F-Theory Model Building with Discrete Symmetry

Discrete 2014

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Overview

- Brief introduction to F-theory
- Discussion of an $SU(5)$ GUT model with A_4 discrete symmetry - based on:

  Discrete Family Symmetry from F-Theory GUTs - A.Karozas, S.F.King, G.K.Leontaris, AM.

  Following from work:

  Neutrino mass textures from F-theory - I. Antoniadis and G. K. Leontaris
- Results and Prospects

Discrete Symmetry in F-theory

F-Theory is a twelve dimensional formulation of Type IIB string theory, with eight dimensions being a Calabi-Yau complex fourfold, which is elliptically fibred over a complex threefold base.

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- Local GUT surface - a D7 Brane
- Points where the GUT surface intersects other D7 Branes give symmetry enhancements, where the maximum enhancement corresponds to a point of E_8

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$$248 \to (24,1) + (1,24) + (10,5) + (\bar{10},\bar{5}) + (5,\bar{10}) + (\bar{5},10) \quad (1)$$

F-Theory - Spectral Cover Equation

The 10s of an $SU(5)$ singularity are described by the Spectral cover equation:

$$\mathcal{C}_5 : \ b_5 + b_4 s + b_3 s^2 + b_2 s^3 + b_1 s^4 + b_0 s^5 = b_0 \prod_{i=1}^{5} (s + t_i)$$

The roots of the spectral cover equation are identified as the weights of the 5 of $SU(5)_\perp$, which in turn specifies the defining equation of the 10 representation of the GUT group:

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Similarly, we have a way to determine our five-curves of the GUT group:

$$\sum_{n=1}^{10} c_n s^{10-n} = b_0 \prod_{i<j}(s - t_i - t_j) = b_3^2 b_4 - b_2 b_3 b_5 + b_0 b_5^2 \,.$$

This can be expressed in terms of the b_k coefficients by identification with the $\mathcal{C}_5$ equation:

An F-theory A_4 model

Motivated by apparent symmetries of the neutrino sector, we might try to use our monodromy group to generate a realistic family symmetry. Historically A_4 has been associated with neutrino mixing, so we have focused on exploiting this case. This gives us a model with:

$$SU(5)_{\mathrm{GUT}} \times A_4 \times U(1) \,.$$

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An A_4 monodromy requires a factorisation to a quartic part and a linear part:

$$\mathcal{C}_4 \times \mathcal{C}_1 : \left(a_1 + a_2 s + a_3 s^2 + a_4 s^3 + a_5 s^4\right) \times \left(a_6 + a_7 s\right) = 0$$

where a_i are necessarily in the same field as the original b_j - which prevents so-called branch cuts.

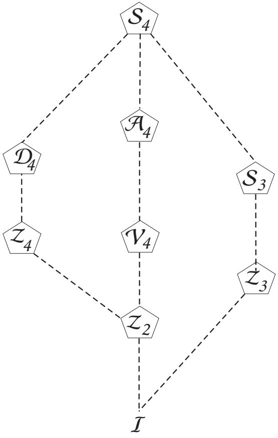
b_i	a_j coefficients for 4+1
b_0	$a_5 a_7$
b_1	$a_5 a_6 + a_4 a_7$
b_2	$a_4 a_6 + a_3 a_7$
b_3	$a_3 a_6 + a_2 a_7$
b_4	$a_2 a_6 + a_1 a_7$
b_5	$a_1 a_6$

An F-theory A_4 model - Which discrete symmetry?

The $\mathcal{C}_4 \times \mathcal{C}_1$ factorisation admits S_4 or any of its discrete subgroups as possible symmetries.

To distinguish between the different cases, we can use Galois theory and examine partially symmetric polynomials of the roots of $\mathcal{C}_4$. See work by G.Leontaris and I.Antoniadis - *arXiv:1308.1581 [hep-th]*

We can also argue from the reducibleness of the representations



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Having asserted an A_4 monodromy group, we must now determine what the matter curve content of the model must be. The defining equation of the ten-curves is:

$$b_5 = a_1 a_6 = 0\,.$$

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b_5	$a_1 a_6$

Before applying any constraints from Galois theory, the defining equation of the fives is:

$$R = \left(a_2^2 a_7 + a_2 a_3 a_6 \mp a_0 a_1 a_6^2\right)\left(a_3 a_6^2 + (a_2 a_6 + a_1 a_7) a_7\right) \,,$$

where we have taken $a_4 = \pm a_0 a_6$ and $a_5 = \mp a_0 a_7$ to satisfy the tracelessness condition of $SU(5)$: $b_1 = 0$

