High Energy Monte Carlo Generators

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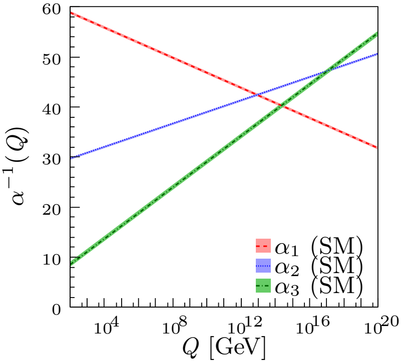
Introduction

Monte Carlo Methods

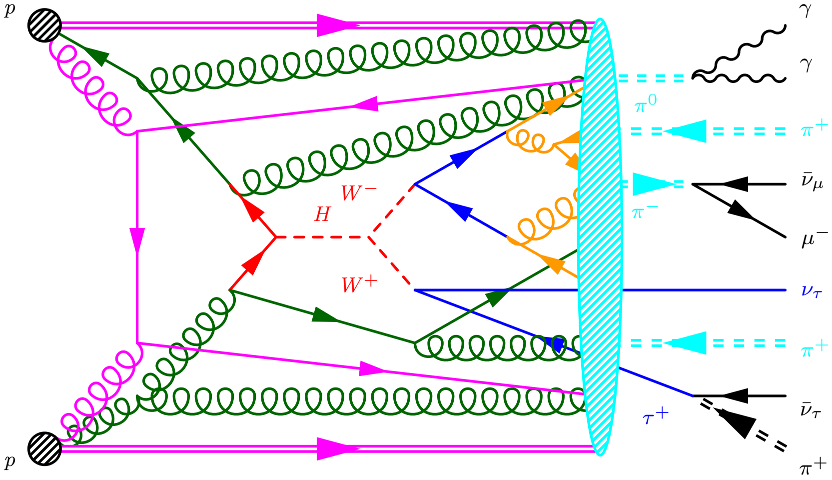
- efficiently perform $3n - 4$ dimensional integrals
- evolve from perturbative QFT to phenomenological models
- provides events for free

1. generate perturbative hard process
2. shower partons to lower energy with perturbative methods
3. apply phenomenological hadronization
4. decay particles



Event Anatomy

1) hard process
3) ISR
5) underlying event
7) particle decays
2) resonance decays
4) FSR
6) hadronization



Tuning Parameters

[T. Sjöstrand]

- hard process
  - PDFs, phase space cut-offs ($\hat{p}_{\mathrm{T}}$, $\hat{m}$), renormalization scale, factorization scale, SM parameters (CKM, $\alpha_s(M_Z)$, $\sin\theta_W$)
- parton showers
  - $\alpha_s(M_Z)$, scales, p_{T} damping, matching parameters, ordering method
- underlying event
  - $\alpha_s(M_Z)$, hard processes, p_{T} damping, beam profile (shape, impact parameter), color reconnection
- hadronization
  - longitudinal momentum sharing, transverse width, flavor composition, vector to pseudo-scalar composition, baryon and meson production ratios

Observables

- test non-perturbative regimes of QCD
- tune multi purpose event generators
- look for new effects to refine models
- hard process
  - inclusive cross-sections
  - differential cross-sections
- ISR
  - light jet thrust ($\alpha_s(M_Z)$)
  - p_{T} from $Z \to \mu\mu$ (primordial k_{T})
- FSR
  - similar to ISR
- underlying event
  - IR safe variables
  - energy flow
- hadronization
  - final state flavor composition
  - IR sensitive variables
  - charge density and multiplicity
- particle decays
  - branching fractions
  - angular and mass distributions, etc.

