Flavour GUT models with $\theta_{13}^{\text{PMNS}} = \theta_C/\sqrt{2}$

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based on arXiv:1305.6612 & arXiv:1306.3984
with S. Antusch, C. Gross and V. Maurer.

Motivation

The measured value of $\theta_{13}^{\text{PMNS}}$ is given by

$$\theta_{13}^{\text{PMNS}} = 8.68^\circ \pm 0.75^\circ .$$

Daya-Bay Collaboration, arXiv:1210.6327 (and also T2K, Minos, Double-Chooz, Reno)

This is close to

$$\theta_{13}^{\text{PMNS}} = \frac{\theta_C}{\sqrt{2}} = 9.21^\circ \pm 0.03^\circ \ ,$$

where θ_C is the Cabibbo angle $\theta_{12}^{\text{CKM}} = 13.02^\circ \pm 0.04^\circ$.

J. Beringer et al. (Particle Data Group), Phys. Rev. D86, 010001 (2012)

Can one explain this characteristic as result from Grand Unified Theories?

Part I

Masses and mixing angles in SU(5) Grand Unification reviewed

$SU(5)$ Grand Unification

- Unification of the Standard Model forces

$$SU(5) \to SU(3) \times SU(2)_L \times U(1)_Y$$

- Unification of the fermions in joint representations of the unifying gauge group.

$$\bar{\mathbf{5}}_i \supset d_i^c, L_i \quad \mathbf{10}_i \supset u_i^c, e_i^c, Q_i$$

$\Rightarrow$ Predictions for quark-lepton mass ratios

$SU(5)$ GUT mass relations

These ratios of down-type quark and charged lepton masses emerge when $SU(5)$ gets broken

Clebsch-Gordan factors c_{ij} from $SU(5)$ Group Theory:

- $b - \tau$ unification: $y_b/y_\tau = c_{33} = 1$

$$y_{33}\bar{\mathbf{5}}_3\mathbf{10}_3 H_{\bar{5}}$$

- Georgi-Jarlskog (GJ) relation: $y_\mu/y_s = c_{22} = 3$

$$y_{22}\bar{\mathbf{5}}_2\mathbf{10}_2 H_{\overline{45}}$$

(and also $y_e/y_d = 1/3$ if $y_{11} = 0$)

H. Georgi and C. Jarlskog, Phys. Lett. B **86** (1979) 297 .

New mass relations from effective operators

Typical Flavour Model:
Fermion mass hierarchy $\leftrightarrow$ effective operators $\frac{\Phi^n}{\Lambda^n}\bar{\mathbf{5}}_i \mathbf{10}_j H_{\bar{5}}$.
C.D. Froggatt and H.B. Nielsen, Nucl. Phys. B **147** (1979) 277 .

- New mass relations can be found from e.g. dimension-5 operators

$$y_{33}\left(\bar{\mathbf{5}}_3 \frac{H_{24}}{\Lambda}\right)\left(\mathbf{10}_3 H_{\bar{5}}\right) \to \frac{y_\tau}{y_b} = \frac{3}{2}$$

$$y_{22}\left(\bar{\mathbf{5}}_2 H_{\bar{5}}\right)\left(\mathbf{10}_2 \frac{H_{24}}{\Lambda}\right) \to \frac{y_\mu}{y_s} = 6$$

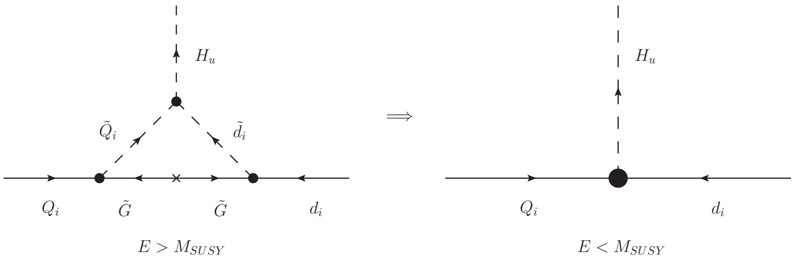
$$y_{22}\left(\bar{\mathbf{5}}_2 \frac{H_{24}}{\Lambda}\right)\left(\mathbf{10}_2 H_{\overline{45}}\right) \to \frac{y_\mu}{y_s} = \frac{9}{2}$$

S. Antusch and M. Spinrath, arXiv:0902.4644 .

UV-completion of the effective theory with messenger fields is required for a predictive model!