An F-theory A_4 model - Matter curves

Curve	Equation	Homology	N	M
10_a	a_1	$\eta - 5c_1 - \chi$	$-N$	M_{10_a}
10_b	a_6	χ	$+N$	M_{10_b}
5_c	$a_2^2 a_7 + a_2 a_3 a_6 \mp a_0 a_1 a_6^2$	$2\eta - 7c_1 - \chi$	$-N$	M_{5_c}
5_d	$a_3 a_6^2 + (a_2 a_6 + a_1 a_7) a_7$	$\eta - 3c_1 + \chi$	$+N$	M_{5_d}

Table : Table of matter curves, their homologies, charges and multiplicities.

We have an A_4 quadruplet and an A_4 singlet for the ten-curves, while for the five-curves we have a sextet and a quadruplet. This summary of our five and ten-curves demonstrates that any attempt to build a model will be difficult.

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An F-theory model - Irreducible representations

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We may then write down the generators of the group for a quadruplet of A_4, then use unitary matrices to block diagonalise into an irreducible basis. Doing this gives a singlet and triplet for the related weights:

$$t_s = t_1 + t_2 + t_3 + t_4 \tag{2}$$

$$\{t_a = t_1 + t_2 - t_3 - t_4, t_b = t_1 - t_2 + t_3 - t_4, t_c = t_1 - t_2 - t_3 + t_4\}$$

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So for the $10s$ of the GUT group we have a triplet and two singlets (one charged under t_5, one under t_s).

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Curve	Rep'n	N	M	Matter content
10_1	$(10,3)_0$	0	M_{T1}	$3\left[M_{T1}Q_L + u_L^c(M_{T1} - N_Y) + e_L^c(M_{T1} + N_Y)\right]$
10_2	$(10,1)_0$	$-N$	M_{T2}	$M_{T2}Q_L + u_L^c(M_{T2} - N_Y) + e_L^c(M_{T2} + N_Y)$
10_3	$(10,1)_{t_5}$	$+N$	M_{T3}	$M_{T3}Q_L + u_L^c(M_{T3} - N_Y) + e_L^c(M_{T3} + N_Y)$
5_1	$(5,3)_0$	0	M_{F1}	$3\left[M_{F1}\bar{d}_L^c + (M_{F1} + N_Y)\bar{L}\right]$
5_2	$(5,3)_0$	$-N$	M_{F2}	$3\left[M_{F2}\bar{\bar{D}} + (M_{F2} + N_Y)\bar{H}_d)\right]$
5_3	$(5,3)_{t_5}$	$+N$	M_{F3}	$3\left[M_{F3}D + (M_{F3} + N_Y)H_u\right]$
5_4	$(5,1)_{t_5}$	0	M_{F4}	$M_{F4}\bar{d}_L^c + (M_{F4} + N_Y)\bar{L}$

Table : Table summarising matter content for an $SU(5) \times A_4$ model

An F-theory A_4 model - $N = 0$

Choosing the simplest possible case, we assign $N = 0$ and select a realistic set of M_i:

$$M_{T1} = M_{F4} = 0$$
$$M_{T2} = 1$$
$$M_{T3} = 2$$
$$M_{F1} = M_{F2} = -M_{F3} = -1$$

This will give us the necessary generations of quarks and leptons.

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Curve	Rep'n	R-sym	Matter content
10_1	$(10, 3)_0$	1	-
$10_2 = T_3$	$(10, 1)_0$	1	$Q_L + u_L^c + e_L^c$
$10_3 = T$	$(10, 1)_{t_5}$	1	$2Q_L + 2u_L^c + 2e_L^c$
$\bar{5}_1 = F$	$(\bar{5}, 3)_0$	1	$3L + 3d_L^c$
$\bar{5}_2 = H_d$	$(\bar{5}, 3)_0$	0	$3\bar{D} + 3H_d$
$5_3 = H_u$	$(5, 3)_{t_5}$	0	$3D + 3H_u$
5_4	$(5, 1)_{t_5}$	1	-
θ_a	$(1, 3)_{-t_5}$	0	Higgs Flavons
θ_b	$(1, 1)_{-t_5}$	0	Flavon
θ_c	$(1, 3)_0$	1	ν_R
θ_d	$(1, 3)_0$	0	Flavons

Table : Table of Matter content in $N = 0$ model

An F-theory A_4 model - Top Quark Couplings

From this model, the Top-type quark couplings are non-renormalisable. This is due to the t_5 charges present on the T and H_u curves, which must be canceled by GUT-singlets to form an invariant coupling.