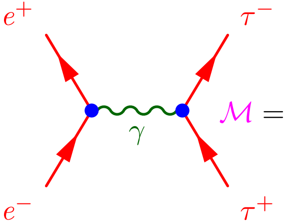
Hard Processes

Methods

- begin with rules from Lagrangian density

$$\mathcal{L}_{\text{QED}} = i\psi^\dagger\gamma_\mu\partial^\mu\psi - m\psi^\dagger\psi - iQg_eA_\mu\psi^\dagger\gamma^\mu\psi \\ - \left(\partial^\mu A^\nu - \partial^\nu A^\mu\right)\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)$$

- build matrix element from diagrams



$$\mathcal{M} = \bar{v}_{e^+}\, iQg_e\gamma^\mu\, u_{e^-}\, \frac{-ig_{\mu\nu}}{q^2}\, \bar{u}_{\tau^-}\, iQg_e\gamma^\nu\, v_{\tau^+}$$

- integrate over phase-space for partonic cross-section

$$\hat{\sigma} = \int \left(\frac{1}{8\pi}\right)^2 \frac{\langle\, |\mathcal{M}|^2\, \rangle}{(E_{e^-} + E_{e^+})} \frac{|\vec{p}_{\mu^-}|}{|\vec{p}_{e^-}|}\, \mathrm{d}\Omega$$

Implementation

- convolute with PDFs for full cross-section

$$\sigma_{a_1 a_2 \to B} = \int\int x_{a_1}\left(x_{p_1}, Q^2, p_1\right) x_{a_2}\left(x_{p_2}, Q^2, p_2\right) \sigma_{p_1 p_2 \to B}\, \mathrm{d}x_{p_1}\, \mathrm{d}x_{p_2}$$

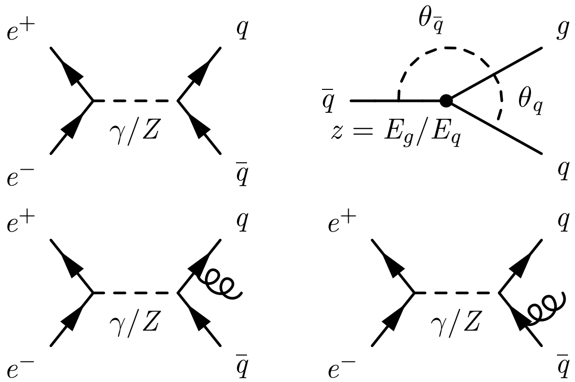
- can use a fully automated process given rules (MadGraph, BlackHat, etc.)
- in practice hard-coded averaged/summed cross-sections in $\mathrm{d}\hat{\sigma}/\mathrm{d}\hat{t}$ are oftentimes used (Pythia, PowhegBox, etc.)

$$\begin{aligned}&\texttt{sigmagmgm} * \texttt{pow2}(\texttt{e1} * \texttt{e2}) + \texttt{sigmagmZ} * \texttt{e1} * \texttt{e2} *\\ &(\texttt{v1} * \texttt{v2} * (1. + \texttt{uH2}/\texttt{sH2}) + \texttt{a1} * \texttt{a2} * \texttt{epsi} * (1. - \texttt{uH2}/\texttt{sH2}))\\ &+\texttt{sigmaZZ} * ((\texttt{v1} * \texttt{v1} + \texttt{a1} * \texttt{a1}) * (\texttt{v2} * \texttt{v2} + \texttt{a2} * \texttt{a2})\\ &*(1. + \texttt{uH2}/\texttt{sH2}) + 4. * \texttt{v1} * \texttt{a1} * \texttt{v2} * \texttt{a2} * \texttt{epsi} * (1. - \texttt{uH2}/\texttt{sH2}))\end{aligned}$$

- $2 \to n$ for large n can challenge phase-space integrators

Showers

Example



$$\mathrm{d}\sigma_{e^+e^-\to q\bar{q}g} \approx \sigma_{e^+e^-\to q\bar{q}} \left(\frac{2\,\mathrm{d}\cos\theta_q}{\sin^2\theta_q}\right)\left(\frac{\alpha_s}{2\pi}\right)\left(\frac{N_c^2-1}{2N_c}\right)\left(\frac{1+(1-z)^2}{z}\right)\mathrm{d}z$$

