FINITE SIZE OF HADRONS AND BOSE-EINSTEIN CORRELATIONS

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STANDARD APPROACH

BOSE-EINSTEIN CORRELATION BETWEEN MOMENTA OF TWO IDENTICAL HADRONS

$$C(p_1, p_2) \equiv \frac{N(p_1, p_2)}{N(p_1)N(p_2)} - 1 \qquad (1)$$

IS USUALLY ANALYZED USING THE FORMULA

$$C(p_1, p_2) = \frac{\tilde{w}(P_{12}; Q)\tilde{w}(P_{12}; -Q)}{w(p_1)w(p_2)} = \frac{|\tilde{w}(P_{12}, Q)|^2}{w(p_1)w(p_2)} \geq 0 \qquad (2)$$

WHERE $w(p.x)$ IS THE SINGLE-PARTICLE "DISTRIBUTION" (WIGNER FUNCTION) AND

$$\tilde{w}(P_{12}; Q) = \int dx\ e^{iQx} w(P_{12}; x); \qquad w(p) = \int dx\ w(p; x)$$

$$P_{12} = (p_1 + p_2)/2; \qquad Q = p_1 - p_2,$$

THIS PROCEDURE ASSUMES THAT HADRONS ARE ***UNCORRELATED*****.**

DATA L3

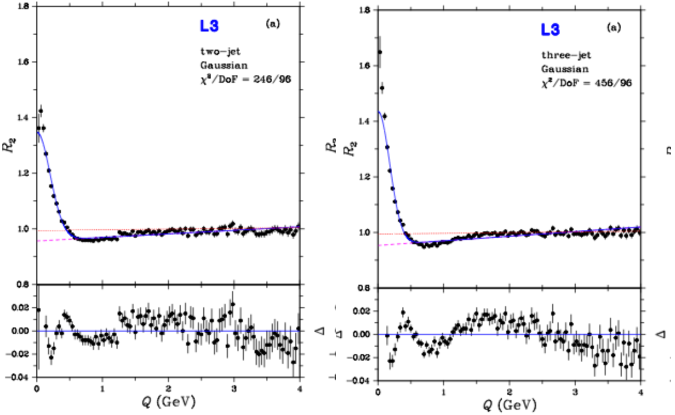


Figure: L3 data for two-jet and three-jet events.

DATA CMS 1

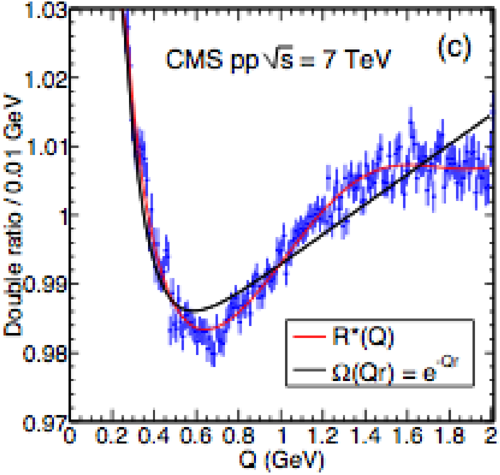


Figure: Two-pion correlation function from CMS (pp at 7 TeV)

GENERAL TWO PARTICLE CORRELATIONS

LET $W(p_1, p_2; x_1, x_2)$ **BE THE MOMENTUM AND SPACE "DISTRIBUTION" OF TWO PARTICLES ("SOURCE FUNCTION"). IF PARTICLES ARE IDENTICAL, THE OBSERVED MOMENTUM DISTRIBUTION IS**

$$\Omega(p_1, p_2) = \int dx_1 dx_2 W(p_1, p_2; x_1, x_2) + \\ + \int dx_1 dx_2 e^{i(x_1 - x_2)Q} W(P_{12}, P_{12}; x_1, x_2) \equiv \\ \equiv \Omega_0(p_1, p_2) \left[1 + C(p_1, p_2)\right] \tag{3}$$

