Bose-Einstein Correlations in e^+e^- annihilation (a review)

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Introduction - BEC

$R_2 = \frac{\rho_2(p_1,p_2)}{\rho_1(p_1)\rho_1(p_2)} \Longrightarrow \frac{\rho_2(Q)}{\rho_0(Q)}$ ρ_0 = 2-particle density of 'reference sample'

Assuming particles produced incoherently
with spatial source density $S(x)$,

$$R_2(Q) = 1 + \lambda|\widetilde{S}(Q)|^2$$

where $\widetilde{S}(Q) = \int \mathrm{d}x\, e^{iQx} S(x)$ – Fourier transform of $S(x)$
$\lambda = 1$ — $\lambda = 0$ if production completely coherent

Assuming $S(x)$ is a spherically symmetric Lévy stable distribution
with radius r, index of stability α ($\alpha = 2$ for a Gaussian) $\Longrightarrow$

$$R_2(Q) = 1 + \lambda\, \mathrm{e}^{-(Qr)^\alpha}$$

Problems with this approach

Assumes

- incoherent average over source
  λ tries to account for
  - partial coherence
  - multiple (distinguishable) sources, long-lived resonances
  - pion purity
- spherical (radius r) Lévy (or Gauss) distribution of particle emitters
  seems unlikely in $\mathrm{e}^+\mathrm{e}^-$ annihilation — jets
- static source, *i*.e., no *t*-dependence
  certainly wrong

Final-State Interactions

1. Coulomb
   - form not certain
     (usually use Gamow factor)
     overcorrects!
   - for R_2, a few % in lowest Q bin
   - double if $+, -$ ref. sample
   - often neglected for R_2
   - but not negligible for R_3
2. Strong interaction - $S = 0$ $\pi\pi$ phase shifts can be incorporated together with Coulomb into the formula for R_2

   Osada, Sano, Biyajima, Z.Phys. C72(1996)285)

   tends to increase λ, decrease r -
   Not used by LEP experiments

Reference Sample

Common choices:

1. $+,-$ pairs
   But different resonances than $+,+$
2. Mixed events – pair particles from different events
   But destroys all correlations, not just BEC

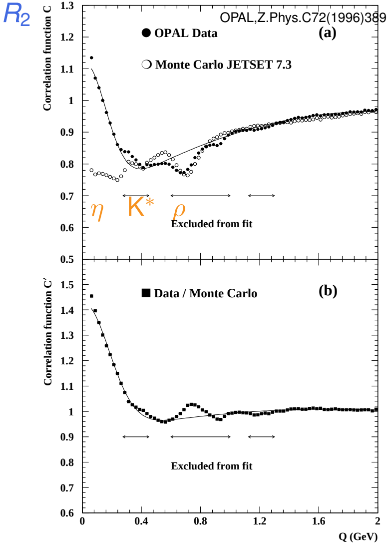
correct by MC (no BEC):

$$\rho_0 \Longrightarrow \rho_0 \frac{\rho_2^{\mathrm{MC}}}{\rho_0^{\mathrm{MC}}}$$

$$R_2 = \frac{\rho_2}{\rho_0} \Longrightarrow \frac{\rho_2}{\rho_0} \Big/ \frac{\rho_2^{\mathrm{MC}}}{\rho_0^{\mathrm{MC}}} \qquad \text{'double ratio'}$$

– But is the MC correct?

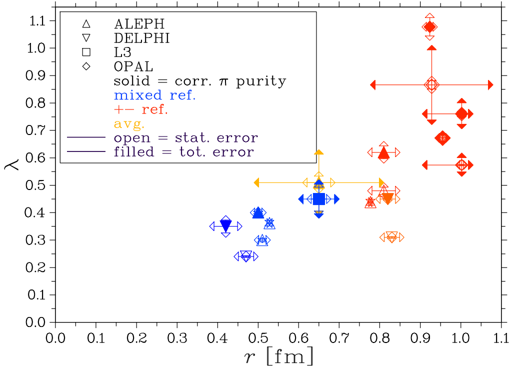
ref. sample, ρ_0, from $+,-$ pairs



Long-range correlations inadequately treated in ref. sample:

$R_2(Q) \propto (1 + \lambda e^{-Q^2 r^2}) \cdot (1 + \delta Q)$ or even $\cdot (1 + \delta Q + \epsilon Q^2)$

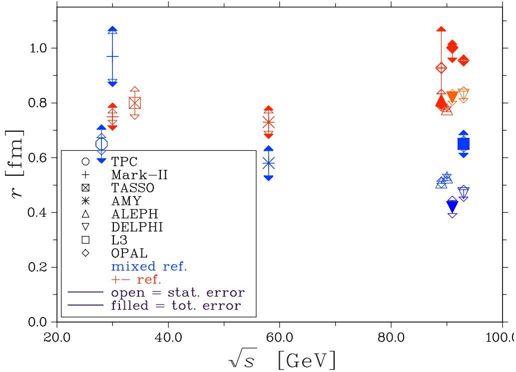
Results from R_2, $\sqrt{s} = M_Z$ (Gaussian parametrization)



– correction for π purity increases λ

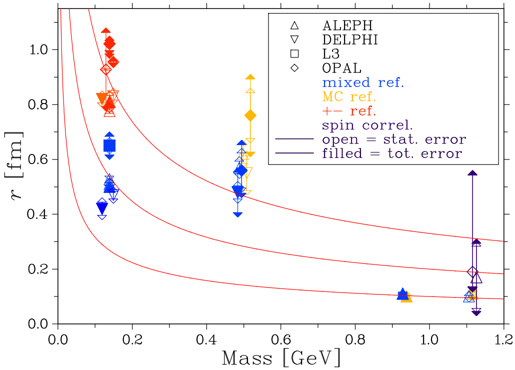
– mixed ref. gives smaller λ, r than $+-$ ref. – Average means little

$\sqrt{s}$ dependence of r



No evidence for $\sqrt{s}$ dependence

Mass dependence of ***r*** — BEC and FDC



No evidence for $r \sim 1/\sqrt{m}$

$r_{\pi-\pi} \approx r_{\text{K-K}}$

$r(\text{mesons}) > r(\text{baryons})$

Disclaimer

- There are many BEC measurements with pions.
- There are also BEC measurements with kaons, and FDC measurements with protons, lambdas, but fewer.
- From here on I will only treat pion results.

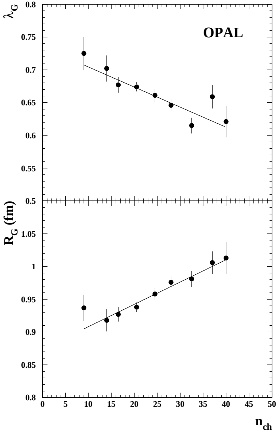
Multiplicity/Jet dependence of λ, r

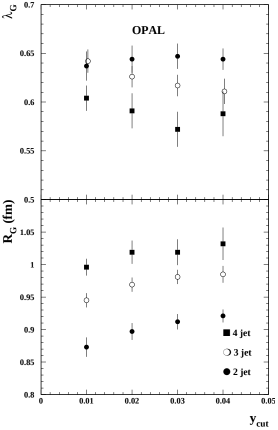
$$R_2(Q) = \gamma(1 + \lambda\, e^{-Q^2 r^2})(1 + \delta Q + \epsilon Q^2)$$

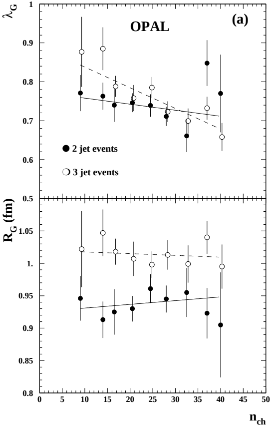
OPAL,Z.Phys.C72(1996)389

λ

r







$\lambda \searrow$ with n_{ch}
$r \nearrow$ with n_{ch}

$\lambda \searrow$ with n_{jet}
$r \nearrow$ with n_{jet}

$\lambda_{\text{n-jet}} \approx$ indep. of n_{ch}
$r_{\text{n-jet}}$ indep. of n_{ch}

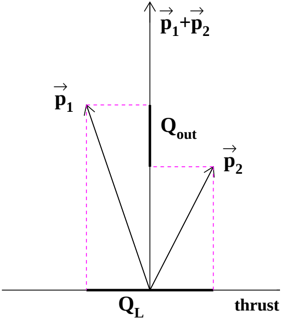
Multiplicity dependence appears to be largely due to number of jets.

Elongation of the source

The usual parametrization assumes a symmetric Gaussian source
But, there is no reason to expect this symmetry in $e^+e^- \to q\bar{q}$.
Therefore, do a 3-dim. analysis in the Longitudinal Center of Mass System
aka Longitudinal Co-Moving System

LCMS:

Boost each π-pair along event axis, e.g., thrust axis



$p_{L1} = -p_{L2}$
$\vec{p}_1 + \vec{p}_2$ defines 'out' axis
$Q_{side} \perp (Q_L, Q_{out})$

the LCMS

Advantages of LCMS:

$$Q^2 = Q_{\mathrm{L}}^2 + Q_{\mathrm{side}}^2 + Q_{\mathrm{out}}^2 - (\Delta E)^2 \qquad \text{where } Q_i^2 = (p_{i\,1} - p_{i\,2})^2$$

$$= Q_{\mathrm{L}}^2 + Q_{\mathrm{side}}^2 + Q_{\mathrm{out}}^2\,(1 - \beta^2) \qquad \text{where } \beta \equiv \frac{p_{\mathrm{out}\,1} + p_{\mathrm{out}\,2}}{E_1 + E_2}$$

Thus, the energy difference,
and therefore the difference in emission time of the pions
couples only to the out-component, Q_{out}.
Thus,
Q_{L} and Q_{side} reflect only spatial dimensions of the source
Q_{out} reflects a mixture of spatial and temporal dimensions.

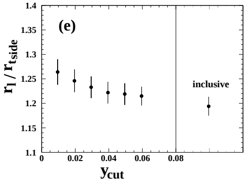
Assuming axial symmetry, source is elliptically shaped with

- r_{L} the logitudinal radius
- r_{side} the transverse radius

Elongation Results

			Gauss / Edgeworth	2-D r_t/r_L	3-D r_{side}/r_L
DELPHI	mixed	2-jet	Gauss	$0.62\pm0.02\pm0.05$	—
ALEPH	mixed	2-jet	Gauss	$0.61\pm0.01\pm0.??$	—
	+−	2-jet	Gauss	$0.91\pm0.02\pm0.??$	—
	mixed	2-jet	Edgeworth	$0.68\pm0.01\pm0.??$	—
	+−	2-jet	Edgeworth	$0.84\pm0.02\pm0.??$	—
OPAL	+−	2-jet	Gauss	—	$0.82\pm0.02\pm^{0.01}_{0.05}$
L3	mixed	all	Gauss	—	$0.80\pm0.02\pm^{0.03}_{0.18}$
	mixed	all	Edgeworth	—	$0.81\pm0.02\pm^{0.03}_{0.19}$

~20% elongation along thrust axis
(ZEUS finds similar results in ep)



OPAL
Elongation larger for narrower jets

3π BEC

Assuming static source density $f(x)$ in space-time, $G(Q) = \int \mathrm{d}x\, e^{iQx} f(x) = G e^{i\phi}$

$$R_2(Q) = \frac{\rho_2(Q)}{\rho_0(Q)} = 1 + \lambda |G(Q)|^2$$

Analog of Q for 3 particles: $\left(Q_3^2 = M_{123}^2 - 9m_\pi^2 = Q_{12}^2 + Q_{23}^2 + Q_{13}^2\right)$

$$R_3(Q_3) = \frac{\rho_3(Q_3)}{\rho_0(Q_3)} = 1 + \underbrace{\lambda\left(|G(Q_{12})|^2 + |G(Q_{23})|^2 + |G(Q_{13})|^2\right)}_{\text{from 2-particle BEC}}$$

$$+ \underbrace{2\lambda^{1.5}\,\Re\{G(Q_{12})G(Q_{23})G(Q_{13})\}}_{\text{from genuine 3-particle BEC}}$$

$$R_3^{\text{genuine}} = 1 + 2\lambda^{1.5}\,\Re\{G(Q_{12})G(Q_{23})G(Q_{13})\}$$

$$\omega = \frac{R_3^{\text{genuine}}(Q_3) - 1}{2\sqrt{(R_2(Q_{12})-1)(R_2(Q_{23})-1)(R_2(Q_{13})-1)}}$$

$$= \cos(\phi_{12} + \phi_{23} + \phi_{13})$$

$$\omega = \frac{R_3^{\text{genuine}}(Q_3) - 1}{2\sqrt{R_2(Q_3) - 1}} \qquad \text{if } f(x) \text{ is Gaussian}$$

If fully incoherent, $\phi_{ij} \neq 0$ only if $f(x)$ asymmetric and $Q_{ij} > 0$

Completely incoherent particle production implies $\lambda = 1$ $\omega = 1$

3π BEC

L3:

from		Gaussian	Edgeworth
R_2 R_3^{genuine}	λ	$0.45 \pm 0.06 \pm 0.03$ $0.47 \pm 0.07 \pm 0.03$	$0.72 \pm 0.08 \pm 0.03$ $0.75 \pm 0.10 \pm 0.03$
R_2 R_3^{genuine}	r (fm)	$0.65 \pm 0.03 \pm 0.03$ $0.65 \pm 0.06 \pm 0.03$	$0.74 \pm 0.06 \pm 0.02$ $0.72 \pm 0.08 \pm 0.03$

Data consistent with $\omega = 1$, *i*.e., fully incoherent.

Values of λ, r from R_2 and R_3^{genuine} are consistent.

expt.		λ	r
MARK-II (29 GeV)	R_2 R_3	$0.45 \pm 0.03 \pm 0.04$ $0.54 \pm 0.06 \pm 0.05$	$1.01 \pm 0.09 \pm 0.06$ $0.90 \pm 0.06 \pm 0.06$
DELPHI	R_2 R_3^{genuine}	$0.24 \pm 0.02 \pm 0.??$ $0.43 \pm 0.05 \pm 0.07$	$0.47 \pm 0.03 \pm 0.??$ $0.93 \pm 0.06 \pm 0.04$
OPAL	R_2 R_3^{genuine}	$0.58 \pm 0.01 \pm 0.??$ $0.63 \pm 0.01 \pm 0.03$	$0.79 \pm 0.02 \pm 0.??$ $0.82 \pm 0.01 \pm 0.04$

Values of λ, r from R_2 and R_3 are fairly consistent.

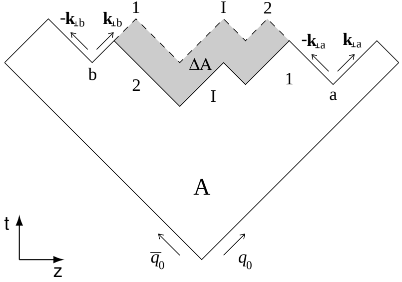
BEC in String Models

Longitudinal BEC

- Different string configurations give same final state
- Matrix element to get a final state depends on area, A:
  $\mathcal{M} = \exp\left[(\imath\kappa - b/2)A\right]$
  where κ is the string tension and b is the decay constant
  $\kappa \approx 1\,\mathrm{GeV/fm}$ and $b \approx 0.3\,\mathrm{GeV/fm}$
- So, must sum all the amplitudes
  But 3-π BEC incoherent ??