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$$m_{u,c,t} = va \begin{pmatrix} y_3 b^2 + y_4 a^2 & y_3 b^2 + y_4 a^2 & y_2 b \\ y_3 b^2 + y_4 a^2 & y_3 b^2 + y_4 a^2 & y_2 b \\ y_2 b & y_2 b & y_1 \end{pmatrix} \quad (3)$$

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Due to the so-called Rank Theorem the lightest quark remains massless It will get a small mass due to non-commutative fluxes and instanton effects.

An F-theory A_4 model - Bottom Quark/Charged Lepton Couplings

The Bottom quark and charged Lepton couplings come from the same operators in $SU(5)$ GUTs. The mass matrix takes the form:

$$m_{1,2,3} = v \begin{pmatrix} y_7 d_2 b + y_{11} d_3 a & y_7 d_2 b + y_{11} d_3 a & y_3 d_2 \\ y_5 a & y_5 a & y_2 d_1 \\ y_4 b & y_4 b & y_1 \end{pmatrix} . \tag{4}$$

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Note: The Standard Model doublets of $SU(2)$ are triplets under A_4. As such we can again argue that the lightest generation gets a mass from some other mechanism.

An F-theory A_4 model - Neutrinos

The Neutrinos are unique in the SM in that they are the only particles that may be Majorana - that is, the neutrino could be its own anti-particle.
The possible couplings allowed in this model are:

	Full coupling
Dirac-type mass	$\theta_c \cdot F \cdot H_u \cdot \theta_a$
	$\theta_c \cdot F \cdot H_u \cdot \theta_a \cdot (\theta_d)^n$
	$\theta_c \cdot F \cdot H_u \cdot \theta_b$
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Right-handed neutrinos	$M\theta_c \cdot \theta_c$
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Right-handed neutrinos	$M\theta_c \cdot \theta_c$
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Reminder: $\langle\theta_a\rangle = (a, 0, 0)^{\mathsf{T}}$. This preserves the S generator, which is associated with the Z_2 part of A_4. Likewise, we have $\langle H_u\rangle = (v, 0, 0)^{\mathsf{T}}$.

An F-theory A_4 model - Neutrinos

The first four operators correspond to Dirac mass terms, coupling left and right-handed neutrinos. Taking the lowest order couplings for simplicity, the Dirac mass matrix is:

$$M_D = \begin{pmatrix} y_0 va & z_3 vd_2 b & z_2 vd_3 b \\ z_1 vd_2 b & y_1 va & y_9 bv \\ z_4 vd_3 b & y_8 bv & y_1 va \end{pmatrix} \qquad (5)$$

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Higher order operators may serve to add small corrections. To the Dirac matrix. Similarly the dominating contribution to the right-handed mass matrix is the diagonal operator:

$$M_R = M \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{6}$$

N.b. $y_0 = y_1 + y_2 + y_3$

In order to reduce the number of free parameters, we observe that the following definitions will simplify our analysis:

$$
\begin{aligned}
Y_1 &= \frac{y_1}{y_0} \leq 1 \\
Y_{2,3} &= \frac{y_{8,9} b}{y_0 a} \\
Z_1 &= \frac{z_1 d_2 b}{y_0 a} \\
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M_{eff} = m_0 \begin{pmatrix} 1 + Z_1^2 + Z_2^2 & Y_1 Z_1 + Y_3 Z_2 + Z_1 & Y_2 Z_1 + Y_1 Z_2 + Z_2 \\ Y_1 Z_1 + Y_3 Z_2 + Z_1 & Y_1^2 + Y_3^2 + Z_1^2 & Y_1(Y_2 + Y_3) + Z_1 Z_2 \\ Y_2 Z_1 + Y_1 Z_2 + Z_2 & Y_1(Y_2 + Y_3) + Z_1 Z_2 & Y_1^2 + Y_2^2 + Z_2^2 \end{pmatrix}, \quad (7)
$$

We proceed to numerically fit for known neutrino parameters, centering our analysis on the value:

$$R = \left| \frac{m_3^2 - m_2^2}{m_2^2 - m_1^2} \right| ,$$

In this way, we are able to make predictions about the absolute neutrino mass scale in our model.

