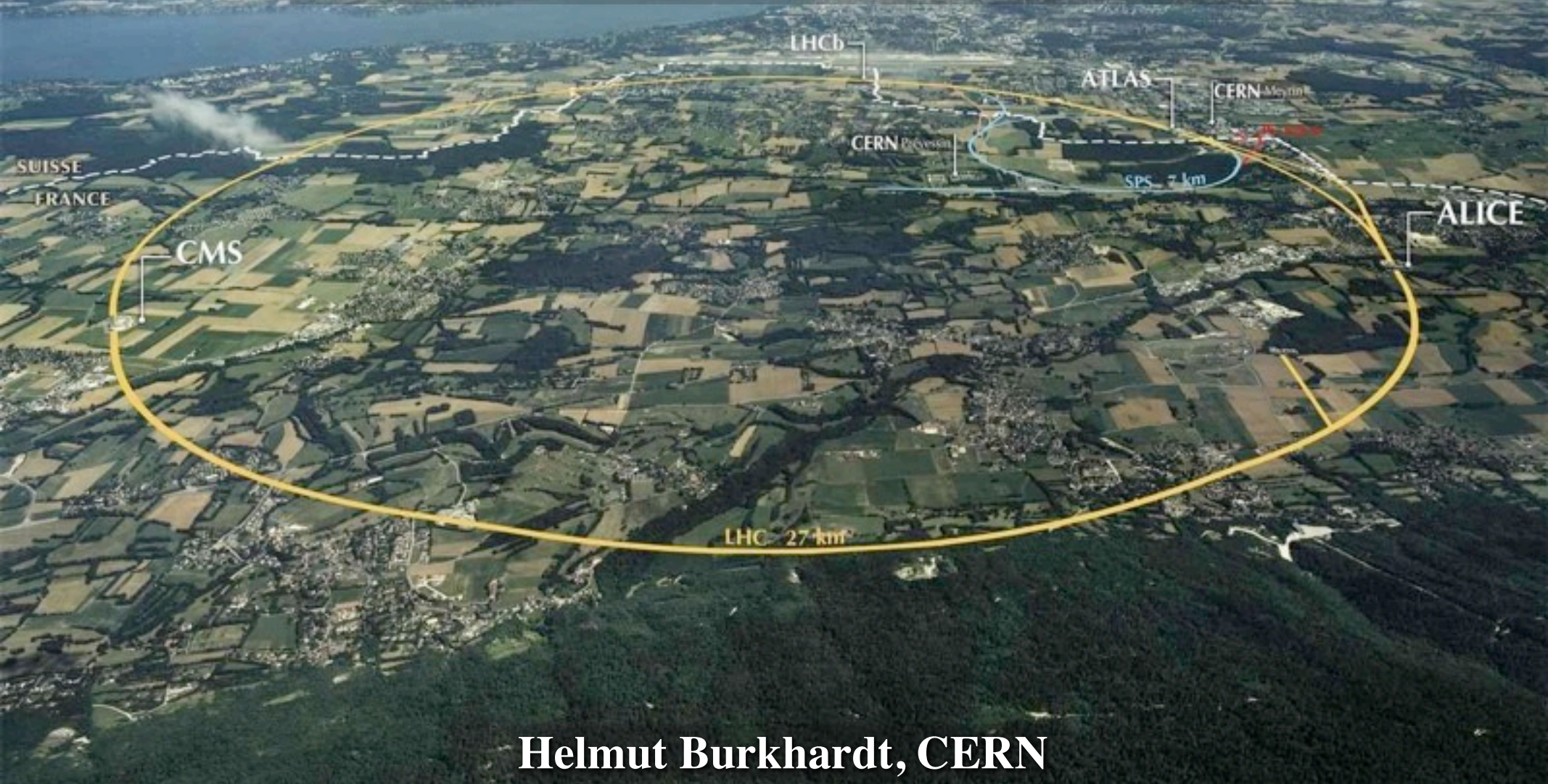


Teilchenbeschleuniger

die grössten wissenschaftlichen Instrumente



Helmut Burkhardt, CERN



Contents

- **Energy Gain, E / B field. Units**
- **Types of accelerators : Ring, Collider, Linac, e+e-, pp ; Cosmic**
- **Components: Source, Magnets, resonant Cavities**
- **Basic machine optics**
- **Energy and Luminosity**
- **Synchrotron Radiation, Vacuum**

- **Examples from CERN with LHC**
- **Plans and ideas for future high energy accelerators**
- **Accelerator applications and R&D**
- **Discussion, questions**

General, introductory refs. and books on Accelerators :

E. D. Courant and H. S. Snyder, *Theory of the Alternating-Gradient Synchrotron*, [pdf](#)

M. Sands, *Physics of Electron Storage Rings*, [SLAC Report No. 121](#); Wiedemann, *Particle Accelerator Physics* Bd. I,II

S.Y. Lee, *Accelerator Physics*, [World Scientific](#); M. Conte, W. MacKay, *Physics of Particle Accelerators*, [World Scientific](#)

CERN CAS yellow reports ; K. Wille, *The physics of particle accelerators*, Oxford University Press, 1996

Accelerators for Particle Physics, H. Burkhardt, in Handbook of Particle Detection and Imaging, [Ed. C. Grupen](#), Oct. 2011

The Large Hadron Collider : O. Brüning, H. Burkhardt, S. Myers, [10.1016/j.ppnp.2012.03.001](#), [CERN-ATS-2012-064](#)

Accelerators and Colliders, Landolt-Börnstein New Series I/21C, [Springer 2013](#)

Accelerators at the Energy Frontier

Livingston plot

Exponential growth
of E_{cm} in **time**

Starting in 60's
with e^+e^- at about 1 GeV

Factor 4 every 10 y

$pp, p\bar{p}$: $E_{cm} / 6$
still $5 \times$ above e^+e^- at
same time

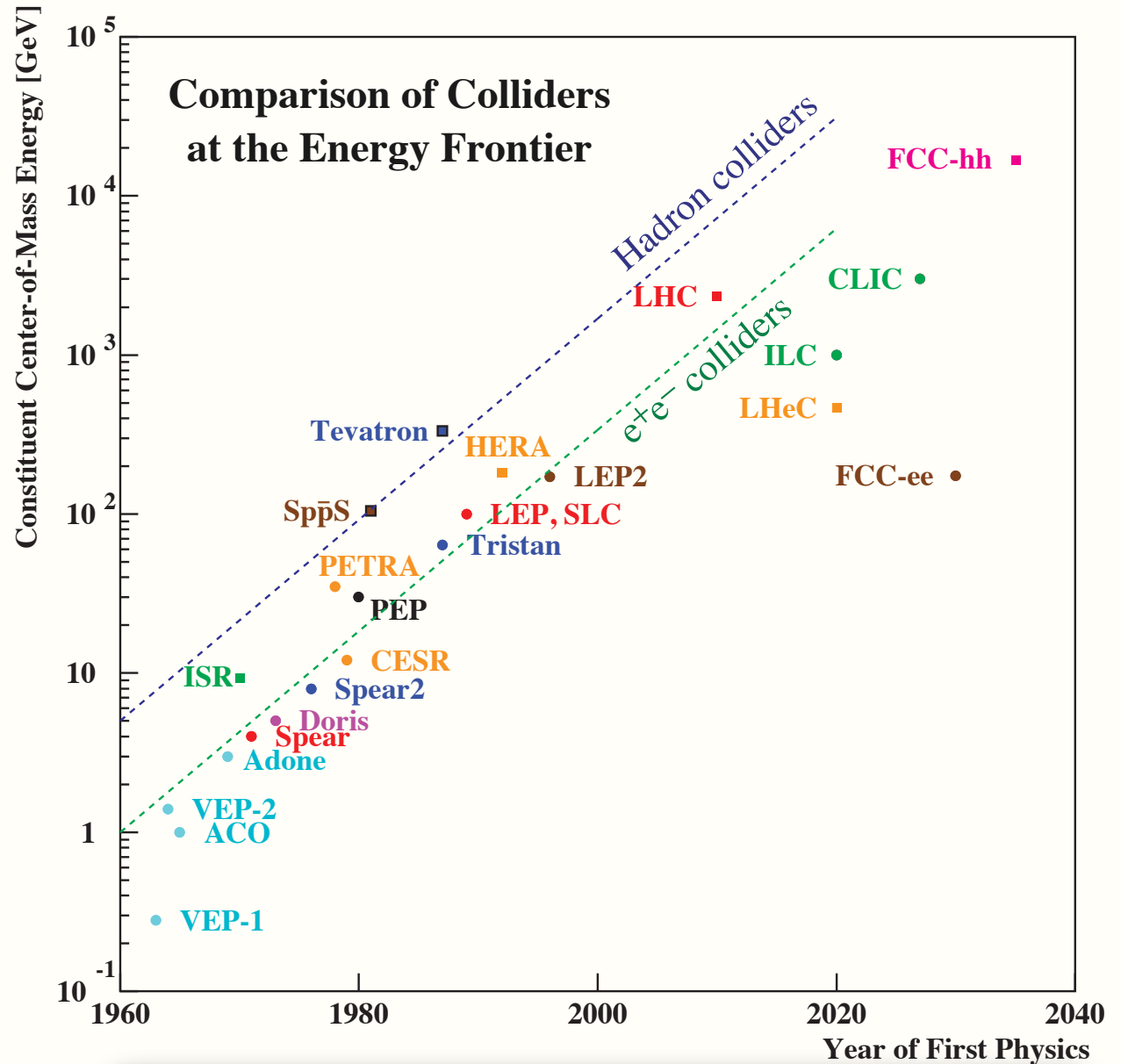
$pp, p\bar{p}$: **discovery**

e^+e^- : **precision**

both required machines

+ ep : hadron structure, QCD

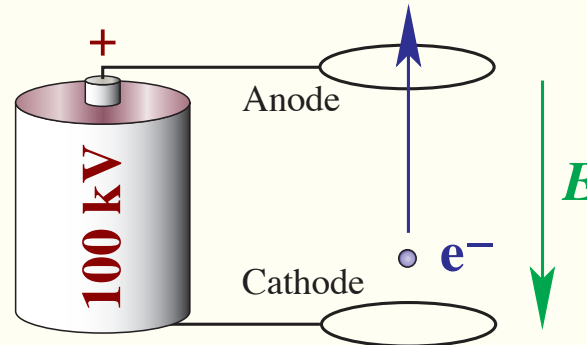
HERA, LHeC



LHC big step --- Higgs discovery
we already look well beyond

Basic concepts and units

Electric field :
Acceleration
 or rather
Energy gain
100 keV



Electric charge **e**
 and electric field **E**

Special relativity, Lorentz transformation

$$E = \gamma m c^2 \quad p = \beta \gamma m c \quad \beta = \frac{v}{c} \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

$$m_e \approx 0.511 \text{ MeV}/c^2 \quad m_p \approx 938 \text{ MeV}/c^2 \quad e \approx 1.602 \times 10^{-19} \text{ C}$$

For $E = 10 \text{ GeV}$:

Electron $\beta = 0.999\,999\,9987 \quad \gamma = 19569.5$

Proton $\beta = 0.995\,588\,4973 \quad \gamma = 10.6579$

Unit conversion

$$\frac{e^2}{4\pi\epsilon_0} = \alpha \hbar c = r_{\text{part}} m_{\text{part}} c^2 = 1.43996 \times 10^{-18} \text{ GeV m}$$

$$\hbar c = 197.327 \times 10^{-18} \text{ GeV m}$$

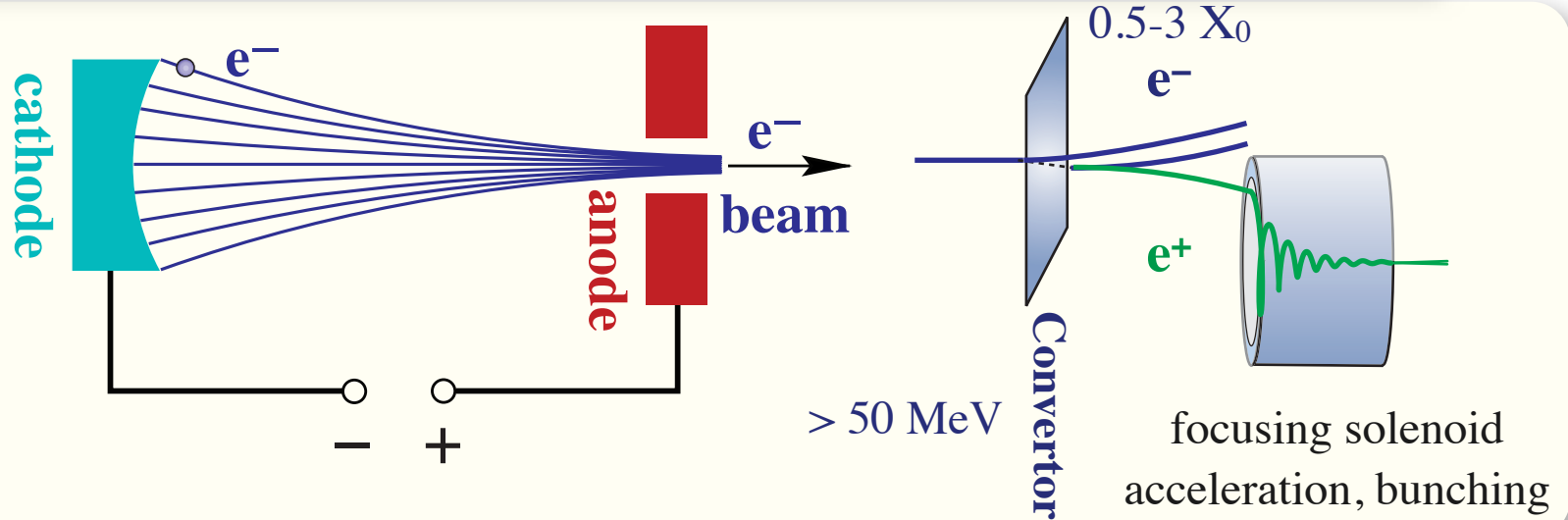
$$(\hbar c)^2 = 3.8938 \times 10^{-32} \text{ GeV}^2 \text{ m}^2 = 3.8938 \times 10^5 \text{ GeV}^2 \text{ nb}$$

for precise numbers see [PDG](#)