Transverse BEC

- Transverse momentum via tunneling, also related to b



Using b from tuning of JETSET, predict

- BEC, including genuine 3-particle BEC
- $r_{\mathrm{t}} < r_{\mathrm{L}}$
- $r(\pi^0\pi^0) < r(\pi^+\pi^+)$

2-particle BEC $\pi^0\pi^0$ and $\pi^\pm\pi^\pm$

- Naively expect same BEC for $\pi^0\pi^0$ and $\pi^\pm\pi^\pm$
- Hadronization with local charge conservation,
  e.g., string, $\Longrightarrow r_{00} < r_{\pm\pm}$
  But most π's from resonances — dilutes this effect.

- Many measurements of BEC with charged π's
- but few with π^0's

  in e^+e^-: L3, P.L. B524 (2002) 55
  OPAL, P.L. B559 (2003) 131

Selection:

OPAL	L3
$p_{\pi^0} > 1.0$ GeV	$E(\pi^0) < 6.0$ GeV
2-jet, $T > 0.9$	all events

2-particle BEC $\pi^0\pi^0$ and $\pi^\pm\pi^\pm$

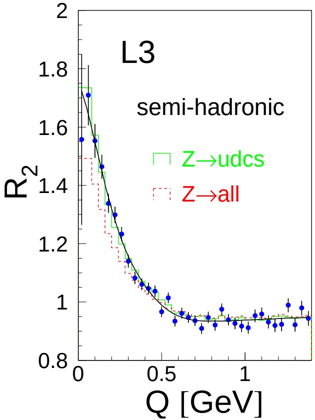
BEC from Z decays
Gaussian parametrization

	Expt.	ρ_0	r (fm)	λ
$\pm\pm$	OPAL	+−	$1.00^{+0.03}_{-0.10}$	0.76 ± 0.06
	L3	mix	0.65 ± 0.04	0.45 ± 0.07
	L3 3-π	mix	0.65 ± 0.07	0.47 ± 0.08
	L3 $E_\pi < 6\,\mathrm{GeV}$	MC	0.46 ± 0.01	0.29 ± 0.03
00	L3 $E_\pi < 6\,\mathrm{GeV}$	MC	0.31 ± 0.10	0.16 ± 0.09
	OPAL $E_\pi > 1$, 2-jet	mix	0.59 ± 0.09	0.55 ± 0.14

- L3: $r_{00} < r_{\pm\pm}$ and $\lambda_{00} < \lambda_{\pm\pm}$, both 1.5σ
- ALEPH, DELPHI find $r_{\pm\pm}(\mathrm{mix})/r_{\pm\pm}(+-) \approx 0.68, 0.51$
  Applying this to OPAL $r_{\pm\pm}$, OPAL $r_{00} \approx r_{\pm\pm}$ and $\lambda_{00} \approx \lambda_{\pm\pm}$
- L3 and OPAL $\pi^0\pi^0$ results disagree by 2σ
- Is the L3-OPAL $\pi^0\pi^0$ difference due to E_π and/or 2-jet selection ???
- OPAL: MC shows that few of selected π^0's are direct from string

Another source of $q\bar{q}$: W

$e^+e^- \to W^+W^- \to q\bar{q}\ell\nu$



BE(W) = BE(Z →light quarks)

$e^+e^- \to W^+W^- \to q\bar{q}q\bar{q}$

If independent decay of W^+W^-,
i.e., no BEC between pions from different W's

$$\begin{aligned} \rho_{4q}(p_1,p_2) = & \quad \rho^+(p_1,p_2) && 1,2 \text{ from } W^+ \\ + & \quad \rho^-(p_1,p_2) && 1,2 \text{ from } W^- \\ + & \quad \rho^+(p_1)\rho^-(p_2) && 1 \text{ from } W^+,\ 2 \text{ from } W^- \\ + & \quad \rho^+(p_2)\rho^-(p_1) && 1 \text{ from } W^-,\ 2 \text{ from } W^+ \end{aligned}$$

Assuming $\rho^+ = \rho^- = \rho_{2q}$, W separation $\sim$ 0.1 fm

$$\rho_{4q}(p_1,p_2) = 2\rho_{2q}(p_1,p_2) + 2\rho_{2q}(p_1)\rho_{2q}(p_2)$$

Inter-W BEC $\Longrightarrow$ W decays *not* independent
$\Longrightarrow$ this relation does *not* hold.

Measure

- $\rho_{4q}(p_1,p_2)$ from $e^+e^- \to W^+W^- \to q\bar{q}q\bar{q}$
- $\rho_{2q}(p_1,p_2)$ from $e^+e^- \to W^+W^- \to q\bar{q}\ell\nu$
- $\rho_{2q}(p_1)\rho_{2q}(p_2)$ from $\rho_{mix}(p_1,p_2)$ obtained by mixing $\ell^+\nu q\bar{q}$ and $q\bar{q}\ell^-\nu$ events

$W^+W^- \to q\bar{q}q\bar{q}$

Measure violation of

$$\rho_{4q}(Q) = 2\rho_{2q}(Q) + 2\rho_{\rm mix}(Q)$$

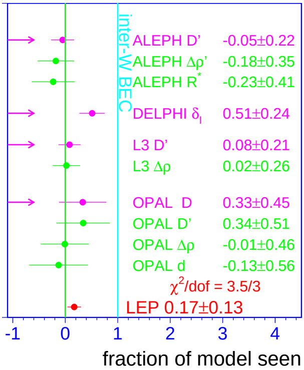
by

$$\Delta\rho(Q) = \rho_{4q}(Q) - [2\rho_{2q}(p_1,p_2) + 2\rho_{\rm mix}(p_1,p_2)]$$

$$D(Q) = \frac{\rho_{4q}(Q)}{2\rho_{2q}(Q) + 2\rho_{\rm mix}(Q)}$$

$$\delta_I(Q) = \frac{\Delta\rho(Q)}{2\rho_{\rm mix}(Q)}$$

Compare to expectation of BE_{32} model in PYTHIA



$\delta_I(Q)$ measures genuine inter-W BEC

DELPHI: $0.51 \pm 0.24 \quad \sim 2\sigma$

average: $0.17 \pm 0.13 \quad \sim 1\sigma$

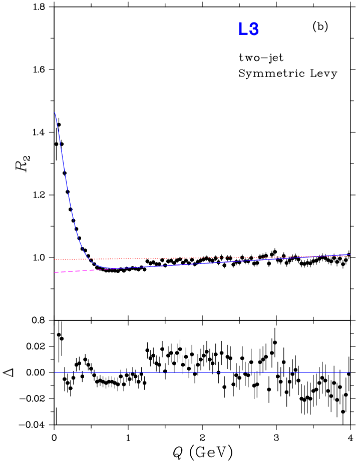
Conclusion: BEC (mostly) between π's from same string

But event selection (4 separated jets) suppresses small Q for π pairs from different strings

Results – ‘Classic’ Parametrizations

$R_2 = \gamma \cdot [1 + \lambda G] \cdot (1 + \epsilon Q)$

- Gaussian
  $G = \exp\left(-(rQ)^2\right)$
- Edgeworth expansion
  $G = \exp\left(-(rQ)^2\right) \cdot \left[1 + \frac{\kappa}{3!} H_3(rQ)\right]$
  Gaussian if $\kappa = 0$
  Fit: $\kappa = 0.71 \pm 0.06$
- symmetric Lévy
  $G = \exp\left(-|rQ|^\alpha\right)$
  $0 < \alpha \leq 2$
  Gaussian if $\alpha = 2$
  Fit: $\alpha = 1.34 \pm 0.04$



	Gauss	Edgew	Lévy
CL:	10^{-15}	10^{-5}	10^{-8}

Poor χ^2. Edgeworth and Lévy better than Gaussian, but poor.
Problem is the dip of R_2 in the region $0.6 < Q < 1.5\,\mathrm{GeV}$

The τ-model

T.Csörgő, W.Kittel, W.J.Metzger, T.Novák, Phys.Lett.**B663**(2008)214
T.Csörgő, J.Zimányi, Nucl.Phys.**A517**(1990)588

- Assume avg. production point is related to momentum:

$$\overline{x}^{\mu}(p^{\mu}) = a\tau p^{\mu}$$

where for 2-jet events, $a = 1/m_{\rm t}$

$\tau = \sqrt{\overline{t}^2 - \overline{r}_z^2}$ is the "longitudinal" proper time

and $m_{\rm t} = \sqrt{E^2 - p_z^2}$ is the "transverse" mass

- Let $\delta_\Delta(x^\mu - \overline{x}^\mu)$ be dist. of prod. points about their mean, and $H(\tau)$ the dist. of τ. Then the emission function is

$$S(x,p) = \int_0^\infty \mathrm{d}\tau H(\tau)\delta_\Delta(x - a\tau p)\rho_1(p)$$

- In the plane-wave approx. F.B.Yano, S.E.Koonin, Phys.Lett.**B78**(1978)556.

$$\rho_2(p_1,p_2) = \int \mathrm{d}^4x_1\mathrm{d}^4x_2 S(x_1,p_1)S(x_2,p_2)\left(1+\cos\left([p_1-p_2][x_1-x_2]\right)\right)$$

- Assume $\delta_\Delta(x^\mu - \overline{x}^\mu)$ is very narrow — a δ-function. Then

$$R_2(p_1,p_2) = 1 + \lambda\,\mathrm{Re}\widetilde{H}\left(\frac{a_1Q^2}{2}\right)\widetilde{H}\left(\frac{a_2Q^2}{2}\right), \quad \widetilde{H}(\omega) = \int \mathrm{d}\tau H(\tau)\exp(i\omega\tau)$$

BEC in the τ-model

- Assume a Lévy distribution for $H(\tau)$
  Since no particle production before the interaction,
  $H(\tau)$ is one-sided.
  Characteristic function is
  $$\widetilde{H}(\omega) = \exp\left[-\frac{1}{2}\left(\Delta\tau|\omega|\right)^{\alpha}\left(1 - i\,\mathrm{sign}(\omega)\tan\left(\frac{\alpha\pi}{2}\right)\right) + i\,\omega\tau_0\right] \;, \quad \alpha \neq 1$$
  where
  - α is the index of stability;
  - τ_0 is the proper time of the onset of particle production;
  - $\Delta\tau$ is a measure of the width of the distribution.
- Then, R_2 depends on Q, a_1, a_2

$$R_2(Q, a_1, a_2) = \gamma\left\{1 + \lambda\cos\left[\frac{\tau_0 Q^2(a_1 + a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha} + a_2^{\alpha}}{2}\right] \cdot\exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha} + a_2^{\alpha}}{2}\right]\right\}\cdot(1 + \epsilon Q)$$

BEC in the τ-model

$$R_2(Q,a_1,a_2) = \gamma\left\{1+\lambda\cos\left[\frac{\tau_0 Q^2(a_1+a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha}+a_2^{\alpha}}{2}\right] \cdot \exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha}+a_2^{\alpha}}{2}\right]\right\}\cdot(1+\epsilon Q)$$

Simplification:

- effective radius, R, defined by $R^{2\alpha} = \left(\frac{\Delta\tau}{2}\right)^{\alpha}\frac{a_1^{\alpha}+a_2^{\alpha}}{2}$
- Particle production begins immediately, $\tau_0 = 0$
- Then

$$R_2(Q) = \gamma\left[1+\lambda\cos\left((R_{\mathrm{a}}Q)^{2\alpha}\right)\exp\left(-(RQ)^{2\alpha}\right)\right]\cdot(1+\epsilon Q)$$

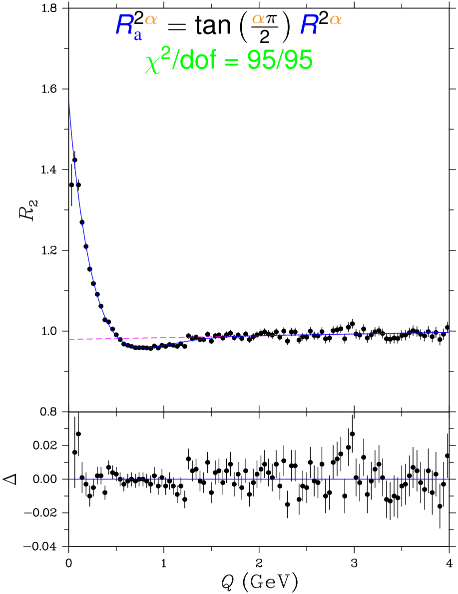
where $R_{\mathrm{a}}^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$

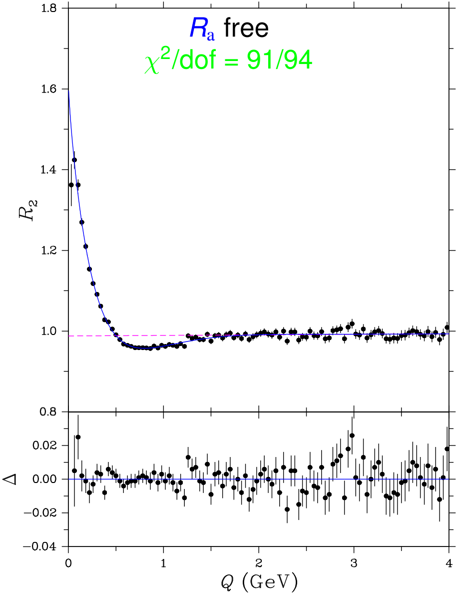
Compare to sym. Lévy parametrization:

$$R_2(Q) = \gamma\left[1+\lambda \qquad \exp\left[-|rQ|^{\alpha}\right]\right](1+\epsilon Q)$$

- R describes the BEC peak
- R_{a} describes the anticorrelation dip
- τ-model: both anticorrelation and BEC are related to 'width' $\Delta\tau$ of $H(\tau)$

2-jet Results on Simplified τ-model from L3 Z decay





Elongation?

- Previous results using fits of Gaussian or Edgeworth found (in LCMS) $r_{\mathrm{side}}/r_{\mathrm{L}} \approx 0.8$ for all events
- But we find that Gaussian and Edgeworth fit $R_2(Q)$ poorly
- τ-model predicts no elongation and fits the data well
- Could the elongation results be an artifact of an incorrect fit function? or is the τ-model in need of modification?
- So, we modify *ad hoc* the τ-model description to allow elongation

Elongation in the Simplified τ-model?