$q \to qg$ Splitting

- diverges for three scenarios
  - $z \to 0$ (soft)
  - $\theta \to 0$ (collinear to q)
  - $\theta \to \pi$ (collinear to $\bar{q}$)
- factorize collinear divergences as independent emissions

$$\mathrm{d}\sigma_{e^+e^-\to q\bar{q}g} \approx \sigma_{e^+e^-\to q\bar{q}} \sum_i \left(\left(\frac{\mathrm{d}\theta_{p_i}^2}{\theta_{p_i}^2} \right) \left(\frac{\alpha_s}{2\pi} \right) \left(\frac{N_c^2 - 1}{2N_c} \right) \left(\frac{1 + (1-z)^2}{z} \right) \mathrm{d}z \right)$$

- generalize for all processes with splitting functions

$$\mathrm{d}\sigma_{A\to Bb_j} \approx \sigma_{A\to B} \sum_i \left(\left(\frac{\mathrm{d}\theta_{b_i}^2}{\theta_{b_i}^2} \right) \mathcal{P}_{b_j b_i}(z, \alpha_s) \, \mathrm{d}z \right)$$

- parton b_i emits parton b_j

Splitting Functions

$$\mathcal{P}_{gq}(z,\alpha_s) = \left(\frac{\alpha_s}{2\pi}\right)\left(\frac{N_c^2-1}{2N_c}\right)\left(\frac{1+(1-z)^2}{z}\right) \qquad (q\to gq)$$

$$\mathcal{P}_{gg}(z,\alpha_s) = \left(\frac{\alpha_s}{2\pi}\right)(2N_c)\left(\frac{1-z}{z}+\frac{z}{1-z}+z(1-z)\right) \qquad (g\to gg)$$

$$\mathcal{P}_{qg}(z,\alpha_s) = \left(\frac{\alpha_s}{2\pi}\right)\left(\frac{1}{2}\right)\left(z^2+(1-z)^2\right) \qquad (g\to q\bar{q})$$

$$\mathcal{P}_{qq}(z,\alpha_s) = \left(\frac{\alpha_s}{2\pi}\right)\left(\frac{N_c^2-1}{2N_c}\right)\left(\frac{1+z^2}{1-z}\right) \qquad (q\to qg)$$

Time and Spacelike Showers

- timelike showers (final state radiation)

$$\Delta_{ij}(q_1^2, q_2^2) = \exp\left(-\int_{q_2^2}^{q_1^2} \frac{1}{q^2} \int_{Q_0^2/q^2}^{1-Q_0^2/q^2} \mathcal{P}_{ji}(z, \alpha_s)\, \mathrm{d}z\, \mathrm{d}q^2\right)$$

  - solve for $\Delta_{ij}(Q^2, q^2) = r \in [0, 1]$ for q^2
  - if $q^2 < Q^2$ (shower cut-off scale) terminate shower
- spacelike showers (initial state radiation)

$$\Delta_{ij}(q_1^2, q_2^2, x) = \exp\Bigg(-\int_{q_2^2}^{q_1^2} \frac{1}{q^2} \int_{Q_0^2/q^2}^{1-Q_0^2/q^2} \mathcal{P}_{ij}(z, \alpha_s) \left(\frac{x}{zx}\right)\left(\frac{f(x/z, q^2, j)}{f(x, q^2, i)}\right) \mathrm{d}z\, \mathrm{d}q^2\Bigg)$$

  - backwards evolution, evolve from high Q^2 and small x to small q^2 and large x

Matching and Merging

Methods

LO n-jet Merging		NLO Merging	
CKKW(L)	MLM	MC@NLO	POWHEG
Catani, Krauss, Kuhn, Webber	Mangano, Moretti, Piccinini, Treccani	Frixione, Webber	Frixione, Nason, Oleari
hep-ph/0109231	hep-ph/0611129	hep-ph/0402116	0709.2092

CKKW(L)