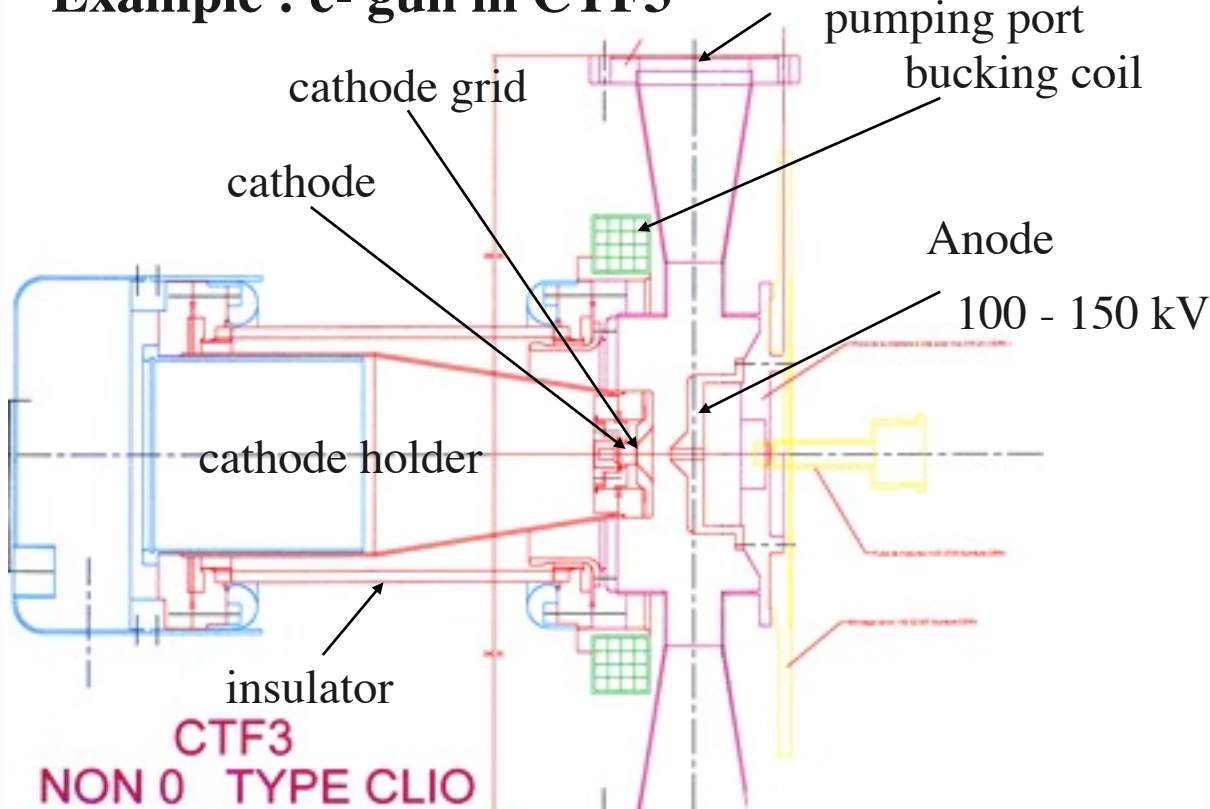
giga G = 10^9 tera T = 10^{12} peta P = 10^{15} exa E = 10^{18} zetta Z = 10^{21} yotta Y = 10^{24}

Particle sources

Thermionic electron source principle same as cathode ray tube



Example : e- gun in CTF3



challenges :

high intensity

polarized e^- sources

damping rings for minimum emittance

undulator polarized e^+ sources

Proton and ion sources

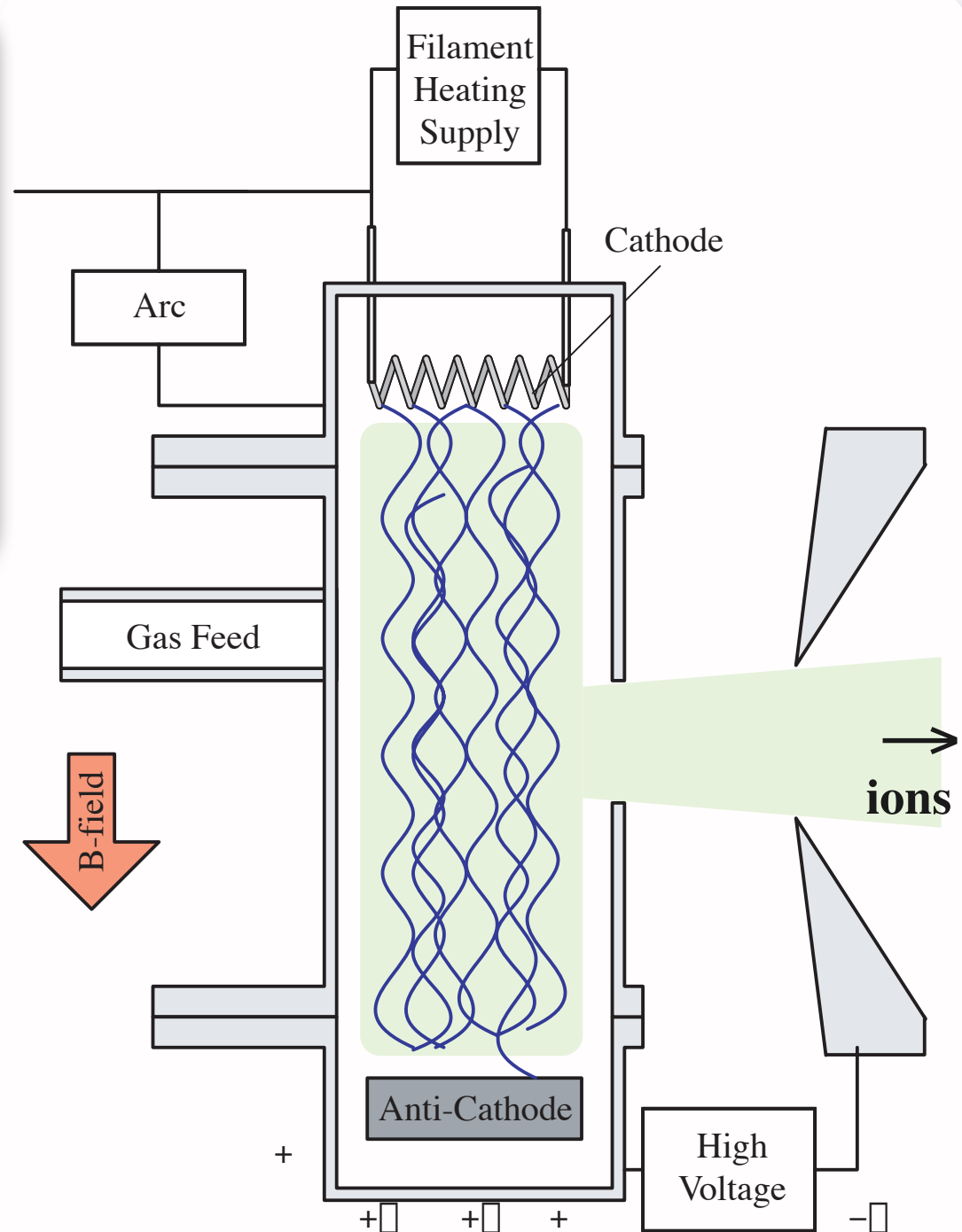
Various methods exist to produce p (H^+), H^- (p with 2 e^-) and heavy ions - heavier atoms, most electrons removed

Typically involves : **low pressure heated gas ionized gas / plasma**, inject H_2 to get protons, **or surface sputtering** and **electric and magnetic fields** to keep the electrons

CERN p-source and 50 MeV Linac



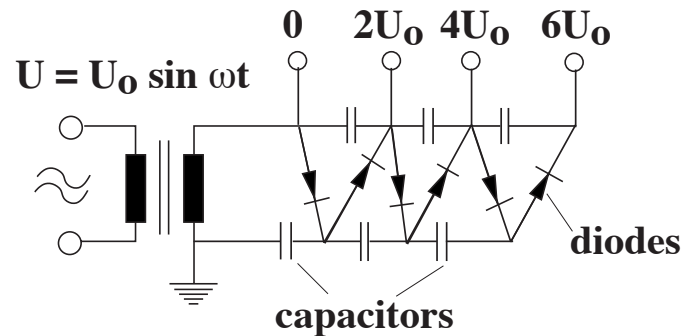
special techniques
 H^- injection
RadioFrequency
Quadrupole



Linear Acceleration with Electrostatic Field

allows for DC, 100 % duty factor
limited by HV-breakdown $\sim 1 \text{ MV} / \text{m}$

**Cockcroft Walton
voltage multiplier**

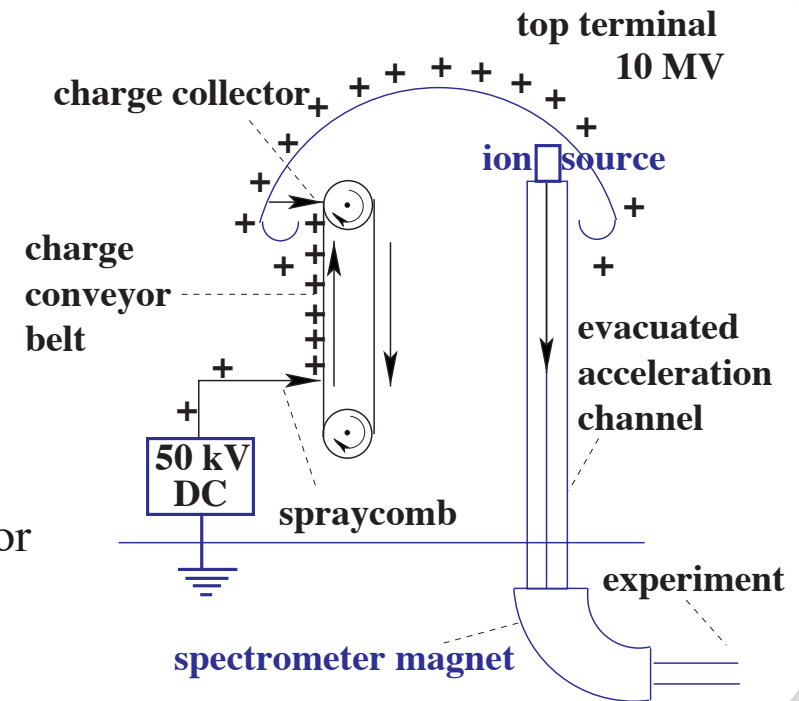


800 kV
proton pre-
injector
used at
CERN
until 1993



Van de Graaff generator
static electricity from belts

Oak Ridge Tandem Van de Graaff generator
reached 25.5 MV using pressurised SF₆



Time Varying Fields

Radio-frequency or short RF acceleration

- allows for multiple passages
- bunched beams, reduced duty cycle
- higher RF frequencies allow for higher acceleration gradients

