BLAST-WAVE MODEL AND HBT RADII IN pp COLLISIONS AT LHC

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OUTLINE

RECENTLY, THE ALICE COLL. PUBLISHED A VAST COLLECTION OF DATA ON HBT RADII MEASURED IN *pp* COLLISIONS AT LHC ENERGIES.

THE GENERAL FEATURES OF THE DATA LOOK, QUALITATIVELY, SIMILAR TO THE ONES OBSERVED EARLIER IN HEAVY ION REACTIONS.

THEREFORE IT IS TEMPTING TO ANALYZE THEM IN TERMS OF THE IDEAS DEVELOPED FOR HEAVY ION COLLISIONS.

WE CONSIDERED THE BLAST WAVE MODEL WHICH SUMMARIZES THE RESULTS OF THE HYDRO CALCULATIONS USING FEW PARAMETERS *WITH A VERY CLEAR PHYSICAL MEANING*.

DATA FROM ALICE

THE MEASUREMENTS GIVE THE HBT RADII OBTAINED FROM FITTING THE MEASURED HBT CORRELATION FUNCTION TO A GAUSSIAN:

$$C(Q^2) \sim e^{-Q^2 R^2} \tag{1}$$

WHERE $Q = P_1 - P_2$ **IS THE MOMENTUM DIFFERENCE WITHIN THE MEASURED PAIR OF PIONS AND** R **IS THE QUOTED RADIUS.**

THE SET CONSISTS OF $6 \times 8 \times 3 = 144$ **DATA POINTS:**

(i) 6 INTERVALS OF TRANSVERSE MOMENTUM from $< 163 >$ **till** $< 650 >$ **MeV**

(ii) 8 INTERVALS OF MULTIPLICITY from ~ 7 **till** ~ 55**.**

(iii) 3 DIRECTIONS: *OUT*, *SIDE* **AND** *LONG***.**

THE BLAST-WAVE MODEL

WE WERE USING THE STANDARD VERSION OF THE MODEL WITH THE SOURCE FUNCTION EXHIBITING:

(i) BOOST INVARIANCE OF THE SYSTEM;

(ii) FREEZE-OUT AT A FIXED (LONGITUDINAL) PROPER TIME $\tau \equiv \sqrt{t^2 - z^2} = \tau_f$**;**

(iii) BOLTZMAN DISTRIBUTION OF THE PRODUCED PARTICLES:

$$dN \sim e^{-\beta E_{rest}} = e^{-\beta u_\mu p^\mu};$$

u_μ **IS THE 4-VELOCITY OF THE FLUID AND** $\beta = 1/T$**.**

(iii) AZIMUTHAL SYMMETRY;

(iv) HUBBLE-LIKE FLOW IN TRANSVERSE DIRECTION:

$$u_\perp \equiv \sinh\theta = \omega r$$

WHERE r **IS THE DISTANCE FROM THE CENTRE;**

(v) COOPER-FRY FORMULA TO ACCOUNT FOR THE EFFECTS OF THE FLOW.

THE SOURCE FUNCTION

WITH THE ASSUMPTIONS LISTED ABOVE, ONE CAN WORK OUT THE EXPLICIT FORM OF THE SOURCE FUNCTION (AT KINETIC FREEZE-OUT)

$$S(p,x)d^4x \sim d\eta \cosh \eta d^2r \; f(r)e^{-\beta p^\mu u_\mu} \tag{2}$$

WHERE η IS THE DIFFERENCE BETWEEN RAPIDITY OF THE PARTICLE AND SPATIAL RAPIDITY OF THE FLUID ELEMENT. $f(r)$ DENOTES A POSITIVE FUNCTION DESCRIBING THE DISTRIBUTION OF THE FLUID IN TRANSVERSE DIRECTION. WE TAKE $f(r)$ IN THE FORM OF A SHIFTED GAUSSIAN:

$$f(r) \sim e^{-(r-R)^2/\delta^2} \tag{3}$$

DESCRIBING EMISSION FROM A SHELL OF THE RADIUS R AND THE WIDTH $\sim 2\delta$. NOTE THAT FOR $R = 0$ THIS DISTRIBUTION BECOMES A SIMPLE GAUSSIAN.