1. calculate Sudakov factor on all lines
2. shower, reject emission using factor

SHERPA

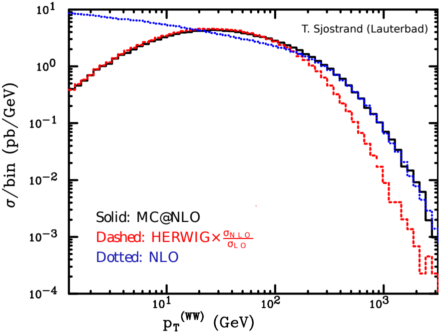
MADGRAPH/PYTHIA

MLM

1. perform shower and cluster jets
2. match jets to partons, reject if $N_p \neq N_{\text{jets}}$

ALPGEN/HERWIG++

MADGRAPH/PYTHIA



MC@NLO

1. calculate NLO corrections to n-body process
2. calculate first shower emission
3. break event into NLO $\mathcal{ME}-$ first emission, remainder
4. apply showers to both parts of event

Advantages:

- NLO variables
- smooth matching with $\mathcal{PS}$
- large number of processes

Disadvantages:

- negative weighting

$pp \to (Z/\gamma^* \to) l_{\mathrm{IL}} \bar{l}_{\mathrm{IL}} + X$
$pp \to (W^+ \to) l^+_{\mathrm{IL}} \nu_{\mathrm{IL}} + X$
$pp \to Z^0 + X$
$pp \to H^0 + X$
$pp \to t/\bar{t} + X$
$pp \to tW^-/\bar{t}W^+ + X$
$pp \to tH^-/\bar{t}H^+ + X$
$pp \to H^0 W^+ + X$
$pp \to H^0 (W^- \to) l^-_i \bar{\nu}_i + X$
$pp \to W^+ W^- + X$
$pp \to W^- Z^0 + X$

$pp \to (Z \to) l_{\mathrm{IL}} \bar{l}_{\mathrm{IL}} + X$
$pp \to (W^- \to) l^-_{\mathrm{IL}} \bar{\nu}_{\mathrm{IL}} + X$
$pp \to W^+ + X$
$pp \to b\bar{b} + X$
$pp \to \bar{t} + X$
$pp \to \bar{t}W^+ + X$
$pp \to \bar{t}H^+ + X$
$pp \to H^0 (W^+ \to) l^+_i \nu_i + X$
$pp \to H^0 Z + X$
$pp \to Z^0 Z^0 + X$

$pp \to (\gamma^* \to) l_{\mathrm{IL}} \bar{l}_{\mathrm{IL}} + X$
$pp \to \gamma^* (\to \sum_i f_i \bar{f}_i) + X$
$pp \to W^- + X$
$pp \to t\bar{t} + X$
$pp \to t + X$
$pp \to tW^- + X$
$pp \to tH^- + X$
$pp \to H^0 W^- + X$
$pp \to H^0 (Z \to) l_i \bar{l}_i + X$
$pp \to W^+ Z^0 + X$

POWHEG

1. pick largest p_T emission from NLO normalized $\mathcal{ME}$
2. evolve shower downwards to p_T scale

Advantages:

- positive weights
- separation of shower

Disadvantages:

- designed for p_T ordered showers

HERWIG++	POWHEGBOX	
$pp \to H$	$pp \to W$	$pp \to Z$
$pp \to HW$	$pp \to W + \text{jet}$	$pp \to Z + \text{jet}$
$pp \to ZH$	$pp \to t$	$pp \to tW$
$pp \to W$	$gg \to H$	$pp \to VV \to H$
$pp \to Z$	$pp \to \text{jet} + \text{jet}$	$pp \to t\bar{t}$
	$pp \to WW + \text{dijet}$	$pp \to WW$
	$pp \to WZ$	$pp \to ZZ$
	$pp \to b\bar{b}WW$	

More Methods

- UMEPS
  - unitarized matrix element and parton shower merging
  - tree-level n-leg merging without inclusive cross-section modification
- NL^3
  - Nils Lavesson and Leif Lönnblad
  - n-leg merging at NLO
- UNLOPS
  - unitarized next-to-leading-order parton shower
  - n-leg merging at NLO but more generalized
- FxFx
  - R. Frederix and S. Frixione
  - merging and matching of MC@NLO