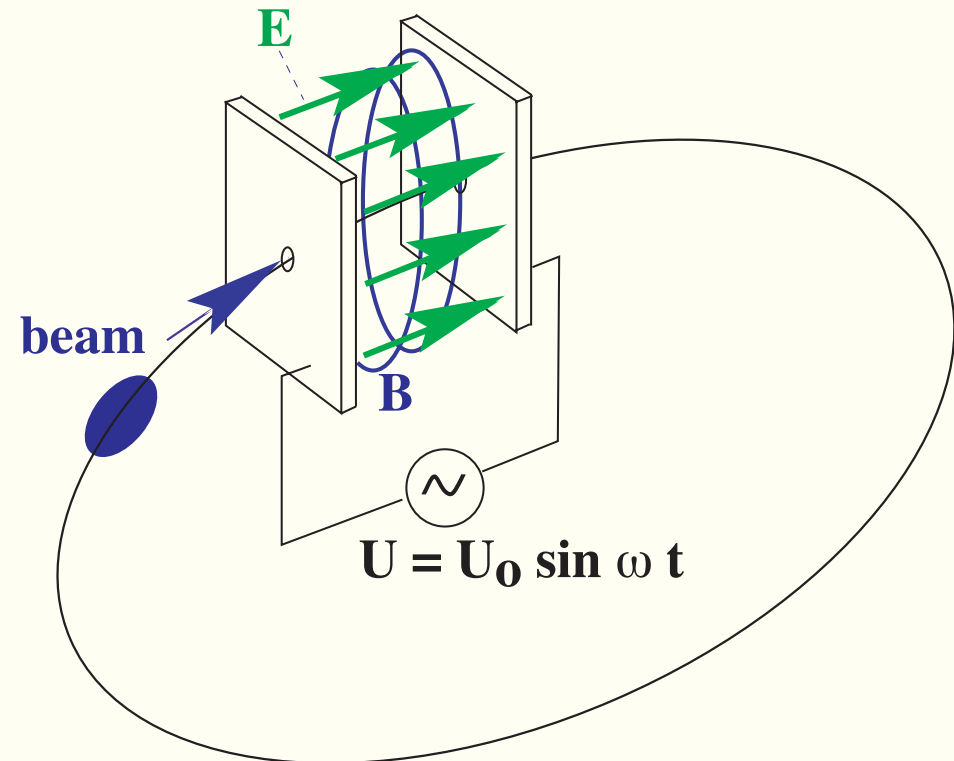
no time for breakdown / flashover

LEP , SC	8 MV / m at 352 MHz
Tesla / ILC, SC	31.5 MV / m at 1.3 GHz
CLIC	100 MV / m at 12 GHz

little gain above 12 GHz

SC limit ~ 50 MV/m, reached for single cell surface gradients higher than acceleration gradients, smooth structures

high f : shorter bunches - collective effects (peak current) and alignment more difficult
less energy stored in structure



Basic parameters, Lorentz Force

$$\mathbf{F} = q (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

charge q , normally $q = e$; $q = Z e$ for ions

- Electric field \mathbf{E} provides the acceleration or rather energy gain
- The magnetic field \mathbf{B} keeps the particles on their path

ρ is the radius of curvature for motion perpendicular to the static magnetic field. Often called

- gyromagnetic or Larmor radius in astroparticle physics
 - bending radius for accelerators
- $B\rho$ known as magnetic rigidity, units Tm

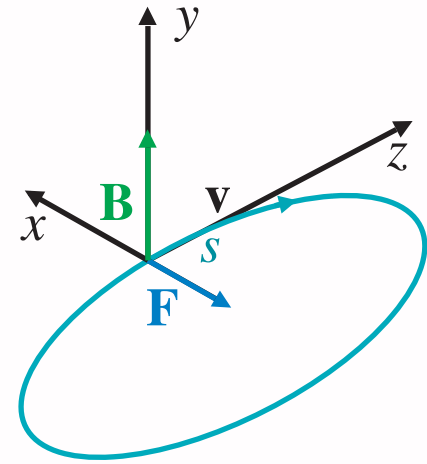
LHC

- Momentum $p = 7 \text{ TeV}/c$
- LHC bending radius $\rho = 2804 \text{ m}$
- Bending field $B = 8.33 \text{ Tesla}$
- magnets at 1.9 K , super-fluid He

Circular motion for

$$\mathbf{E} = 0$$

$$\mathbf{v} \perp \mathbf{B}$$



$$B = \frac{p}{q \rho}$$

for $q = e$ numerically
 $B \text{ [T]} = p \text{ [GeV}/c] \cdot 3.336 \text{ m} / \rho$
high energy, $v = c$ “ $p = E$ ”
 $E < E_H = q B \rho$ Hillas criterion

Astroparticle

units $10^{-4}\text{T} = 1\text{Gauss}$; a.u. = $1.5 \times 10^{11}\text{m}$

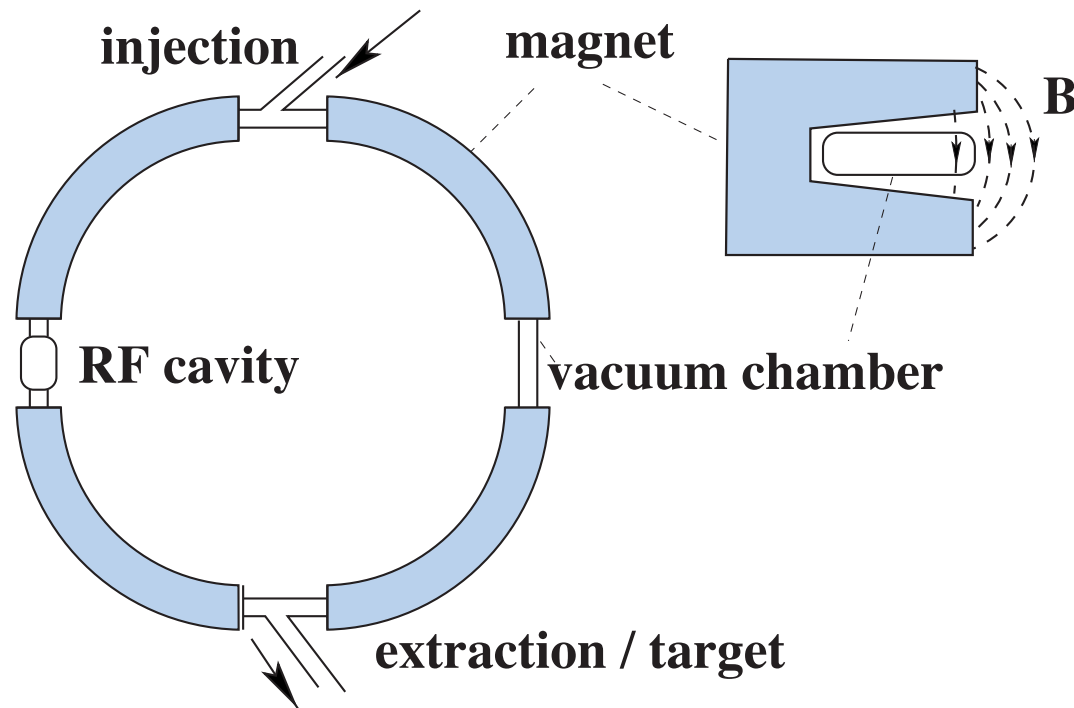
Solar system $B = 10 \mu\text{G}$ $E = 5 \text{ TeV}$ $\rho = 11 \text{ a.u.}$

Intergalactic $B = 1 \text{ nG}$ $E = 5 \text{ PeV (knee)}$

$\rho = 1.7 \times 10^{19}\text{m}$ (4 % of galaxy-radius)

Circular Accelerator

- **Cyclotron** : constant rf-frequency. Magnetic field radius ρ increases with energy. Used for smaller machines
- **Synchrotron** : $\rho = \text{const.}$ **B increased with energy.** RF-frequency adjusted slightly ($\beta = 0.999 \dots 1.0$). Most HEP and all CERN ring accelerators PS, SPS, LEP, LHC of this type. Principle same for e, p, heavy-ion – PS, SPS – accelerate(d) all of these, in some cases switching within seconds



Phase stability I

acceleration,
ramping up in energy :

- allow for enough RF-voltage
- ramp up magnets
- particle adjust themselves in radius and phase to gain on average the right amount of energy

LHC nominal RF parameters

Voltage at injection 8 MV

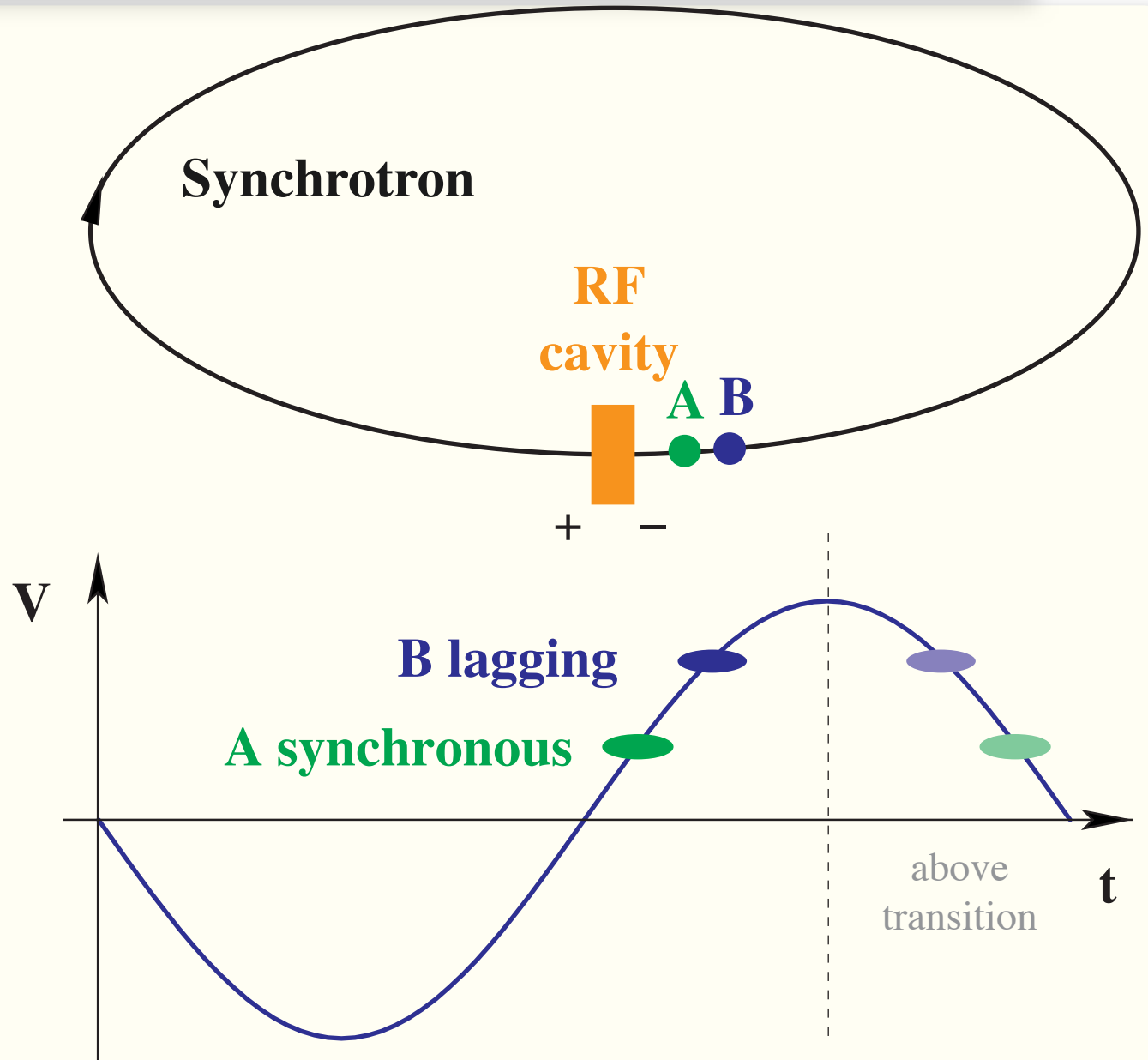
top energy 16 MV

Revolution frequency $f_{rf} = h f_{rev}$

Circumference $L = v / f_{rev} = \beta c / f_{rev}$

$h = 35\,640$ $f_{rf} = 400.7896$ MHz $L = 26658.864$ m

$f_{rev} = 11.2455$ kHz 1 turn in 88.92446 μ s



Magnets and Power Consumption

Why super conducting magnets ?

