

Overview on transverse-momentum resummation

Giancarlo Ferrera

Milan University & INFN, Milan



REF 2014 – Antwerp – Dec. 10th 2014

Outline

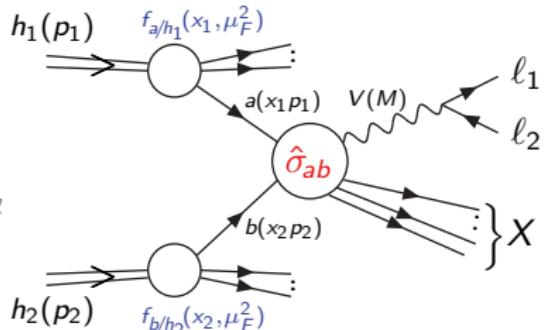
- 1 q_T resummation in Drell-Yan process
- 2 $q\bar{q}$ annihilation vs gluon fusion process
- 3 Universality in q_T resummation
- 4 q_T resummation for DY and Higgs: numerical results
- 5 Non perturbative intrinsic k_T effects
- 6 q_T resummation for heavy-quark hadroproduction
- 7 Conclusions

Drell–Yan q_T distribution

$$h_1(p_1) + h_2(p_2) \rightarrow V(M) + X \rightarrow \ell_1 + \ell_2 + X$$

where $V = \gamma^*, Z^0, W^\pm$ and $\ell_1 \ell_2 = \ell^+ \ell^-, \ell \nu_\ell$

pQCD collinear factorization formula ($M \gg \Lambda_{QCD}$):



$$\frac{d\sigma}{dq_T^2}(q_T, M, s) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) \frac{d\hat{\sigma}_{ab}}{dq_T^2}(q_T, M, \hat{s}; \alpha_S, \mu_R^2, \mu_F^2).$$

Fixed-order perturbative expansion not reliable for $q_T \ll M$:

$$\int_0^{q_T^2} d\bar{q}_T^2 \frac{d\hat{\sigma}_{q\bar{q}}}{d\bar{q}_T^2} \stackrel{q_T \ll M}{\sim} 1 + \alpha_S \left[c_{12} \ln^2 \frac{M^2}{q_T^2} + c_{11} \ln \frac{M^2}{q_T^2} + c_{10} \right] + \dots$$

$\alpha_S \ln(M^2/q_T^2) \gg 1$: need for resummation of large logs.

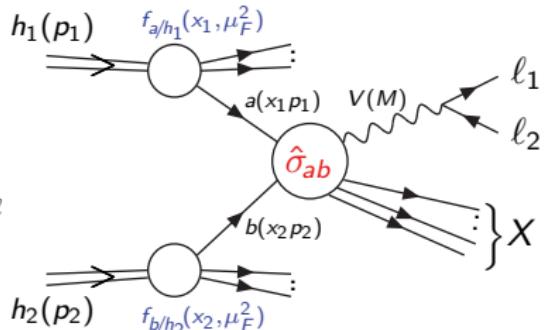
$$\frac{d\sigma}{dq_T^2} = \frac{d\sigma^{(res)}}{dq_T^2} + \frac{d\sigma^{(fin)}}{dq_T^2}; \quad \begin{aligned} \int_0^{q_T^2} d\bar{q}_T^2 \frac{d\hat{\sigma}^{(fin)}}{d\bar{q}_T^2} &\stackrel{q_T \rightarrow 0}{=} 0 \\ \int_0^{q_T^2} d\bar{q}_T^2 \frac{d\hat{\sigma}^{(res)}}{d\bar{q}_T^2} &\stackrel{q_T \rightarrow 0}{\sim} 1 + \sum_n \sum_{m=0}^{2n} c_{nm} \alpha_S^n \ln^m \frac{M^2}{q_T^2} \end{aligned}$$

Drell–Yan q_T distribution

$$h_1(p_1) + h_2(p_2) \rightarrow V(M) + X \rightarrow \ell_1 + \ell_2 + X$$

where $V = \gamma^*, Z^0, W^\pm$ and $\ell_1 \ell_2 = \ell^+ \ell^-, \ell \nu_\ell$

pQCD *collinear factorization formula* ($M \gg \Lambda_{QCD}$):



$$\frac{d\sigma}{dq_T^2}(q_T, M, s) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) \frac{d\hat{\sigma}_{ab}}{dq_T^2}(q_T, M, \hat{s}; \alpha_S, \mu_R^2, \mu_F^2).$$

Fixed-order perturbative expansion *not reliable for* $q_T \ll M$:

$$\int_0^{q_T^2} d\bar{q}_T^2 \frac{d\hat{\sigma}_{q\bar{q}}}{d\bar{q}_T^2} \stackrel{q_T \ll M}{\sim} 1 + \alpha_S \left[c_{12} \ln^2 \frac{M^2}{q_T^2} + c_{11} \ln \frac{M^2}{q_T^2} + c_{10} \right] + \dots$$

$\alpha_S \ln(M^2/q_T^2) \gg 1$: need for resummation of large logs.

$$\frac{d\sigma}{dq_T^2} = \frac{d\sigma^{(res)}}{dq_T^2} + \frac{d\sigma^{(fin)}}{dq_T^2}; \quad \begin{aligned} \int_0^{q_T^2} d\bar{q}_T^2 \frac{d\hat{\sigma}^{(fin)}}{d\bar{q}_T^2} &\stackrel{q_T \rightarrow 0}{=} 0 \\ \int_0^{q_T^2} d\bar{q}_T^2 \frac{d\hat{\sigma}^{(res)}}{d\bar{q}_T^2} &\stackrel{q_T \rightarrow 0}{\sim} 1 + \sum_n \sum_{m=0}^{2n} c_{nm} \alpha_S^n \ln^m \frac{M^2}{q_T^2} \end{aligned}$$

Drell–Yan q_T distribution

$$h_1(p_1) + h_2(p_2) \rightarrow V(M) + X \rightarrow \ell_1 + \ell_2 + X$$

where $V = \gamma^*, Z^0, W^\pm$ and $\ell_1 \ell_2 = \ell^+ \ell^-, \ell \nu_\ell$

pQCD collinear factorization formula ($M \gg \Lambda_{QCD}$):

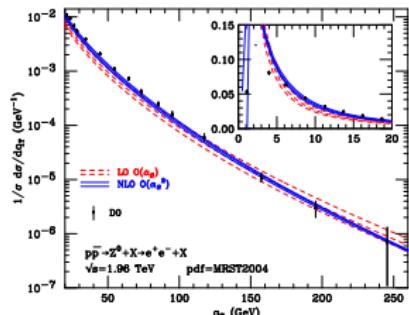
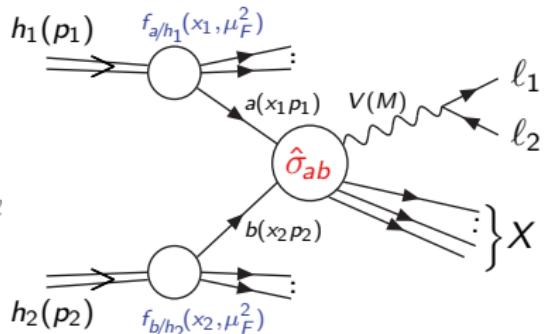
$$\frac{d\sigma}{dq_T^2}(q_T, M, s) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) \frac{d\hat{\sigma}_{ab}}{dq_T^2}(q_T, M, \hat{s}; \alpha_S, \mu_R^2, \mu_F^2).$$

Fixed-order perturbative expansion **not reliable for $q_T \ll M$** :

$$\int_0^{q_T^2} d\bar{q}_T^2 \frac{d\hat{\sigma}_{q\bar{q}}}{d\bar{q}_T^2} \stackrel{q_T \ll M}{\sim} 1 + \alpha_S \left[c_{12} \ln^2 \frac{M^2}{q_T^2} + c_{11} \ln \frac{M^2}{q_T^2} + c_{10} \right] + \dots$$

$\alpha_S \ln(M^2/q_T^2) \gg 1$: need for resummation of large logs.

$$\frac{d\sigma}{dq_T^2} = \frac{d\sigma^{(res)}}{dq_T^2} + \frac{d\sigma^{(fin)}}{dq_T^2}; \quad \begin{aligned} \int_0^{q_T^2} d\bar{q}_T^2 \frac{d\hat{\sigma}^{(fin)}}{d\bar{q}_T^2} &\stackrel{q_T \rightarrow 0}{=} 0 \\ \int_0^{q_T^2} d\bar{q}_T^2 \frac{d\hat{\sigma}^{(res)}}{d\bar{q}_T^2} &\stackrel{q_T \rightarrow 0}{\sim} 1 + \sum_n \sum_{m=0}^{2n} c_{nm} \alpha_S^m \ln^m \frac{M^2}{q_T^2} \end{aligned}$$



Soft gluon exponentiation

Sudakov resummation feasible when:
dynamics AND kinematics factorize
 \Rightarrow exponentiation.

- Dynamics factorization: general property of QCD matrix element for soft emissions.

$$dw_n(q_1, \dots, q_n) \simeq \frac{1}{n!} \prod_{i=1}^n dw_i(q_i)$$

- Kinematics factorization: not valid in general. For q_T distribution it holds in the impact parameter space (Fourier transform)

$$\int d^2\mathbf{q}_T \exp(-i\mathbf{b} \cdot \mathbf{q}_T) \delta\left(\mathbf{q}_T - \sum_{j=1}^n \mathbf{q}_{T_j}\right) = \exp\left(-i\mathbf{b} \cdot \sum_{j=1}^n \mathbf{q}_{T_j}\right) = \prod_{j=1}^n \exp(-i\mathbf{b} \cdot \mathbf{q}_{T_j}).$$

- Exponentiation holds in the impact parameter space. Results have then to be transformed back to the physical space: $q_T \ll M \Leftrightarrow Mb \gg 1$,
 $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$.