Multi Parton Interactions

Models

hard → soft model (HEP)	soft → hard model (air-shower)

hard → soft model (HEP)

- begin with t-channel $2 \to 2$ QCD

$$\mathrm{d}\hat{\sigma}_{2\to 2} \propto \mathrm{d}p_{\mathrm{T}}^2 \frac{\alpha_s^2(p_{\mathrm{T}}^2)}{p_{\mathrm{T}}^4}$$

- divergent in p_{T}, cut-off or damp

$$\frac{\alpha_s^2({p_{\mathrm{T}}}_0^2 + p_{\mathrm{T}}^2)}{\alpha_s^2(p_{\mathrm{T}}^2)} \frac{p_{\mathrm{T}}^4}{({p_{\mathrm{T}}}_0^2 + p_{\mathrm{T}}^2)^2}$$

- models color screening and saturation effects
- number of interactions also depends on impact parameter

$$f(x, b) = f(x)g(b)$$

soft → hard model (air-shower)

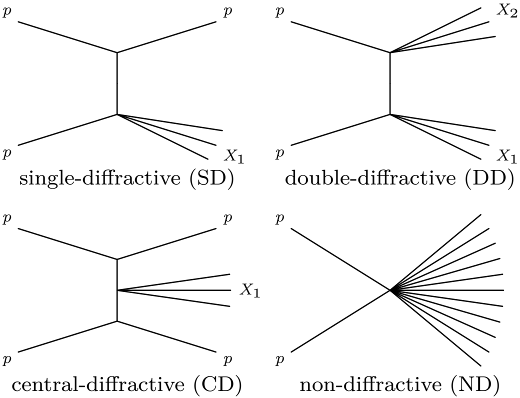
- begin with Regge effective field theory

$$\mathrm{d}\sigma \propto \frac{\mathrm{d}M^2}{M^2}$$

- M is mass of the diffractive system
- exchange of color-singlet pomeron between hadrons
    - leading structure is $f\bar{f}$ or gg
    - at high energy primarily gg
- include hard structure by resolving pomeron constituents
- requires some smooth transition between the two regimes

Event Classification

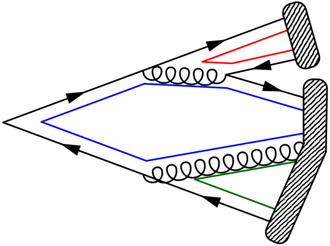
$$\sigma_{\text{inelastic}} = \sigma_{\text{SD}} + \sigma_{\text{DD}} + \sigma_{\text{CD}} + \sigma_{\text{ND}}$$



Hadronization

Models

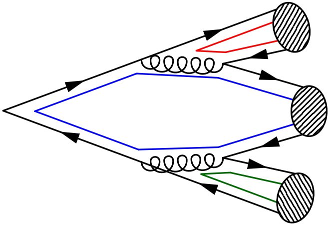
string model (PYTHIA)	cluster model (HERWIG)



- linear confinement

$$V \approx \kappa r - \frac{4\alpha_s}{3r}$$

- split strings into hadrons
- kinematics well modeled
- poor final state flavor description

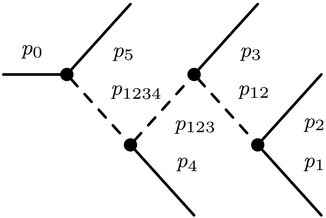


- pre-confinement
  - clusters independent of hard process scale
  - dependent on QCD and shower scale
- decay clusters into hadrons
- kinematics not as well modeled
- better final state flavor description