$$P = R I^2$$

LEP

B = 0.1 T LEP2 ~ 100 GeV

(half) cells with each three 11.55 m long dipole magnets

I = 4500 A together **R = 1 mΩ** **P = 20 kW / cell**

488 cells

P = 10 MW

if we would have kept the same magnets for the LHC

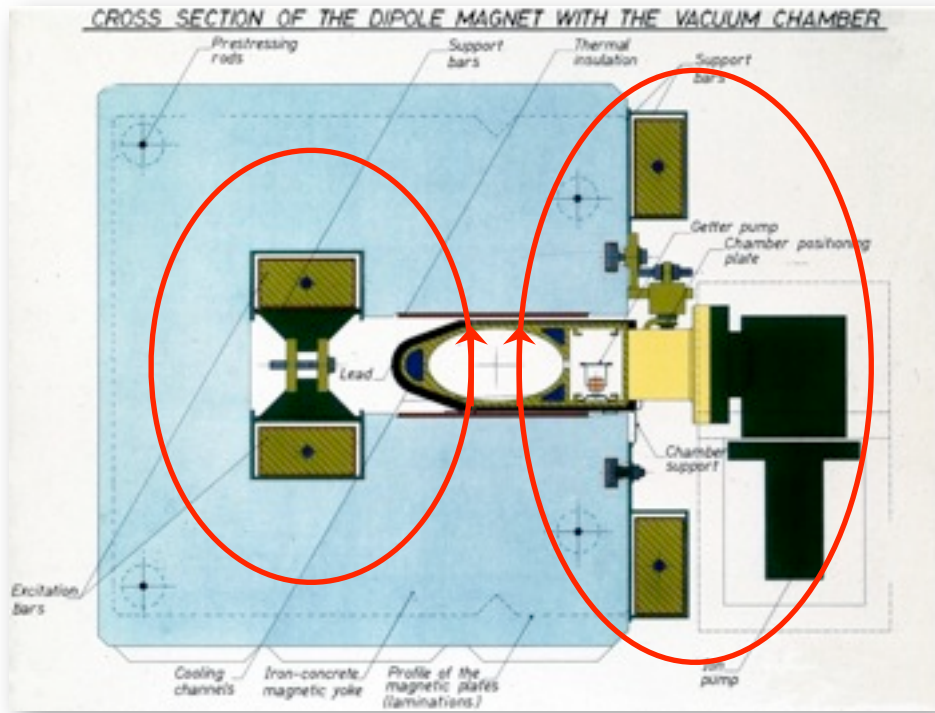
LHC **B ∝ I** **B = 8.38 T**

would need now **I = 280 kA** with LEP magnets **R = 1 mΩ**

P = 78 MW / cell × **488 cells** **total power P = 38 GW**

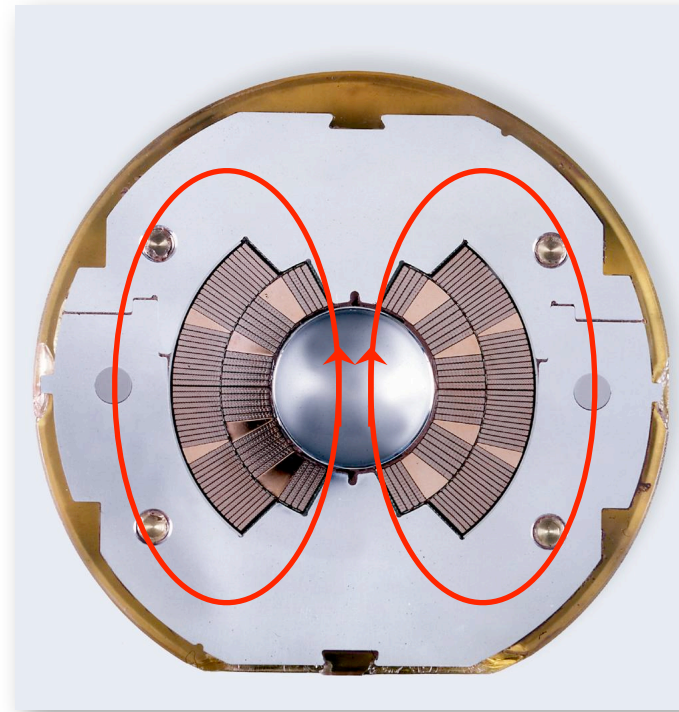
Magnet technology

warm

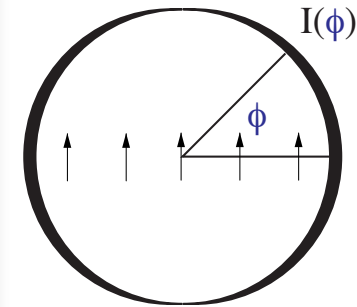


- field quality given by pole face geometry
- field amplified by Ferromagnetic material
- hysteresis and saturation ~ 2 T
- Ohmic losses for high magnet currents

cold



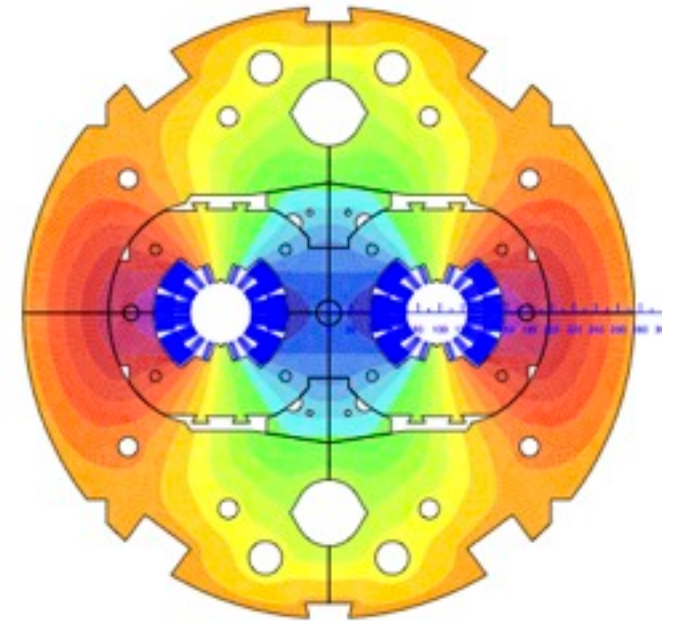
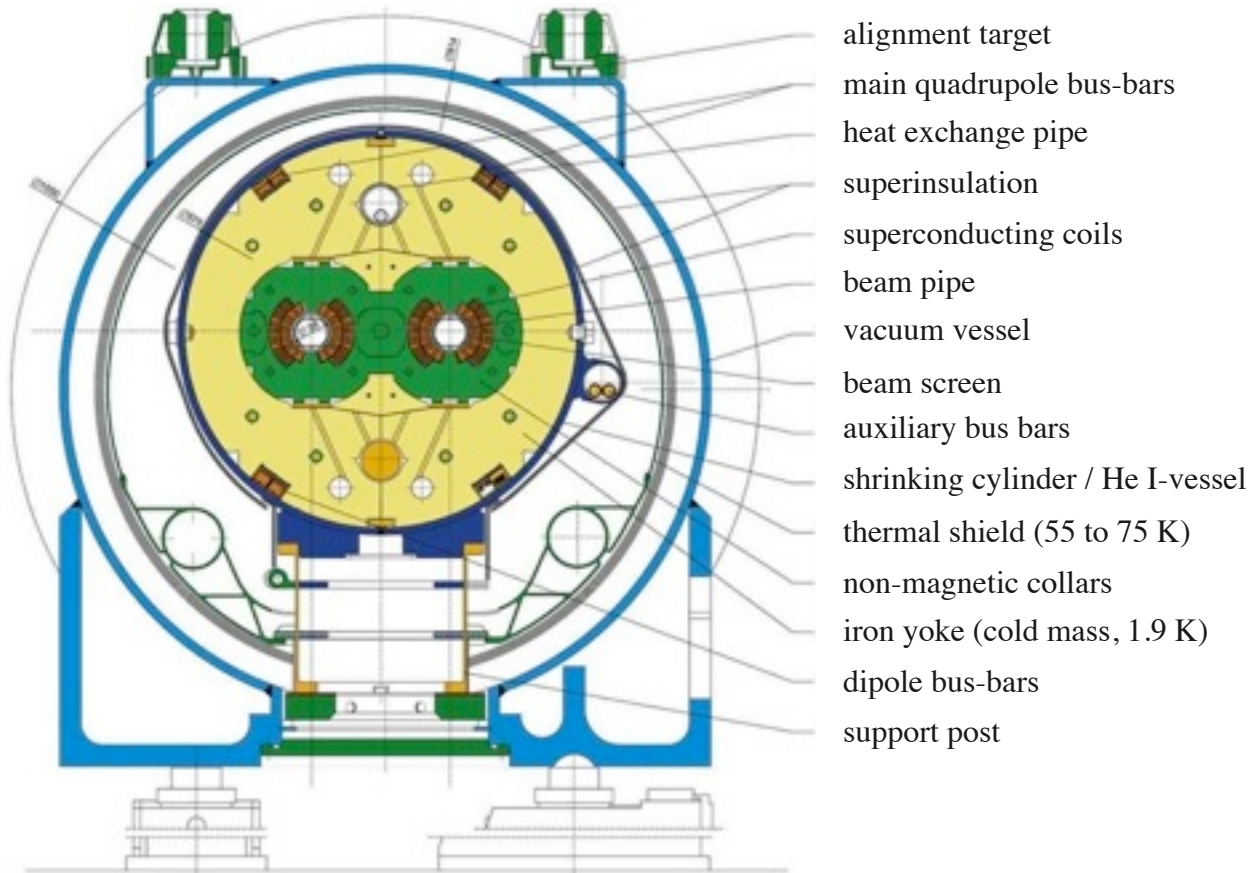
Dipole current distribution
 $I(\Phi) = I_0 \cos(\Phi)$



- field quality given by coil geometry
- requires cooling to cryogenic temperatures
- persistent currents and snap back
- risk of magnet quenches

LHC dipole magnet

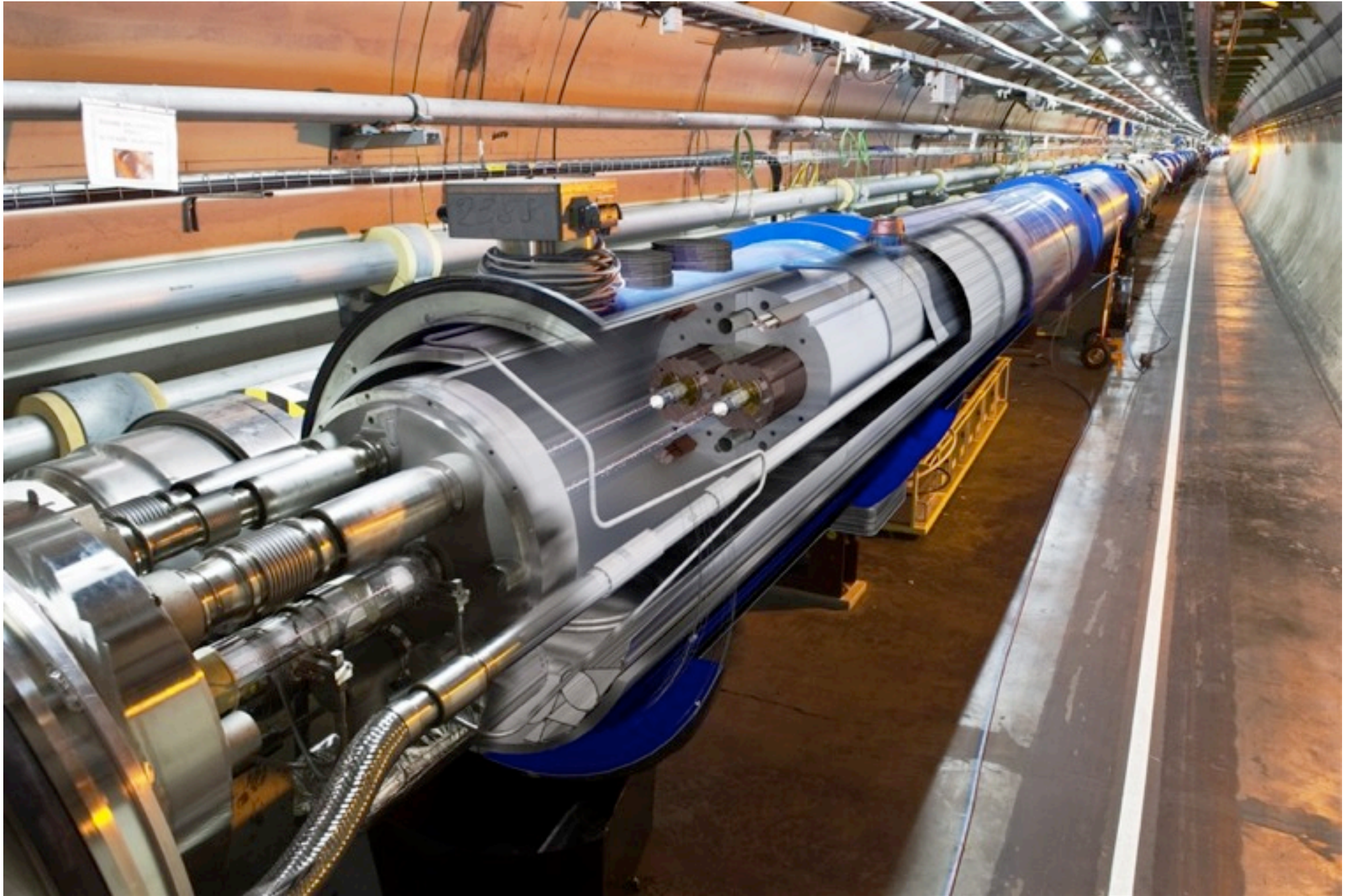
2-in-1 dipole magnet, 8.33 T field, 15 m long, mass 30 ton