State of the art: q_T resummation

- Method to resum large q_T logarithms is known [Dokshitzer, Diakonov, Troian ('78)], [Parisi, Petronzio ('79)], [Kodaira, Trentadue ('82)], [Collins, Soper, Sterman ('85)], [Altarelli et al. ('84)], [Catani, d'Emilio, Trentadue ('88)], [Catani, de Florian, Grazzini ('01)], [Catani, Grazzini ('10)], [Catani, Grazzini, Torre ('14)]
- Various phenomenological studies [ResBos: Balasz, Yuan, Nadolsky et al. ('97, '02)], [Ellis et al. ('97)], [Kulesza et al. ('02)], [Guzzi, Nadolsky, Wang ('13)].
- Results for q_T resummation in the framework of Effective Theories [Gao, Li, Liu ('05)], [Idilbi, Ji, Yuan ('05)], [Mantry, Petriello ('10)], [Becher, Neubert ('10)], [Echevarria, Idilbi, Scimemi ('11)].
- Studies within transverse-momentum dependent (TMD) factorization and TMD parton densities [D'Alesio, Murgia ('04)], [Roger, Mulders ('10)], [Collins ('11)], [D'Alesio, Echevarria, Melis, Scimemi ('14)], [Ceccopieri, Trentadue ('14)].
- Effective q_T -resummation obtained with Parton Shower algorithms POWHEG/MC@NLO [Barze et al. ('12, '13)], [Hoeche, Li, Prestel ('14)], [Karlberg, Re, Zanderighi ('14)].

q_T resummation: $q\bar{q}$ -annihilation processes

Hadroproduction of a system F of *colourless* particles initiated at Born level by $q_f \bar{q}_{f'} \rightarrow F$.

$$\frac{d\sigma_F^{(res)}(p_1, p_2; q_T, M, y, \Omega)}{d^2 q_T dM^2 dy d\Omega} = \frac{M^2}{s} \sum_{c=q,\bar{q}} \left[d\sigma_{c\bar{c},F}^{(0)} \right] \int \frac{d^2 b}{(2\pi)^2} e^{ib \cdot q_T} S_q(M, b) \\ \times \sum_{a_1, a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} \left[H^F C_1 C_2 \right]_{c\bar{c}; a_1 a_2} f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) f_{a_2/h_2}(x_2/z_2, b_0^2/b^2) ,$$

$$b_0 = 2e^{-\gamma_E} (\gamma_E = 0.57\dots), \quad x_{1,2} = \frac{M}{\sqrt{s}} e^{\pm y}, \quad L \equiv \ln Mb \quad [\text{Collins, Soper, Sterman ('85)],} \\ [\text{Catani, de Florian, Grazzini ('01)]}$$

$$S_q(M, b) = \exp \left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_q(\alpha_S(q^2)) \ln \frac{M^2}{q^2} + B_q(\alpha_S(q^2)) \right] \right\} .$$

$$\left[H^F C_1 C_2 \right]_{q\bar{q}; a_1 a_2} = H_q^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) C_{qa_1}(z_1; \alpha_S(b_0^2/b^2)) C_{\bar{q}a_2}(z_2; \alpha_S(b_0^2/b^2)) ,$$

$$A_q(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n A_c^{(n)}, \quad B_q(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n B_c^{(n)},$$

$$H_q^F(\alpha_S) = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n H_q^{F(n)}, \quad C_{qa}(z; \alpha_S) = \delta_{qa} \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n C_{qa}^{(n)}(z) .$$

$$\text{LL}(\sim \alpha_S^n L^{n+1}): A_q^{(1)}; \quad \text{NLL}(\sim \alpha_S^n L^n): A_q^{(2)}, B_q^{(1)}, H_q^{F(1)}, C_{qa}^{(1)}; \quad \text{NNLL}(\sim \alpha_S^n L^{n-1}): A_q^{(3)}, B_q^{(2)}, H_q^{F(2)}, C_{qa}^{(2)}$$

q_T resummation: $q\bar{q}$ -annihilation processes

Hadroproduction of a system F of *colourless* particles initiated at Born level by $q_f \bar{q}_{f'} \rightarrow F$.

$$\frac{d\sigma_F^{(res)}(p_1, p_2; \mathbf{q}_T, M, y, \Omega)}{d^2\mathbf{q}_T \, dM^2 \, dy \, d\Omega} = \frac{M^2}{s} \sum_{c=q,\bar{q}} \left[d\sigma_{c\bar{c},F}^{(0)} \right] \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{q}_T} S_q(M, b) \\ \times \sum_{a_1, a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} \left[H^F C_1 C_2 \right]_{c\bar{c}; a_1 a_2} f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) f_{a_2/h_2}(x_2/z_2, b_0^2/b^2) ,$$

$$b_0 = 2e^{-\gamma_E} (\gamma_E = 0.57\dots), \quad x_{1,2} = \frac{M}{\sqrt{s}} e^{\pm y}, \quad L \equiv \ln Mb \quad [\text{Collins, Soper, Sterman ('85)],} \\ [\text{Catani, de Florian, Grazzini ('01)]}$$

$$S_q(M, b) = \exp \left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_q(\alpha_S(q^2)) \ln \frac{M^2}{q^2} + B_q(\alpha_S(q^2)) \right] \right\} .$$

$$\left[H^F C_1 C_2 \right]_{q\bar{q}; a_1 a_2} = H_q^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) C_{qa_1}(z_1; \alpha_S(b_0^2/b^2)) C_{\bar{q}a_2}(z_2; \alpha_S(b_0^2/b^2)) ,$$

$$A_q(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n A_c^{(n)}, \quad B_q(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n B_c^{(n)},$$

$$H_q^F(\alpha_S) = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n H_q^{F(n)}, \quad C_{qa}(z; \alpha_S) = \delta_{qa} \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n C_{qa}^{(n)}(z) .$$

$$\text{LL}(\sim \alpha_S^n L^{n+1}): A_q^{(1)}; \quad \text{NLL}(\sim \alpha_S^n L^n): A_q^{(2)}, B_q^{(1)}, H_q^{F(1)}, C_{qa}^{(1)}; \quad \text{NNLL}(\sim \alpha_S^n L^{n-1}): A_q^{(3)}, B_q^{(2)}, H_q^{F(2)}, C_{qa}^{(2)}$$

q_T resummation: $q\bar{q}$ -annihilation processes

Hadroproduction of a system F of *colourless* particles initiated at Born level by $q_f \bar{q}_{f'} \rightarrow F$.

$$\frac{d\sigma_F^{(res)}(p_1, p_2; \mathbf{q}_T, M, y, \Omega)}{d^2\mathbf{q}_T \, dM^2 \, dy \, d\Omega} = \frac{M^2}{s} \sum_{c=q,\bar{q}} \left[d\sigma_{c\bar{c},F}^{(0)} \right] \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{q}_T} S_q(M, b) \\ \times \sum_{a_1, a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} \left[H^F C_1 C_2 \right]_{c\bar{c}; a_1 a_2} f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) f_{a_2/h_2}(x_2/z_2, b_0^2/b^2) ,$$

$$b_0 = 2e^{-\gamma_E} (\gamma_E = 0.57\dots), \quad x_{1,2} = \frac{M}{\sqrt{s}} e^{\pm y}, \quad L \equiv \ln Mb \quad [\text{Collins, Soper, Sterman ('85)],} \\ [\text{Catani, de Florian, Grazzini ('01)]}$$

$$S_q(M, b) = \exp \left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_q(\alpha_S(q^2)) \ln \frac{M^2}{q^2} + B_q(\alpha_S(q^2)) \right] \right\} .$$

$$\left[H^F C_1 C_2 \right]_{q\bar{q}; a_1 a_2} = H_q^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) \, C_{qa_1}(z_1; \alpha_S(b_0^2/b^2)) \, C_{\bar{q}a_2}(z_2; \alpha_S(b_0^2/b^2)) ,$$

$$A_q(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n A_c^{(n)}, \quad B_q(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n B_c^{(n)},$$

$$H_q^F(\alpha_S) = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n H_q^{F(n)}, \quad C_{qa}(z; \alpha_S) = \delta_{qa} \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n C_{qa}^{(n)}(z) .$$

$$\text{LL}(\sim \alpha_S^n L^{n+1}): A_q^{(1)}; \quad \text{NLL}(\sim \alpha_S^n L^n): A_q^{(2)}, B_q^{(1)}, H_q^{F(1)}, C_{qa}^{(1)}; \quad \text{NNLL}(\sim \alpha_S^n L^{n-1}): A_q^{(3)}, B_q^{(2)}, H_q^{F(2)}, C_{qa}^{(2)}$$

q_T resummation: $q\bar{q}$ -annihilation processes

Hadroproduction of a system F of *colourless* particles initiated at Born level by $q_f \bar{q}_{f'} \rightarrow F$.