Parameter	Central value	Min $\to$ Max
$\theta_{12}/^\circ$	33.57	32.82$\to$34.34
$\theta_{23}/^\circ$	41.9	41.5$\to$42.4
$\theta_{13}/^\circ$	8.73	8.37$\to$9.08
$\Delta m_{21}^2/10^{-5}$eV	7.45	7.29 $\to$ 7.64
$\Delta m_{31}^2/10^{-3}$eV	2.417	2.403 $\to$ 2.431
$R = \frac{\Delta m_{31}^2}{\Delta m_{21}^2}$	32.0	31.1 $\to$ 33.0

Table : Summary of neutrino parameters, using best fit values as found at nu-fit.org .

An F-theory A_4 model - Neutrinos

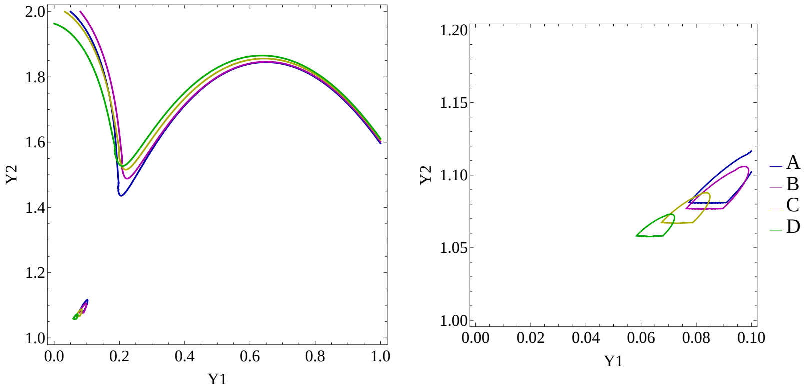

Figure : Plots of lines with the best fit value of $R = 32$ in the parameter space of (Y_1, Y_2). Left: The full range of the space examined. Right: A close plot of a small portion of the parameter space taken from the full plot. The curves have (Y_3, Z_1, Z_2) values set as follows: $A = (1.08, 0.05, 0.02)$, $B = (1.08, 0.0, 0.08)$, $C = (1.07, 0.002, 0.77)$, and $D = (1.06, 0.01, 0.065)$.

An F-theory A_4 model - Neutrinos

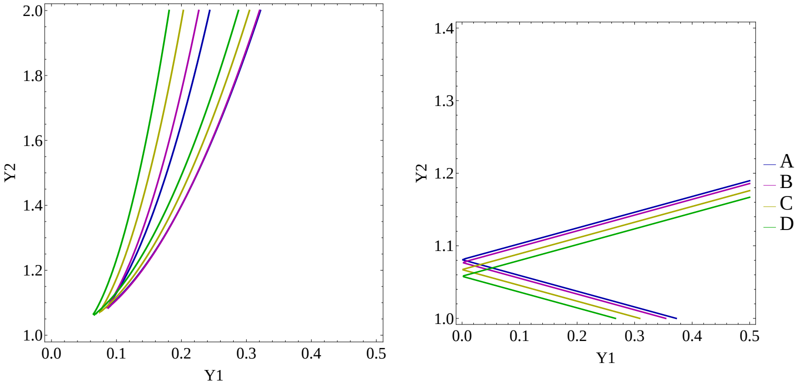

Figure : The figures show plots of two large neutrino mixing angles at their current best fit values. Left: Plot of $\sin^2(\theta_{12}) = 0.306$, Right: Plot of $\sin^2(\theta_{23}) = 0.446$. The curves have (Y_3, Z_1, Z_2) values set as follows: $A = (1.08, 0.05, 0.02)$, $B = (1.08, 0.0, 0.08)$, $C = (1.07, 0.002, 0.77)$, and $D = (1.06, 0.01, 0.065)$.