Remarks

- Which ratios are phenomenological viable?



Threshold corrections depend on supersymmetry parameters.

- There are also GUT relations for the mixing angles θ^d_{ij}, θ^e_{ij}

$\theta_{13}^{\mathrm{PMNS}} \approx \theta_C/\sqrt{2}$ from GUT mixing angle relations

$$
\begin{tikzcd}
m_\nu \arrow[rr, leftrightarrow, "U_{\mathrm{PMNS}} = U_e^\dagger U_\nu"] & & Y_e \arrow[d, leftrightarrow, "SU(5)"] \\
Y_u \arrow[rr, leftrightarrow, "U_{\mathrm{CKM}} = U_u^\dagger U_d"] & & Y_d
\end{tikzcd}
$$

$$\theta_{12}^e = \frac{c_{12}}{c_{22}} \theta_{12}^d$$

- $\theta_{13}^{\mathrm{PMNS}}$ from $U_e^\dagger$ $(\theta_{13}^e \approx \theta_{13}^\nu \approx 0)$
- $c_{12} = c_{22}$
- $\theta_{12}^d \approx \theta_C$

$$\theta_{13}^{\mathrm{PMNS}} \approx \theta_{12}^e \sin\left(\theta_{23}^{\mathrm{PMNS}}\right) \approx \frac{\theta_C}{\sqrt{2}}$$

S. Antusch, C. Gross, V. Maurer and CS, arXiv:1205.1051 .

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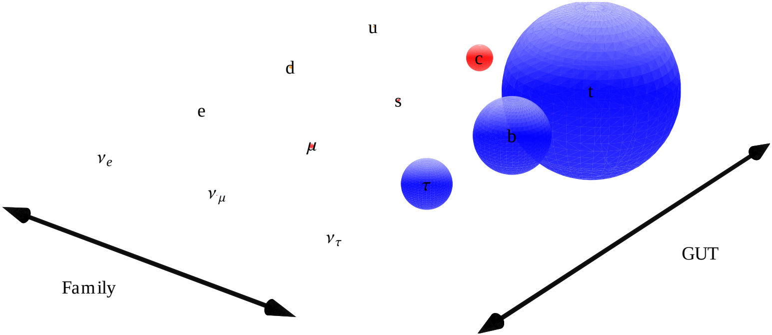
Part II

Constructing a flavour GUT model with $\theta_{13}^{\text{PMNS}} = \frac{\theta_C}{\sqrt{2}}$

Family symmetries allow unification in a 'Double Sense'



A_4 family symmetry

Smallest discrete group with irreducible **3** representation is A_4

$$
\begin{array}{lcccc}
 & F = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} & T_1, T_2, T_3 & N_1, N_2 & H_{R_i} \\
SU(5) \times A_4 & \bar{\mathbf{5}} \times \mathbf{3} & \mathbf{10} \times \mathbf{1} & \mathbf{1} \times \mathbf{1} & \mathbf{R_i} \times \mathbf{1}
\end{array}
$$

$$R_i = 5, \bar{5}, \overline{45}, 24$$

Yukawa matrices and Flavon fields

A_4 is spontaneously broken by flavour Higgs fields (flavons) in $\mathbf{3}$ of A_4.

$$\frac{\langle\phi_2\rangle}{\Lambda} \propto \epsilon_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \frac{\langle\phi_3\rangle}{\Lambda} \propto \epsilon_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \frac{\langle\phi_{ab}\rangle}{\Lambda} \propto \epsilon_{ab} \begin{pmatrix} \cos\theta_{ab} \\ -i\sin\theta_{ab} \\ 0 \end{pmatrix}$$

$$\frac{\langle\phi_{N_1}\rangle}{\Lambda} \propto \epsilon_{N_1} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \quad \frac{\langle\phi_{N_2}\rangle}{\Lambda} \propto \epsilon_{N_2} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

In addition CP symmetry is spontaneously broken.