LCMS: $$Q^2 = Q_\mathrm{L}^2 + Q_\mathrm{side}^2 + Q_\mathrm{out}^2 - (\Delta E)^2$$
$$= Q_\mathrm{L}^2 + Q_\mathrm{side}^2 + Q_\mathrm{out}^2 \left(1 - \beta^2\right) , \qquad \beta = \frac{p_{1\mathrm{out}} + p_{2\mathrm{out}}}{E_1 + E_2}$$

Replace $$R^2 Q^2 \Longrightarrow A^2 = R_\mathrm{L}^2 Q_\mathrm{L}^2 + R_\mathrm{side}^2 Q_\mathrm{side}^2 + \rho_\mathrm{out}^2 Q_\mathrm{out}^2$$

Then in τ-model,

$$R_2(Q_\mathrm{L}, Q_\mathrm{side}, Q_\mathrm{out}) = \gamma \left[1 + \lambda \cos\left(\tan\left(\frac{\alpha\pi}{2}\right) A^{2\alpha}\right) \exp\left(-A^{2\alpha}\right)\right]$$
$$\cdot \left(1 + \epsilon_\mathrm{L} Q_\mathrm{L} + \epsilon_\mathrm{side} Q_\mathrm{side} + \epsilon_\mathrm{out} Q_\mathrm{out}\right)$$

for 2-jet events:

		χ^2/dof	CL
τ-model	$R_\mathrm{side}/R_\mathrm{L} = 0.61 \pm 0.02$	14847/14921	66%
Edgeworth	$r_\mathrm{side}/r_\mathrm{L} = 0.64 \pm 0.02$	14891/14919	56%
	consistent		

Elongation is real

α_s

- LLA parton shower leads to a fractal in momentum space
  fractal dimension, α, is related to α_s — Gustafson *et* al.
- Lévy dist. arises naturally from a fractal, or random walk,
  or anomalous diffusion — Metzler and Klafter, Phys.Rep.**339**(2000)1.
- strong momentum-space/configuration space correlation of τ-model $\Longrightarrow$
  fractal in configuration space with same α
- generalized LPHD suggests particle dist. has same properties as gluon dist.
- Putting this all together leads to — Csörgő *et* al.

$$\alpha_s = \frac{2\pi}{3}\alpha^2$$

- Using our value of $\alpha = 0.47 \pm 0.04$ yields $\alpha_s = 0.46 \pm 0.04$
- This value is reasonable for a scale of 1–2 GeV,
  where production of hadrons takes place
  cf., from τ decays $\alpha_s(m_\tau \approx 1.8\,\mathrm{GeV}) = 0.34 \pm 0.03$ — PDG

Multiplicity/Jet/rapidity dependence in τ-model

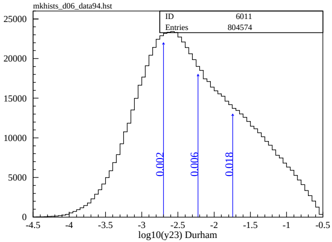
Use simplified τ-model, $\tau_0 = 0$
to investigate multiplicity and jet dependence

To stabilize fits against large correlation of parameters α and *R*, fix $\alpha = 0.44$

Jets

Jets — JADE and Durham algorithms

- force event to have 3 jets:
  - normally stop combining when all 'distances' between jets are $> y_{\text{cut}}$
  - instead, stop combining when there are only 3 jets left
  - y_{23} is the smallest 'distance' between any 2 of the 3 jets
- y_{23} is value of y_{cut} where number of jets changes from 2 to 3



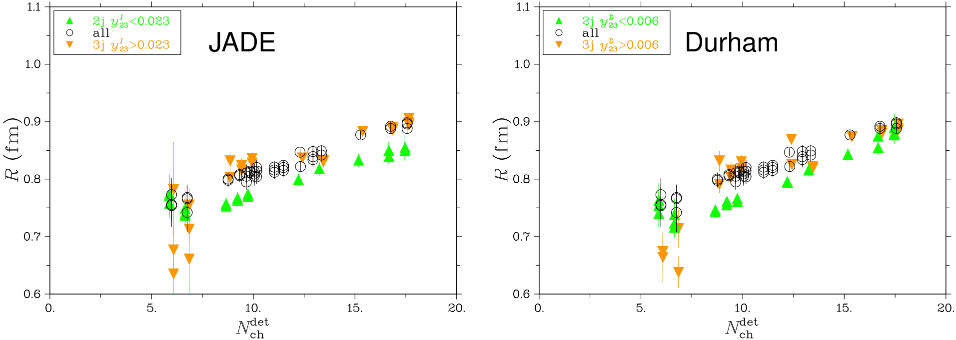
define regions of y_{23}^{D} (Durham):

$y_{23}^{\mathrm{D}} < 0.002$	narrow two-jet	or	
$0.002 < y_{23}^{\mathrm{D}} < 0.006$	less narrow two-jet	$y_{23}^{\mathrm{D}} < 0.006$	two-jet
$0.006 < y_{23}^{\mathrm{D}} < 0.018$	narrow three-jet	$0.006 < y_{23}^{\mathrm{D}}$	three-jet
$0.018 < y_{23}^{\mathrm{D}}$	wide three-jet		

and similarly for y_{23}^{J} (JADE): 0.009, 0.023, 0.056

Multiplicity/Jet dependence in τ-model

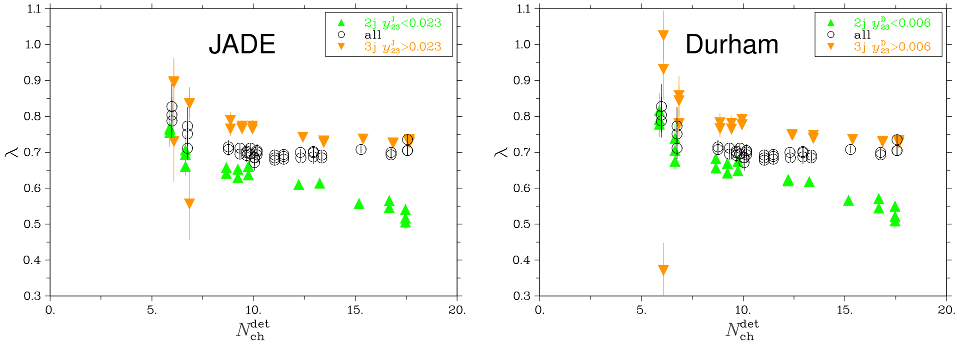
Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$ L3 PRELIMINARY



- R increases with N_{ch} and with number of jets
  whereas OPAL found $r_{n\text{-jet}}$ approx. indep. of N_{ch}
- Increase of R with N_{ch} similar for 2- and 3-jet events
- However, $R_{3\text{-jet}} \approx R_{all}$

Multiplicity/Jet dependence in τ-model

Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$ L3 PRELIMINARY



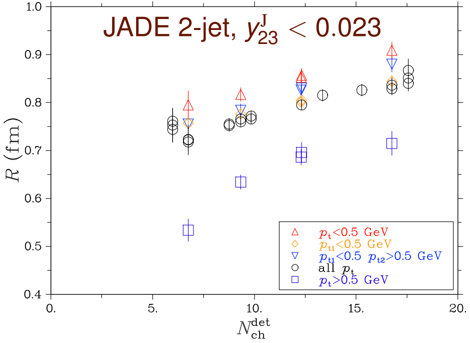
- $\lambda_{\text{3-jet}} > \lambda_{\text{2-jet}}$ opposite of OPAL
- λ initially decreases with N_{ch}
- then λ_{all} and $\lambda_{\text{3-jet}}$ approx. constant
  while $\lambda_{\text{2-jet}}$ continues to decrease, but more slowly
- whereas OPAL found λ_{all} decreasing approx. linearly with N_{ch}

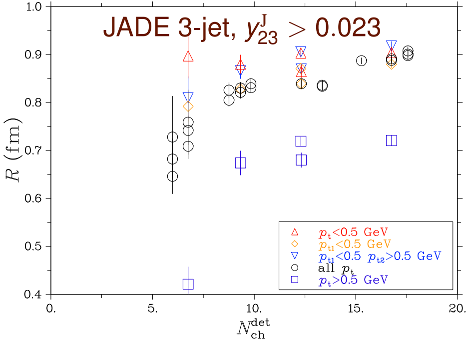
m_t dependence in τ-model

Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$

L3 PRELIMINARY

and cutting on $p_t = 0.5\,\mathrm{GeV}$ ($m_t = 0.52\,\mathrm{GeV}$)



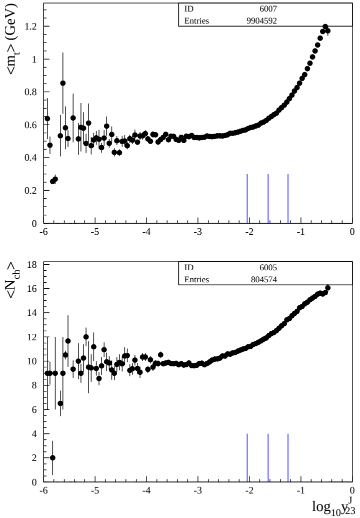


- R decreases with m_t for all N_{ch}
  smallest when both particles at high p_t

On what do r, R, λ depend?

- r, R increase with N_{ch}
- r, R increase with N_{jets}
- for fixed number of jets, R increases with N_{ch} but r is constant (OPAL)
- r, R decrease with m_t
- Although m_t, N_{ch}, N_{jets} are correlated, each contributes to the increase/decrease of R but only m_t, N_{jets} contribute to the increase/decrease of r

- λ decreases with N_{ch}, N_{jets} though somewhat differently for τ-model, Gaussian (OPAL)
- λ decreases with m_t

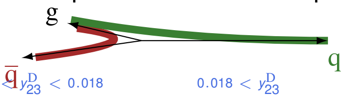


Jets and Rapidity

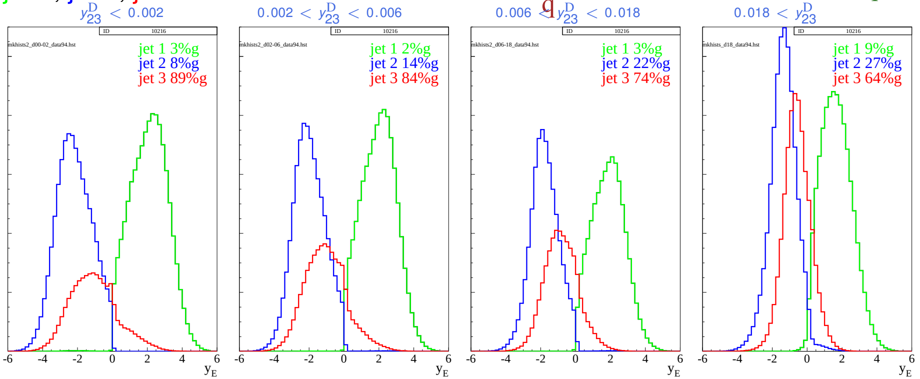
order jets by energy: $E_1 > E_2 > E_3$

Note: thrust only defines axis $|\vec{n}_\mathrm{T}|$, not its direction.

Choose positive thrust direction such that jet 1 is in positive thrust hemisphere



rapidity, y_E, of particles from
jet 1, jet 2, jet 3:

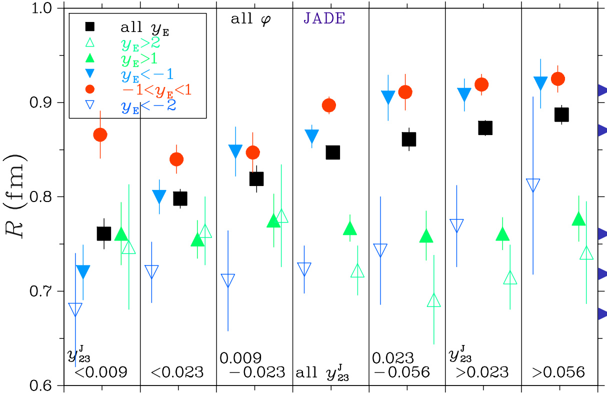


- $y_\mathrm{E} > 1$ almost all jet 1 — almost all quark
- $y_\mathrm{E} < -1$ mostly jet 2, some jet 3 — mostly quark
- $-1 < y_\mathrm{E} < 1$ jet-3 enriched — largely gluon

Jets and Rapidity – simplified τ-model – L3 preliminary

To stabilize fits against large correlation of α, R, fix $\alpha = 0.44$

Select particle pairs by rapidity of pair



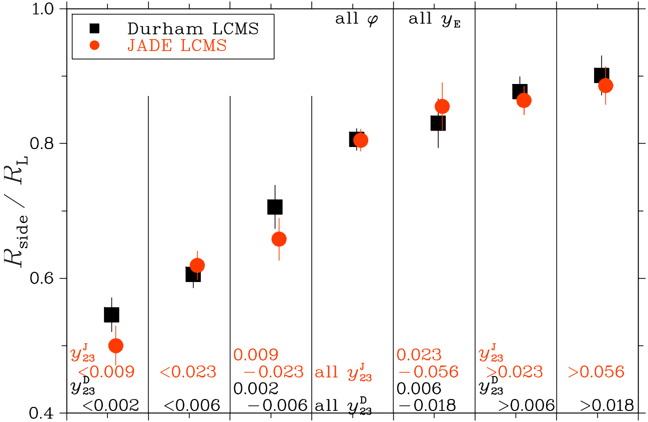
With y_{23}^J,

- all y: R increases
- 'pure' q jet, $y_E > 1$, or $y_E < -1$ & y_{23}^J small, or $y_E < -2$: R const.
- $R_{-1<y_E<1} > R_{\text{'pure' } q}$
- $R_{y_E<-1}$ increases
- at large y_{23}^D $R_{-1<y_E<1} = R_{y_E<-1}$

Conclusion (Durham agrees):

Increase in R with y_{23}^J is due to appearance of gluon jet

τ-model elongation – L3 preliminary



- Durham, JADE agree
- Elongation decreases with y_{23}, $R_{side} \approx 0.5$–$0.9\ R_{long}$
- agrees with Gaussian/Edgeworth

Conclusions/Comments/Lessons

1. Ref. sample is important
   - Comparison of results using different ρ_0 is very problematic
   - Agreement among LHC expts. would facilitate comparisons, *e*.g., central rapidity *v*s. forward rapidity
2. Ratios, *e*.g., $r_{\mathrm{side}}/r_{\mathrm{L}}$ are robust to differences in ρ_0, parametrization (Gauss, Lévy, τ-model)
3. Look beyond $Q = 2\,\mathrm{GeV}$ – at least to 3, preferably 4 GeV
4. τ-model
   - τ-model is closely related to a string picture
     - strong *x*-*p* correlation
     - fractal - Lévy distribution
   - CMS finds BEC in pp at 0.9 and 7 TeV are described by simplified τ-model formula JHEP 05 (2011) 029
   - suggests that BEC in pp is (mostly) from string fragmentation
5. Anticorrelation region is important
   - On what does it depend, N_{ch}, rapidity, m_{t}, ...?
   - Is the τ-model the correct explanation?
6. R, r depends on N_{jets}, N_{ch}, m_{t}.
   Also on (mini)jets, color reconnection, N_{strings}, color ropes?