$$\frac{d\sigma_F^{(res)}(p_1, p_2; \mathbf{q}_T, M, y, \Omega)}{d^2\mathbf{q}_T \, dM^2 \, dy \, d\Omega} = \frac{M^2}{s} \sum_{c=q,\bar{q}} \left[d\sigma_{c\bar{c},F}^{(0)} \right] \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{q}_T} S_q(M, b) \\ \times \sum_{a_1, a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} \left[H^F C_1 C_2 \right]_{c\bar{c}; a_1 a_2} f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) f_{a_2/h_2}(x_2/z_2, b_0^2/b^2) ,$$

$$b_0 = 2e^{-\gamma_E} (\gamma_E = 0.57\dots), \quad x_{1,2} = \frac{M}{\sqrt{s}} e^{\pm y}, \quad L \equiv \ln Mb \quad [\text{Collins, Soper, Sterman ('85)],} \\ [\text{Catani, de Florian, Grazzini ('01)]}]$$

$$S_q(M, b) = \exp \left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_q(\alpha_S(q^2)) \ln \frac{M^2}{q^2} + B_q(\alpha_S(q^2)) \right] \right\} .$$

$$\left[H^F C_1 C_2 \right]_{q\bar{q}; a_1 a_2} = H_q^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) \, C_{qa_1}(z_1; \alpha_S(b_0^2/b^2)) \, C_{\bar{q}a_2}(z_2; \alpha_S(b_0^2/b^2)) ,$$

$$A_q(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n A_c^{(n)}, \quad B_q(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n B_c^{(n)},$$

$$H_q^F(\alpha_S) = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n H_q^{F(n)}, \quad C_{qa}(z; \alpha_S) = \delta_{qa} \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n C_{qa}^{(n)}(z) .$$

$$\text{LL}(\sim \alpha_S^n L^{n+1}): A_q^{(1)}; \quad \text{NLL}(\sim \alpha_S^n L^n): A_q^{(2)}, B_q^{(1)}, H_q^{F(1)}, C_{qa}^{(1)}; \quad \text{NNLL}(\sim \alpha_S^n L^{n-1}): A_q^{(3)}, B_q^{(2)}, H_q^{F(2)}, C_{qa}^{(2)}$$

\mathbf{q}_T resummation: $q\bar{q}$ -annihilation processes

Hadroproduction of a system F of *colourless* particles initiated at Born level by $q_f \bar{q}_{f'} \rightarrow F$.

$$\frac{d\sigma_F^{(res)}(p_1, p_2; \mathbf{q}_T, M, y, \Omega)}{d^2\mathbf{q}_T dM^2 dy d\Omega} = \frac{M^2}{s} \sum_{c=q,\bar{q}} \left[d\sigma_{c\bar{c},F}^{(0)} \right] \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{q}_T} S_q(M, b) \\ \times \sum_{a_1, a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} \left[H^F C_1 C_2 \right]_{c\bar{c}; a_1 a_2} f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) f_{a_2/h_2}(x_2/z_2, b_0^2/b^2) ,$$

$$b_0 = 2e^{-\gamma_E} (\gamma_E = 0.57\dots), \quad x_{1,2} = \frac{M}{\sqrt{s}} e^{\pm y}, \quad L \equiv \ln Mb \quad [\text{Collins, Soper, Sterman ('85)],} \\ [\text{Catani, de Florian, Grazzini ('01)]}$$

$$S_q(M, b) = \exp \left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_q(\alpha_S(q^2)) \ln \frac{M^2}{q^2} + B_q(\alpha_S(q^2)) \right] \right\} .$$

$$\left[H^F C_1 C_2 \right]_{q\bar{q}; a_1 a_2} = H_q^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) C_{qa_1}(z_1; \alpha_S(b_0^2/b^2)) C_{\bar{q}a_2}(z_2; \alpha_S(b_0^2/b^2)) ,$$

$$A_q(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n A_c^{(n)}, \quad B_q(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n B_c^{(n)},$$

$$H_q^F(\alpha_S) = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n H_q^{F(n)}, \quad C_{qa}(z; \alpha_S) = \delta_{qa} \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n C_{qa}^{(n)}(z) .$$

$$\text{LL}(\sim \alpha_S^n L^{n+1}): A_q^{(1)}; \quad \text{NLL}(\sim \alpha_S^n L^n): A_q^{(2)}, B_q^{(1)}, H_q^{F(1)}, C_{qa}^{(1)}; \quad \text{NNLL}(\sim \alpha_S^n L^{n-1}): A_q^{(3)}, B_q^{(2)}, H_q^{F(2)}, C_{qa}^{(2)}$$

q_T resummation: $q\bar{q}$ -annihilation processes

Hadroproduction of a system F of *colourless* particles initiated at Born level by $q_f \bar{q}_{f'} \rightarrow F$.

$$\frac{d\sigma_F^{(res)}(p_1, p_2; \mathbf{q}_T, M, y, \Omega)}{d^2\mathbf{q}_T \, dM^2 \, dy \, d\Omega} = \frac{M^2}{s} \sum_{c=q,\bar{q}} \left[d\sigma_{c\bar{c},F}^{(0)} \right] \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{q}_T} S_q(M, b) \\ \times \sum_{a_1, a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} \left[H^F C_1 C_2 \right]_{c\bar{c}; a_1 a_2} f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) f_{a_2/h_2}(x_2/z_2, b_0^2/b^2) ,$$

$$b_0 = 2e^{-\gamma_E} (\gamma_E = 0.57\dots), \quad x_{1,2} = \frac{M}{\sqrt{s}} e^{\pm y}, \quad L \equiv \ln Mb \quad [\text{Collins, Soper, Sterman ('85)],} \\ [\text{Catani, de Florian, Grazzini ('01)]}$$

$$S_q(M, b) = \exp \left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_q(\alpha_S(q^2)) \ln \frac{M^2}{q^2} + B_q(\alpha_S(q^2)) \right] \right\} .$$

$$\left[H^F C_1 C_2 \right]_{q\bar{q}; a_1 a_2} = H_q^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) \, C_{qa_1}(z_1; \alpha_S(b_0^2/b^2)) \, C_{\bar{q}a_2}(z_2; \alpha_S(b_0^2/b^2)) ,$$

$$A_q(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n A_c^{(n)}, \quad B_q(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n B_c^{(n)},$$

$$H_q^F(\alpha_S) = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n H_q^{F(n)}, \quad C_{qa}(z; \alpha_S) = \delta_{qa} \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n C_{qa}^{(n)}(z) .$$

$$\text{LL}(\sim \alpha_S^n L^{n+1}): A_q^{(1)}; \quad \text{NLL}(\sim \alpha_S^n L^n): A_q^{(2)}, B_q^{(1)}, H_q^{F(1)}, C_{qa}^{(1)}; \quad \text{NNLL}(\sim \alpha_S^n L^{n-1}): A_q^{(3)}, B_q^{(2)}, H_q^{F(2)}, C_{qa}^{(2)}$$

q_T resummation: $q\bar{q}$ -annihilation processes

Hadroproduction of a system F of *colourless* particles initiated at Born level by $q_f \bar{q}_{f'} \rightarrow F$.

$$\frac{d\sigma_F^{(res)}(p_1, p_2; \mathbf{q}_T, M, y, \Omega)}{d^2\mathbf{q}_T \, dM^2 \, dy \, d\Omega} = \frac{M^2}{s} \sum_{c=q,\bar{q}} \left[d\sigma_{c\bar{c},F}^{(0)} \right] \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{q}_T} S_q(M, b) \\ \times \sum_{a_1, a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} \left[H^F C_1 C_2 \right]_{c\bar{c}; a_1 a_2} f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) f_{a_2/h_2}(x_2/z_2, b_0^2/b^2) ,$$

$$b_0 = 2e^{-\gamma_E} (\gamma_E = 0.57\dots), \quad x_{1,2} = \frac{M}{\sqrt{s}} e^{\pm y}, \quad L \equiv \ln Mb \quad [\text{Collins, Soper, Sterman ('85)],} \\ [\text{Catani, de Florian, Grazzini ('01)]}$$

$$S_q(M, b) = \exp \left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_q(\alpha_S(q^2)) \ln \frac{M^2}{q^2} + B_q(\alpha_S(q^2)) \right] \right\} .$$

$$\left[H^F C_1 C_2 \right]_{q\bar{q}; a_1 a_2} = H_q^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) \, C_{qa_1}(z_1; \alpha_S(b_0^2/b^2)) \, C_{\bar{q}a_2}(z_2; \alpha_S(b_0^2/b^2)) ,$$

$$A_q(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n A_c^{(n)}, \quad B_q(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n B_c^{(n)},$$

$$H_q^F(\alpha_S) = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n H_q^{F(n)}, \quad C_{qa}(z; \alpha_S) = \delta_{qa} \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n C_{qa}^{(n)}(z) .$$

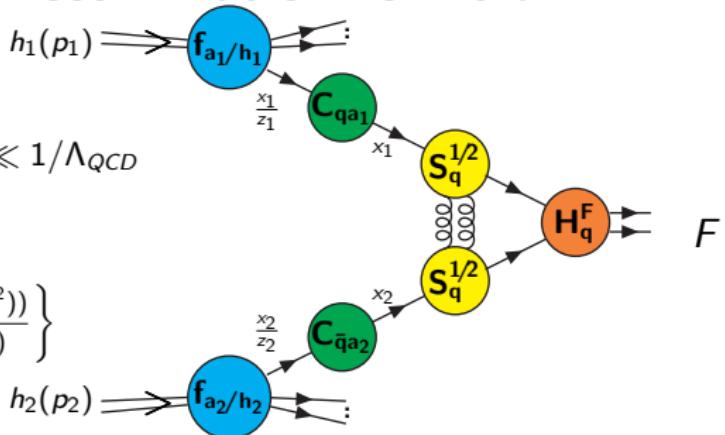
$$\text{LL}(\sim \alpha_S^n L^{n+1}): A_q^{(1)}; \quad \text{NLL}(\sim \alpha_S^n L^n): A_q^{(2)}, B_q^{(1)}, H_q^{F(1)}, C_{qa}^{(1)}; \quad \text{NNLL}(\sim \alpha_S^n L^{n-1}): A_q^{(3)}, B_q^{(2)}, H_q^{F(2)}, C_{qa}^{(2)}$$