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After A_4 and $SU(5)$ are broken to the SM gauge group the Flavon VEVs become rows/columns of Y_d/Y_e

$$\left(T_3 H_{\overline{5}}\right)\left(F\frac{H_{24}}{\Lambda}\right)\frac{\langle\phi_3\rangle}{\Lambda} \longrightarrow \tilde{\epsilon}_3 Q_3 d_3^c H_d - \frac{3}{2}\tilde{\epsilon}_3 L_3 e_3^c H_d$$

In order to predict Yukawa structure: UV complete effective operators with messenger fields and additional shaping symmetries

- Specific contraction of matter fields
- Alignment of flavon vevs $\langle\phi_i\rangle$

More details in our paper arXiv:1305.6612

Numerical fit to measured observables

- We use one-loop RGEs for the MSSM and SM, respectively, to evolve the model from M_{GUT} to $m_t(m_t)$.
- SUSY threshold corrections are included at the matching of MSSM to SM at $\Lambda_{\mathrm{SUSY}} = 1$ TeV.
- 14 parameters fitted to 18 observables.
- In addition, our model predicts δ^{PMNS} and φ^{PMNS} $\Rightarrow$ Our model makes 6 predictions.
- We find a best fit point with reduced χ^2 of $\chi^2/d.o.f. = 2.0$.
- Uncertainties are obtained from a Markov Chain Monte Carlo analysis.

Results

Observable		Value at m_t		Best fit result	Uncertainty
m_u	in MeV	1.22	${}^{+0.48}_{-0.40}$	1.22	${}^{+0.49}_{-0.40}$
m_c	in GeV	0.59	± 0.08	0.59	± 0.08
m_t	in GeV	162.9	± 2.8	162.89	${}^{+2.62}_{-2.36}$
m_d	in MeV	2.76	${}^{+1.19}_{-1.14}$	2.73	${}^{+0.30}_{-0.70}$
m_s	in MeV	52	± 15	51.66	${}^{+5.60}_{-13.68}$
m_b	in GeV	2.75	± 0.09	2.75	± 0.09
m_e	in MeV	0.485	$\pm 1\%$	0.483	± 0.005
m_μ	in MeV	102.46	$\pm 1\%$	102.83	${}^{+1.01}_{-0.98}$
m_τ	in MeV	1742	$\pm 1\%$	1741.75	${}^{+17.38}_{-17.10}$
$\sin\theta_C$		0.2254	± 0.0007	0.2255	± 0.0007
$\sin\theta_{23}^{\rm CKM}$		0.0421	± 0.0006	0.0422	± 0.0006
$\sin\theta_{13}^{\rm CKM}$		0.0036	± 0.0001	0.0036	± 0.0001
$\delta^{\rm CKM}$	in $^\circ$	69.2	± 3.1	65.65	${}^{+1.78}_{-0.53}$
$\sin^2\theta_{12}^{\rm PMNS}$		0.306	± 0.012	0.317	± 0.006
$\sin^2\theta_{23}^{\rm PMNS}$		0.437	${}^{+0.061}_{-0.031}$	0.387	${}^{+0.017}_{-0.023}$
$\sin^2\theta_{13}^{\rm PMNS}$		0.0231	${}^{+0.0023}_{-0.0022}$	0.0269	${}^{+0.0011}_{-0.0015}$
$\delta^{\rm PMNS}$	in $^\circ$	-		268.79	${}^{+1.32}_{-1.72}$
$\varphi_2^{\rm PMNS}$	in $^\circ$	-		297.34	${}^{+8.66}_{-10.01}$
$\Delta m^2_{\rm sol}$	in 10^{-5} eV^2	7.45	${}^{+0.19}_{-0.16}$	7.45	${}^{+0.18}_{-0.17}$
$\Delta m^2_{\rm atm}$	in 10^{-3} eV^2	2.421	${}^{+0.022}_{-0.023}$	2.421	${}^{+0.022}_{-0.023}$