An F-theory A_4 model - Neutrinos

Inputs				
Y_1	0.08	0.09	0.09	0.10
Y_2	1.09	1.10	1.10	1.11
Y_3	1.07	1.08	1.08	1.09
Z_1	0.01	0.01	0.00	0.01
Z_2	0.07	0.08	0.08	0.08
m_0	54.0meV	51.6meV	50.3meV	47.8meV
Outputs				
θ_{12}	33.5	33.2	33.1	32.8
θ_{13}	8.70	8.82	9.05	9.05
θ_{23}	41.9	41.7	41.7	41.5
m_1	53.4meV	51.1meV	49.8meV	47.3meV
m_2	54.1meV	51.8meV	50.5meV	48.1meV
m_3	73.2meV	71.5meV	70.8meV	69.1meV

Table : Table of Benchmark values in the Parameter space, where all experimental constraints are satisfied within errors. These point are samples of the space of all possible points, where we assume θ_{23} is in the first octant. All inputs are given to two decimal places, while the outputs are given to 3s.f.

An F-theory A_4 model - Neutrinos

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Parameter	Central value	Min $\rightarrow$ Max
$\theta_{12}/^\circ$	33.57	32.82$\rightarrow$34.34
$\theta_{23}/^\circ$	41.9	41.5$\rightarrow$42.4
$\theta_{13}/^\circ$	8.73	8.37$\rightarrow$9.08
$\Delta m_{21}^2/10^{-5}$eV	7.45	7.29 $\rightarrow$ 7.64
$\Delta m_{31}^2/10^{-3}$eV	2.417	2.403 $\rightarrow$ 2.431
$R = \frac{\Delta m_{31}^2}{\Delta m_{21}^2}$	32.0	31.1 $\rightarrow$ 33.0

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These benchmark values show that it is possible to fine-tune the free parameters of this model to satisfy the constraints on neutrino parameters from experiment.

- Model disfavours inverted hierarchy

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- Model disfavours inverted hierarchy
- Model prefers first octant θ_{23}
- The absolute scale for the neutrino mass is $\geq 40meV$, with most values being $\sim 50meV$
- The sum of neutrino masses is predicted to be $< 200meV$

Concluding Remarks

- F-theory provides a natural frame work to generate discrete groups
- The $SU(5) \times A_4$ model discussed is able to match known neutrino parameters, while also predicting an absolute mass scale for neutrinos of about $m_1 > 45 mev$
- The model insists upon a normal ordered hierarchy, with θ_{23} in the first octant
- In our next work we plan to examine D_4 models in F-theory

References

1 Discrete Family Symmetry from F-Theory GUTs - A.Karozas, S.F.King, G.K.Leontaris, AM. *ArXiv:1406.6290*

2 Neutrino mass textures from F-theory - I. Antoniadis and G. K. Leontaris, Eur. Phys. J. C 73 (2013) 2670 [arXiv:1308.1581 [hep-th]].

3 Aspects of F-Theory GUTs - G.K.Leontaris,

F-Theory - Spectral Cover Equation

Elliptically fibred spaces are described by the Weierstrass equation:

$$y^2 + a_1 xy + a_3 y = x^3 + a_2 x^2 + a_4 x + a_6 \tag{8}$$

Since we are assuming an $SU(5)$ unifying symmetry, the coefficients of the Weierstrass equation are constrained by the Kodaira classification of elliptic fibration to have vanishing orders:

$$a_1 = -b_5,\ a_2 = b_4 z,\ a_3 = -b_3 z^2,\ a_4 = b_2 z^3,\ a_6 = b_0 z^5.$$

The resulting equation can be simplified to the so-called spectral cover equation by means of well chosen homogenous coordinates ($z \to U$, $x \to V^2$, $y \to V^3$), which may be written in terms of an affine parameter, $s = \frac{U}{V}$:

$$\mathcal{C}_5 :\ b_5 + b_4 s + b_3 s^2 + b_2 s^3 + b_1 s^4 + b_0 s^5 \tag{9}$$

F-Theory - Monodromy and Discrete Symmetry

Monodromy groups are discrete groups that **relate the roots** of the spectral cover equation. These are required by F-theory in order to allow for a tree level Top quark Yukawa:

$$5_H \times 10_M \times 10_M$$
$$t_j + t_k - 2t_i = 0$$

In order for the charges to cancel, two of the weights must be identified by some monodromy action - Z_2 being the minimal case. This amounts to requiring that two of the roots of the spectral cover equation must not factorise:

$$(a_1 + a_2 s + a_3 s^2)(a_4 + a_5 s)(a_6 + a_7 s)(a_8 + a_9 s) = 0$$

F-Theory - Monodromy and Discrete Symmetry

The monodromy is best understood by closely examining the quadratic part of the factorised spectral cover:

$$(a_1 + a_2 s + a_3 s^2) = 0$$

$$s_{\pm} = \frac{-a_2 \pm \sqrt{a_2 - 4a_1 a_3}}{2a_3}$$

we see that since $\sqrt{a_2 - 4a_1 a_3} = e^{i\theta/2}\sqrt{|a_2 - 4a_1 a_3|}$, under $\theta \to \theta + 2\pi$, the two solutions interchange.

