GUT}}}{\langle \tilde{m}_u \rangle^2_{RR(12)}|_{M_W}}$$

$$\approx \frac{\sqrt{(\tilde{m}^2_u)^{11}_{RR}|_{M_{GUT}} (\tilde{m}^2_u)^{22}_{RR}|_{M_{GUT}}}}{\sqrt{\left((\tilde{m}^2_u)^{11}_{RR}|_{M_{GUT}} + 6.15\ m^2_{1/2}\right)\left((\tilde{m}^2_u)^{22}_{RR}|_{M_{GUT}} + 6.15\ m^2_{1/2}\right)}}$$

$$\boxed{S^{u(12)}_{RR} \approx \frac{\frac{b_{01}}{k_{01}}}{\frac{b_{01}}{k_{01}} + 6.15\ \frac{m^2_{1/2}}{m^2_0}}}$$

Fermion masses & mixings

- Fermion mass terms generated through <u>Yukawa</u> couplings to Higgs:

$$-y_u \bar{Q}_L \Phi^c u_R + \text{h.c.} \xrightarrow{\text{SSB}} -(y_u \upsilon)\bar{u}_L u_R + \text{h.c.}$$

- Additional generations $\longrightarrow$ coupling terms that mix them

$$-(Y_u)^{ij} \bar{Q}_L{}^i \Phi^c u_R{}^j + \text{h.c.} \qquad -\bar{u}_L{}^i (Y_u)^{ij} u_R{}^j\, \upsilon + \text{h.c.}$$

$$\Downarrow$$

$$-\bar{u}_L{}^i \underbrace{(U^{u\dagger}{}_L Y_u U^u{}_R)}_{Y^u\text{: diag}}{}^{ij} u_R{}^j \upsilon + \text{h.c.}$$

- Then, $m_u{}^i \sim (Y^u{}_{diag})^{ii}$,
- while off-diag. elements of $(Y_u)^{ij}$ are related to mixing between the three generations.

Understanding the pattern of fermion masses =
=Understanding Yukawa matrices.