Transverse-momentum resummation formula

$$M \gg \Lambda_{QCD}, \quad b \gg 1/M, \quad b \ll 1/\Lambda_{QCD}$$

$$C(\alpha_S(b_0^2/b^2)) = C(\alpha_S(M^2))$$

$$\times \exp \left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \beta(\alpha_S(q^2)) \frac{d \ln C(\alpha_S(q^2))}{d \ln \alpha_S(q^2)} \right\}$$



$$\frac{d\sigma_F^{(res)}}{d^2\mathbf{q}_T \, dM^2 \, dy \, d\Omega} = \frac{M^2}{s} \left[d\sigma_{q\bar{q},F}^{(0)} \right] H_q^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) \sum_{a_1, a_2} \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{q}_T} S_q(M, b)$$

$$\times \int_{x_1}^1 \frac{dz_1}{z_1} C_{qa_1}(z_1; \alpha_S(b_0^2/b^2)) f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \int_{x_2}^1 \frac{dz_2}{z_2} C_{\bar{q}a_2}(z_2; \alpha_S(b_0^2/b^2)) f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

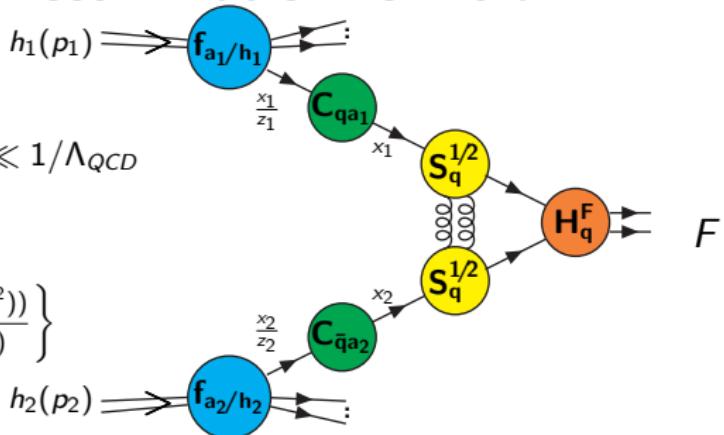
$$\tilde{F}_{q_f/h}(x, b, M) = \sum_a \int_x^1 \frac{dz}{z} \sqrt{S_q(M, b)} C_{q_f a}(z; \alpha_S(b_0^2/b^2)) f_{a/h}(x/z, b_0^2/b^2)$$

Transverse-momentum resummation formula

$$M \gg \Lambda_{QCD}, \quad b \gg 1/M, \quad b \ll 1/\Lambda_{QCD}$$

$$C(\alpha_S(b_0^2/b^2)) = C(\alpha_S(M^2))$$

$$\times \exp \left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \beta(\alpha_S(q^2)) \frac{d \ln C(\alpha_S(q^2))}{d \ln \alpha_S(q^2)} \right\}$$



$$\frac{d\sigma_F^{(res)}}{d^2\mathbf{q}_T \, dM^2 \, dy \, d\Omega} = \frac{M^2}{s} \left[d\sigma_{q\bar{q},F}^{(0)} \right] H_q^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) \sum_{a_1, a_2} \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{q}_T} S_q(M, b)$$

$$\times \int_{x_1}^1 \frac{dz_1}{z_1} C_{qa_1}(z_1; \alpha_S(b_0^2/b^2)) f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \int_{x_2}^1 \frac{dz_2}{z_2} C_{\bar{q}a_2}(z_2; \alpha_S(b_0^2/b^2)) f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

$$\tilde{F}_{q_f/h}(x, b, M) = \sum_a \int_x^1 \frac{dz}{z} \sqrt{S_q(M, b)} C_{q_f a}(z; \alpha_S(b_0^2/b^2)) f_{a/h}(x/z, b_0^2/b^2)$$

q_T resummation: gluon fusion processes

In processes initiated at Born level by the gluon fusion channel ($gg \rightarrow F$), collinear radiation from gluons leads to spin and azimuthal correlations [Catani, Grazzini ('11)].

$$\begin{aligned} \left[H^F C_1 C_2 \right]_{gg; a_1 a_2} &= H^F_{g; \mu_1 \nu_1, \mu_2 \nu_2}(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) \\ &\times C_{ga_1}^{\mu_1 \nu_1}(z_1; p_1, p_2, \mathbf{b}; \alpha_S(b_0^2/b^2)) C_{ga_2}^{\mu_2 \nu_2}(z_2; p_1, p_2, \mathbf{b}; \alpha_S(b_0^2/b^2)). \end{aligned}$$

where $H_g^{F\mu_1 \nu_1, \mu_2 \nu_2}(\alpha_S) = \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n H_g^{F(n)\mu_1 \nu_1, \mu_2 \nu_2}$,

$$C_{ga}^{\mu\nu}(z; p_1, p_2, \mathbf{b}; \alpha_S) = d^{\mu\nu}(p_1, p_2) C_{ga}(z; \alpha_S) + D^{\mu\nu}(p_1, p_2; \mathbf{b}) G_{ga}(z; \alpha_S) ,$$

$$d^{\mu\nu}(p_1, p_2) = -g^{\mu\nu} + \frac{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu}{p_1 \cdot p_2} , \quad D^{\mu\nu}(p_1, p_2; \mathbf{b}) = d^{\mu\nu}(p_1, p_2) - 2 \frac{b^\mu b^\nu}{b^2} ,$$

$$C_{ga}(z; \alpha_S) = \delta_{ga} \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n C_{ga}^{(n)}(z) , \quad G_{ga}(z; \alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n G_{ga}^{(n)}(z) .$$

- Unlike $q\bar{q}$ annih. $[H^F C_1 C_2]$ does depend on the azimuthal angle $\phi(\mathbf{b})$, this leads to azimuthal correlations with respect to the azimuthal angle $\phi(q_T)$ (consistent with [Mulders, Rodrigues ('00)], [Henneman et al. ('02)]).
- Small- q_T cross section expressed in terms of $\phi(q_T)$ -independent plus $\cos(2\phi(q_T))$, $\sin(2\phi(q_T))$, $\cos(4\phi(q_T))$ and $\sin(4\phi(q_T))$ dependent contributions.

q_T resummation: gluon fusion processes

In processes initiated at Born level by the gluon fusion channel ($gg \rightarrow F$), collinear radiation from gluons leads to spin and azimuthal correlations [Catani, Grazzini ('11)].

$$\begin{aligned} \left[H^F C_1 C_2 \right]_{gg; a_1 a_2} &= H^F_{g; \mu_1 \nu_1, \mu_2 \nu_2}(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) \\ &\times C_{ga_1}^{\mu_1 \nu_1}(z_1; p_1, p_2, \mathbf{b}; \alpha_S(b_0^2/b^2)) C_{ga_2}^{\mu_2 \nu_2}(z_2; p_1, p_2, \mathbf{b}; \alpha_S(b_0^2/b^2)). \end{aligned}$$

where $H_g^{F\mu_1 \nu_1, \mu_2 \nu_2}(\alpha_S) = \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n H_g^{F(n)\mu_1 \nu_1, \mu_2 \nu_2}$,

$$C_{ga}^{\mu\nu}(z; p_1, p_2, \mathbf{b}; \alpha_S) = d^{\mu\nu}(p_1, p_2) C_{ga}(z; \alpha_S) + D^{\mu\nu}(p_1, p_2; \mathbf{b}) G_{ga}(z; \alpha_S) ,$$

$$d^{\mu\nu}(p_1, p_2) = -g^{\mu\nu} + \frac{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu}{p_1 \cdot p_2} , \quad D^{\mu\nu}(p_1, p_2; \mathbf{b}) = d^{\mu\nu}(p_1, p_2) - 2 \frac{b^\mu b^\nu}{\mathbf{b}^2} ,$$

$$C_{ga}(z; \alpha_S) = \delta_{ga} \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n C_{ga}^{(n)}(z) , \quad G_{ga}(z; \alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n G_{ga}^{(n)}(z) .$$

- Unlike $q\bar{q}$ annih. $[H^F C_1 C_2]$ does depend on the azimuthal angle $\phi(\mathbf{b})$, this leads to azimuthal correlations with respect to the azimuthal angle $\phi(q_T)$ (consistent with [Mulders, Rodrigues ('00)], [Henneman et al. ('02)]).
- Small- q_T cross section expressed in terms of $\phi(q_T)$ -independent plus $\cos(2\phi(q_T))$, $\sin(2\phi(q_T))$, $\cos(4\phi(q_T))$ and $\sin(4\phi(q_T))$ dependent contributions.

q_T resummation: gluon fusion processes

In processes initiated at Born level by the gluon fusion channel ($gg \rightarrow F$), collinear radiation from gluons leads to spin and azimuthal correlations [Catani, Grazzini ('11)].