BACKUP

Introduction — Correlations

q-particle density $$\rho_q(p_1, ..., p_q) = \frac{1}{\sigma_{\rm tot}} \frac{{\rm d}^q\sigma_q(p_1,\ldots,p_q)}{{\rm d}p_1\ldots{\rm d}p_q}$$

where σ_q is inclusive cross section

Normalization:
$$\int \rho_1(p)\,{\rm d}p = \langle n\rangle$$
$$\int \rho_2(p_1,p_2)\,{\rm d}p_1\,{\rm d}p_2 = \langle n(n-1)\rangle$$

In terms of 'factorial cumulants', C
$$\rho_1(p_1) = C_1(p_1)$$
$$\rho_2(p_1,p_2) = C_1(p_1)C_1(p_2) + C_2(p_1,p_2)$$
$$\rho_3(p_1,p_2,p_3)) = C_1(p_1)C_1(p_2)C_1(p_3)$$

"trivial" 3-particle correlations $$+\sum_{3\text{ perms}} C_1(p_1)C_2(p_2,p_3)$$

"genuine" 3-particle correlations $$+C_3(p_1,p_2,p_3)$$

2-particle correlations $$C_2 = \rho_2(p_1,p_2) - C_1(p_1)C_1(p_2)$$

Convenient to normalize $$R_q = \frac{\rho_q}{\prod_{i=1}^{q}\rho_1(p_i)} \qquad K_q = \frac{C_q}{\prod_{i=1}^{q}\rho_1(p_i)}$$

e.g., $$R_2 = 1 + \frac{C_2}{\rho_1(p_1)\rho_1(p_2)} = 1 + K_2$$

Introduction — BEC

To study BEC, not other correlations, replace $\prod_{i=1}^{q} \rho_1(p_i)$ by $\rho_0(p_1, \ldots, p_q)$, the q-particle density if no BEC (reference sample)

e.g., 2-particle BEC are studied in terms of

$$R_2(p_1, p_2) = \frac{\rho(p_1, p_2)}{\rho_0(p_1, p_2)}$$

Since 2-π BEC only at small $Q = \sqrt{-(p_1 - p_2)^2} = \sqrt{M_{12}^2 - 4m_\pi^2}$, integrate over other variables

$$R_2(Q) = \frac{\rho(Q)}{\rho_0(Q)}$$

Assuming incoherent particle production and spatial source density $S(x)$,

$$R_2(Q) = 1 + |G(Q)|^2$$

where $G(Q) = \int \mathrm{d}x\, e^{iQx} S(x)$ is the Fourier transform of $S(x)$

Assuming $S(x)$ is a Gaussian with radius r $\Longrightarrow$

$$R_2(Q) = 1 + e^{-Q^2 r^2}$$

$R_2(Q) \propto 1 + \lambda e^{-Q^2 r^2}$

Assumes

- incoherent average over source
  λ tries to account for
  - partial coherence
  - multiple (distinguishable) sources, long-lived resonances
  - pion purity
- spherical (radius r) Gaussian density of particle emitters
  seems unlikely in $\mathrm{e}^+\mathrm{e}^-$ annihilation — jets
- static source, *i.e.*, no *t*-dependence
  certainly wrong

Nevertheless, this Gaussian formula is the most often used parametrization

And it works fairly well

But what do the values of λ and r actually mean?

When Gaussian parametrization does not fit well,

- can expand about the Gaussian (Edgeworth expansion).
  Keeping only the lowest-order non-Gaussian term,
  $\exp\left(-Q^2 r^2\right)$ becomes
  $$\exp\left(-Q^2 r^2\right) \cdot \left[1 + \frac{\kappa}{3!} H_3(Qr)\right]$$
  (H_3 is third-order Hermite polynomial)
- Assume source radius is a symmetric Lévy distribution rather than Gaussian.
  Then $\exp\left(-Q^2 r^2\right)$ becomes
  $$\exp\left(-Q^2 r^\alpha\right) \quad , 0 < \alpha \leq 2$$
  α is the Lévy index of stability

Experimental Problems I

I. Pion purity

1. mis-identified pions – K, p
   – correct by MC.
   But is the MC correct?
2. resonances
   - long-lived affect λ
   BEC peak narrower than resolution
   - short-lived, *e*.g., ρ, - affect *r*
   – correct by MC.
   But is the MC correct?
3. weak decays
   $\sim$ 20% of Z decays are $\mathrm{b\bar{b}}$

   like long-lived resonances, decrease λ

- per Z: 17.0 $\pi^\pm$, 2.3 K$^\pm$, 1.0 p
  (15% non-π)

Origin of π^+ in Z decay	(%) (JETSET 7.4)
direct (string fragmentation)	16
decay (short-lived resonances) $\Gamma > 6.7\,\mathrm{MeV}, \tau < 30$ fm (ρ, ω, K*, Δ, ...)	62
decay (long-lived resonances) $\Gamma < 6.7\,\mathrm{MeV}, \tau > 30$ fm	22

Experimental Problems II

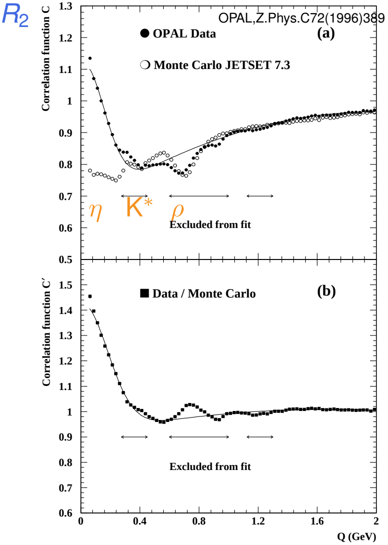
II. Reference Sample, ρ_0

— it does NOT exist

Common choices:

1. $+,-$ pairs
   But different resonances than $+,+$
   – correct by MC. – But is it correct?
2. Monte Carlo — But is it correct?
3. Mixed events – pair particles from different events
   But destroys all correlations, not just BEC
   – correct by MC. – But is it correct?
4. Mixed hemispheres (for 2-jet events)
   – pair particle with particle reflected from opposite hemisphere
   But destroys all correlations
   – correct by MC. – But is it correct?

ref. sample, ρ_0, from $+,-$ pairs



Experimental Problems III, IV

III. Final-State Interactions

1. Coulomb
   - form not certain
     (usually use Gamow factor)
     overcorrects!
   - for R_2, a few % in lowest Q bin
   - double if $+, -$ ref. sample
   - often neglected for R_2
   - but not negligible for R_3
2. Strong interaction - $S = 0$ $\pi\pi$ phase shifts can be incorporated together with Coulomb into the formula for R_2

   Osada, Sano, Biyajima, Z.Phys. C72(1996)285)

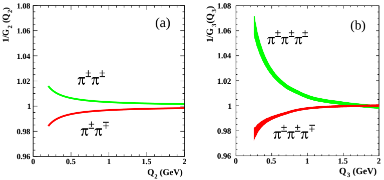
   tends to increase λ, decrease r

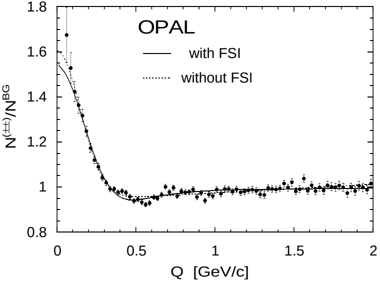
   e.g., OPAL data:

   $\lambda_{\mathrm{noFSI}} = 0.71$, $\lambda_{\mathrm{FSI}} = 1.04$

   $r_{\mathrm{noFSI}} = 1.34$, $r_{\mathrm{FSI}} = 1.09\,\mathrm{fm}$

   - Not used by experimental groups



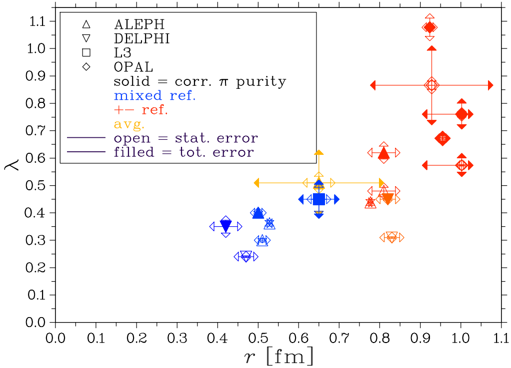


IV. Long-range correlations

inadequately treated in ref. sample:

$R_2(Q) \propto (1 + \lambda e^{-Q^2 r^2})(1 + \delta Q)$

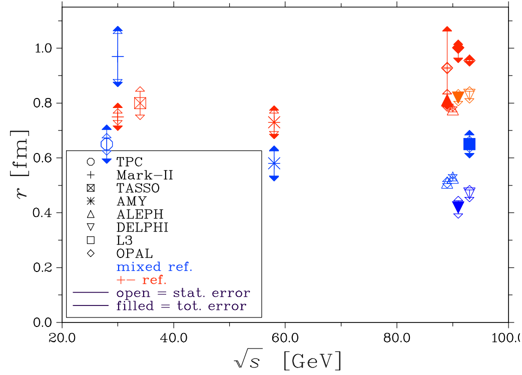
Results from R_2, $\sqrt{s} = M_Z$ (Gaussian parametrization)



– correction for π purity increases λ

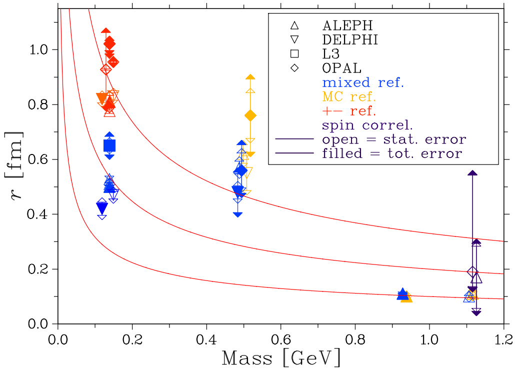
– mixed ref. gives smaller λ, r than $+-$ ref. – Average means little

$\sqrt{s}$ dependence of r



No evidence for $\sqrt{s}$ dependence

Mass dependence of r — BEC and FDC



No evidence for $r \sim 1/\sqrt{m}$

$r(\text{mesons}) > r(\text{baryons})$

$r_{\pi-\pi} \approx r_{\text{K-K}}$

Disclaimer

- There are many BEC measurements with pions.
- There are also BEC measurements with kaons, and FDC measurements with protons, lambdas, but fewer.
- From here on I will only treat pion results.

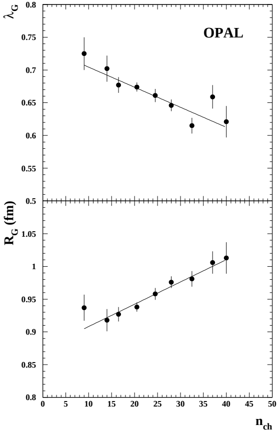
Multiplicity/Jet dependence of λ, r

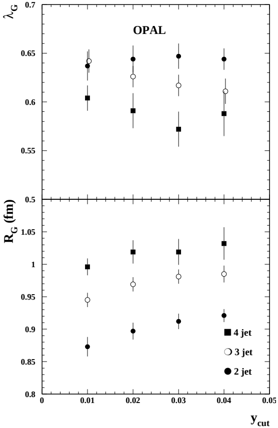
$$R_2(Q) = \gamma(1 + \lambda\, e^{-Q^2 r^2})(1 + \delta Q + \epsilon Q^2)$$

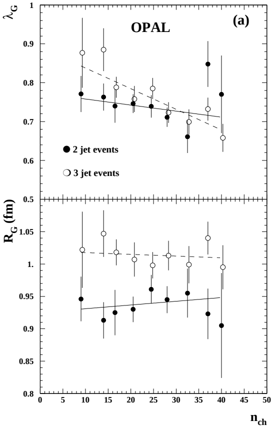
OPAL,Z.Phys.C72(1996)389

λ

r







$\lambda \searrow$ with n_{ch}

$r \nearrow$ with n_{ch}

$\lambda \searrow$ with n_{jet}

$r \nearrow$ with n_{jet}

$\lambda_{\text{n-jet}} \approx$ indep. of n_{ch}

$r_{\text{n-jet}}$ indep. of n_{ch}

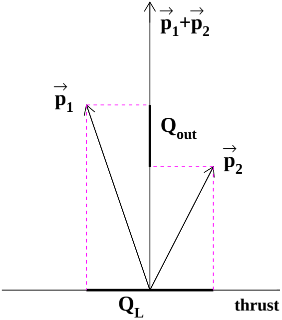
Multiplicity dependence appears to be largely due to number of jets.

Elongation of the source

The usual parametrization assumes a symmetric Gaussian source
But, there is no reason to expect this symmetry in $e^+e^- \to q\bar{q}$.
Therefore, do a 3-dim. analysis in the Longitudinal Center of Mass System
aka Longitudinal Co-Moving System

LCMS:

Boost each π-pair along event axis, e.g., thrust axis



$p_{L1} = -p_{L2}$
$\vec{p}_1 + \vec{p}_2$ defines 'out' axis
$Q_{side} \perp (Q_L, Q_{out})$

the LCMS

Advantages of LCMS:

$$Q^2 = Q_{\mathrm{L}}^2 + Q_{\mathrm{side}}^2 + Q_{\mathrm{out}}^2 - (\Delta E)^2 \qquad \text{where } Q_i^2 = (p_{i\,1} - p_{i\,2})^2$$

$$= Q_{\mathrm{L}}^2 + Q_{\mathrm{side}}^2 + Q_{\mathrm{out}}^2\,(1 - \beta^2) \qquad \text{where } \beta \equiv \frac{p_{\mathrm{out}\,1} + p_{\mathrm{out}\,2}}{E_1 + E_2}$$

Thus, the energy difference,
and therefore the difference in emission time of the pions
couples only to the out-component, Q_{out}.
Thus,
Q_{L} and Q_{side} reflect only spatial dimensions of the source
Q_{out} reflects a mixture of spatial and temporal dimensions.

Assuming axial symmetry, source is elliptically shaped with

- r_{L} the logitudinal radius
- r_{side} the transverse radius

Parametrization of R_2

Writing R_2 in terms of $\vec{Q} = (Q_\mathrm{L}, Q_\mathrm{side}, Q_\mathrm{out})$: $\quad R_2(\vec{Q}) = \frac{\rho(\vec{Q})}{\rho_0(\vec{Q})}$

We parametrize $R_2(\vec{Q})$ by a 3-dimensional Gaussian

$$R_2(Q_\mathrm{L}, Q_\mathrm{out}, Q_\mathrm{side}) = \gamma \cdot (1 + \lambda G) \cdot B$$

where

- γ = normalization (≈ 1)
- λ = "incoherence", or strength of BE effect
- G = azimuthally symmetric Gaussian:

$$G \quad = \quad \exp\left(-r_\mathrm{L}^2 Q_\mathrm{L}^2 - r_\mathrm{out}^2 Q_\mathrm{out}^2 - r_\mathrm{side}^2 Q_\mathrm{side}^2 + 2\rho_{\mathrm{L,out}} R_\mathrm{L} R_\mathrm{out} Q_\mathrm{L} Q_\mathrm{out}\right)$$

  longitudinal sym. $\Longrightarrow \rho_{\mathrm{L,out}} = 0$ (do not tag $\mathrm{q},\bar{\mathrm{q}}$, and fragment the same)
- Or G = Edgeworth expansion about azimuthally symmetric Gaussian:

$$\exp(-r_i^2 Q_i^2) \longrightarrow \exp(-r_i^2 Q_i^2)\cdot\left[1 + \frac{\kappa_i}{3!} H_3(r_i Q_i)\right] \;,\; H_3 = 3^\mathrm{rd} \text{ order Hermite polynomial}$$

- $B = (1 + \delta Q_\mathrm{L} + \varepsilon Q_\mathrm{out} + \xi Q_\mathrm{side})$ describes large Q (long-range correlations)

Elongation Results (L3)

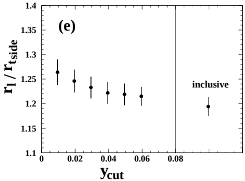
parameter	Gaussian	Edgeworth
λ	$0.41 \pm 0.01^{+0.02}_{-0.19}$	$0.54 \pm 0.02^{+0.04}_{-0.26}$
R_L (fm) R_out (fm) R_side (fm)	$0.74 \pm 0.02^{+0.04}_{-0.03}$ $0.53 \pm 0.02^{+0.05}_{-0.06}$ $0.59 \pm 0.01^{+0.03}_{-0.13}$	$0.69 \pm 0.02^{+0.04}_{-0.03}$ $0.44 \pm 0.02^{+0.05}_{-0.06}$ $0.56 \pm 0.02^{+0.03}_{-0.12}$
$R_\mathrm{out}/R_\mathrm{L}$ $R_\mathrm{side}/R_\mathrm{L}$	$0.71 \pm 0.02^{+0.05}_{-0.08}$ $0.80 \pm 0.02^{+0.03}_{-0.18}$	$0.65 \pm 0.03^{+0.06}_{-0.09}$ $0.81 \pm 0.02^{+0.03}_{-0.19}$
κ_L κ_out κ_side	– – –	$0.5 \pm 0.1^{+0.1}_{-0.2}$ $0.8 \pm 0.1 \pm 0.3$ $0.1 \pm 0.1 \pm 0.3$
δ ϵ ξ	$0.025 \pm 0.005^{+0.014}_{-0.015}$ $0.005 \pm 0.005^{+0.034}_{-0.012}$ $-0.035 \pm 0.005^{+0.031}_{-0.024}$	$0.036 \pm 0.007^{+0.012}_{-0.023}$ $0.011 \pm 0.005^{+0.037}_{-0.012}$ $-0.022 \pm 0.006^{+0.020}_{-0.025}$
χ^2/DoF C.L. (%)	2314/2189 3.1	2220/2186 30

- Edgeworth fit significantly better than Gaussian
- $R_\mathrm{side}/R_\mathrm{L} < 1$ more than 5 std. dev. Elongation along thrust axis
- Models which assume a spherical source are too simple.