COUNTING THE PARAMETERS

THE MODEL IS DESCRIBED IN TERMS OF 5 PARAMETERS:
(i) TEMPERATURE AT KINETIC FREEZE-OUT, $T = 1/\beta$;
(ii) THE FLOW PARAMETER, ω;
(iii) THE PROPER TIME, τ_f;
(iv) TWO PARAMETERS DESCRIBING THE TRANSVERSE GEOMETRY OF THE SYSTEM, R AND δ.

THE TEMPERATURE WAS FIXED AT T=100 MeV.

THE FLOW PARAMETER ω WAS DETERMINED IN TERMS OF R and δ BY DEMANDING THAT THE AVERAGE TRANSVERSE MOMENTUM EVALUATED FROM THE MODEL REPRODUCES THE CMS DATA.

THUS WE ARE LEFT WITH 3 PARAMETERS τ_f, R and δ AT EACH MULTIPLICITY. THESE THREE PARAMETERS MUST ACCOUNT FOR 3×6 DATA POINTS.

TRANSVERSE MOMENTUM

THE DISTRIBUTION OF TRANSVERSE MOMENTUM IS OBTAINED BY INTEGRATING THE SOURCE FUNCTION OVER THE SPATIAL VARIABLES. THE RESULT IS

$$\frac{dN}{dp_\perp^2} \equiv W(p_\perp) = \int d^4x S(p,x) \sim$$
$$\sim m_\perp \int r dr f(r) K_1(\beta m_\perp \cosh\theta) I_0(\beta p_\perp \sinh\theta) \tag{4}$$

DEMANDING THAT THE AVERAGE TRANSVERSE MOMENTUM FROM THIS FORMULA REPRODUCES THE CMS DATA, GIVES ONE CONSTRAINT ON THE PARAMETERS

HBT CORRELATION FUNCTION

THE HBT CORRELATION FUNCTION IS EXPRESSED IN TERMS OF THE FOURIER TRANSFORM OF THE SOURCE FUNCTION:

$$C(p_1, p_2) = \frac{H(P,Q)H(P,-Q)}{W(p_1)W(p_2)} = \frac{|H(P,Q)|^2}{W(p_1)W(p_2)} \tag{5}$$

WHERE $P = (p_1 + p_2)/2$ **AND** $Q = p_1 - p_2$ **WITH**

$$H(P,Q) \sim \int d^4x S(P,x) e^{iQ_\mu x^\mu} \tag{6}$$

THE RESULT DEPENDS ON THE DIRECTION OF Q**. TRADITIONALLY ONE SELECTS THREE PERPENDICULAR DIRECTIONS:** ***LONG*****,** ***OUT*** **AND** ***SIDE*****. THE CORRESPONDING FORMULAE ARE KNOWN; see e.g. AB, W.Florkowski and K.Zalewski, Acta Phys. Pol. B45 (2014) 1883.**

THE HBT RADII

THE HBT RADII ARE DETERMINED IN THE ALICE EXPERIMENT BY FITTING THE HBT CORRELATION FUNCTION TO THE FORMULA

$$C(p_1, p_2) \sim e^{-Q^2 R^2} \tag{7}$$

THIS MEANS THAT, THEORETICALLY, ONE CAN EVALUATE R FROM THE FORMULA

$$R^2 = -\frac{d}{dQ^2} \log[C(p_1, p_2)] = -\frac{dC(p_1, p_2)/dQ^2}{C(p_1.p_2)} \tag{8}$$

WE DERIVED THE CORRESPONDING FORMULAE AND USED THEM TO EVALUATE R_{long}, R_{side} AND R_{out} AT EACH $p_\perp$ AND MULTIPLICITY MEASURED BY ALICE COLL.

DETERMINATION OF THE PARAMETERS

AT EACH MULTIPLICITY THERE ARE 3×6 DATA POINTS TO BE DESCRIBED BY THE THREE PARAMETERS: τ_f, R AND δ. WE SEARCHED FOR THE MINIMUM OF χ^2. IT TURNED OUT THAT, ACTUALLY, THE DATA ARE NOT PRECISE ENOUGH TO DETERMINE ALL THREE PARAMETERS.
THEREFORE WE DECIDED TO FIX δ AT THE VALUE OF 0.75 fm FOR ALL MULTIPLICITIES. THIS IS A TENTATIVE ASSUMPTION WHICH MAY TURN OUT INCORRECT WHEN DATA OF BETTER QUALITY ARE AVAILABLE.
IT IS, HOWEVER, RATHER UNLIKELY THAT THE ACTUAL VALUE OF δ IS SMALLER THAN THIS NUMBER, BECAUSE THEN THE CORRELATION FUNCTIONS THEMSELVES REVEAL SOME STRANGE FEATURES AT LARGE VALUES OF Q^2: EITHER LONG TAILS OR LARGE OSCILLATIONS .