Particle Decays

Phase Space

- RAMBO
  - random momenta and boost
  - requires massless decay products
- m-generator
  - mass generator
  - two-body decays through intermediate masses



$$
\begin{aligned}
\mathrm{d}\Phi_2(q_0, q_1, q_2) &= \left(\frac{1}{(2\pi)^2 2^2}\right) \delta(q_0 - q_1 - q_2) \frac{\mathrm{d}\vec{q}_1}{E_1} \frac{\mathrm{d}\vec{q}_2}{E_2} \\
\mathrm{d}\Phi_3(q_0, q_1, q_2, q_3) &= \left(\frac{1}{(2\pi)^5 2^3}\right) \delta(q_0 - q_1 - q_2 - q_3) \frac{\mathrm{d}\vec{q}_1}{E_1} \frac{\mathrm{d}\vec{q}_2}{E_2} \frac{\mathrm{d}\vec{q}_3}{E_3} \\
&\quad \delta(q_{12} - q_1 - q_2)\delta(q_{12}^2 - m_{12}^2)\, \mathrm{d}q_{12}\, \mathrm{d}m_{12}^2 \\
&= \left(\frac{2}{\pi}\right) \mathrm{d}\Phi_2(q_0, q_{12}, q_3) m_{12}\, \mathrm{d}m_{12}\, \mathrm{d}\Phi_2(q_{12}, q_1, q_2)
\end{aligned}
$$

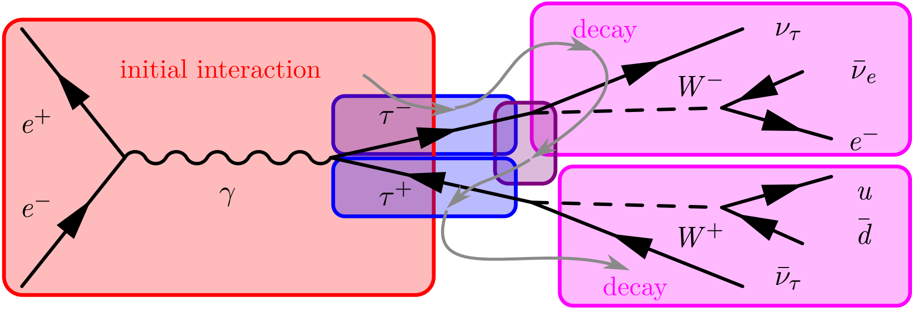
Helicity Correlations

- algorithm by Collins and Knowles, expanded by Richardson
- $D \equiv$ decay matrix for each particle, $D_{\text{initial}} = \mathbb{I}$
- $\mathcal{M} \equiv$ matrix element, $\rho \equiv$ density matrix

1. Calculate $\mathcal{M}$ for the initial interaction.
2. Find ρ for an outgoing particle using the interaction $\mathcal{M}$ and D's of the remaining outgoing particles.
3. Decay the particle using its $\mathcal{M}$, ρ, and the D's of its decay products.
4. Repeat 2 - 3 until all decay products are stable.
5. Calculate D for the particle.
6. Go up a decay and perform 2 - 5 on the undecayed particles.
7. Repeat 2 - 6 until all particles are decayed.

Example

2. $\rho^{j}_{\lambda_j\lambda'_j} = \rho^{1}_{\kappa_1\kappa'_1}\rho^{2}_{\kappa_2\kappa'_2}\mathcal{M}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_n}\mathcal{M}^{*}_{\kappa'_1\kappa'_2;\lambda'_1\ldots\lambda'_n}\prod_{k\neq j} D^{k}_{\lambda_k\lambda'_k}$
3. $\mathcal{W}_{\text{decay}} = \rho_{\lambda_0\lambda'_0}\mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^{*}_{\lambda'_0;\lambda'_1\ldots\lambda'_n}\prod_{k=1,n} D^{k}_{\lambda_k\lambda'_k}$
5. $D_{\lambda_0\lambda'_0} = \mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^{*}_{\lambda'_0;\lambda'_1\ldots\lambda'_n}\prod_{l=1,n} D^{l}_{\lambda_l\lambda'_l}$



Tools
(by no means exhaustive)