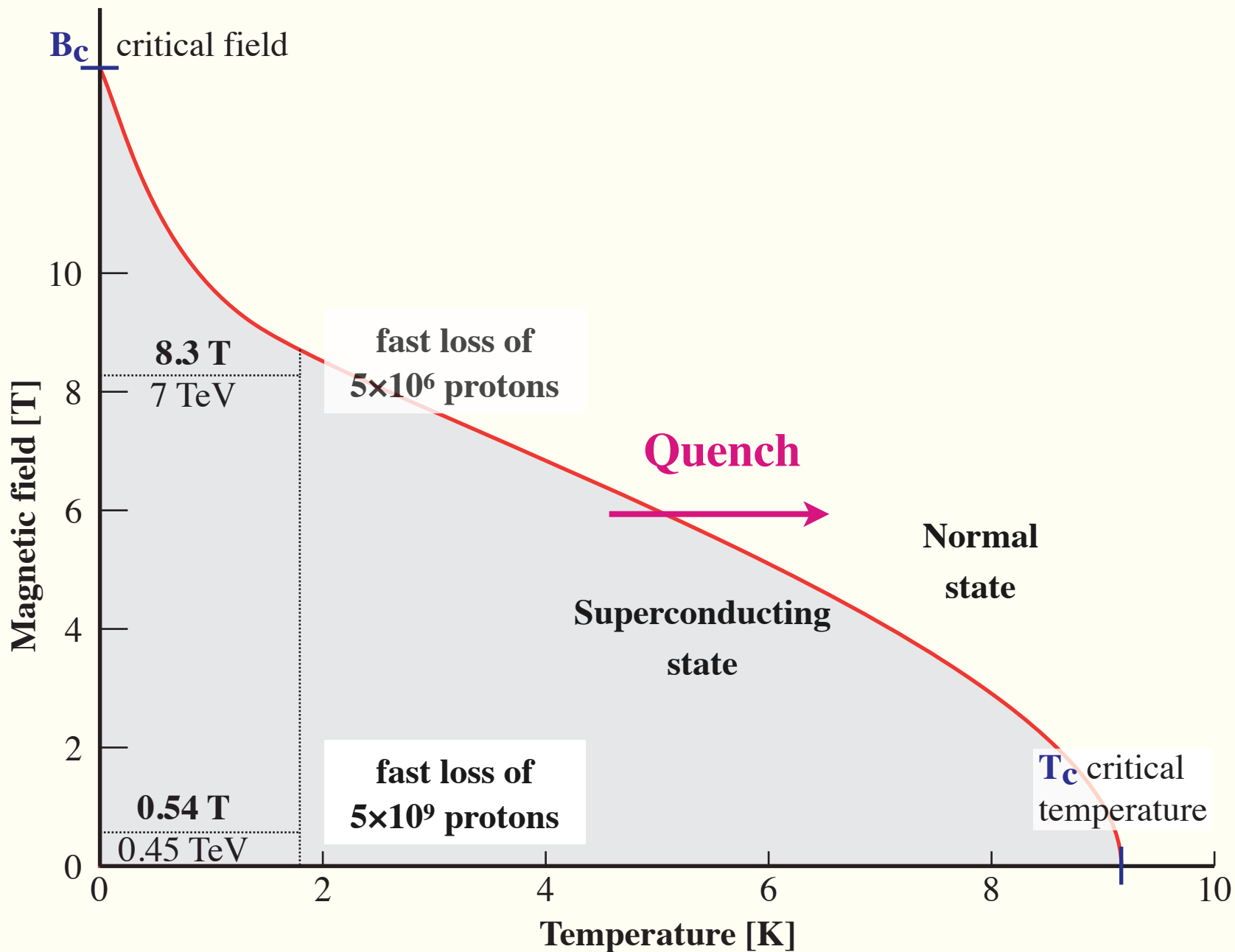
current distribution

LHC dipole magnet cross-section

LHC magnets installed in the tunnel



Operational margin of a superconducting LHC dipole



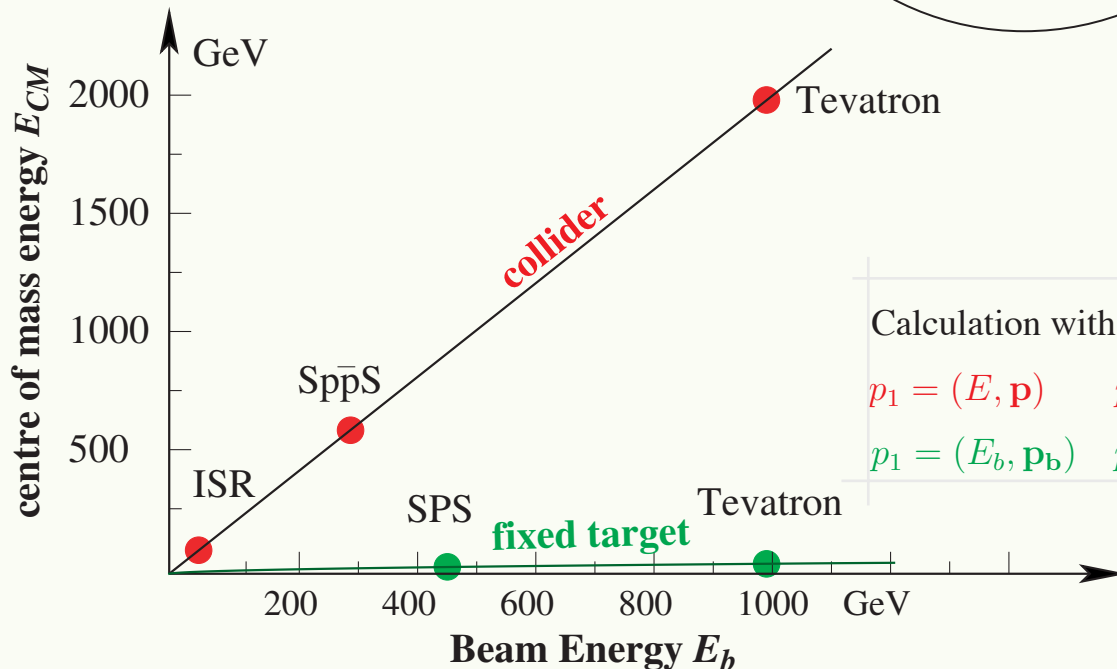
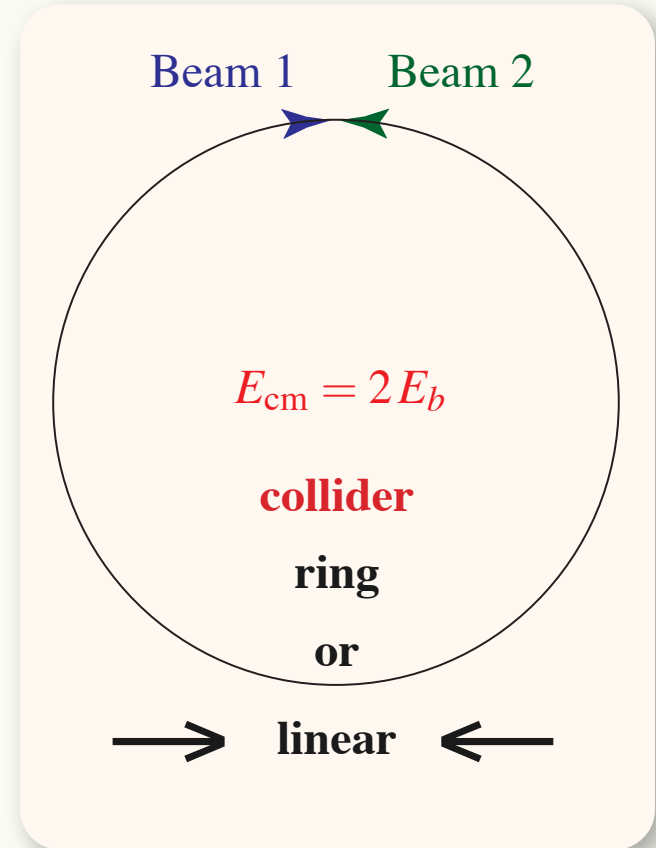
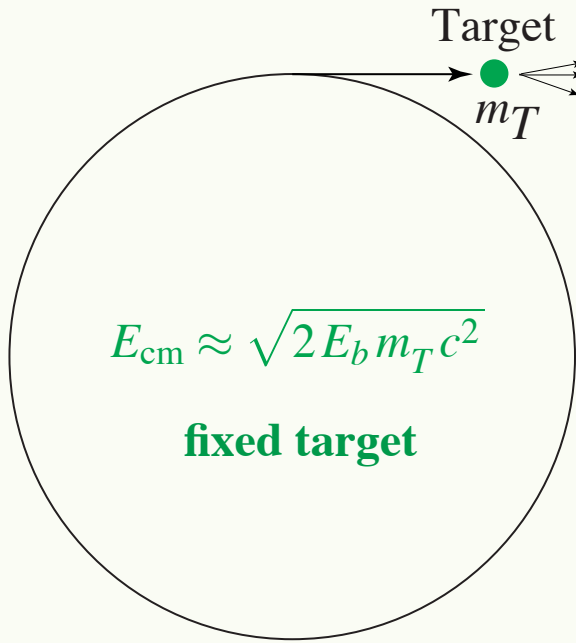
Fixed Target vs Collider

Fixed target, high energy collisions :

Energy “lost” as kinetic energy

High Energy $e+e^-$ and very high energy pp gain a lot from **colliders**

Gain for LHC is by **$\times 122$**
(14 TeV / 114.6 GeV)



Calculation with four vectors for $c = 1$ $E_{CM} = \sqrt{s}$ $s = (p_1 + p_2)^2$

$p_1 = (E, \mathbf{p})$ $p_2 = (E, -\mathbf{p})$ $s = 2m^2 + 2E^2 + 2p^2 = 4E^2$ **collider**

$p_1 = (E_b, \mathbf{p}_b)$ $p_2 = (m_T, \mathbf{0})$ $s = m_b^2 + m_T^2 + 2m_T E_b$ **fixed target**

Primary cosmic ray spectrum

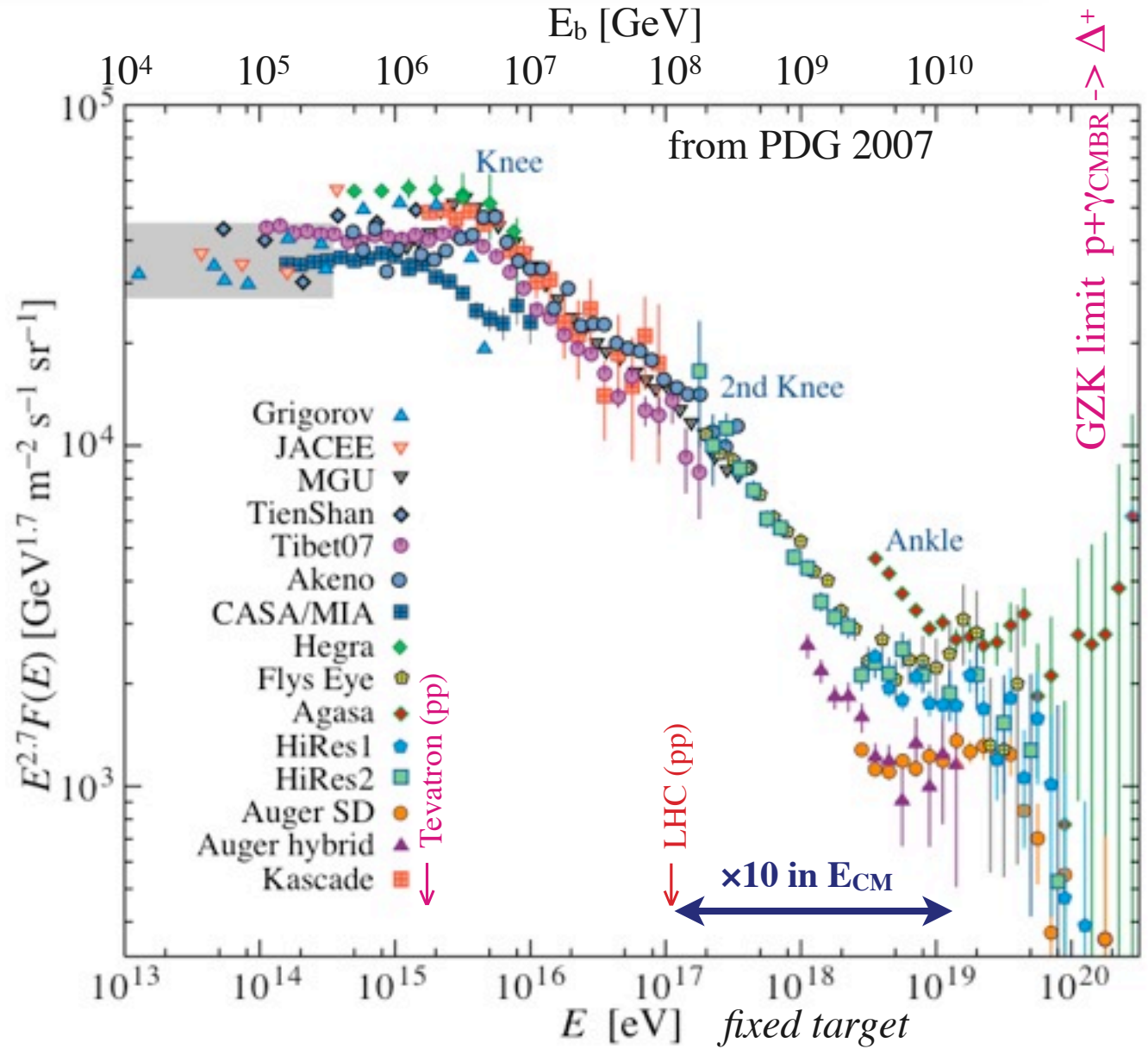
E spectrum falls as $E^{-2.7}$
 to knee at $E \approx 5e15 \text{ eV}$
 $= 5 \times 10^6 \text{ GeV}$
 $\sim 1 \text{ particle/m}^2 \text{ and year}$
 origin galactic

above $\sim E^{-3}$

back to $E^{-2.7}$ at very
 highest energies

conversion to E_{cm}

E_b [eV]	E_{cm} [TeV]
10^{13}	0.137
10^{15}	1.370
10^{17}	13.70 \approx LHC pp
10^{19}	137.0 \leftarrow LHC ions
10^{21}	1370.