Elongation Results

			Gauss / Edgeworth	2-D r_t/r_L	3-D r_{side}/r_L
DELPHI	mixed	2-jet	Gauss	$0.62 \pm 0.02 \pm 0.05$	—
ALEPH	mixed	2-jet	Gauss	$0.61 \pm 0.01 \pm 0.??$	—
	$+-$	2-jet	Gauss	$0.91 \pm 0.02 \pm 0.??$	—
	mixed	2-jet	Edgeworth	$0.68 \pm 0.01 \pm 0.??$	—
	$+-$	2-jet	Edgeworth	$0.84 \pm 0.02 \pm 0.??$	—
OPAL	$+-$	2-jet	Gauss	—	$0.82 \pm 0.02 \pm^{0.01}_{0.05}$
L3	mixed	all	Gauss	—	$0.80 \pm 0.02 \pm^{0.03}_{0.18}$
	mixed	all	Edgeworth	—	$0.81 \pm 0.02 \pm^{0.03}_{0.19}$

$\sim$20% elongation along thrust axis
(ZEUS finds similar results in ep)



OPAL
Elongation larger for narrower jets

3π BEC

Recall

"trivial" 3-particle correlations
"genuine" 3-particle correlations

$$\begin{aligned} \rho_3(p_1,p_2,p_3)) &= C_1(p_1)C_1(p_2)C_1(p_3) \\ &+ \textstyle\sum_{\text{3 perms}} C_1(p_1)C_2(p_2,p_3) \\ &+ C_3(p_1,p_2,p_3) \end{aligned}$$

or

$$\begin{aligned} \rho_3(p_1,p_2,p_3) &= \rho_1(p_1)\rho_1(p_2)\rho_1(p_3) \\ &+ \sum_{\substack{3\\ \text{perm}}} \left[\rho_1(p_1)\left(\rho_2(p_2,p_3)-\rho_1(p_2)\rho_1(p_3)\right)\right] \\ &+ C_3(p_1,p_2,p_3) \end{aligned}$$

3-particle BEC are studied in terms of

$$R_3(p_1,p_2,p_3) = \frac{\rho_3(p_1,p_2,p_3)}{\rho_0(p_1,p_2,p_3)}$$

3π BEC

Since BEC at small Q_3 $\quad \left(Q_3^2 = M_{123}^2 - 9m_\pi^2 = Q_{12}^2 + Q_{23}^2 + Q_{13}^2\right)$

we use $R_3(Q_3) = \dfrac{\rho(Q_3)}{\rho_0(Q_3)}$ and $R_2 = \dfrac{\rho(Q)}{\rho_0(Q)}$

$$R_3^{\text{nongen}}(Q_3) = 1 + \sum_{\substack{3\ \text{perm} \\ Q_3}} \frac{\rho_1\rho_2}{\rho_0} - 3 = 1 + \sum_{\substack{3\ \text{perm} \\ Q_3}} \left[R_2(Q_{12}) - 1\right]$$

$$R_3^{\text{genuine}}(Q_3) = 1 + \frac{C_3(Q_3)}{\rho_0(Q_3)} = 1 + R_3(Q_3) - R_3^{\text{nongen}}(Q_3)$$

3π BEC

Assuming static source density $f(x)$ in space-time, $G(Q) = \int \mathrm{d}x\, e^{iQx} f(x) = Ge^{i\phi}$

$$R_2(Q) = \frac{\rho_2(Q)}{\rho_0(Q)} = 1 + \lambda |G(Q)|^2$$

Analog of Q for 3 particles: $\left(Q_3^2 = M_{123}^2 - 9m_\pi^2 = Q_{12}^2 + Q_{23}^2 + Q_{13}^2\right)$

$$R_3(Q_3) = \frac{\rho_3(Q_3)}{\rho_0(Q_3)} = 1 + \underbrace{\lambda \left(|G(Q_{12})|^2 + |G(Q_{23})|^2 + |G(Q_{13})|^2\right)}_{\text{from 2-particle BEC}}$$

$$+ \underbrace{2\lambda^{1.5}\,\Re\{G(Q_{12})G(Q_{23})G(Q_{13})\}}_{\text{from genuine 3-particle BEC}}$$

$$R_3^{\text{genuine}} = 1 + 2\lambda^{1.5}\,\Re\{G(Q_{12})G(Q_{23})G(Q_{13})\}$$

$$\omega = \frac{R_3^{\text{genuine}}(Q_3) - 1}{2\sqrt{(R_2(Q_{12})-1)(R_2(Q_{23})-1)(R_2(Q_{13})-1)}}$$

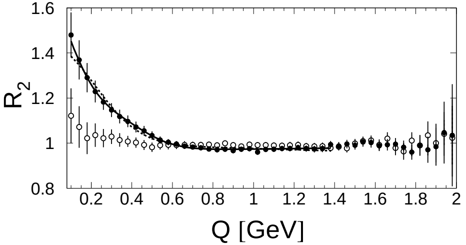
$$= \cos(\phi_{12} + \phi_{23} + \phi_{13})$$

$$\omega = \frac{R_3^{\text{genuine}}(Q_3) - 1}{2\sqrt{R_2(Q_3) - 1}} \qquad \text{if } f(x) \text{ is Gaussian}$$

If fully incoherent, $\phi_{ij} \neq 0$ only if $f(x)$ asymmetric and $Q_{ij} > 0$

Completely incoherent particle production implies $\lambda = 1$ $\omega = 1$

3π BEC



$R_2 \propto 1 + \lambda \exp(-Q^2 r^2)$

- - - Gaussian $\chi^2 = 60$, 29 dof

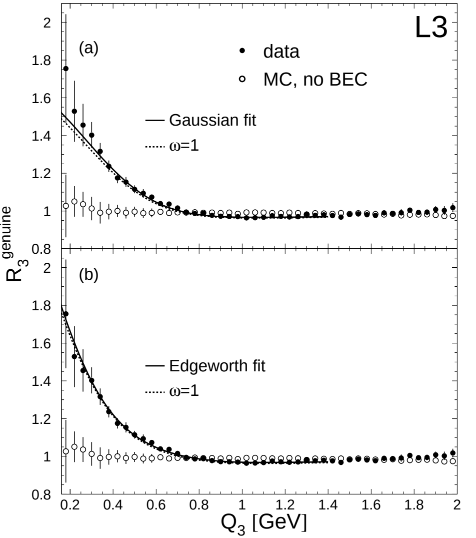
——Edgeworth $\chi^2 = 26$, 28 dof

$R_3^{\mathrm{genuine}} \propto 1 + 2\lambda^{1.5} \exp(-Q^2 r^2/2)$

- - - Gaussian $\chi^2 = 30$, 27 dof

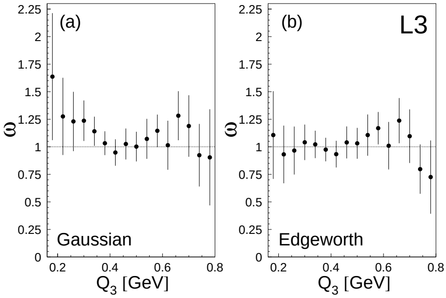
——Edgeworth $\chi^2 = 18$, 26 dof

L3, PLB540 (2002) 185



$$\omega = \frac{R_3^{\text{genuine}}(Q_3) - 1}{2\sqrt{R_2(Q_3) - 1}}$$

Using R_3^{genuine} from data, R_2 from fit



Conclusion: Data consistent with $\omega = 1$,

i.e., with completely incoherent pion production

3π BEC

L3:

from		Gaussian	Edgeworth
R_2 R_3^{genuine}	λ	$0.45 \pm 0.06 \pm 0.03$ $0.47 \pm 0.07 \pm 0.03$	$0.72 \pm 0.08 \pm 0.03$ $0.75 \pm 0.10 \pm 0.03$
R_2 R_3^{genuine}	r (fm)	$0.65 \pm 0.03 \pm 0.03$ $0.65 \pm 0.06 \pm 0.03$	$0.74 \pm 0.06 \pm 0.02$ $0.72 \pm 0.08 \pm 0.03$

Data consistent with $\omega = 1$, *i*.e., fully incoherent.

Values of λ, r from R_2 and R_3^{genuine} are consistent.

expt.		λ	r
MARK-II (29 GeV)	R_2 R_3	$0.45 \pm 0.03 \pm 0.04$ $0.54 \pm 0.06 \pm 0.05$	$1.01 \pm 0.09 \pm 0.06$ $0.90 \pm 0.06 \pm 0.06$
DELPHI	R_2 R_3^{genuine}	$0.24 \pm 0.02 \pm 0.??$ $0.43 \pm 0.05 \pm 0.07$	$0.47 \pm 0.03 \pm 0.??$ $0.93 \pm 0.06 \pm 0.04$
OPAL	R_2 R_3^{genuine}	$0.58 \pm 0.01 \pm 0.??$ $0.63 \pm 0.01 \pm 0.03$	$0.79 \pm 0.02 \pm 0.??$ $0.82 \pm 0.01 \pm 0.04$

Values of λ, r from R_2 and R_3 are fairly consistent.

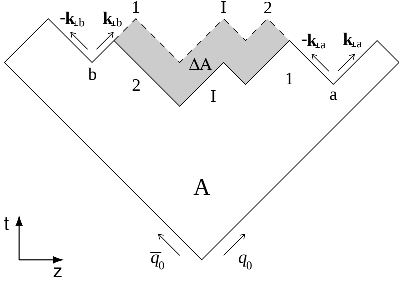
BEC in String Models

Longitudinal BEC

- Different string configurations give same final state
- Matrix element to get a final state depends on area, A:
  $\mathcal{M} = \exp\left[(\imath\kappa - b/2)A\right]$
  where κ is the string tension and b is the decay constant
  $\kappa \approx 1\,\mathrm{GeV/fm}$ and $b \approx 0.3\,\mathrm{GeV/fm}$
- So, must sum all the amplitudes
  But 3-π BEC incoherent ??

Transverse BEC

- Transverse momentum via tunneling, also related to b



Using b from tuning of JETSET, predict

- BEC,
  including genuine 3-particle BEC
- $r_\mathrm{t} < r_\mathrm{L}$
- $r(\pi^0\pi^0) < r(\pi^+\pi^+)$

2-particle BEC $\pi^0\pi^0$ and $\pi^\pm\pi^\pm$

- Naively expect same BEC for $\pi^0\pi^0$ and $\pi^\pm\pi^\pm$
- Hadronization with local charge conservation,
  e.g., string, $\Longrightarrow r_{00} < r_{\pm\pm}$
  But most π's from resonances — dilutes this effect.

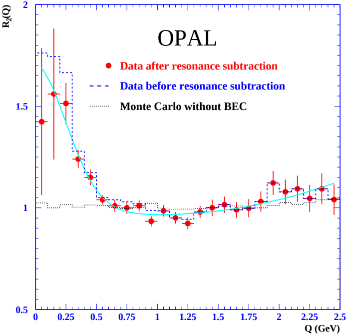
- Many measurements of BEC with charged π's
- but few with π^0's

  in e^+e^-: L3, P.L. B524 (2002) 55
  OPAL, P.L. B559 (2003) 131

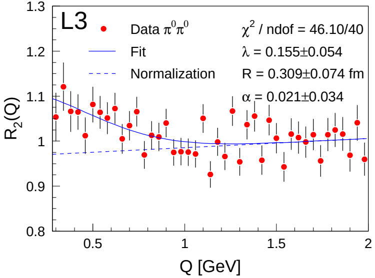
Selection:

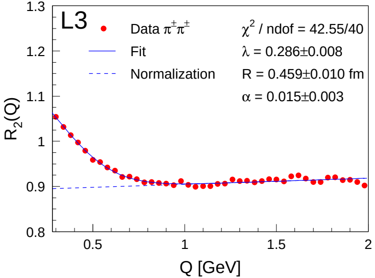
OPAL	L3
$p_{\pi^0} > 1.0$ GeV	$E(\pi^0) < 6.0$ GeV
2-jet, $T > 0.9$	all events

2-particle BEC $\pi^0\pi^0$ and $\pi^\pm\pi^\pm$



λ=0.55±0.10±0.10
r=0.59±0.08±0.05 fm





2-particle BEC $\pi^0\pi^0$ and $\pi^{\pm}\pi^{\pm}$

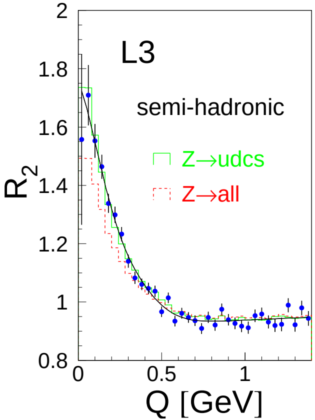
BEC from Z decays
Gaussian parametrization

	Expt.	ρ_0	r (fm)	λ
±±	OPAL	+−	$1.00^{+0.03}_{-0.10}$	0.76 ± 0.06
	L3	mix	0.65 ± 0.04	0.45 ± 0.07
	L3 3-π	mix	0.65 ± 0.07	0.47 ± 0.08
	L3 $E_\pi < 6\,\mathrm{GeV}$	MC	0.46 ± 0.01	0.29 ± 0.03
00	L3 $E_\pi < 6\,\mathrm{GeV}$	MC	0.31 ± 0.10	0.16 ± 0.09
	OPAL $E_\pi > 1$, 2-jet	mix	0.59 ± 0.09	0.55 ± 0.14

- L3: $r_{00} < r_{\pm\pm}$ and $\lambda_{00} < \lambda_{\pm\pm}$, both 1.5σ
- ALEPH, DELPHI find $r_{\pm\pm}(\mathrm{mix})/r_{\pm\pm}(+-) \approx 0.68, 0.51$
  Applying this to OPAL $r_{\pm\pm}$, OPAL $r_{00} \approx r_{\pm\pm}$ and $\lambda_{00} \approx \lambda_{\pm\pm}$
- L3 and OPAL $\pi^0\pi^0$ results disagree by 2σ
- Is the L3-OPAL $\pi^0\pi^0$ difference due to E_π and/or 2-jet selection ???
- OPAL: MC shows that few of selected π^0's are direct from string

Another source of $q\bar{q}$: W

$e^+e^- \to W^+W^- \to q\bar{q}\ell\nu$



BE(W) = BE(Z →light quarks)

$e^+e^- \to W^+W^- \to q\bar{q}q\bar{q}$

If independent decay of W^+W^-,
i.e., no BEC between pions from different W's

$$\begin{aligned}\rho_{4q}(p_1,p_2) = &\ \rho^+(p_1,p_2) && 1,2 \text{ from } W^+ \\ + &\ \rho^-(p_1,p_2) && 1,2 \text{ from } W^- \\ + &\ \rho^+(p_1)\rho^-(p_2) && 1 \text{ from } W^+,\ 2 \text{ from } W^- \\ + &\ \rho^+(p_2)\rho^-(p_1) && 1 \text{ from } W^-,\ 2 \text{ from } W^+\end{aligned}$$

Assuming $\rho^+ = \rho^- = \rho_{2q}$, W separation $\sim$ 0.1 fm

$$\rho_{4q}(p_1,p_2) = 2\rho_{2q}(p_1,p_2) + 2\rho_{2q}(p_1)\rho_{2q}(p_2)$$

Inter-W BEC $\Longrightarrow$ W decays *not* independent
$\Longrightarrow$ this relation does *not* hold.