WHERE $P_{12} = (p_1 + p_2)/2$, $Q = p_1 - p_2$, **AND**

$$\Omega_0(p_1, p_2) = \int dx_1 dx_2 W(p_1, p_2; x_1, x_2) \tag{4}$$

ONE SEES THAT $C(p_1, p_2)$ **GIVES INFORMATION ONLY ON THE DISTRIBUTION OF** $x_1 - x_2$**.**

NO CORRELATIONS BETWEEN PARTICLES

IF THERE ARE NO CORRELATIONS BETWEEN PARTICLES,

$$W(p_1, p_2; x_1, x_2) = w(p_1, x_1)w(p_2, x_2)$$

THEN $\Omega(p_1, p_2) = w(p_1)w(p_2) + |\tilde{w}(P_{12}, Q)|^2$**,**

WHERE $\tilde{w}(P_{12}, Q) = \int dx\ w(P_{12}, x)e^{ixQ}.$

THUS THE CORRELATION FUNCTION IS

$$C_2(p_1, p_2) = \frac{|\tilde{w}(P_{12}, Q)|^2}{w(p_1)w(p_2)} \geq 0!!!! \quad (5)$$

THIS IS THE COMMONLY USED FORMULA.

FROM $\tilde{w}(P_{12}, Q)$ **ONE CAN RECOVER** $w(P_{12}, x)$**.**

BUT: *THIS IS VALID ONLY IF THERE ARE NO INTER-PARTICLE CORRELATIONS.*

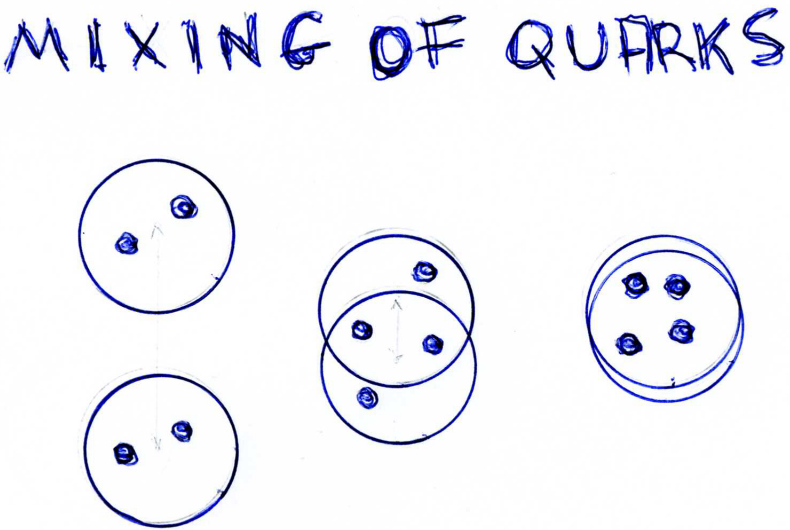
CORRELATIONS IN SPACE (1)

**IDEA: WHEN PIONS ARE TOO CLOSE TO EACH OTHER THEY ARE *NOT* PIONS ANYMORE!!!
(BECAUSE THEIR CONSTITUENTS ARE MIXING AND THEIR WAVE FUNCTIONS ARE NOT WELL-DETERMINED).**

SINCE HBT EXPERIMENTS MEASURE QUANTUM INTERFERENCE BETWEEN THE WAVE FUNCTIONS OF PIONS, THEY CANNOT SEE PIONS WHICH ARE TOO CLOSE TO EACH OTHER.

THEREFORE $W(P_{12}, P_{12}; x_1, x_2)$ **MUST VANISH AT SMALL** $|x_1 - x_2|$**, IMPLYING** *CORRELATION* **BETWEEN POSITIONS OF TWO PIONS.**

PICTURE



CORRELATIONS IN SPACE (2)

Repeat: $W(P_{12}, P_{12}; x_1, x_2)$ **MUST VANISH AT** $|x_1 - x_2| \approx 0$**, MEANING** *CORRELATION* **BETWEEN POSITIONS OF TWO PIONS. THIS IS THE** *NECESSARY* **CONSEQUENCE OF THE FUNDAMENTAL PROPERTY OF HADRONS: THEY ARE** *NOT* **POINT-LIKE.**