Nature has much larger and more powerful **cosmic accelerators** than we can ever built.

With colliders we can get to these collision energies in clean laboratory conditions.

The LHC already gets us to within 1-2 orders of magnitude of the very highest cosmic rays.

Luminosity and collision rates

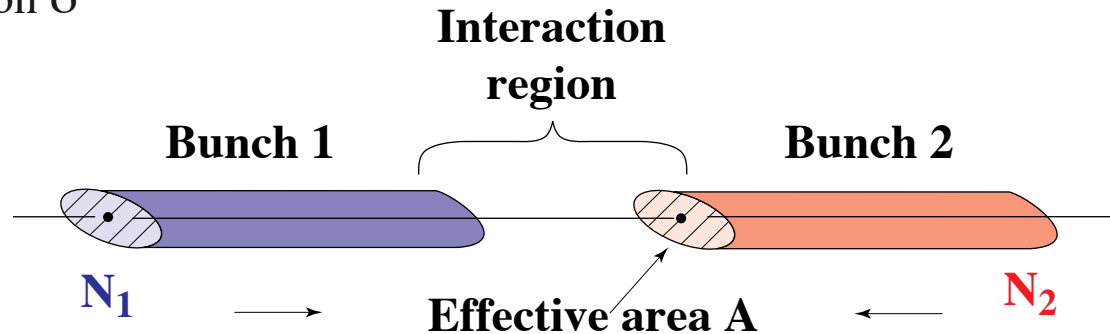
Event rate for process with cross section σ

$$\dot{n} = \mathcal{L} \sigma$$

Luminosity from bunch

crossings at frequency $f = f_{\text{rev}} n_b$

$$\mathcal{L} = \frac{N_1 N_2 f}{A}$$



for Gaussian bunches with rms sizes $\sigma_x \sigma_y$ $A = 4 \pi \sigma_x \sigma_y$

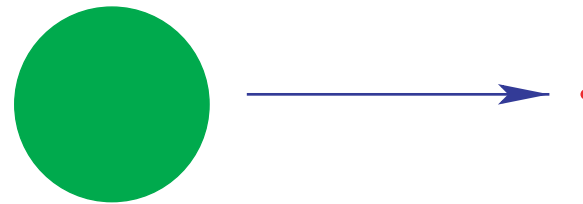
High **Luminosity** : $N \uparrow$ collide many particles, $A \downarrow$ squeezed in small bunches

LHC 1.15×10^{11} protons, $n_b = 2808$ ($f \uparrow$ crossings at 25 ns intervals)

Beams squeezed using strong large aperture quadrupoles around the interaction points

from ~ 0.2 mm to

$$\sigma_x = \sigma_y = 17 \mu\text{m}$$



$$\langle \beta \rangle_{\text{arc}} = 80 \text{ m}$$

$$\beta_{\text{IP}} = 0.5 \text{ m}$$

Rare new processes, like Higgs production can have very small cross section, like $1 \text{ fb} = 10^{-39} \text{ cm}^2$. LHC designed for very high Luminosity $\mathbf{L = 10^{34} \text{ cm}^{-2}\text{s}^{-1}}$

Event rate for such rare processes : ~ 1 new particle every 28h.

Instead pp $\sigma_{\text{tot}} \approx 0.1$ barn 30 / crossing

Alternate gradient focusing

**Quadrupole lens
focusing in x,
defocusing in y
or vice versa**

$$\mathbf{F} = e (\mathbf{v} \times \mathbf{B})$$

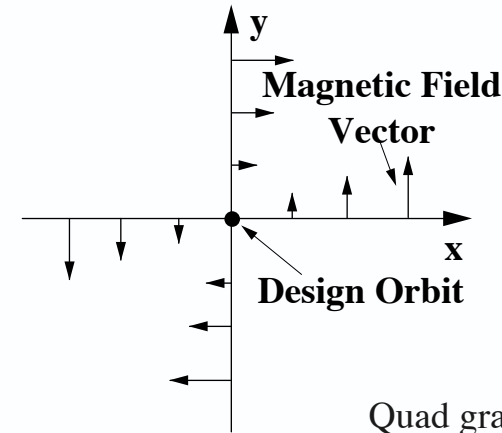
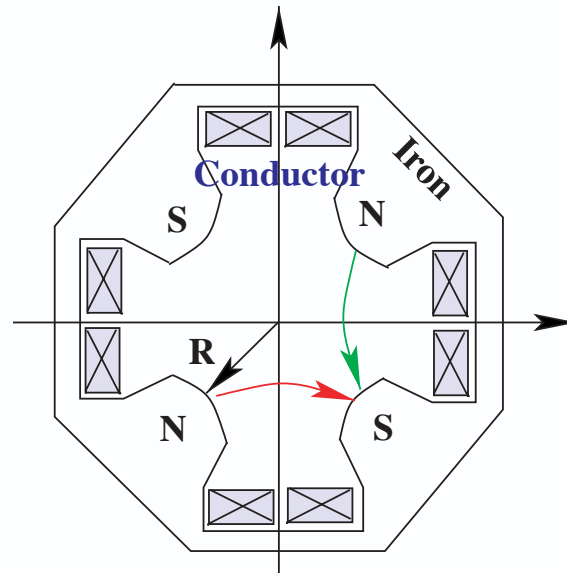
here

$$\mathbf{F} = e (0, 0, v) \times (B_x, B_y, 0)$$

$$= e (-v B_y, +v B_x, 0)$$

Combine F D
Defocusing when at
small amplitude
Overall focusing

Normal (light) optics :
Focal length of two lenses
at distance D
 $1/f = 1/f_1 + 1/f_2 - D/f_1 f_2$
is overall focusing
with $1/f = D/f^2$
for $f = f_1 = -f_2$



$$B_x = k y$$

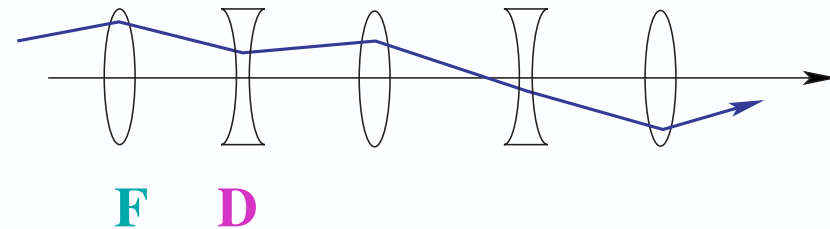
$$B_y = k x$$

$$B_z = 0$$

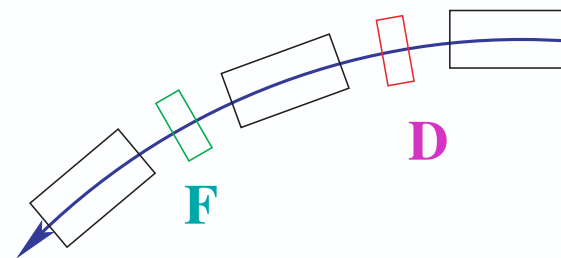
$$\nabla \times \mathbf{B} = \mathbf{0}$$

Quad gradients in the LHC
 $K = 1/B_0 \partial B_y / \partial x \approx 200 \text{ T/m}$

**alternate gradient
focusing**



**together with
bending magnets
FODO lattice**



N. C. Christofilos, unpublished manuscript in 1950 and patent

Courant, Snyder in 1952, Phys. Rev. 88, pp 1190 - 1196 + longer review in Annals of Physics 3 (1958)

Betatron motion

Equation of motion of particles in a ring (with bending fields) **and quadrupoles** (field gradients $\propto \partial B / \partial r$)

In both transverse planes, here written with x for x, y : known as Mathieu-Hill equation

$$x''(s) + k(s) x(s) = 0, \quad \text{derived in 1801 to describe planetary motion}$$

Generalised oscillator equation with position dependent, periodic restoring force $k(L+s) = k(s)$ given by the quadrupole gradients (+ the small weakly focusing bending term in the ring plane)

Solution : $x(s) = \sqrt{\epsilon \beta(s)} \cos(\mu(s) + \phi)$

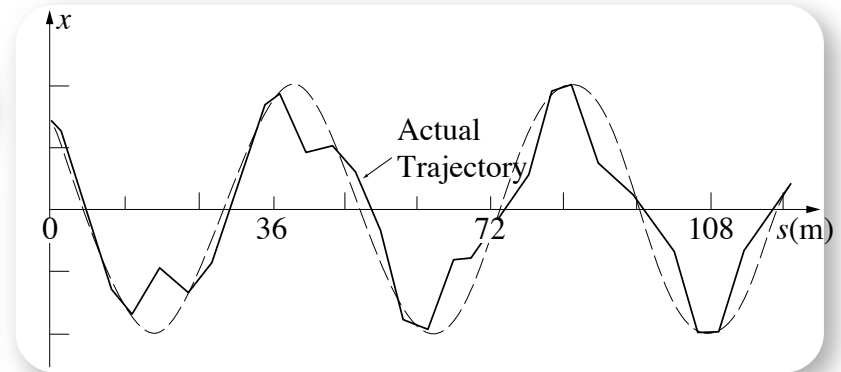
Phase advance

Lyapunov-Floquet Transformation

Tune # of betatron oscillations

$$\mu(s) = \int_0^s \frac{ds}{\beta(s)}$$

$$Q = \mu / 2\pi$$



*motion $x/\sqrt{\beta}$ plotted with phase advance
normalised coordinates - becomes simple cos*

$\beta(s)$ **beta function**, describes the focusing properties of the magnetic lattice

\mathcal{E} invariant, together with $\beta(s)$ amplitude. “single particle emittance”

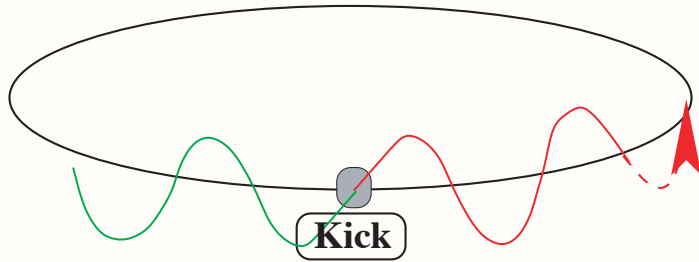
Motion conveniently described in phase space (x, x') where $x' = p_x / p$

and linear optics elements as matrices ; with simple case for M, applies for IP to IP

$$\begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = \mathbf{M} \begin{pmatrix} x(s_0) \\ x'(s_0) \end{pmatrix} \quad \mathbf{M} = \begin{pmatrix} \cos 2\pi Q & \beta \sin 2\pi Q \\ -\frac{1}{\beta} \sin 2\pi Q & \cos 2\pi Q \end{pmatrix}$$

Accelerator design : starts with magnet lattice based on linear beam optics ; MAD program