$$\begin{aligned} [H^F C_1 C_2]_{gg; a_1 a_2} &= H_{g;\mu_1\nu_1,\mu_2\nu_2}^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) \\ &\times C_{ga_1}^{\mu_1\nu_1}(z_1; p_1, p_2, \mathbf{b}; \alpha_S(b_0^2/b^2)) C_{ga_2}^{\mu_2\nu_2}(z_2; p_1, p_2, \mathbf{b}; \alpha_S(b_0^2/b^2)). \end{aligned}$$

where $H_g^{F\mu_1\nu_1,\mu_2\nu_2}(\alpha_S) = \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n H_g^{F(n)\mu_1\nu_1,\mu_2\nu_2}$,

$$C_{ga}^{\mu\nu}(z; p_1, p_2, \mathbf{b}; \alpha_S) = d^{\mu\nu}(p_1, p_2) C_{ga}(z; \alpha_S) + D^{\mu\nu}(p_1, p_2; \mathbf{b}) G_{ga}(z; \alpha_S) ,$$

$$d^{\mu\nu}(p_1, p_2) = -g^{\mu\nu} + \frac{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu}{p_1 \cdot p_2} , \quad D^{\mu\nu}(p_1, p_2; \mathbf{b}) = d^{\mu\nu}(p_1, p_2) - 2 \frac{b^\mu b^\nu}{\mathbf{b}^2} ,$$

$$C_{ga}(z; \alpha_S) = \delta_{ga} \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n C_{ga}^{(n)}(z) , \quad G_{ga}(z; \alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n G_{ga}^{(n)}(z) .$$

- Unlike $q\bar{q}$ annih. $[H^F C_1 C_2]$ does depend on the azimuthal angle $\phi(\mathbf{b})$, this leads to azimuthal correlations with respect to the azimuthal angle $\phi(q_T)$ (consistent with [Mulders, Rodrigues ('00)], [Henneman et al. ('02)]).
- Small- q_T cross section expressed in terms of $\phi(q_T)$ -independent plus $\cos(2\phi(q_T))$, $\sin(2\phi(q_T))$, $\cos(4\phi(q_T))$ and $\sin(4\phi(q_T))$ dependent contributions.

\mathbf{q}_T resummation: gluon fusion processes

In processes initiated at Born level by the gluon fusion channel ($gg \rightarrow F$), collinear radiation from gluons leads to spin and azimuthal correlations [Catani, Grazzini ('11)].

$$\begin{aligned} [H^F C_1 C_2]_{gg; a_1 a_2} &= H_{g;\mu_1\nu_1,\mu_2\nu_2}^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) \\ &\times C_{ga_1}^{\mu_1\nu_1}(z_1; p_1, p_2, \mathbf{b}; \alpha_S(b_0^2/b^2)) C_{ga_2}^{\mu_2\nu_2}(z_2; p_1, p_2, \mathbf{b}; \alpha_S(b_0^2/b^2)). \end{aligned}$$

where $H_g^{F\mu_1\nu_1,\mu_2\nu_2}(\alpha_S) = \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n H_g^{F(n)\mu_1\nu_1,\mu_2\nu_2}$,

$$C_{ga}^{\mu\nu}(z; p_1, p_2, \mathbf{b}; \alpha_S) = d^{\mu\nu}(p_1, p_2) C_{ga}(z; \alpha_S) + D^{\mu\nu}(p_1, p_2; \mathbf{b}) G_{ga}(z; \alpha_S) ,$$

$$d^{\mu\nu}(p_1, p_2) = -g^{\mu\nu} + \frac{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu}{p_1 \cdot p_2} , \quad D^{\mu\nu}(p_1, p_2; \mathbf{b}) = d^{\mu\nu}(p_1, p_2) - 2 \frac{b^\mu b^\nu}{\mathbf{b}^2} ,$$

$$C_{ga}(z; \alpha_S) = \delta_{ga} \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n C_{ga}^{(n)}(z) , \quad G_{ga}(z; \alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n G_{ga}^{(n)}(z) .$$

- Unlike $q\bar{q}$ annih. $[H^F C_1 C_2]$ does depend on the azimuthal angle $\phi(\mathbf{b})$, this leads to azimuthal correlations with respect to the azimuthal angle $\phi(\mathbf{q}_T)$ (consistent with [Mulders, Rodrigues ('00)], [Henneman et al. ('02)]).
- Small- q_T cross section expressed in terms of $\phi(\mathbf{q}_T)$ -independent plus $\cos(2\phi(\mathbf{q}_T))$, $\sin(2\phi(\mathbf{q}_T))$, $\cos(4\phi(\mathbf{q}_T))$ and $\sin(4\phi(\mathbf{q}_T))$ dependent contributions.

Universality in q_T resummation

The resummation formula is invariant under the *resummation scheme* transformations [Catani, de Florian, Grazzini ('01)] (for $h_c(\alpha_S) = 1 + \sum_{n=1}^{\infty} \alpha_S^n h_c^{(n)}$):

$$\begin{aligned} H_c^F(\alpha_S) &\rightarrow H_c^F(\alpha_S) [h_c(\alpha_S)]^{-1}, \\ B_c(\alpha_S) &\rightarrow B_c(\alpha_S) - \beta(\alpha_S) \frac{d \ln h_c(\alpha_S)}{d \ln \alpha_S}, \\ C_{cb}(z, \alpha_S) &\rightarrow C_{cb}(z, \alpha_S) [h_c(\alpha_S)]^{1/2}. \end{aligned}$$

- This implies that H_c^F, S_c (B_c) and C_{cb} not unambiguously computable separately.
- **Resummation scheme:** define H_c^F (or C_{ab}) for *single* processes (one for $q\bar{q} \rightarrow F$ one for $gg \rightarrow F$) and unambiguously determine the process-dependent H_c^F and the universal (process-independent) S_c and C_{ab} for any other process.
- **DY/H resummation scheme:** $H_q^{DY}(\alpha_S) \equiv 1$, $H_g^H(\alpha_S) \equiv 1$.
Hard resummation scheme: $C_{ab}^{(n)}(z)$ for $n \geq 1$ do not contain any hard-virtual term i.e. any $\delta(1-z)$ term (other plus distributions) i.e. .
- $H_c^F(\alpha_S) = 1$ (i.e. $h_c(\alpha_S) = H_c^F(\alpha_S)$) does not correspond to a resummation scheme (S_c^F and C_{ab}^F would be process dependent, [de Florian, Grazzini ('00)]).

Universality in q_T resummation

The resummation formula is invariant under the *resummation scheme* transformations [Catani, de Florian, Grazzini ('01)] (for $h_c(\alpha_S) = 1 + \sum_{n=1}^{\infty} \alpha_S^n h_c^{(n)}$):

$$\begin{aligned} H_c^F(\alpha_S) &\rightarrow H_c^F(\alpha_S) [h_c(\alpha_S)]^{-1}, \\ B_c(\alpha_S) &\rightarrow B_c(\alpha_S) - \beta(\alpha_S) \frac{d \ln h_c(\alpha_S)}{d \ln \alpha_S}, \\ C_{cb}(z, \alpha_S) &\rightarrow C_{cb}(z, \alpha_S) [h_c(\alpha_S)]^{1/2}. \end{aligned}$$

- This implies that H_c^F, S_c (B_c) and C_{cb} not unambiguously computable separately.
- **Resummation scheme:** define H_c^F (or C_{ab}) for *single* processes (one for $q\bar{q} \rightarrow F$ one for $gg \rightarrow F$) and unambiguously determine the process-dependent H_c^F and the universal (process-independent) S_c and C_{ab} for any other process.
- *DY/H resummation scheme:* $H_q^{DY}(\alpha_S) \equiv 1$, $H_g^H(\alpha_S) \equiv 1$.
Hard resummation scheme: $C_{ab}^{(n)}(z)$ for $n \geq 1$ do not contain any hard-virtual term i.e. any $\delta(1-z)$ term (other plus distributions) i.e. .
- $H_c^F(\alpha_S) = 1$ (i.e. $h_c(\alpha_S) = H_c^F(\alpha_S)$) does not correspond to a resummation scheme (S_c^F and C_{ab}^F would be process dependent, [de Florian, Grazzini ('00)]).

Universality in q_T resummation

The resummation formula is invariant under the *resummation scheme* transformations [Catani, de Florian, Grazzini ('01)] (for $h_c(\alpha_S) = 1 + \sum_{n=1}^{\infty} \alpha_S^n h_c^{(n)}$):

$$\begin{aligned} H_c^F(\alpha_S) &\rightarrow H_c^F(\alpha_S) [h_c(\alpha_S)]^{-1}, \\ B_c(\alpha_S) &\rightarrow B_c(\alpha_S) - \beta(\alpha_S) \frac{d \ln h_c(\alpha_S)}{d \ln \alpha_S}, \\ C_{cb}(z, \alpha_S) &\rightarrow C_{cb}(z, \alpha_S) [h_c(\alpha_S)]^{1/2}. \end{aligned}$$

- This implies that H_c^F, S_c (B_c) and C_{cb} not unambiguously computable separately.
- **Resummation scheme:** define H_c^F (or C_{ab}) for *single* processes (one for $q\bar{q} \rightarrow F$ one for $gg \rightarrow F$) and unambiguously determine the process-dependent H_c^F and the universal (process-independent) S_c and C_{ab} for any other process.
- **DY/H resummation scheme:** $H_q^{DY}(\alpha_S) \equiv 1$, $H_g^H(\alpha_S) \equiv 1$.
Hard resummation scheme: $C_{ab}^{(n)}(z)$ for $n \geq 1$ do not contain any hard-virtual term i.e. any $\delta(1-z)$ term (other plus distributions) i.e. .
- $H_c^F(\alpha_S) = 1$ (i.e. $h_c(\alpha_S) = H_c^F(\alpha_S)$) does not correspond to a resummation scheme (S_c^F and C_{ab}^F would be process dependent, [de Florian, Grazzini ('00)]).