Measure

- $\rho_{4q}(p_1,p_2)$ from $e^+e^- \to W^+W^- \to q\bar{q}q\bar{q}$
- $\rho_{2q}(p_1,p_2)$ from $e^+e^- \to W^+W^- \to q\bar{q}\ell\nu$
- $\rho_{2q}(p_1)\rho_{2q}(p_2)$ from $\rho_{mix}(p_1,p_2)$ obtained by mixing $\ell^+\nu q\bar{q}$ and $q\bar{q}\ell^-\nu$ events

$W^+W^- \to q\bar{q}q\bar{q}$

Measure violation of

$$\rho_{4q}(Q) = 2\rho_{2q}(Q) + 2\rho_{mix}(Q)$$

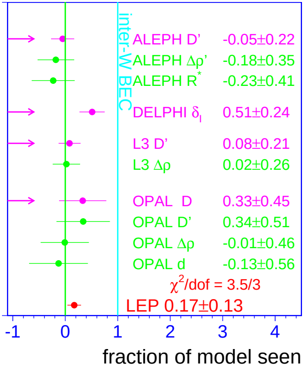
by

$$\Delta\rho(Q) = \rho_{4q}(Q) - [2\rho_{2q}(p_1, p_2) + 2\rho_{mix}(p_1, p_2)]$$

$$D(Q) = \frac{\rho_{4q}(Q)}{2\rho_{2q}(Q) + 2\rho_{mix}(Q)}$$

$$\delta_I(Q) = \frac{\Delta\rho(Q)}{2\rho_{mix}(Q)}$$

Compare to expectation of BE_{32} model in PYTHIA



$\delta_I(Q)$ measures genuine inter-W BEC

DELPHI: $0.51 \pm 0.24 \quad \sim 2\sigma$

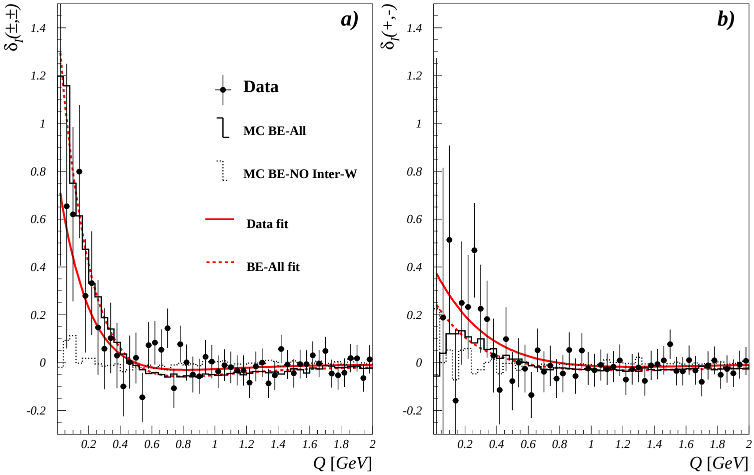
average: $0.17 \pm 0.13 \quad \sim 1\sigma$

Conclusion: BEC (mostly) between π's from same string

But event selection (4 separated jets) suppresses small Q for π pairs from different strings

$W^+W^- \to q\bar{q}q\bar{q}$

DELPHI

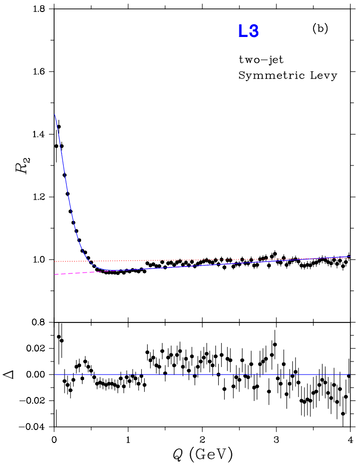


But conclusions are tricky: Also effect in $(+,-)$

Results – 'Classic' Parametrizations

$R_2 = \gamma \cdot [1 + \lambda G] \cdot (1 + \epsilon Q)$

- Gaussian
  $G = \exp\left(-(rQ)^2\right)$
- Edgeworth expansion
  $G = \exp\left(-(rQ)^2\right) \cdot \left[1 + \frac{\kappa}{3!} H_3(rQ)\right]$
  Gaussian if $\kappa = 0$
  Fit: $\kappa = 0.71 \pm 0.06$
- symmetric Lévy
  $G = \exp\left(-|rQ|^{\alpha}\right)$
  $0 < \alpha \leq 2$
  Gaussian if $\alpha = 2$
  Fit: $\alpha = 1.34 \pm 0.04$



	Gauss	Edgew	Lévy
CL:	10^{-15}	10^{-5}	10^{-8}

Poor χ^2. Edgeworth and Lévy better than Gaussian, but poor.
Problem is the dip of R_2 in the region $0.6 < Q < 1.5\,\mathrm{GeV}$

The τ-model

T.Csörgő, W.Kittel, W.J.Metzger, T.Novák, Phys.Lett.**B663**(2008)214
T.Csörgő, J.Zimányi, Nucl.Phys.**A517**(1990)588

- Assume avg. production point is related to momentum:

$$\overline{x}^{\mu}(p^{\mu}) = a\tau p^{\mu}$$

where for 2-jet events, $a = 1/m_{\mathrm{t}}$

$\tau = \sqrt{\overline{t}^2 - \overline{r}_z^2}$ is the "longitudinal" proper time

and $m_{\mathrm{t}} = \sqrt{E^2 - p_z^2}$ is the "transverse" mass

- Let $\delta_{\Delta}(x^{\mu} - \overline{x}^{\mu})$ be dist. of prod. points about their mean, and $H(\tau)$ the dist. of τ. Then the emission function is

$$S(x,p) = \int_0^{\infty} \mathrm{d}\tau H(\tau)\delta_{\Delta}(x - a\tau p)\rho_1(p)$$

- In the plane-wave approx. F.B.Yano, S.E.Koonin, Phys.Lett.**B78**(1978)556.

$$\rho_2(p_1,p_2) = \int \mathrm{d}^4x_1\mathrm{d}^4x_2 S(x_1,p_1)S(x_2,p_2)\left(1 + \cos\left([p_1 - p_2][x_1 - x_2]\right)\right)$$

- Assume $\delta_{\Delta}(x^{\mu} - \overline{x}^{\mu})$ is very narrow — a δ-function. Then

$$R_2(p_1,p_2) = 1 + \lambda\,\mathrm{Re}\widetilde{H}\left(\frac{a_1Q^2}{2}\right)\widetilde{H}\left(\frac{a_2Q^2}{2}\right), \quad \widetilde{H}(\omega) = \int \mathrm{d}\tau H(\tau)\exp(i\omega\tau)$$

BEC in the τ-model

- Assume a Lévy distribution for $H(\tau)$
  Since no particle production before the interaction,
  $H(\tau)$ is one-sided.
  Characteristic function is

$$\widetilde{H}(\omega) = \exp\left[-\frac{1}{2}\left(\Delta\tau|\omega|\right)^{\alpha}\left(1 - i\,\mathrm{sign}(\omega)\tan\left(\frac{\alpha\pi}{2}\right)\right) + i\,\omega\tau_0\right] \;, \quad \alpha \neq 1$$

  where
  - α is the index of stability;
  - τ_0 is the proper time of the onset of particle production;
  - $\Delta\tau$ is a measure of the width of the distribution.
- Then, R_2 depends on Q, a_1, a_2

$$\begin{aligned} R_2(Q, a_1, a_2) \quad = \quad & \gamma\left\{1 + \lambda\cos\left[\frac{\tau_0 Q^2(a_1 + a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha} + a_2^{\alpha}}{2}\right]\right. \\ & \left.\cdot\exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha} + a_2^{\alpha}}{2}\right]\right\}\cdot(1 + \epsilon Q) \end{aligned}$$

BEC in the τ-model

$$R_2(Q, a_1, a_2) = \gamma \left\{ 1 + \lambda \cos\left[\frac{\tau_0 Q^2 (a_1 + a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right) \left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{a_1^\alpha + a_2^\alpha}{2} \right] \cdot \exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{a_1^\alpha + a_2^\alpha}{2} \right] \right\} \cdot (1 + \epsilon Q)$$

Simplification:

- effective radius, R, defined by $R^{2\alpha} = \left(\frac{\Delta\tau}{2}\right)^{\alpha} \frac{a_1^\alpha + a_2^\alpha}{2}$
- Particle production begins immediately, $\tau_0 = 0$
- Then

$$R_2(Q) = \gamma \left[1 + \lambda \cos\left((R_\mathrm{a} Q)^{2\alpha} \right) \exp\left(-(RQ)^{2\alpha} \right) \right] \cdot (1 + \epsilon Q)$$

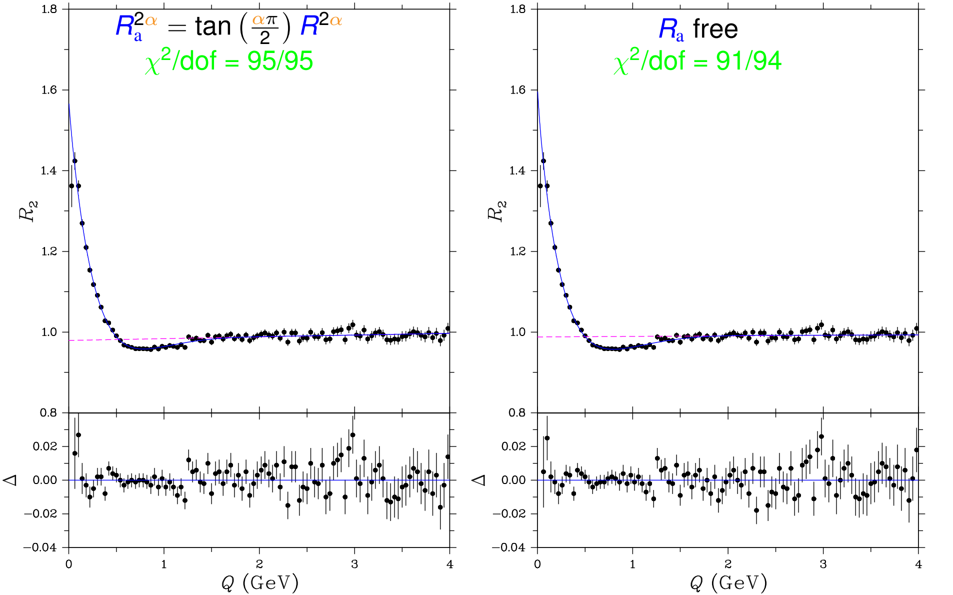
where $R_\mathrm{a}^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$

Compare to sym. Lévy parametrization:

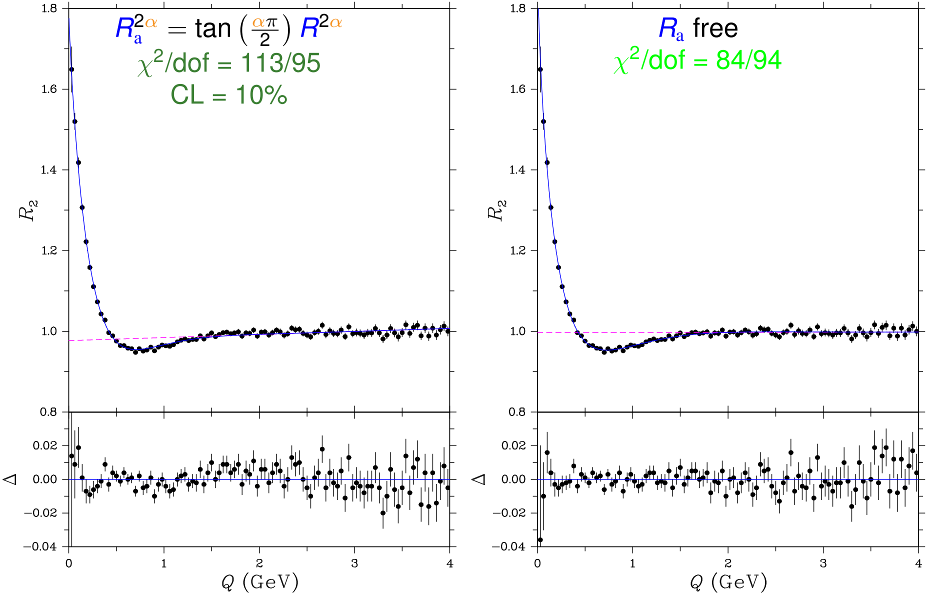
$$R_2(Q) = \gamma \left[1 + \lambda \qquad\qquad \exp\left[-|rQ|^{\alpha} \right] \right] (1 + \epsilon Q)$$

- R describes the BEC peak
- R_a describes the anticorrelation dip
- τ-model: both anticorrelation and BEC are related to 'width' $\Delta\tau$ of $H(\tau)$