THE RESULT (1)

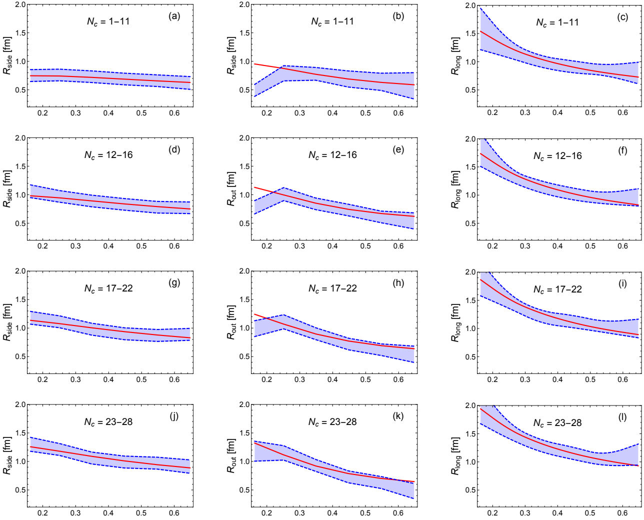

Figure: The model results (solid curves) compared to the experiment results (central points of the bands). The width of the bands represents the experimental error.

THE RESULT (2)

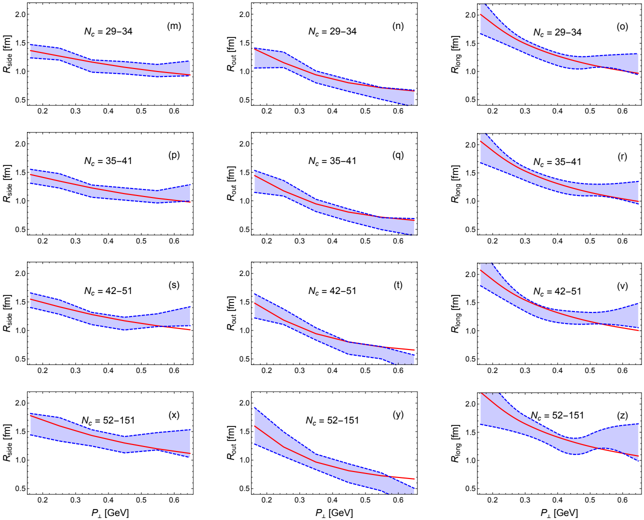

Figure: The model results (solid curves) compared to the experiment results (central points of the bands). The width of the bands represents the experimental error.

RADIUS AND PROPER TIME

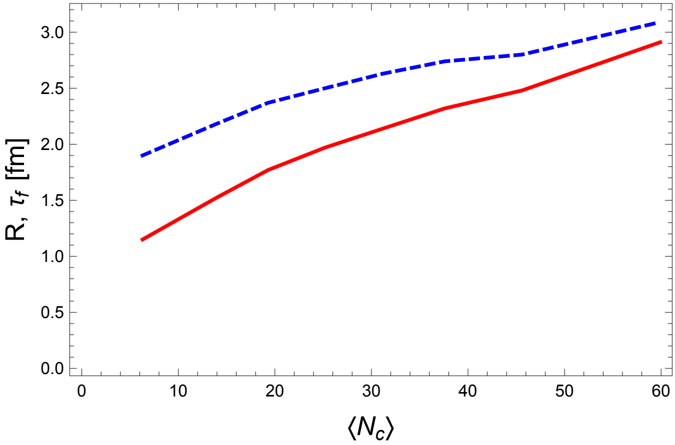

Figure: The fitted values of R (solid line) and τ_f (dashed line) as functions of the mean multiplicity.