Since we do not know anything about the global geometry, in semi-local F-theory we must choose our monodromy group.

An F-theory A_4 model - Rank Theorem

Rank theorem (*arXiv: 0811.2417*) - In F-theory, the Yukawa couplings of quarks and leptons to the Higgs are given by triple overlap integral:

$$\lambda^{ij} = \int_S \Lambda \Psi^i \Phi^j \,,$$

over their intersection in the GUT surface - S. Since this constitutes points of intsections (p), it amounts to a sum of the products of the wavefunctions at those points,

$$\lambda^{ij} = \sum_p \Lambda(p) \Psi^i(p) \Phi^j(p) \,.$$

The triple-intersection points of interest are those corresponding to the superpotential terms $10_M \cdot 10_M \cdot 5_{H_u}$ and $10_M \cdot \bar{5}_M \cdot \bar{5}_{H_d}$. Multiple triple-intersection points may exist. However, the minimal case will only have **one intersection** and so only **one Yukawa interaction**. We expect the sub-matrix of Yukawa interactions to have only one non-zero entry and as such to be trivially **rank one**.

Extra Stuff - Table of Operators

Coupling type	Generations	Full coupling
Top-type	Third generation	$T_3 \cdot T_3 \cdot H_u \cdot \theta_a$
	Third-First/Second generation	$T \cdot T_3 \cdot H_u \cdot \theta_a \cdot \theta_b$
		$T \cdot T_3 \cdot H_u \cdot (\theta_a)^2$
	First/Second generation	$T \cdot T \cdot H_u \cdot \theta_a \cdot (\theta_b)^2$
		$T \cdot T \cdot H_u \cdot (\theta_a)^2 \cdot \theta_b$
		$T \cdot T \cdot H_u \cdot (\theta_a)^3$
Bottom-type/Charged Leptons	Third generation	$F \cdot H_d \cdot T_3$
		$F \cdot H_d \cdot T_3 \cdot \theta_d$
	First/Second generation	$F \cdot H_d \cdot T \cdot \theta_b$
		$F \cdot H_d \cdot T \cdot \theta_a$
		$F \cdot H_d \cdot T \cdot \theta_a \cdot \theta_d$
		$F \cdot H_d \cdot T \cdot \theta_b \cdot \theta_d$
Neutrinos	Dirac-type mass	$\theta_c \cdot F \cdot H_u \cdot \theta_a$
		$\theta_c \cdot F \cdot H_u \cdot \theta_a \cdot \theta_d$
		$\theta_c \cdot F \cdot H_u \cdot \theta_b$
		$\theta_c \cdot F \cdot H_u \cdot \theta_b \cdot \theta_d$
	Right-handed neutrinos	$M\theta_c \cdot \theta_c$
		$(\theta_d)^n \cdot \theta_c \cdot \theta_c$

Table : Table of all mass operators for $N = 0$ model.

Extra Stuff - Basis

The generators for the triplets of A_4 are:

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

The bases of the triplets are such that for two triplet $3_a = (a_1, a_2, a_3)^\mathsf{T}$ and $3_b = (b_1, b_2, b_3)^\mathsf{T}$ the product of those triplet $3_a \times 3_b = 1 + 1' + 1'' + 3_1 + 3_2$, behaves as:

$$1 = a_1 b_2 + a_2 b_2 + a_3 b_3$$

$$1' = a_1 b_2 + \omega a_2 b_2 + \omega^2 a_3 b_3$$

$$1'' = a_1 b_2 + \omega^2 a_2 b_2 + \omega a_3 b_3$$

$$3_1 = (a_2 b_3, a_3 b_1, a_1 b_2)^\mathsf{T}$$

$$3_2 = (a_3 b_2, a_1 b_3, a_2 b_1)^\mathsf{T}$$