2-jet Results on Simplified τ-model from L3 Z decay

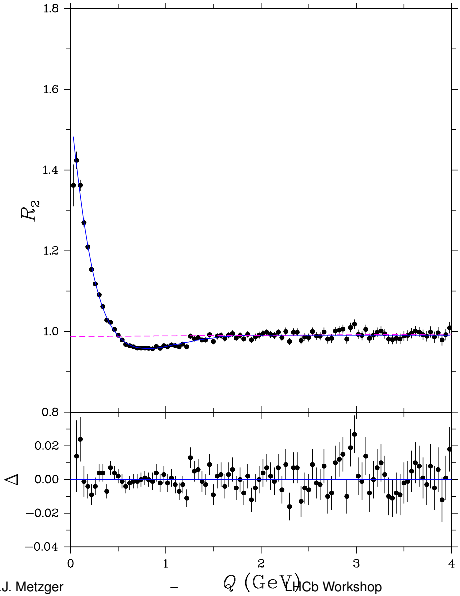


3-jet Results on Simplified τ-model from L3 Z decay



Full τ-model for 2-jet events — $a = 1/m_t$

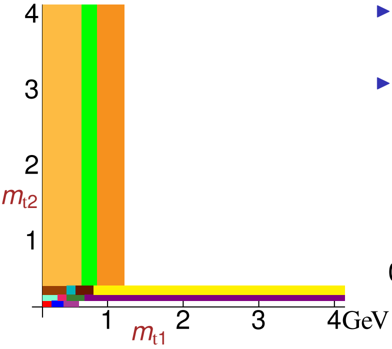
$$R_2(Q, m_{t1}, m_{t2}) = \gamma \left\{1 + \lambda \cos\left[\frac{\tau_0 Q^2 (m_{t1}+m_{t2})}{2(m_{t1} m_{t2})} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{m_{t1}^{\alpha}+m_{t2}^{\alpha}}{2(m_{t1} m_{t2})^{\alpha}}\right] \cdot \exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{m_{t1}^{\alpha}+m_{t2}^{\alpha}}{2(m_{t1} m_{t2})^{\alpha}}\right]\right\} \cdot (1 + \epsilon Q)$$



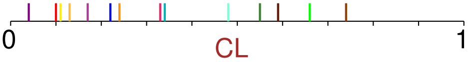
- Fit $R_2(Q)$ using avg m_{t1}, m_{t2} in each Q bin, $m_{t1} > m_{t2}$
- $\tau_0 = 0.00 \pm 0.02$ so fix to 0
- χ^2/dof = 90/95

Full τ-model for 2-jet events

- τ-model predicts dependence on m_t, $R_2(Q, m_{t1}, m_{t2})$
- Parameters α, $\Delta\tau$, τ_0 are independent of m_t
- λ (strength of BEC) can depend on m_t



- divide m_{t1}-m_{t2} plane in regions (equal statistics)
- in each region fit $R_2(Q)$
  using avg m_{t1}, m_{t2} in each Q bin
  with α, $\Delta\tau$, fixed to values found for entire plane and $\tau_0 = 0$



Summary of τ-model

- τ-model with a one-sided Lévy proper-time distribution describes BEC well
  - in simplified form it provides a new parametrization of $R_2(Q)$ for both 2- and 3-jet events,
  - in full form for 2-jet events, $R_2(Q, m_{t1}, m_{t2})$
    - both Q- and m_t-dependence described correctly
    - Note: we found $\Delta\tau$ to be independent of m_t
      $\Delta\tau$ enters R_2 as $\Delta\tau Q^2/m_t$
      In Gaussian parametrization, r enters R_2 as $r^2 Q^2$
      Thus $\Delta\tau$ independent of m_t corresponds to $r \propto 1/\sqrt{m_t}$
- BUT, what about elongation?

Elongation?

- Previous results using fits of Gaussian or Edgeworth found (in LCMS) $r_{\mathrm{side}}/r_{\mathrm{L}} \approx 0.8$ for all events
- But we find that Gaussian and Edgeworth fit $R_2(Q)$ poorly
- τ-model predicts no elongation and fits the data well
- Could the elongation results be an artifact of an incorrect fit function? or is the τ-model in need of modification?
- So, we modify *ad hoc* the τ-model description to allow elongation

Elongation in the Simplified τ-model?

LCMS: $$Q^2 = Q_{\mathrm{L}}^2 + Q_{\mathrm{side}}^2 + Q_{\mathrm{out}}^2 - (\Delta E)^2$$
$$= Q_{\mathrm{L}}^2 + Q_{\mathrm{side}}^2 + Q_{\mathrm{out}}^2 \left(1 - \beta^2\right) , \qquad \beta = \frac{p_{1\mathrm{out}} + p_{2\mathrm{out}}}{E_1 + E_2}$$

Replace $$R^2 Q^2 \Longrightarrow A^2 = R_{\mathrm{L}}^2 Q_{\mathrm{L}}^2 + R_{\mathrm{side}}^2 Q_{\mathrm{side}}^2 + \rho_{\mathrm{out}}^2 Q_{\mathrm{out}}^2$$

Then in τ-model,

$$R_2(Q_{\mathrm{L}}, Q_{\mathrm{side}}, Q_{\mathrm{out}}) = \gamma \left[1 + \lambda \cos\left(\tan\left(\frac{\alpha\pi}{2}\right) A^{2\alpha}\right) \exp\left(-A^{2\alpha}\right)\right] \cdot \left(1 + \epsilon_{\mathrm{L}} Q_{\mathrm{L}} + \epsilon_{\mathrm{side}} Q_{\mathrm{side}} + \epsilon_{\mathrm{out}} Q_{\mathrm{out}}\right)$$

for 2-jet events:

		χ^2/dof	CL
τ-model	$R_{\mathrm{side}}/R_{\mathrm{L}} = 0.61 \pm 0.02$	14847/14921	66%
Edgeworth	$r_{\mathrm{side}}/r_{\mathrm{L}} = 0.64 \pm 0.02$	14891/14919	56%
	consistent		

Elongation is real

Direct Test of Q^2-only Dependence

1. $Q^2 = Q^2_{\rm LE} + Q^2_{\rm side} + Q^2_{\rm out}$ where $Q^2_{\rm LE} = Q^2_{\rm L} - (\Delta E)^2$
   inv. boosts along thrust axis
2. $Q^2 = Q^2_{\rm L} + Q^2_{\rm side} + q^2_{\rm out}$ where $q_{\rm out}$ = $Q_{\rm out}$ boosted (β) along
   out direction to rest frame of pair

In τ-model, for case 1

$$R_2(Q_{\rm LE}, Q_{\rm side}, Q_{\rm out}) = \gamma \left[1 + \lambda \cos\left(\tan\left(\frac{\alpha\pi}{2}\right) B^{2\alpha}\right) \exp\left(-B^{2\alpha}\right)\right] b$$

$$\text{where } B^2 = R^2_{\rm LE} Q^2_{\rm LE} + R^2_{\rm side} Q^2_{\rm side} + R^2_{\rm out} Q^2_{\rm out}$$

$$b = 1 + \epsilon_{\rm LE} Q_{\rm LE} + \epsilon_{\rm side} Q_{\rm side} + \epsilon_{\rm out} Q_{\rm out}$$

and comparable expression for case 2, $R_2(Q_{\rm L}, Q_{\rm side}, q_{\rm out})$

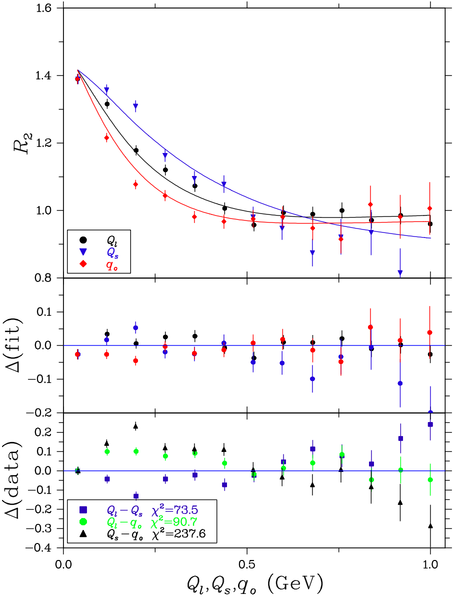
Direct Test of Q^2-only Dependence

Compare fits with all ‘radii’ free
to fits with all ‘radii’ constrained to be equal

case 1	α	0.46 ± 0.01	0.46 ± 0.01	
	R_{LE} (fm)	0.84 ± 0.04	0.71 ± 0.04	
	$R_{\mathrm{side}}/R_{\mathrm{LE}}$	0.60 ± 0.02	1	
	$R_{\mathrm{out}}/R_{\mathrm{LE}}$	0.986 ± 0.003	1	difference
	χ^2/DoF	$14590/14538$	$14886/14540$	$\Delta\chi^2 = 296/2$
	CL	38%	2%	≈ 0
case 2	α	0.41 ± 0.01	0.44 ± 0.01	
	R_{L} (fm)	0.96 ± 0.05	0.82 ± 0.04	
	$R_{\mathrm{side}}/R_{\mathrm{L}}$	0.62 ± 0.02	1	
	$r_{\mathrm{out}}/R_{\mathrm{L}}$	1.23 ± 0.03	1	difference
	χ^2/DoF	$10966/10647$	$11430/10649$	$\Delta\chi^2 = 464/2$
	CL	2%	10^{-7}	≈ 0

Dependence on components of Q is strongly preferred.

Q Dependence



$R_2(Q_\mathrm{L}, Q_\mathrm{side}, q_\mathrm{out})$ *vs.*

Q_L for $Q_\mathrm{side}, q_\mathrm{out} < 0.08\,\mathrm{GeV}$

Q_side for $Q_\mathrm{L}, q_\mathrm{out} < 0.08\,\mathrm{GeV}$

q_out for $Q_\mathrm{L}, Q_\mathrm{side} < 0.08\,\mathrm{GeV}$

Dependence on components of Q is preferred.

$r_\mathrm{out} > R_\mathrm{L} > R_\mathrm{side}$

Not azimuthally symmetric

Summary

- R_2 depends, to some degree, separately on components of Q, *i.e.*, on $\vec{Q}$
- contradicts τ-model, where dependence is on Q, not on $\vec{Q}$
- Nevertheless, τ-model with a one-sided Lévy proper-time distribution succeeds:
  - Simplified, provides a new parametrization of $R_2(Q)$ which works well
  - $R_2(Q, m_{t1}, m_{t2})$ successfully fits R_2 for 2-jet events both Q- and m_t-dependence described correctly
- But dependence of R_2 on components of Q implies τ-model is in need of modification
  Perhaps, a should be different for transverse/longitudinal

$$\overline{x}^{\mu}(p^{\mu}) = a\,\tau p^{\mu}, \qquad a = 1/m_t \text{ for 2-jet}$$

Emission Function of 2-jet Events.

In the τ-model, the emission function in configuration space is

$$S(\vec{x},\tau) = \frac{1}{\overline{n}}\frac{\mathrm{d}^4 n}{\mathrm{d}\tau \mathrm{d}\vec{x}} = \frac{1}{\overline{n}}\left(\frac{m_\mathrm{t}}{\tau}\right)^3 H(\tau)\rho_1\left(\vec{p} = \frac{m_\mathrm{t}\vec{x}}{\tau}\right)$$

For simplicity, assume $\rho_1(\vec{p}) = \rho_\mathrm{y}(y)\rho_{p_\mathrm{t}}(p_\mathrm{t})/\overline{n}$

(ρ_1, ρ_y, ρ_{p_t} are inclusive single-particle distributions)

Then $S(\vec{x},\tau) = \frac{1}{\overline{n}^2} H(\tau)G(\eta)I(r)$

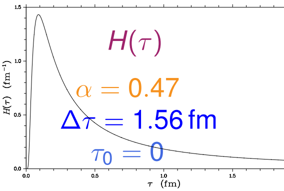
Strongly correlated $x, p \Longrightarrow$

$$\eta = y \qquad\qquad r = p_\mathrm{t}\tau/m_\mathrm{t}$$

$$G(\eta) = \rho_\mathrm{y}(\eta) \qquad I(r) = \left(\frac{m_\mathrm{t}}{\tau}\right)^3 \rho_{p_\mathrm{t}}(rm_\mathrm{t}/\tau)$$

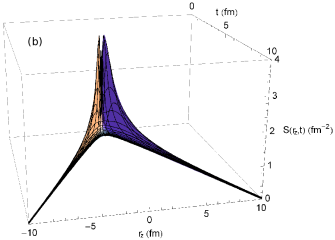
So, using experimental $\rho_\mathrm{y}(y)$, $\rho_{p_\mathrm{t}}(p_\mathrm{t})$ dists.
and $H(\tau)$ from BEC fits,
we can reconstruct S.

expt. –
Factorization OK

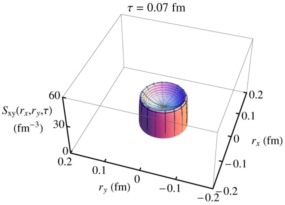


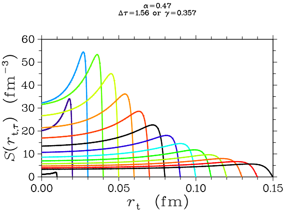
Emission Function of 2-jet Events.

Integrating over *r*,



Integrating over *z*,



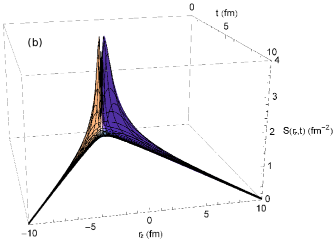


“Boomerang” shape

Particle production is close to the light-cone

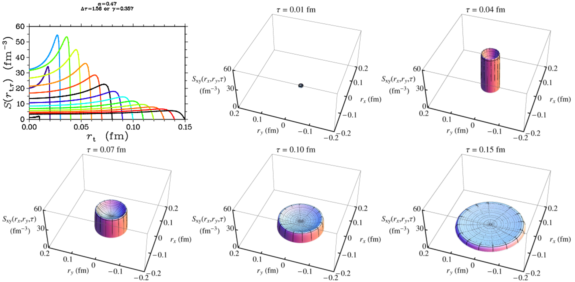
Emission Function of 2-jet Events.