Orbit stability and tune



Misalignments and dipole field errors

→ **orbit perturbations**

would add up on successive turns

for integer tune $Q = N$

Higher order field errors,

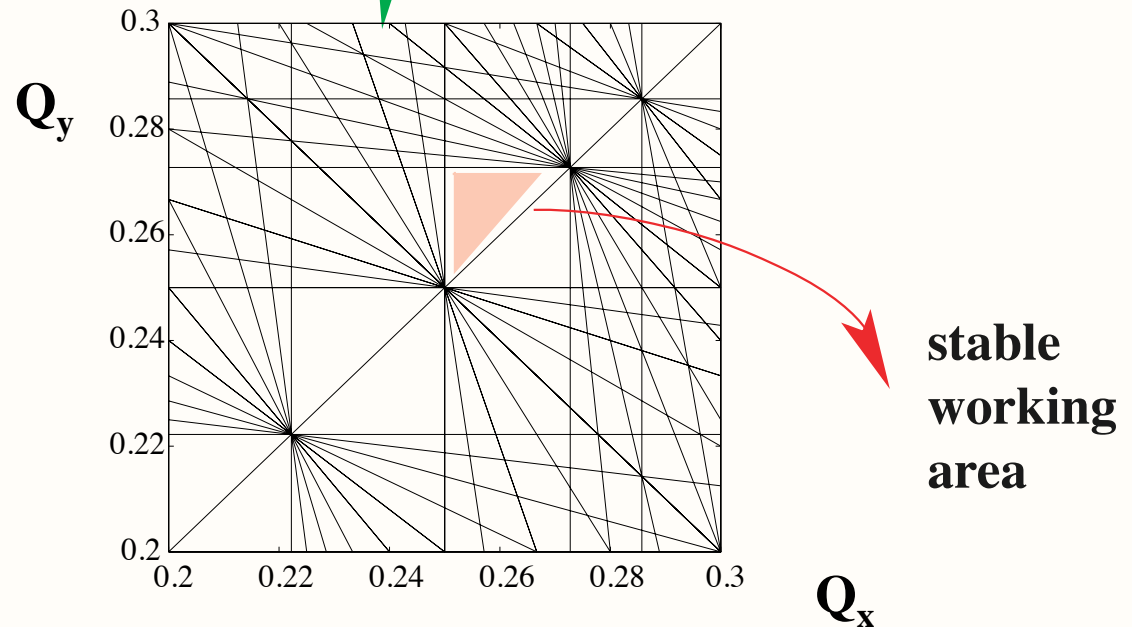
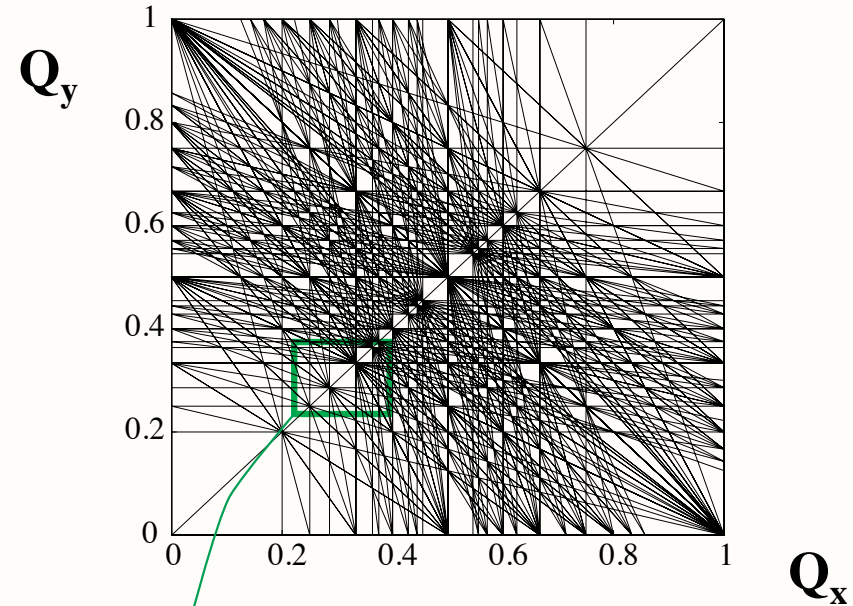
Quad., Sext. perturbations.

Avoid simple fractional tunes

$nQ_x + mQ_y + mQ_s = \text{int.}$

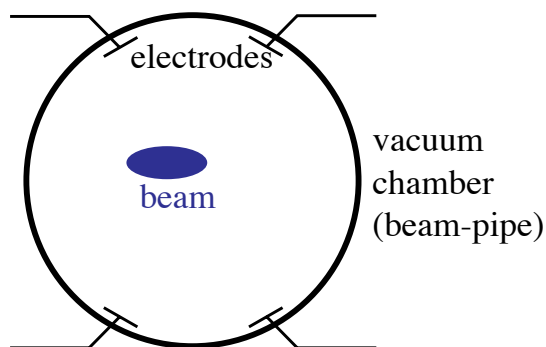
Minimise field and alignment

errors

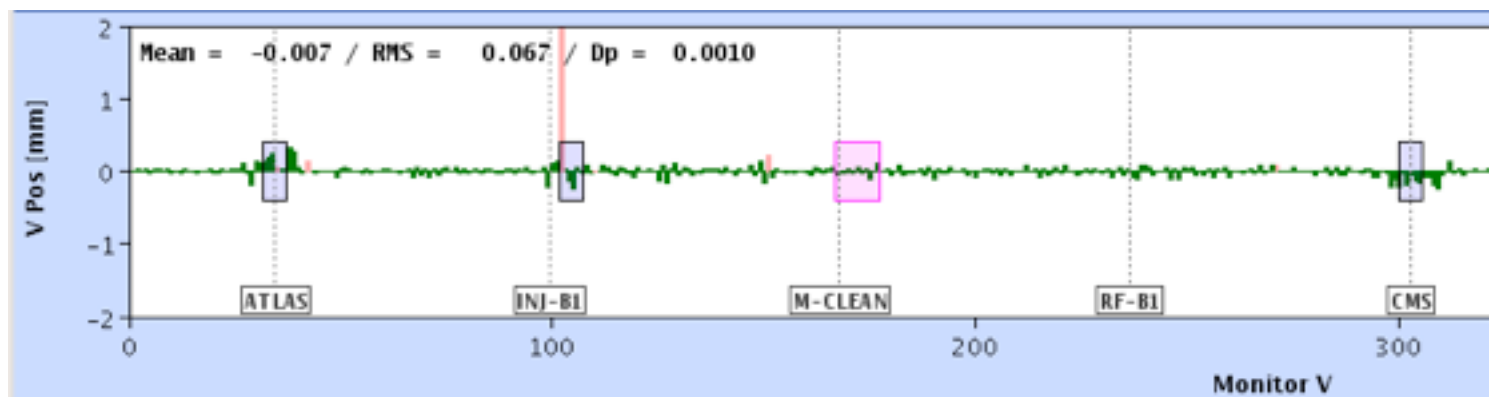


Orbit, tune measurement and peak beam current

vertical orbit, June 2011, 1st half of LHC shown



Beam Pickup Monitor



$\langle I_b \rangle$ average ring
and
 \hat{I} local peak
current

$$\hat{I} = \frac{\langle I_b \rangle L}{\sqrt{2\pi} \sigma_z}$$

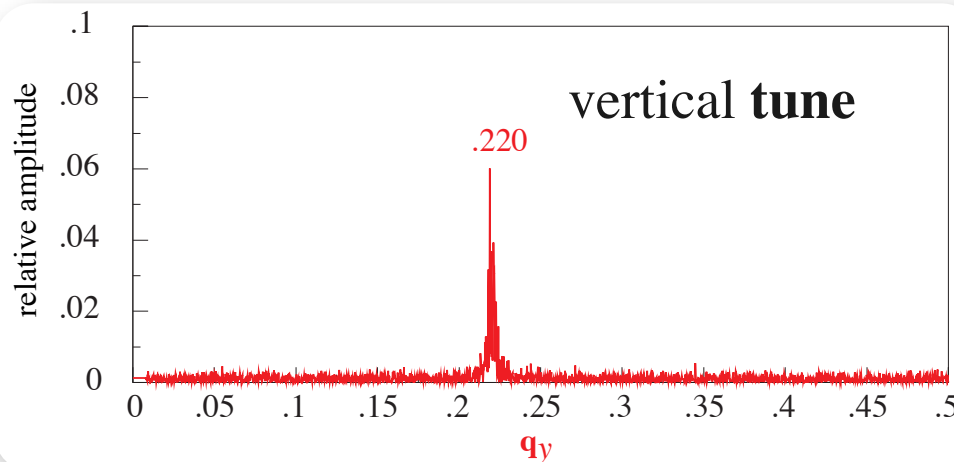
Typical numbers, for a single bunch $\langle I_b \rangle = n e f_{\text{rev}}$

LEP	$n = 4 \times 10^{11}$	$\langle I_b \rangle = 0.72 \text{ mA}$	$\sigma_z = 2 \text{ cm}$	$\hat{I} = 960 \text{ A}$
LHC	$n = 1.15 \times 10^{11}$	$\langle I_b \rangle = 0.21 \text{ mA}$	$\sigma_z = 7.55 \text{ cm}$	$\hat{I} = 73.2 \text{ A}$

$f_{\text{rev}} = 11245 \text{ kHz}, \quad L = 26658.9 \text{ m}$

Bunch peak currents are many Amperes !
Strong signals, used to monitor beam position and oscillations

Also source of undesirable effects :
wake fields, heating, instabilities



Transverse beam size and emittance

consider : beam of many particles on stable orbit and

simple case : dispersion and slope $\beta' = 0$ by default at IP - relevant for experiments

beam size, r.m.s. $\sigma(s) = \sqrt{\varepsilon \beta(s)}$

beam divergence, r.m.s. $\theta(s) = \sqrt{\varepsilon / \beta(s)}$

product $\varepsilon = \sigma(s) \theta(s)$

β - function : local machine quantity - focusing of lattice

Emittance ε : beam quantity - the average action

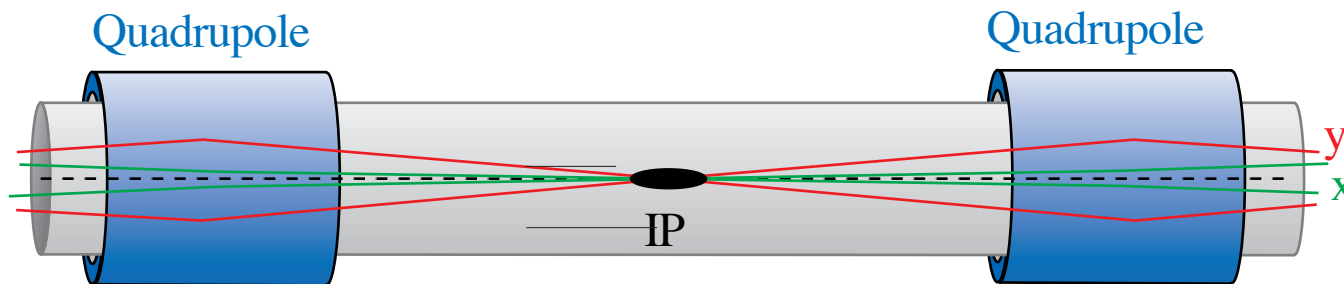
related to phase space density or kind of beam temperature

given by initial conditions (injected beam)

or equilibrium of quantum excitation and damping - 2nd lecture

in ideal machine : x, y, z motion uncoupled, 3 emittances $\varepsilon_x, \varepsilon_y, \varepsilon_z$

IP: squeeze β to a minimum, called β^* \Rightarrow maximum of divergence, needs aperture



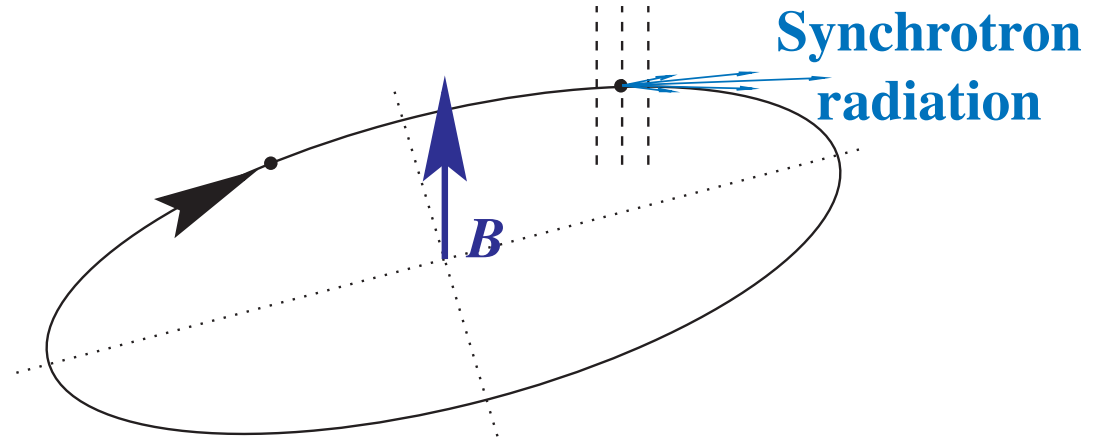
LHC $\varepsilon_N = \varepsilon \beta \gamma = 3.75 \mu\text{m}$, at top $E_b = 7 \text{ TeV}$: $\varepsilon = 0.503 \text{ nm}$, $\beta^* = 0.55 \text{ m}$, $\sigma^* = 16.63 \mu\text{m}$, $\theta^* = 30 \mu\text{rad}$

Standard Synchrotron Radiation

$$E_c = \frac{3}{2} \frac{\hbar c \gamma^3}{\rho} = 2.96 \times 10^{-7} \text{ eV m} \frac{\gamma^3}{\rho}$$

$$U_0 = \frac{e^2}{3\epsilon_0} \frac{\gamma^4}{\rho} \approx 6.0317 \cdot 10^{-9} \text{ eV m} \frac{\gamma^4}{\rho}$$

$$P_b = \frac{U_0 I_b}{e}$$



		E GeV	γ	ϱ m	U_0 MeV	E_c keV	τ_d s	N 10^{12}	I mA	P_b MW	B T
RHIC	Au	A×100	107.4	242.8	21×10^{-6}	1.5×10^{-6}	4.9×10^6	0.06	60	1.3×10^{-12}	3.42
LHC	p	7000	7460.5	2804	0.0067	0.044	61729	646	1163	0.0072	8.33
LEP1	e	45.6	89237	3026	126	69.5	23×10^{-3}	2.22	4	0.5	0.05
LEP2	e	104.5	204501	3026	3490	836	1.9×10^{-3}	2.8	5	18	0.115