FLOW PARAMETERS

$$u_\perp \equiv \sinh\theta = \omega r \quad (9)$$

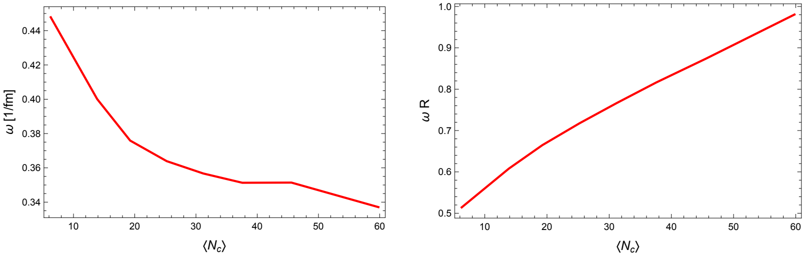

Figure: The calculated flow parameters ω (left) and ωR (right) plotted vs. N_c.

EFFECTIVE VOLUME

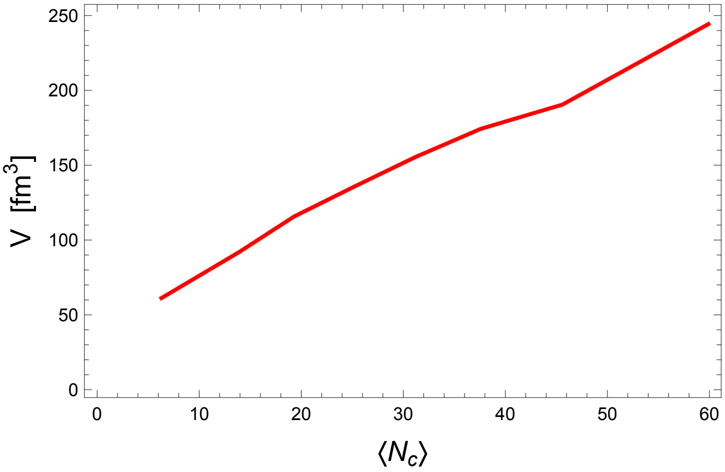

Figure: Effective volume of the system as function of the multiplicity N_c.

NON-GAUSSIAN CORRELATION FUNCTION

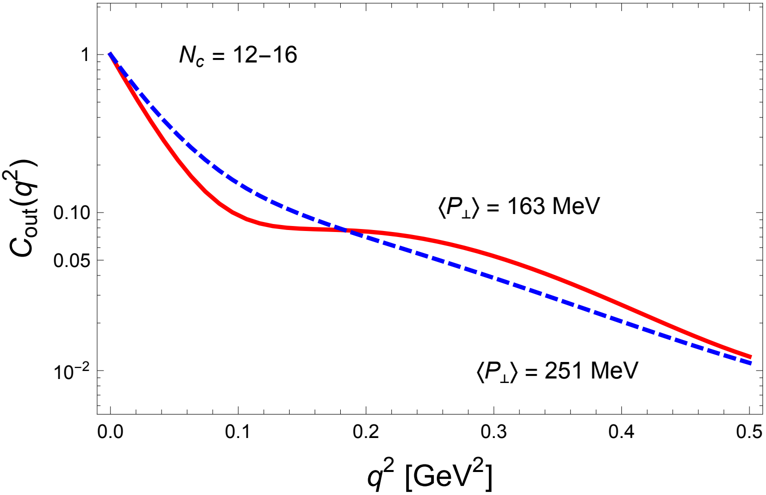

Figure: The function $\log[C_{out}]$ for $12 \leq N_c \leq 18$, plotted vs q^2, to illustrate deviations from the Gaussian behaviour. Full line: $\langle P_\perp \rangle$=163 MeV. Dashed line: $\langle P_\perp \rangle$=251 MeV.