Integrating over *r*,



"Boomerang" shape

Integrating over *z*,



Expanding ring

Particle production is close to the light-cone

α_s

- LLA parton shower leads to a fractal in momentum space
  fractal dimension, α, is related to α_s — Gustafson *et* al.
- Lévy dist. arises naturally from a fractal, or random walk,
  or anomalous diffusion — Metzler and Klafter, Phys.Rep.**339**(2000)1.
- strong momentum-space/configuration space correlation of τ-model $\Longrightarrow$
  fractal in configuration space with same α
- generalized LPHD suggests particle dist. has same properties as gluon dist.
- Putting this all together leads to — Csörgő *et* al.

$$\alpha_s = \frac{2\pi}{3}\alpha^2$$

- Using our value of $\alpha = 0.47 \pm 0.04$ yields $\alpha_s = 0.46 \pm 0.04$
- This value is reasonable for a scale of 1–2 GeV,
  where production of hadrons takes place
  cf., from τ decays $\alpha_s(m_\tau \approx 1.8\,\mathrm{GeV}) = 0.34 \pm 0.03$ — PDG

A Comment

- τ-model is closely related to a string picture
  - strong *x*-*p* correlation
  - fractal - Lévy distribution
- CMS finds BEC in pp at 0.9 and 7 TeV are described by simplified τ-model formula JHEP 05 (2011) 029
- suggests that BEC in pp is (mostly) from string fragmentation

Summary

- $R_2(Q)$, not $R_2(\vec{Q})$ is a reasonably good approximation
- But sym. Gaussian, Edgeworth, Lévy $R_2(Q)$ do not fit well
- τ-model with a one-sided Lévy proper-time distribution
  - Simplified, it provides a new parametrization of R_2:
    - Works well with eff. R, R_a for all events;
    - with only eff. R for 2-jet events.
  - $R_2(Q, m_t)$ successfully fits R_2 for 2-jet events
    - both Q- and m_t-dependence described correctly
    - Note: we found $\Delta\tau$ to be independent of m_t
      $\Delta\tau$ enters R_2 as $\Delta\tau Q^2/m_t$
      In Gaussian parametrization, r enters R_2 as $r^2 Q^2$
      Thus $\Delta\tau$ independent of m_t corresponds to $r \propto 1/\sqrt{m_t}$
- Emission function shaped like a boomerang in z-t
  and an expanding ring in x-y
  Particle production is close to the light-cone

Multiplicity/Jet/rapidity dependence in τ-model

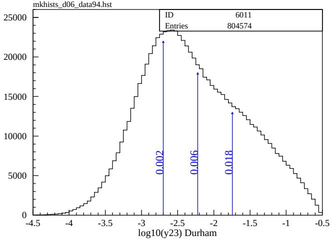
Use simplified τ-model, $\tau_0 = 0$
to investigate multiplicity and jet dependence

To stabilize fits against large correlation of parameters α and *R*, fix $\alpha = 0.44$

Jets

Jets — JADE and Durham algorithms

- force event to have 3 jets:
  - normally stop combining when all 'distances' between jets are $> y_{\text{cut}}$
  - instead, stop combining when there are only 3 jets left
  - y_{23} is the smallest 'distance' between any 2 of the 3 jets



- y_{23} is value of y_{cut} where number of jets changes from 2 to 3

define regions of y_{23}^{D} (Durham):

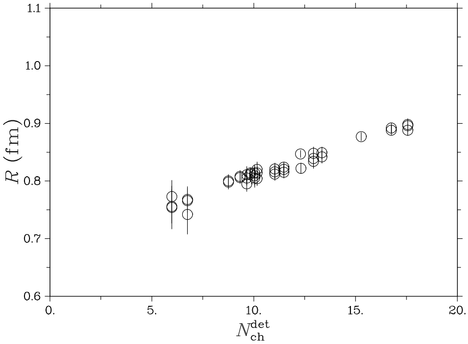
$y_{23}^{\text{D}} < 0.002$	narrow two-jet	or	
$0.002 < y_{23}^{\text{D}} < 0.006$	less narrow two-jet	$y_{23}^{\text{D}} < 0.006$	two-jet
$0.006 < y_{23}^{\text{D}} < 0.018$	narrow three-jet	$0.006 < y_{23}^{\text{D}}$	three-jet
$0.018 < y_{23}^{\text{D}}$	wide three-jet		

and similarly for y_{23}^{J} (JADE): 0.009, 0.023, 0.056

Multiplicity dependence in τ-model

Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$

L3 PRELIMINARY

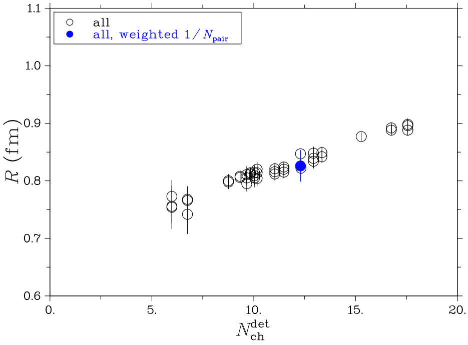


R increases with multiplicity

Multiplicity dependence in τ-model

Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$

L3 PRELIMINARY



R not constant

$\Longrightarrow$ R from fit is an average

But maybe not the average we want

To get R at avg. multiplicity of sample, should weight pairs by $1/N_{\text{pairs in event}}$ or calculate average multiplicity as

$$\frac{\sum_{\text{events}} N_{\text{event}} N_{\text{pairs in event}}}{N_{\text{pairs}}}$$

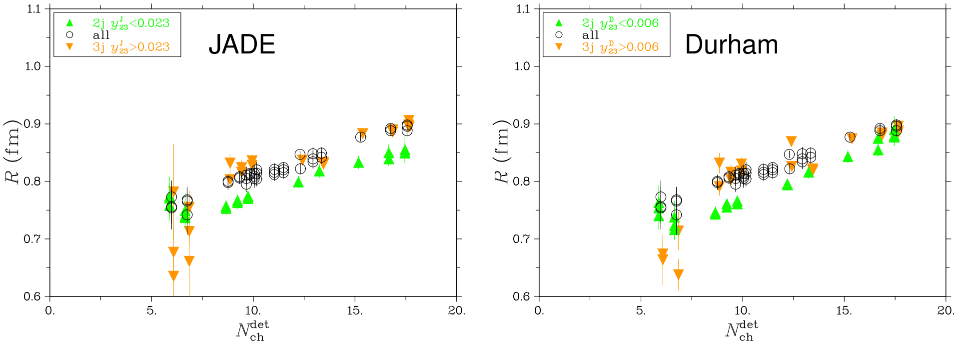
But the difference is small

So I ignore it.

R increases with multiplicity

Multiplicity/Jet dependence in τ-model

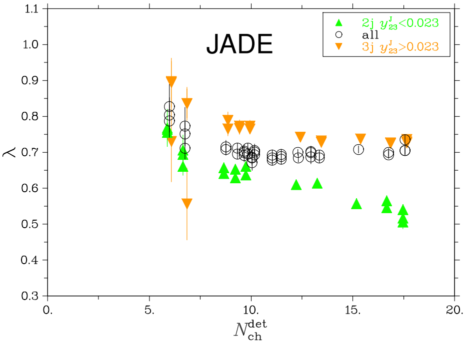
Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$ L3 PRELIMINARY

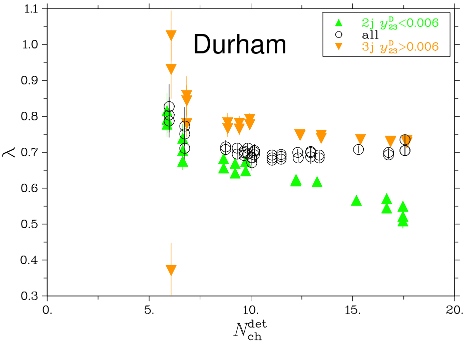


- R increases with N_{ch} and with number of jets
  whereas OPAL found $r_{n\text{-jet}}$ approx. indep. of N_{ch}
- Increase of R with N_{ch} similar for 2- and 3-jet events
- However, $R_{3\text{-jet}} \approx R_{all}$

Multiplicity/Jet dependence in τ-model

Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$ L3 PRELIMINARY





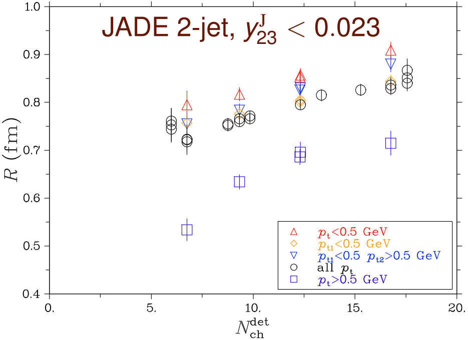
- $\lambda_{\text{3-jet}} > \lambda_{\text{2-jet}}$ opposite of OPAL
- λ initially decreases with N_{ch}
- then λ_{all} and $\lambda_{\text{3-jet}}$ approx. constant
  while $\lambda_{\text{2-jet}}$ continues to decrease, but more slowly
- whereas OPAL found λ_{all} decreasing approx. linearly with N_{ch}

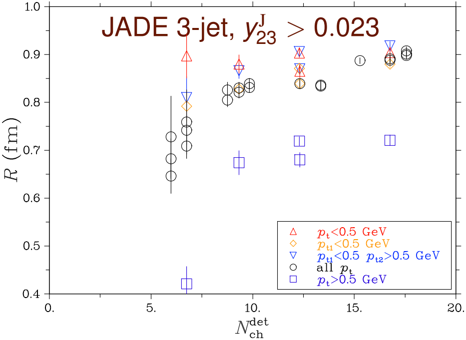
m_t dependence in τ-model

Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$

L3 PRELIMINARY

and cutting on $p_t = 0.5\,\mathrm{GeV}$ ($m_t = 0.52\,\mathrm{GeV}$)





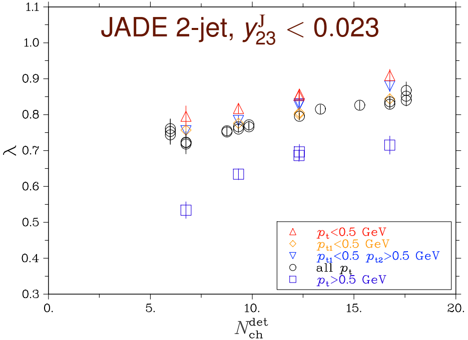
- R decreases with m_t for all N_{ch}
  smallest when both particles at high p_t

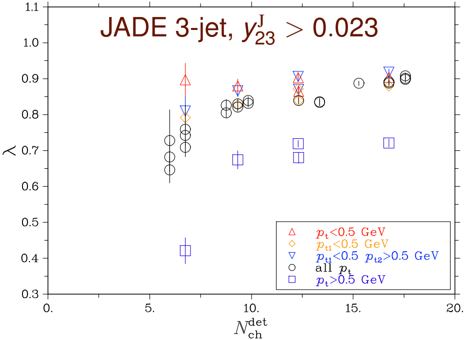
m_t dependence in τ-model

Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$

L3 PRELIMINARY

and cutting on $p_t = 0.5\,\mathrm{GeV}$ ($m_t = 0.52\,\mathrm{GeV}$)

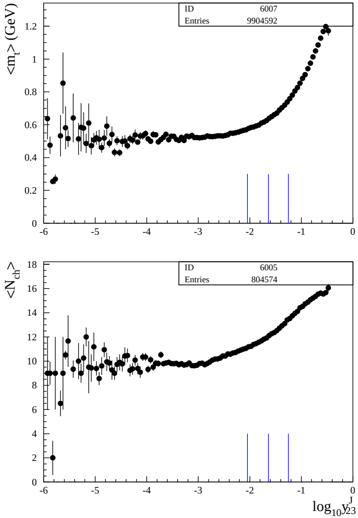




- λ decreases with m_t
  smallest when both particles at high p_t

On what do r, R, λ depend?

- r, R increase with N_{ch}
- r, R increase with N_{jets}
- for fixed number of jets, R increases with N_{ch} but r is constant (OPAL)
- r, R decrease with m_t
- Although m_t, N_{ch}, N_{jets} are correlated, each contributes to the increase/decrease of R but only m_t, N_{jets} contribute to the increase/decrease of r
- λ decreases with N_{ch}, N_{jets} though somewhat differently for τ-model, Gaussian (OPAL)
- λ decreases with m_t



Jets and Rapidity

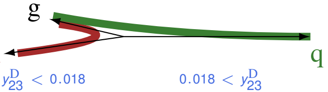
order jets by energy: $E_1 > E_2 > E_3$

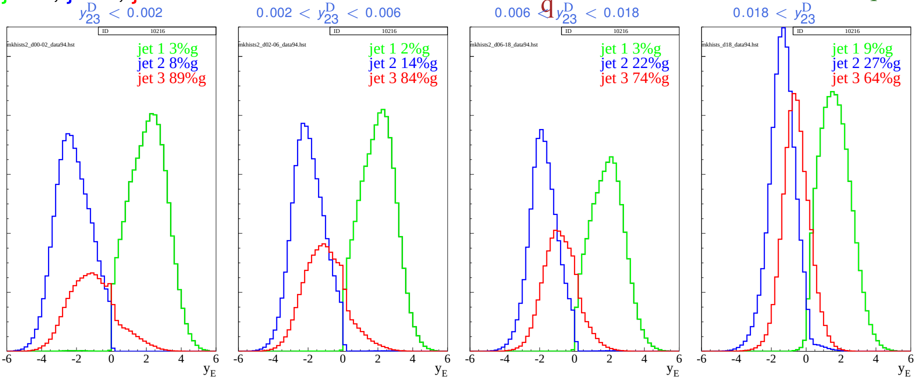
Note: thrust only defines axis $|\vec{n}_\mathrm{T}|$, not its direction.

Choose positive thrust direction such that jet 1 is in positive thrust hemisphere

rapidity, y_E, of particles from

jet 1, jet 2, jet 3:



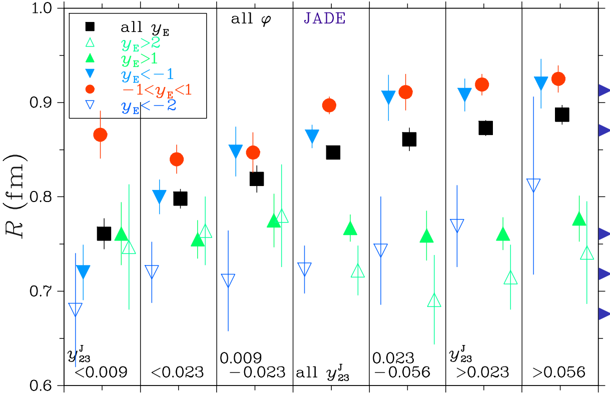


- $y_\mathrm{E} > 1$ almost all jet 1 — almost all quark
- $y_\mathrm{E} < -1$ mostly jet 2, some jet 3 — mostly quark
- $-1 < y_\mathrm{E} < 1$ jet-3 enriched — largely gluon

Jets and Rapidity – simplified τ-model – L3 preliminary

To stabilize fits against large correlation of α, R, fix $\alpha = 0.44$

Select particle pairs by rapidity of pair



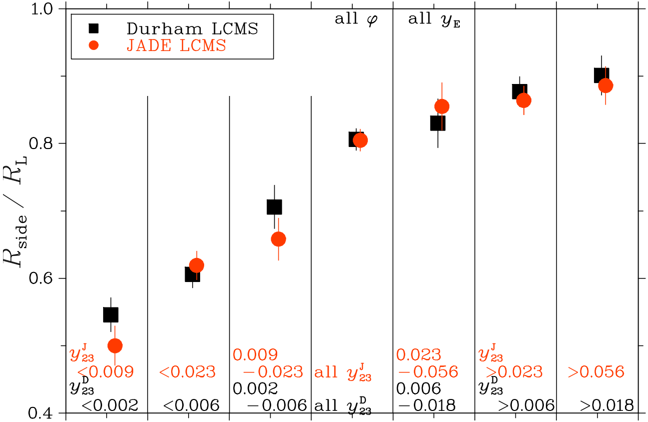
With y^J_{23},

- all y: R increases
- 'pure' q jet, $y_E > 1$, or $y_E < -1$ & y^J_{23} small, or $y_E < -2$: R const.
- $R_{-1<y_E<1} > R_{\text{'pure' q}}$
- $R_{y_E<-1}$ increases
- at large y^D_{23} $R_{-1<y_E<1} = R_{y_E<-1}$

Conclusion (Durham agrees):

Increase in R with y^J_{23} is due to appearance of gluon jet

τ-model elongation – L3 preliminary



- Durham, JADE agree
- Elongation decreases with y_{23}, $R_{\text{side}} \approx 0.5$–$0.9\ R_{\text{long}}$
- agrees with Gaussian/Edgeworth