THUS THE TWO-PION DISTRIBUTION IS OF THE FORM

$$W(P_{12}, P_{12}; x_1, x_2) = w(P_{12}; x_1)w(P_{12}; x_2)[1 - D(x_1 - x_2)]. \quad (6)$$

WHERE THE FUNCTION $D(x_1 - x_2)$ **EQUALS 1 AT SMALL** $(x_1 - x_2)$ **(BELOW, SAY, 1 fm) AND VANISHES AT LARGER DISTANCES.**

CORRELATIONS IN SPACE (3)

THE HBT CORRELATION FUNCTION BECOMES:

$$C(P_{12}, Q) = \frac{|\tilde{w}(P_{12}, Q)|^2}{w(p_1)w(p_2)} - C_{corr}(p_1, p_2);$$

$$C_{corr} = \frac{\int dx_1 dx_2 e^{i(x_1-x_2)Q} w(P_{12}; x_1) w(P_{12}; x_2) D(x_1 - x_2)}{w(p_1)w(p_2)} \tag{7}$$

ONE SEES THAT THE CORRELATED PART IS NEGATIVE. MOREOVER, SINCE IT GETS CONTRIBUTION ONLY FROM THE REGION OF SMALL x**, IT EXTENDS TO LARGER** Q **THAN THE FIRST, UNCORRELATED, PART. CONSEQUENTLY, THE TOTAL HBT CORRELATION FUNCTION IS EXPECTED TO BE** *NEGATIVE* **AT LARGE** Q**.**

EXAMPLE

FOR ILLUSTRATION, TAKE

$$\Delta(x_1 - x_2) = \Theta[r_{cut}^2 - |\vec{x}_1 - \vec{x}_2|^2 - (t_1 - t_2)^2];$$

$$w(P,x) = e^{-|\vec{x}|^2/R^2} e^{-t^2/\tau^2} f(P)$$

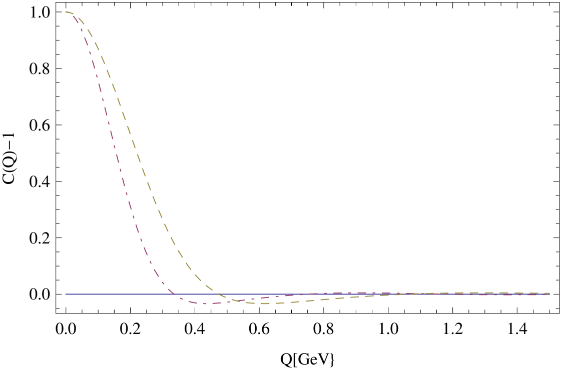


Figure: Oscillating two-pion correlation function. $R = r_{cut} = \tau = 1$ fm.

DATA CMS 2

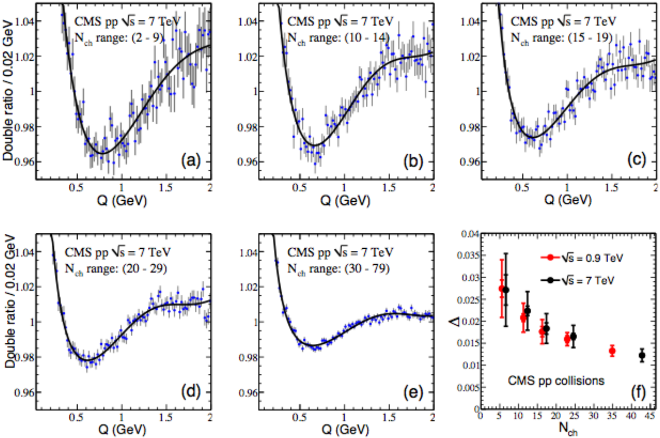


Figure: Two-pion correlation function for various multiplicities from CMS (pp at 7 TeV)

COMMENTS

(i) The presented qualitative argument shows that the observed negative values of the HBT correlation function are not accidental but reflect the fundamental fact that hadrons are NOT POINT-LIKE. Therefore this region of Q^2 deserves special attention in data analysis. It seems that the effect simply MUST BE THERE and the real experimental challenge is to determine its position and its size. Precise measurements should allow to determine the distance at which the hadron structure is affected by its neighbors and thus also the density at which the hadron gas starts melting into quarks and gluons.