Hard-collinear coefficients at NNLO

- Resummation coefficients in Sudakov form factor known since some time up to $\mathcal{O}(\alpha_S^2)$ ($A_c^{(1,2)}$, $B_c^{(1,2)}$), $A_c^{(3)}$ derived more recently [Becher,Neubert('11)]
- Explicit NNLO *analytic* calculations of the q_T cross section (at small- q_T):
 - (i) SM Higgs boson production [Catani,Grazzini('07,'12)] and
 - (ii) DY process [Catani,Cieri,de Florian,G.F.,Grazzini('09,'12)].
- These calculations provide complete knowledge of the process-independent *collinear* coeff. $C_{ca}(z, \alpha_S)$ up to $\mathcal{O}(\alpha_S^2)$ ($G_{ga}(z, \alpha_S)$ up to $\mathcal{O}(\alpha_S)$), and of the *hard-virtual* factor $H_c^F(\alpha_S)$ up to $\mathcal{O}(\alpha_S^2)$ for DY/H processes. In the *hard scheme*:

$$C_{qq}^{(1)}(z) = \frac{C_F}{2}(1-z), \quad C_{gq}^{(1)}(z) = \frac{C_F}{2}z, \quad C_{qg}^{(1)}(z) = \frac{z}{2}(1-z),$$

$$C_{gg}^{(1)}(z) = C_{q\bar{q}}^{(1)}(z) = C_{qq'}^{(1)}(z) = C_{q\bar{q}'}^{(1)}(z) = 0, \quad G_{ga}^{(1)}(z) = C_a \frac{1-z}{z} \quad (a = q, g).$$

$$H_q^{DY(1)} = C_F \left(\frac{\pi^2}{2} - 4 \right), \quad H_g^{H(1)} = C_A \pi^2 / 2 + \frac{11}{2}.$$

Analogous (bit longer) expressions for : $C_{qq}^{(2)}(z)$, $C_{qg}^{(2)}(z)$, $C_{gg}^{(2)}(z)$, $C_{gq}^{(2)}(z)$, $H_q^{DY(2)}$, $H_g^{H(2)}$.

- Explicit independent computation of the hard-collinear coefficients in a TMD factorization approach in full agreement [Gehrmann,Lubbert,Yang('12,'14)].

Hard-collinear coefficients at NNLO

- Resummation coefficients in Sudakov form factor known since some time up to $\mathcal{O}(\alpha_S^2)$ ($A_c^{(1,2)}$, $B_c^{(1,2)}$), $A_c^{(3)}$ derived more recently [Becher,Neubert('11)]
- Explicit NNLO *analytic* calculations of the q_T cross section (at small- q_T):
 - (i) SM Higgs boson production [Catani,Grazzini('07,'12)] and
 - (ii) DY process [Catani,Cieri,de Florian,G.F.,Grazzini('09,'12)].
- These calculations provide complete knowledge of the process-independent *collinear* coeff. $C_{ca}(z, \alpha_S)$ up to $\mathcal{O}(\alpha_S^2)$ ($G_{ga}(z, \alpha_S)$ up to $\mathcal{O}(\alpha_S)$), and of the *hard-virtual* factor $H_c^F(\alpha_S)$ up to $\mathcal{O}(\alpha_S^2)$ for DY/H processes. In the *hard scheme*:

$$C_{qq}^{(1)}(z) = \frac{C_F}{2}(1-z), \quad C_{gq}^{(1)}(z) = \frac{C_F}{2}z, \quad C_{qg}^{(1)}(z) = \frac{z}{2}(1-z),$$

$$C_{gg}^{(1)}(z) = C_{q\bar{q}}^{(1)}(z) = C_{qq'}^{(1)}(z) = C_{q\bar{q}'}^{(1)}(z) = 0, \quad G_{ga}^{(1)}(z) = C_a \frac{1-z}{z} \quad (a = q, g).$$

$$H_q^{DY(1)} = C_F \left(\frac{\pi^2}{2} - 4 \right), \quad H_g^{H(1)} = C_A \pi^2 / 2 + \frac{11}{2}.$$

Analogous (bit longer) expressions for : $C_{qq}^{(2)}(z)$, $C_{qg}^{(2)}(z)$, $C_{gg}^{(2)}(z)$, $C_{gq}^{(2)}(z)$, $H_q^{DY(2)}$, $H_g^{H(2)}$.

- Explicit independent computation of the hard-collinear coefficients in a TMD factorization approach in full agreement [Gehrmann,Lubbert,Yang('12,'14)].

Hard-collinear coefficients at NNLO

- Resummation coefficients in Sudakov form factor known since some time up to $\mathcal{O}(\alpha_S^2)$ ($A_c^{(1,2)}$, $B_c^{(1,2)}$), $A_c^{(3)}$ derived more recently [Becher, Neubert ('11)]
- Explicit NNLO *analytic* calculations of the q_T cross section (at small- q_T):
 - (i) SM Higgs boson production [Catani, Grazzini ('07, '12)] and
 - (ii) DY process [Catani, Cieri, de Florian, G.F., Grazzini ('09, '12)].
- These calculations provide complete knowledge of the process-independent *collinear* coeff. $C_{ca}(z, \alpha_S)$ up to $\mathcal{O}(\alpha_S^2)$ ($G_{ga}(z, \alpha_S)$ up to $\mathcal{O}(\alpha_S)$), and of the *hard-virtual* factor $H_c^F(\alpha_S)$ up to $\mathcal{O}(\alpha_S^2)$ for DY/H processes. In the *hard scheme*:

$$C_{qq}^{(1)}(z) = \frac{C_F}{2}(1-z), \quad C_{gq}^{(1)}(z) = \frac{C_F}{2}z, \quad C_{qg}^{(1)}(z) = \frac{z}{2}(1-z),$$

$$C_{gg}^{(1)}(z) = C_{q\bar{q}}^{(1)}(z) = C_{qq'}^{(1)}(z) = C_{q\bar{q}'}^{(1)}(z) = 0, \quad G_{ga}^{(1)}(z) = C_a \frac{1-z}{z} \quad (a = q, g).$$

$$H_q^{DY(1)} = C_F \left(\frac{\pi^2}{2} - 4 \right), \quad H_g^{H(1)} = C_A \pi^2 / 2 + \frac{11}{2}.$$

Analogous (bit longer) expressions for : $C_{qq}^{(2)}(z)$, $C_{qg}^{(2)}(z)$, $C_{gg}^{(2)}(z)$, $C_{gq}^{(2)}(z)$, $H_q^{DY(2)}$, $H_g^{H(2)}$.

- Explicit independent computation of the hard-collinear coefficients in a TMD factorization approach in full agreement [Gehrmann, Lubbert, Yang ('12, '14)].

Universality of hard factors at all orders

- *Process-dependence* is fully encoded in the hard-virtual factor $H_c^F(\alpha_S)$.
- However $H_c^F(\alpha_S)$ has an *all-order universal* structure: it can be directly related to the virtual amplitude of the corresponding process $c(\hat{p}_1) + \bar{c}(\hat{p}_2) \rightarrow F(\{q_i\})$.

$$\mathcal{M}_{c\bar{c}\rightarrow F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = \alpha_S^k \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{2\pi} \right)^n \mathcal{M}_{c\bar{c}\rightarrow F}^{(n)}(\hat{p}_1, \hat{p}_2; \{q_i\}), \quad \text{renormalized virtual amplitude (UV finite but IR divergent).}$$

$$\tilde{I}_c(\epsilon, M^2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{2\pi} \right)^n \tilde{I}_c^{(n)}(\epsilon), \quad \begin{aligned} &\text{IR subtraction } \textit{universal} \text{ operators} \\ &\text{(contain IR } \epsilon\text{-poles and IR finite terms)} \end{aligned}$$

$$\widetilde{\mathcal{M}}_{c\bar{c}\rightarrow F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = [1 - \tilde{I}_c(\epsilon, M^2)] \mathcal{M}_{c\bar{c}\rightarrow F}(\hat{p}_1, \hat{p}_2; \{q_i\}), \quad \text{hard-virtual subtracted amplitude (IR finite).}$$

Hard factor is directly related to the all-loop virtual amplitude:

$$\alpha_S^{2k}(M^2) H_q^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\rightarrow F}(x_1 p_1, x_2 p_2; \{q_i\})|^2}{|\mathcal{M}_{q\bar{q}\rightarrow F}^{(0)}(x_1 p_1, x_2 p_2; \{q_i\})|^2},$$

(α_S^k is the overall α_S power (e.g. $k = 0$ for DY, $k = 1$ for $gg \rightarrow H$)).