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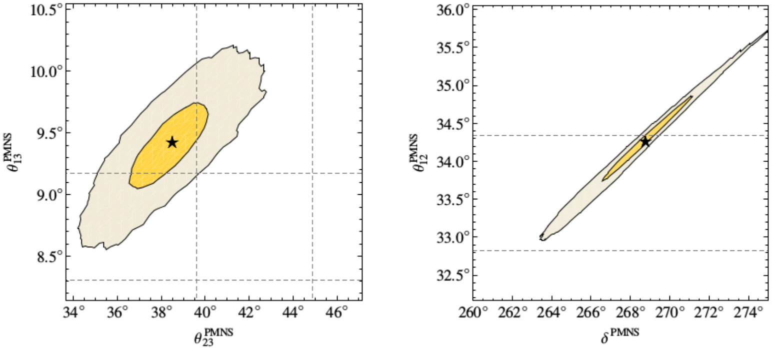

$$\theta_{13}^{\mathrm{PMNS}} \simeq \theta_{12}^{e} \sin(\theta_{23}^{\mathrm{PMNS}}) \simeq \theta_C/\sqrt{2}$$

$$\theta_{12}^{\mathrm{PMNS}} \simeq \theta_{12}^{\nu} + \theta_{12}^{e} \cos(\theta_{23}^{\mathrm{PMNS}}) \cos(\delta^{\mathrm{PMNS}})$$

The yellow and grey regions give 1σ and 3σ highest posterior density regions of the MCMC. The star marks the best fit point. Dashed grey lines indicate the 1σ intervals of the measured observables.

A second model with IH ($\chi^2/d.o.f. = 1.1$)

- $\sin^2(\theta_{23}^{\text{PMNS}}) < 0.5$

- $\delta^{\text{PMNS}} = \begin{cases} 268.79^{\circ}{}^{+1.32^{\circ}}_{-1.72^{\circ}} & \text{NH} \\ 180^{\circ} & \text{IH} \end{cases}$

- $\theta_{12}^{\text{PMNS}} = \begin{cases} 34.29^{\circ}{}^{+0.35^{\circ}}_{-0.39^{\circ}} & \text{NH} \\ 33.38^{\circ}{}^{+0.30^{\circ}}_{-0.28^{\circ}} & \text{IH} \end{cases}$

- $m_{\beta\beta} = \begin{cases} (2.31^{+0.12}_{-0.09}) \cdot 10^{-3}\text{eV} & \text{NH} \\ (1.83^{+0.05}_{-0.06}) \cdot 10^{-2}\text{eV} & \text{IH} \end{cases}$

Summary and Conclusion

- The relation $\theta_{13}^{\mathrm{PMNS}} = \frac{\theta_C}{\sqrt{2}}$ could be a footprint of a Grand Unified Theory.

- Using an A_4 family symmetry we have constructed two viable flavour GUT models, with $\Delta m^2_{\mathrm{atm}} > 0$ and < 0, respectively.

- Each model makes 6 predictions, allowing them to be tested and distinguished in the next round of neutrino oscillation experiments, $0\nu\beta\beta$ decay experiments and precision measurements of δ^{CKM}.

End of the talk

Thank you for your attention!

Towards predictive flavour models in SUSY SU(5) GUTs with doublet-triplet splitting

Heavy triplet in H_5 leads to proton decay:

$$W_{\cancel{B}} = \frac{1}{M_T^{\text{eff}}} \left[\frac{1}{2} Y_{qq}^{ij} Y_{ql}^{mn} Q_i Q_j Q_m L_n + Y_{ue}^{ij} Y_{ud}^{mn} \bar{U}_i \bar{E}_j \bar{U}_m \bar{D}_n \right]$$

Double Missing Partner Mechanism with two H_{24}'s and predictive Yukawa mass ratios:

- $\frac{y_\tau}{y_b} = -\frac{3}{2}$, $\frac{y_\mu}{y_s} = 6$, $\frac{y_e}{y_d} = -\frac{1}{2}$
- $M_T^{\text{eff}} \gtrsim 10^{17}$ GeV
- Full control over all Yukawa matrices

S. Antusch, I. de Medeiros Varzielas, V. Maurer, CS and M. Spinrath arXiv: 1405.6962