Resources

- HepForge
  - https://www.hepforge.org/
  - collection of high energy physics tools
  - primarily MC related, not exhaustive but most complete available list
- *General-purpose event generators for LHC physics*
  - http://arxiv.org/abs/1101.2599
  - Buckley, Butterworth, Gieseke, Grellsheid, Höche, Hoeth, Krauss, Lönnblad, Nurse, Richardson, Schumann, Seymour, Sjöstrand, Skands, Webber
  - excellent review of current (relatively) MC techniques
- *QCD and Collider Physics* (the big pink book)
  - http://www.hep.phy.cam.ac.uk/theory/webber/QCDupdates.html
  - Ellis, Stirling, Webber
  - more detailed theoretical study of phenomenology

Legacy General Purpose

- Pythia 6
  - https://pythia6.hepforge.org/
  - named after the oracle of Delphi
  - written in Fortran
  - most recent version is 6.4.28 released 05/09/2013
  - Lund string fragmentation model
  - no longer actively developed, now superseded by Pythia 8
- Herwig
  - http://www.hep.phy.cam.ac.uk/theory/webber/Herwig/
  - Hadron Emission Reactions With Interfering Gluons
  - written in Fortran
  - most recent version is 6.5.21 released 11/03/2013
  - interface to Mc@Nlo
  - no longer actively developed, now superseded by Herwig++

Current General Purpose

- PYTHIA 8
  - http://home.thep.lu.se/~torbjorn/Pythia.html
  - written in C++
  - most recent version is 8.201 released 14/10/2014
  - comprehensive merging and matching schemes, interfaces well with most hard process generators
- HERWIG++ and THEPEG
  - https://herwig.hepforge.org/
  - written in C++
  - most recent version is 2.7.1/1.9.2 released 07/07/2014
  - fully automated LO BSM with helicity correlations
- SHERPA
  - https://sherpa.hepforge.org
  - written in C++
  - most recent version is 2.1.1 released 06/06/2014
  - automated LO n-leg hard processes with COMIX and AMEGIC

Hard Coded Hard Process

- PowhegBox
    - http://powhegbox.mib.infn.it/
    - written in Fortran
    - most recent version is 2 released 29/03/2011 (but rolling in SVN)
    - structure for user written processes, large number of processes
- Mc@Nlo
    - http://www.hep.phy.cam.ac.uk/theory/webber/MCatNLO/
    - written in Fortran
    - most recent version is 4.10 released 17/07/2013
    - large number of processes, cohesive user interface
    - aMc@Nlo focus of current development
- Mcfm
    - http://mcfm.fnal.gov/
    - Monte Carlo for FeMtobarn processes
    - written in Fortran
    - most recent version is 6.8 released 26/04/2014

Automated Hard Process

- BlackHat
  - http://blackhat.hepforge.org/
  - written in C++
  - still in beta
  - n-leg with one-loop in QCD
- MadGraph 5
  - http://madgraph.hep.uiuc.edu/
  - written in Fortran, Python, and C++
  - most recent version is 2.2.1 released 25/09/2014
  - n-leg with one-loop in QCD and QED with aMc@Nlo
  - can be showered with Pythia 8 using FxFx

After-burners

- TAUOLA
  - http://tauolapp.web.cern.ch/tauolapp/
  - written in FORTRAN and C++
  - most recent version is 1.1.5 release 04/06/2014
  - tau decays
- PHOTOS
  - http://photospp.web.cern.ch/photospp/
  - written in FORTRAN and C++
  - most recent version is 3.65 released 03/05/2014
  - QED FSR emissions
- EVTGEN
  - http://evtgen.warwick.ac.uk/
  - written in C++
  - most recent version is 1.3.0 released 01/10/2014
  - B and D decays

Conclusions

MC at LHCb

- generation was primarily through Pythia 6, migrated to Pythia 8
- use case very different from other experiments, care primarily about B and D decays
- mainly use minbias with EvtGen on top
- focus not on hard processes, but do have PowhegBox and Alpgen interfaced with Pythia 6 showers
- also have Herwig++ interfaced

Outlook

- very active MC community and development
- large variety of tools available
- pick your tools carefully!
- exciting developments in automated process calculation and shower matching/merging