Universality of hard factors at all orders

- *Process-dependence* is fully encoded in the hard-virtual factor $H_c^F(\alpha_S)$.
- However $H_c^F(\alpha_S)$ has an *all-order universal* structure: it can be directly related to the virtual amplitude of the corresponding process $c(\hat{p}_1) + \bar{c}(\hat{p}_2) \rightarrow F(\{q_i\})$.

$$\mathcal{M}_{c\bar{c}\rightarrow F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = \alpha_S^k \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{2\pi} \right)^n \mathcal{M}_{c\bar{c}\rightarrow F}^{(n)}(\hat{p}_1, \hat{p}_2; \{q_i\}), \quad \text{renormalized virtual amplitude (UV finite but IR divergent).}$$

$$\tilde{I}_c(\epsilon, M^2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{2\pi} \right)^n \tilde{I}_c^{(n)}(\epsilon), \quad \begin{aligned} &\text{IR subtraction } \textit{universal} \text{ operators} \\ &\text{(contain IR } \epsilon\text{-poles and IR finite terms)} \end{aligned}$$

$$\widetilde{\mathcal{M}}_{c\bar{c}\rightarrow F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = [1 - \tilde{I}_c(\epsilon, M^2)] \mathcal{M}_{c\bar{c}\rightarrow F}(\hat{p}_1, \hat{p}_2; \{q_i\}), \quad \text{hard-virtual subtracted amplitude (IR finite).}$$

Hard factor is directly related to the all-loop virtual amplitude:

$$\alpha_S^{2k}(M^2) H_q^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\rightarrow F}(x_1 p_1, x_2 p_2; \{q_i\})|^2}{|\mathcal{M}_{q\bar{q}\rightarrow F}^{(0)}(x_1 p_1, x_2 p_2; \{q_i\})|^2},$$

(α_S^k is the overall α_S power (e.g. $k = 0$ for DY, $k = 1$ for $gg \rightarrow H$)).

Universality of hard factors at all orders

- *Process-dependence* is fully encoded in the hard-virtual factor $H_c^F(\alpha_S)$.
- However $H_c^F(\alpha_S)$ has an *all-order universal* structure: it can be directly related to the virtual amplitude of the corresponding process $c(\hat{p}_1) + \bar{c}(\hat{p}_2) \rightarrow F(\{q_i\})$.

$$\mathcal{M}_{c\bar{c}\rightarrow F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = \alpha_S^k \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{2\pi} \right)^n \mathcal{M}_{c\bar{c}\rightarrow F}^{(n)}(\hat{p}_1, \hat{p}_2; \{q_i\}), \quad \text{renormalized virtual amplitude (UV finite but IR divergent).}$$

$$\tilde{I}_c(\epsilon, M^2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{2\pi} \right)^n \tilde{I}_c^{(n)}(\epsilon), \quad \begin{array}{l} \text{IR subtraction } \textit{universal} \text{ operators} \\ (\text{contain IR } \epsilon\text{-poles and IR finite terms}) \end{array}$$

$$\widetilde{\mathcal{M}}_{c\bar{c}\rightarrow F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = [1 - \tilde{I}_c(\epsilon, M^2)] \mathcal{M}_{c\bar{c}\rightarrow F}(\hat{p}_1, \hat{p}_2; \{q_i\}), \quad \text{hard-virtual subtracted amplitude (IR finite).}$$

Hard factor is directly related to the all-loop virtual amplitude:

$$\alpha_S^{2k}(M^2) H_q^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\rightarrow F}(x_1 p_1, x_2 p_2; \{q_i\})|^2}{|\mathcal{M}_{q\bar{q}\rightarrow F}^{(0)}(x_1 p_1, x_2 p_2; \{q_i\})|^2},$$

(α_S^k is the overall α_S power (e.g. $k = 0$ for DY, $k = 1$ for $gg \rightarrow H$)).

Universality of hard factors at all orders

- *Process-dependence* is fully encoded in the hard-virtual factor $H_c^F(\alpha_S)$.
- However $H_c^F(\alpha_S)$ has an *all-order universal* structure: it can be directly related to the virtual amplitude of the corresponding process $c(\hat{p}_1) + \bar{c}(\hat{p}_2) \rightarrow F(\{q_i\})$.

$$\mathcal{M}_{c\bar{c}\rightarrow F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = \alpha_S^k \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{2\pi} \right)^n \mathcal{M}_{c\bar{c}\rightarrow F}^{(n)}(\hat{p}_1, \hat{p}_2; \{q_i\}), \quad \text{renormalized virtual amplitude (UV finite but IR divergent).}$$

$$\tilde{I}_c(\epsilon, M^2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{2\pi} \right)^n \tilde{I}_c^{(n)}(\epsilon), \quad \begin{aligned} &\text{IR subtraction } \textit{universal} \text{ operators} \\ &\text{(contain IR } \epsilon\text{-poles and IR finite terms)} \end{aligned}$$

$$\widetilde{\mathcal{M}}_{c\bar{c}\rightarrow F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = [1 - \tilde{I}_c(\epsilon, M^2)] \mathcal{M}_{c\bar{c}\rightarrow F}(\hat{p}_1, \hat{p}_2; \{q_i\}), \quad \text{hard-virtual subtracted amplitude (IR finite).}$$

Hard factor is directly related to the all-loop virtual amplitude:

$$\alpha_S^{2k}(M^2) H_q^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\rightarrow F}(x_1 p_1, x_2 p_2; \{q_i\})|^2}{|\mathcal{M}_{q\bar{q}\rightarrow F}^{(0)}(x_1 p_1, x_2 p_2; \{q_i\})|^2},$$

(α_S^k is the overall α_S power (e.g. $k = 0$ for DY, $k = 1$ for $gg \rightarrow H$)).

Universality of hard factors at all orders

- *Process-dependence* is fully encoded in the hard-virtual factor $H_c^F(\alpha_S)$.
- However $H_c^F(\alpha_S)$ has an *all-order universal* structure: it can be directly related to the virtual amplitude of the corresponding process $c(\hat{p}_1) + \bar{c}(\hat{p}_2) \rightarrow F(\{q_i\})$.

$$\mathcal{M}_{c\bar{c}\rightarrow F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = \alpha_S^k \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{2\pi} \right)^n \mathcal{M}_{c\bar{c}\rightarrow F}^{(n)}(\hat{p}_1, \hat{p}_2; \{q_i\}), \quad \text{renormalized virtual amplitude (UV finite but IR divergent).}$$

$$\tilde{I}_c(\epsilon, M^2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{2\pi} \right)^n \tilde{I}_c^{(n)}(\epsilon), \quad \begin{aligned} &\text{IR subtraction } \textit{universal} \text{ operators} \\ &\text{(contain IR } \epsilon\text{-poles and IR finite terms)} \end{aligned}$$

$$\widetilde{\mathcal{M}}_{c\bar{c}\rightarrow F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = [1 - \tilde{I}_c(\epsilon, M^2)] \mathcal{M}_{c\bar{c}\rightarrow F}(\hat{p}_1, \hat{p}_2; \{q_i\}), \quad \text{hard-virtual subtracted amplitude (IR finite).}$$

Hard factor is directly related to the all-loop virtual amplitude:

$$\alpha_S^{2k}(M^2) H_q^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\rightarrow F}(x_1 p_1, x_2 p_2; \{q_i\})|^2}{|\mathcal{M}_{q\bar{q}\rightarrow F}^{(0)}(x_1 p_1, x_2 p_2; \{q_i\})|^2},$$

(α_S^k is the overall α_S power (e.g. $k = 0$ for DY, $k = 1$ for $gg \rightarrow H$)).

Hard factors at NNLO

- The previous all-order factorization formula was explicitly evaluated up to NNLO: we know the explicit expression of the *universal* subtraction operators up to two-loops $\tilde{I}_c^{(1)}(\epsilon)$, $\tilde{I}_c^{(2)}(\epsilon)$.
- We can straightforward apply the factorization formula to determine the NNLO hard-virtual factors from the knowledge of the two-loop amplitudes.
- E.g. diphoton production: we rederived the result for $H_q^{\gamma\gamma(1)}$ [Balazs et al. ('98)] and (using the two-loop amplitudes [Anastasiou et al. ('02)]) we obtained the $H_q^{\gamma\gamma(2)}$ [Catani, Cieri, de Florian, GF, Grazzini ('12)]

$$H_q^{\gamma\gamma(1)} = \frac{C_F}{2} \left\{ (\pi^2 - 7) + \frac{(1 - v)^2 + 1}{(1 - v)^2 + v^2} \left[\ln^2(1 - v) + v(v + 2) \ln(1 - v) + (v^2 + 1) \ln^2 v + (1 - v)(3 - v) \ln v \right] \right\} .$$

$$\begin{aligned} H_q^{\gamma\gamma(2)} &= \frac{1}{4\mathcal{A}_{LO}} \left[\mathcal{F}_{init, q\bar{q}\gamma\gamma;s}^{0\times 2} + \mathcal{F}_{init, q\bar{q}\gamma\gamma;s}^{1\times 1} \right] + 3\zeta_2 C_F H_q^{\gamma\gamma(1)} - \frac{45}{4} \zeta_4 C_F^2 + C_F N_f \left(-\frac{41}{162} - \frac{97}{72} \zeta_2 + \frac{17}{72} \zeta_3 \right) \\ &\quad + C_F C_A \left(\frac{607}{324} + \frac{1181}{144} \zeta_2 - \frac{187}{144} \zeta_3 - \frac{105}{32} \zeta_4 \right), \quad \text{where } v = -(p_q - p_\gamma)^2/M^2. \end{aligned}$$

- Analogous results were obtained for ZZ , $W\gamma$, $Z\gamma$ [Grazzini et al. ('14)], [Cascioli et al. ('14)], [Gehrmann et al. ('14)] and $b\bar{b} \rightarrow H$ production [Harlander et al. ('14)].