(ii) More serious calculations, as well as a detailed comparison with data are clearly needed and are in progress (together with W.Florkowski). The preliminary results indicate that the effect significantly depends on the orientation of Q. This points to necessity of separate measurements in *side*, *out* and *long* directions.

DERIVATION OF THE HBT FORMULA (I)

Density matrix in momentum space:

$$\rho(p_1, p_2; p_1', p_2') =$$

$$= \int dx_1 dx_2 e^{i(p_1 x_1 + p_2 x_2)} \int dx_1' dx_2' e^{-i(p_1' x_1' + p_2' x_2')} \rho(x_1, x_2; x_1', x_2') \quad (8)$$

The particle distribution is

$$\Omega(p_1, p_2) = \rho(p_1, p_2; p_1, p_2) \quad (9)$$

The Wigner function:

$$W(p_1, p_2; x_1^+, x_2^+) = \int dx_1^- dx_2^- e^{i(p_1 x_1^- + p_2 x_2^-)} \rho(x_1, x_2; x_1', x_2') \quad (10)$$

with

$$x^+ = (x + x')/2; \quad x^- = x - x' \quad (11)$$

DERIVATION OF THE HBT FORMULA (II)

Symmetrization:

$$\rho(p_1, p_2; p_1', p_2') \to \rho(p_1, p_2; p_1', p_2') + \rho(p_1 p_2; p_2', p_1') \quad (12)$$

$$\Omega(p_1, p_2) = \rho(p_1, p_2; p_1, p_2) + \rho(p_1 p_2; p_2, p_1) =$$
$$= \int dx_1 dx_2 e^{i(p_1 x_1 + p_2 x_2)} \int dx_1' dx_2' e^{-i(p_1 x_1' + p_2 x_2')} \rho(x_1, x_2; x_1', x_2') +$$
$$+ \int dx_1 dx_2 e^{i(p_1 x_1 + p_2 x_2)} \int dx_1' dx_2' e^{-i(p_2 x_1' + p_1 x_2')} \rho(x_1, x_2; x_1', x_2') \quad (13)$$

$$dx_1 dx_1' = dx_1^+ dx_1^-; \quad dx_1 dx_1' = dx_2^+ dx_2^-$$

$$p_1 x_1 + p_2 x_2 - p_1 x_1' - p_2 x_2' = p_1 x_1^- - p_2 x_2^-$$
$$p_1 x_1 + p_2 x_2 - p_2 x_1' - p_1 x_2' = P_{12} x_1^- + P_{12} x_2^- + Q(x_1^+ - x_2^+) \quad (14)$$

$$P_{12} = (p_1 + p_2)/2; \quad Q = p_1 - p_2$$

DERIVATION OF THE HBT FORMULA (III)

$$\Omega(p_1,p_2) = \int dx_1^+ dx_2^+ \int dx_1^- dx_2^- e^{i(p_1 x_1^- - p_2 x_2^-)} \rho(x_1,x_2;x_1',x_2') +$$
$$+ \int dx_1^+ dx_2^+ e^{iQ(x_1^+ - x_2^+)} \int dx_1^- dx_2^- e^{i(P_{12} x_1^- + P_{12} x_2^-)} \rho(x_1,x_2;x_1',x_2') =$$
$$= \int dx_1^+ dx_2^+ W(p_1,p_2;x_1^+,x_2^+) +$$
$$+ \int dx_1^+ dx_2^+ e^{iQ(x_1^+ - x_2^+)} W(P_{12},P_{12};x_1^+,x_2^+) \qquad (15)$$

If particles are uncorrelated, i.e.

$$W(p_1,p_2;x_1,x_2) = W(p_1,x_1)W(p_2,x_2)$$

one obtains

$$\Omega(p_1,p_2) = \Omega(p_1)\Omega(p_2) + \tilde{W}(P_{12},Q)\tilde{W}^*(P_{12},Q) \qquad (16)$$