SUMMARY AND QUESTIONS

(i) THE BLAST-WAVE MODEL WORKS IN pp **COLLISIONS; DOES IT REALLY MEAN THE PRESENCE OF FLOW? WILL IT SURVIVE WHEN SYSTEMATIC ERRORS ARE REDUCED?**

(ii) THE TRANSVERSE GEOMETRY AT FREEZE-OUT IS *NOT* **GAUSSIAN BUT REPRESENTS RATHER AN EMISSION OF PARTICLES FROM A SHELL. THIS FEATURE IS ESSENTIAL FOR THE CORRECT DESCRIPTION OF THE RATIO** R_{out}/R_{side}**. WILL IT BE PRESENT ALSO IN** $p-Pb$ **COLLISIONS?**

(iii) AS EXPECTED, BOTH R **AND** τ_f **INCREASE WITH INCREASING MULTIPLICITY. BUT THE PARTICLE DENSITY AT THE FREEZE-OUT ALSO INCREASES (reaching up to** $\sim 1/4$ **fm**^{3}**).**

(iv) THE MODEL GIVES A HOST OF PREDICTIONS FOR THE CORRELATION FUNCTIONS AT LARGER Q **(to be checked with future data).**

DERIVATION OF THE HBT FORMULA (I)

Density matrix in momentum space:

$$\rho(p_1, p_2; p_1', p_2') =$$

$$= \int dx_1 dx_2 e^{i(p_1 x_1 + p_2 x_2)} \int dx_1' dx_2' e^{-i(p_1' x_1' + p_2' x_2')} \rho(x_1, x_2; x_1', x_2') \quad (10)$$

The particle distribution is

$$\Omega(p_1, p_2) = \rho(p_1, p_2; p_1, p_2) \quad (11)$$

The Wigner function:

$$W(p_1, p_2; x_1^+, x_2^+) = \int dx_1^- dx_2^- e^{i(p_1 x_1^- + p_2 x_2^-)} \rho(x_1, x_2; x_1', x_2') \quad (12)$$

with

$$x^+ = (x + x')/2; \quad x^- = x - x' \quad (13)$$

DERIVATION OF THE HBT FORMULA (II)

Symmetrization:

$$\rho(p_1, p_2; p_1', p_2') \to \rho(p_1, p_2; p_1', p_2') + \rho(p_1 p_2; p_2', p_1') \tag{14}$$

$$\Omega(p_1, p_2) = \rho(p_1, p_2; p_1, p_2) + \rho(p_1 p_2; p_2, p_1) =$$
$$= \int dx_1 dx_2 e^{i(p_1 x_1 + p_2 x_2)} \int dx_1' dx_2' e^{-i(p_1 x_1' + p_2 x_2')} \rho(x_1, x_2; x_1', x_2') +$$
$$+ \int dx_1 dx_2 e^{i(p_1 x_1 + p_2 x_2)} \int dx_1' dx_2' e^{-i(p_2 x_1' + p_1 x_2')} \rho(x_1, x_2; x_1', x_2') \tag{15}$$

$$dx_1 dx_1' = dx_1^+ dx_1^-; \quad dx_1 dx_1' = dx_2^+ dx_2^-$$

$$p_1 x_1 + p_2 x_2 - p_1 x_1' - p_2 x_2' = p_1 x_1^- - p_2 x_2^-$$
$$p_1 x_1 + p_2 x_2 - p_2 x_1' - p_1 x_2' = P_{12} x_1^- + P_{12} x_2^- + Q(x_1^+ - x_2^+) \tag{16}$$

$$P_{12} = (p_1 + p_2)/2; \quad Q = p_1 - p_2$$

DERIVATION OF THE HBT FORMULA (III)

$$\Omega(p_1,p_2) = \int dx_1^+ dx_2^+ \int dx_1^- dx_2^- e^{i(p_1 x_1^- - p_2 x_2^-)} \rho(x_1,x_2;x_1',x_2') +$$

$$+ \int dx_1^+ dx_2^+ e^{iQ(x_1^+ - x_2^+)} \int dx_1^- dx_2^- e^{i(P_{12} x_1^- + P_{12} x_2^-)} \rho(x_1,x_2;x_1',x_2') =$$

$$= \int dx_1^+ dx_2^+ W(p_1,p_2;x_1^+,x_2^+) +$$

$$+ \int dx_1^+ dx_2^+ e^{iQ(x_1^+ - x_2^+)} W(P_{12},P_{12};x_1^+,x_2^+) \qquad (17)$$

If particles are uncorrelated, i.e.

$$W(p_1,p_2;x_1,x_2) = W(p_1,x_1)W(p_2,x_2)$$

one obtains

$$\Omega(p_1,p_2) = \Omega(p_1)\Omega(p_2) + \tilde{W}(P_{12},Q)\tilde{W}^*(P_{12},Q) \qquad (18)$$