Same beam energy E and radius ϱ : electron instead of proton $U_0 \sim \gamma^4$: $(m_p/m_e)^4 = 1.13 \times 10^{13}$

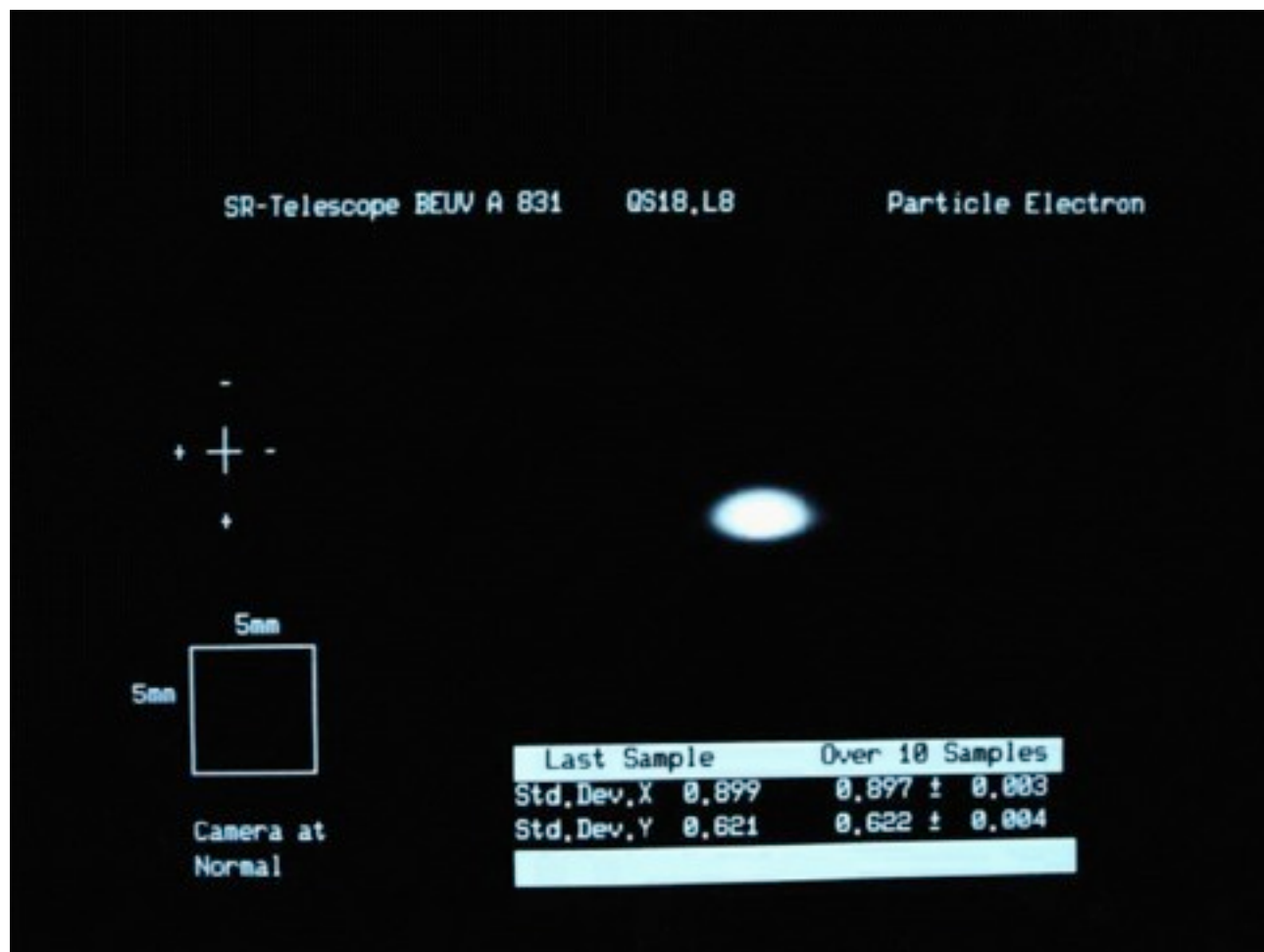
Electrons, $E \gg 100$ GeV needs linear collider (ILC / CLIC)

Damping time E / U_0 turns or $\tau_d = t_{\text{rev}} E / U_0$ revolution time LEP/LHC $t_{\text{rev}} = 88.9 \mu\text{s}$

Gold ions Au⁷⁹⁺ A=197 $\langle E_\gamma \rangle = 8/(15\sqrt{3}) E_c$ $8/(15\sqrt{3}) \approx 0.308$

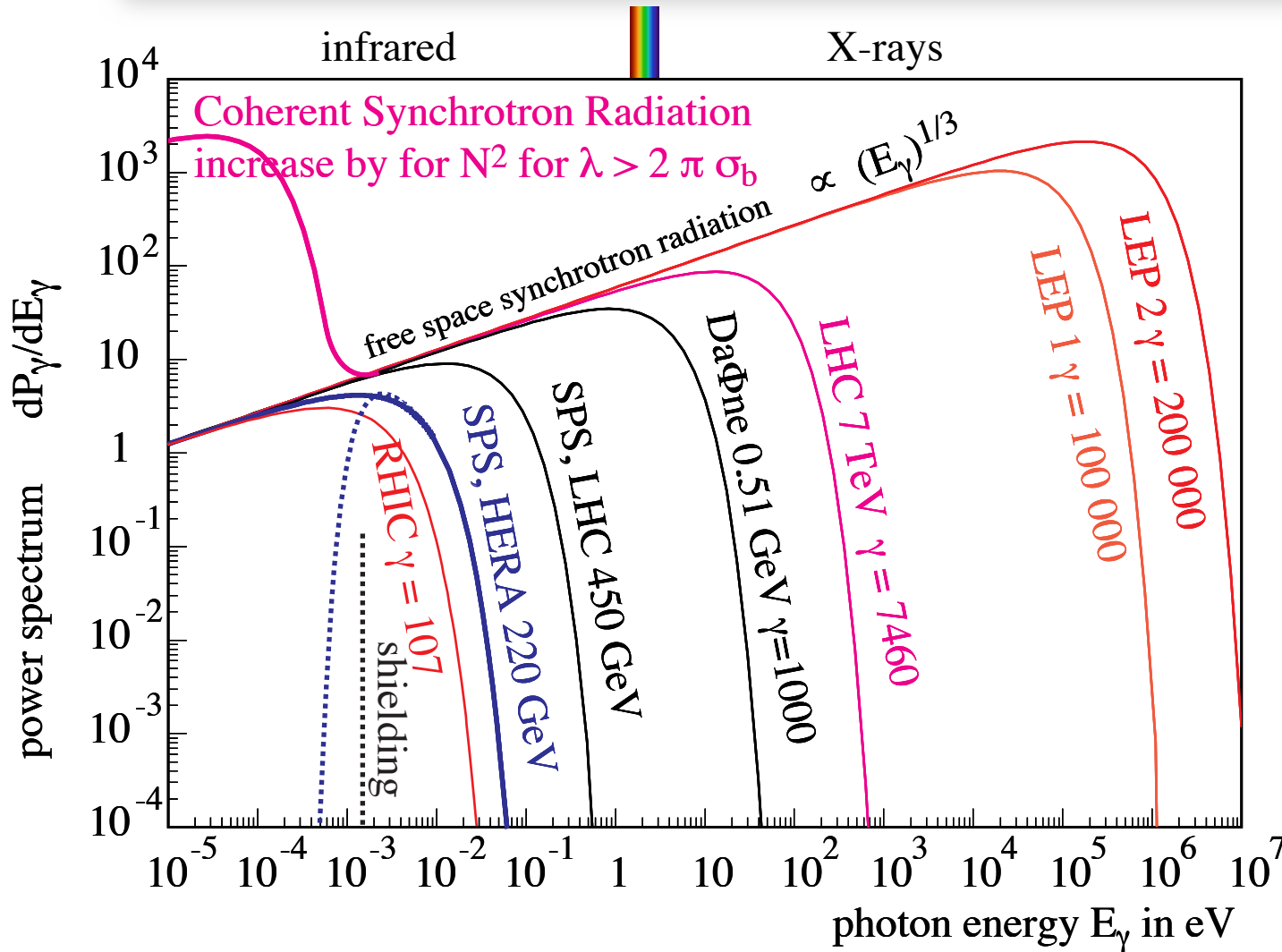
Synchrotron light monitor

Picture from
LEP. Typical
transverse
rms beam size
0.15 mm vertical
1.5 mm horiz.



Mirror, small slit, telescope and camera : beams continuously visible.
Now also used for protons in the LHC.

Power Spectrum, Free space, Cutoff and CSR



$$\frac{f_{\text{cutoff}}}{f_{\text{rev}}} = \sqrt{\frac{2}{3}} \left(\frac{\pi\rho}{h} \right)^{3/2}$$

ρ bending radius
 h chamber height
 cutoff relevant
 for $\gamma \approx 100$

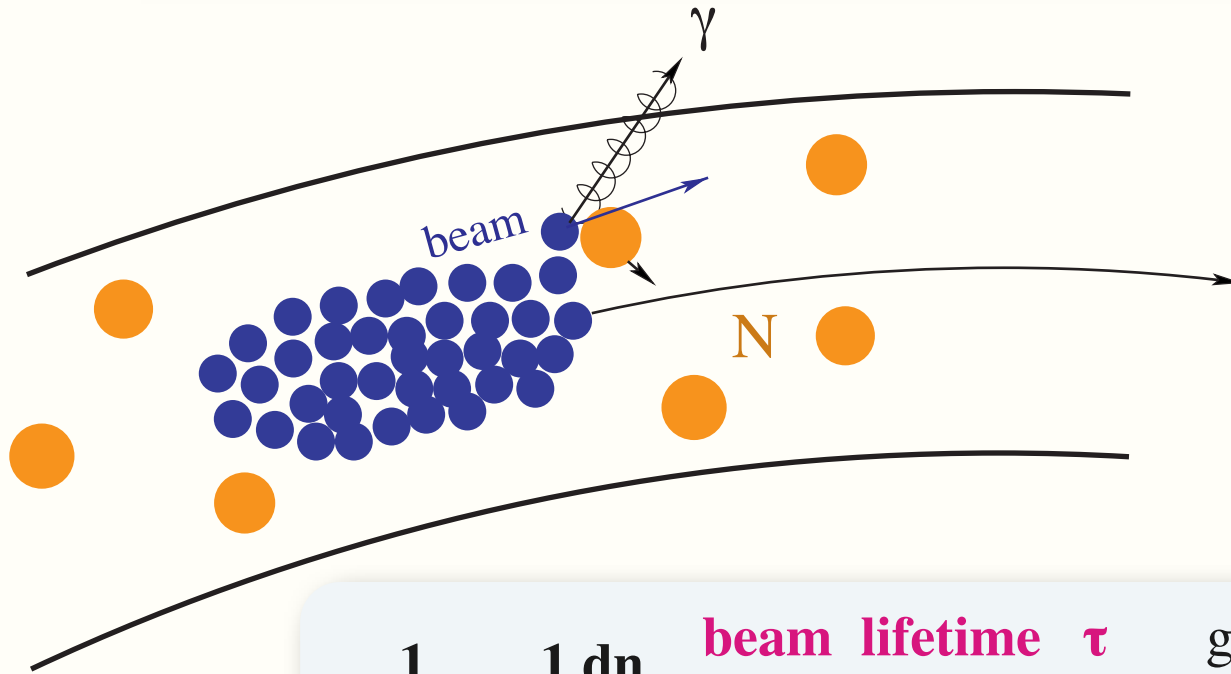
12 orders of magnitude
 in E_γ and λ
 10^{-5} eV $\lambda = 0.124$ m
 10^{+7} eV $\lambda = 124$ fm

Effects which can modify the low energy, long wavelength spectrum :

- i) **Coherent Synchrotron Radiation CSR** increases radiation and loss
- ii) **Boundary conditions - cutoff by conducting chamber** decreases radiation and loss

Energy Loss of Gold Ions in RHIC, [EPAC 2008](#)

Vacuum, beam Gas - lifetime



Beam blow up, core + halo
Background to experiments
loss, radiation, beam and
Luminosity lifetime

Minimize effect :
Good vacuum
O(nTorr or 10^{-9} mb)
Collimation

$$\frac{1}{\tau} = - \frac{1}{n} \frac{dn}{dt}$$

beam lifetime τ general expression
average time between collisions leading to beam loss
inverse normalised loss rate

$$p = 1 \text{ ntorr} = 1.33 \times 10^{-7} \text{ Pa}$$

$$\rho_m = \frac{p}{kT} = 3.26 \times 10^{13} \text{ molecules / m}^3$$

$$\text{typical cross section } \sigma = 6 \text{ barn} = 6 \times 10^{-28} \text{ m}^2$$

$$\text{collision probability } P_{\text{coll}} = \sigma \rho_m = 1.96 \times 10^{-14} / \text{m}$$

$$\tau = \frac{1}{P_{\text{coll}} c} = 1.7 \times 10^5 \text{ s} = 47 \text{ hours} \quad \text{for } v \approx c$$