Hard factors at NNLO

- The previous all-order factorization formula was explicitly evaluated up to NNLO: we know the explicit expression of the *universal* subtraction operators up to two-loops $\tilde{l}_c^{(1)}(\epsilon)$, $\tilde{l}_c^{(2)}(\epsilon)$.
- We can straightforward apply the factorization formula to determine the NNLO hard-virtual factors from the knowledge of the two-loop amplitudes.
- E.g. diphoton production: we rederived the result for $H_q^{\gamma\gamma(1)}$ [Balazs et al. ('98)] and (using the two-loop amplitudes [Anastasiou et al. ('02)]) we obtained the $H_q^{\gamma\gamma(2)}$ [Catani, Cieri, de Florian, GF, Grazzini ('12)]

$$H_q^{\gamma\gamma(1)} = \frac{C_F}{2} \left\{ (\pi^2 - 7) + \frac{(1 - v)^2 + 1}{(1 - v)^2 + v^2} \left[\ln^2(1 - v) + v(v + 2) \ln(1 - v) + (v^2 + 1) \ln^2 v + (1 - v)(3 - v) \ln v \right] \right\}$$
$$H_q^{\gamma\gamma(2)} = \frac{1}{4\mathcal{A}_{LO}} \left[\mathcal{F}_{init, q\bar{q}\gamma\gamma; s}^{0 \times 2} + \mathcal{F}_{init, q\bar{q}\gamma\gamma; s}^{1 \times 1} \right] + 3\zeta_2 C_F H_q^{\gamma\gamma(1)} - \frac{45}{4} \zeta_4 C_F^2 + C_F N_f \left(-\frac{41}{162} - \frac{97}{72} \zeta_2 + \frac{17}{72} \zeta_3 \right) + C_F C_A \left(\frac{607}{324} + \frac{1181}{144} \zeta_2 - \frac{187}{144} \zeta_3 - \frac{105}{32} \zeta_4 \right), \quad \text{where } v = -(p_q - p_\gamma)^2/M^2.$$

- Analogous results were obtained for ZZ , $W\gamma$, $Z\gamma$ [Grazzini et al. ('14)], [Cascioli et al. ('14)], [Gehrmann et al. ('14)] and $b\bar{b} \rightarrow H$ production [Harlander et al. ('14)].

The q_T resummation formalism

Distinctive features of the formalism [Catani et al ('01)], [Bozzi et al.('03,'06)]:

- Resummed effects exponentiated in a universal of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp \left\{ - \int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2} \gamma_N(\alpha_S(q^2)) \right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)], [Catani et al.('96)].
- Introduction of resummation scale $Q \sim M$: variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(M^2 b^2) \rightarrow \ln(Q^2 b^2) + \ln(M^2/Q^2)$$

- Perturbative unitarity constraint:

$$\ln(Q^2 b^2) \rightarrow \tilde{L} \equiv \ln(Q^2 b^2 + 1)$$

- avoids unjustified higher-order contributions in the small- b region.
- recover exactly the total cross-section (upon integration on q_T)

The q_T resummation formalism

Distinctive features of the formalism [Catani et al ('01)], [Bozzi et al.('03,'06)]:

- Resummed effects exponentiated in a **universal** of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp \left\{ - \int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2} \gamma_N(\alpha_S(q^2)) \right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)], [Catani et al.('96)].
- Introduction of **resummation scale $Q \sim M$** : variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(M^2 b^2) \rightarrow \ln(Q^2 b^2) + \ln(M^2/Q^2)$$

- Perturbative **unitarity constraint**:

$$\ln(Q^2 b^2) \rightarrow \tilde{L} \equiv \ln(Q^2 b^2 + 1)$$

- avoids unjustified higher-order contributions in the small- b region.
- recover *exactly* the total cross-section (upon integration on q_T)

The q_T resummation formalism

Distinctive features of the formalism [Catani et al ('01)], [Bozzi et al.('03,'06)]:

- Resummed effects exponentiated in a **universal** of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp \left\{ - \int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2} \gamma_N(\alpha_S(q^2)) \right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)], [Catani et al.('96)].
- Introduction of **resummation scale $Q \sim M$** : variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(M^2 b^2) \rightarrow \ln(Q^2 b^2) + \ln(M^2/Q^2)$$

- Perturbative **unitarity constraint**:

$$\ln(Q^2 b^2) \rightarrow \tilde{L} \equiv \ln(Q^2 b^2 + 1)$$

- avoids unjustified higher-order contributions in the small- b region.
- recover *exactly* the total cross-section (upon integration on q_T)

The q_T resummation formalism

Distinctive features of the formalism [Catani et al ('01)], [Bozzi et al.('03,'06)]:

- Resummed effects exponentiated in a **universal** of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp \left\{ - \int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2} \gamma_N(\alpha_S(q^2)) \right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)], [Catani et al.('96)].
- Introduction of **resummation scale $Q \sim M$** : variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(M^2 b^2) \rightarrow \ln(Q^2 b^2) + \ln(M^2/Q^2)$$

- Perturbative **unitarity constraint**:

$$\ln(Q^2 b^2) \rightarrow \tilde{L} \equiv \ln(Q^2 b^2 + 1)$$

- avoids unjustified higher-order contributions in the small- b region.
- recover *exactly* the total cross-section (upon integration on q_T)

The q_T resummation formalism

Distinctive features of the formalism [Catani et al ('01)], [Bozzi et al.('03,'06)]:

- Resummed effects exponentiated in a **universal** of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp \left\{ - \int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2} \gamma_N(\alpha_S(q^2)) \right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)], [Catani et al.('96)].
- Introduction of **resummation scale $Q \sim M$** : variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(M^2 b^2) \rightarrow \ln(Q^2 b^2) + \ln(M^2/Q^2)$$

- Perturbative **unitarity constraint**:

$$\ln(Q^2 b^2) \rightarrow \tilde{L} \equiv \ln(Q^2 b^2 + 1)$$

- avoids unjustified higher-order contributions in the small- b region.
- recover *exactly* the total cross-section (upon integration on q_T)

The q_T resummation formalism

Distinctive features of the formalism [Catani et al ('01)], [Bozzi et al.('03,'06)]:

- Resummed effects exponentiated in a **universal** of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp \left\{ - \int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2} \gamma_N(\alpha_S(q^2)) \right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)], [Catani et al.('96)].
- Introduction of **resummation scale $Q \sim M$** : variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(M^2 b^2) \rightarrow \ln(Q^2 b^2) + \ln(M^2/Q^2)$$

- Perturbative **unitarity constraint**:

$$\ln(Q^2 b^2) \rightarrow \tilde{L} \equiv \ln(Q^2 b^2 + 1)$$

- avoids unjustified higher-order contributions in the small- b region.
- recover *exactly* the total cross-section (upon integration on q_T)

The q_T resummation formalism

Distinctive features of the formalism [Catani et al ('01)], [Bozzi et al. ('03, '06)]:

- Resummed effects exponentiated in a **universal** of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp \left\{ - \int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2} \gamma_N(\alpha_S(q^2)) \right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al. ('00)], [Catani et al. ('96)].
- Introduction of **resummation scale $Q \sim M$** : variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(M^2 b^2) \rightarrow \ln(Q^2 b^2) + \ln(M^2/Q^2)$$

- Perturbative **unitarity constraint**:

$$\ln(Q^2 b^2) \rightarrow \tilde{L} \equiv \ln(Q^2 b^2 + 1) \Rightarrow \exp \{ \alpha_S^n \tilde{L}^k \} \Big|_{b=0} = 1$$

- avoids unjustified higher-order contributions in the small- b region.
- recover *exactly* the total cross-section (upon integration on q_T)

The q_T resummation formalism

Distinctive features of the formalism [Catani et al ('01)], [Bozzi et al. ('03, '06)]:

- Resummed effects exponentiated in a **universal** of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp \left\{ - \int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2} \gamma_N(\alpha_S(q^2)) \right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al. ('00)], [Catani et al. ('96)].
- Introduction of **resummation scale $Q \sim M$** : variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(M^2 b^2) \rightarrow \ln(Q^2 b^2) + \ln(M^2/Q^2)$$

- Perturbative **unitarity constraint**:

$$\ln(Q^2 b^2) \rightarrow \tilde{L} \equiv \ln(Q^2 b^2 + 1) \Rightarrow \exp \left\{ \alpha_S^n \tilde{L}^k \right\} \Big|_{b=0} = 1 \Rightarrow \int_0^\infty dq_T^2 \left(\frac{d\hat{\sigma}}{dq_T^2} \right) = \hat{\sigma}^{(tot)};$$

- avoids unjustified higher-order contributions in the small- b region.
- recover *exactly* the total cross-section (upon integration on q_T)

HqT/DYqT: q_T -resummation at NNLL

- We have performed the resummation up to **NNLL(NNLO)+NLO**. It means that our complete formula includes:
 - **NNLL** logarithmic contributions to all orders (i.e. $\mathcal{O}(\alpha_S^n L^{n-1})$ in the exponent);
 - **NNLO** corrections (i.e. $\mathcal{O}(\alpha_S^2)$) at small q_T ;
 - **NLO** corrections (i.e. $\mathcal{O}(\alpha_S^2)$) at large q_T ;
 - **NNLO** result (i.e. $\mathcal{O}(\alpha_S^2)$) for the total cross section (upon integration over q_T).
- NLO+PS generators (MC@NLO/POWHEG) reach NLL+LO accuracy (NLO for total cross section).
- The calculation of the resummed q_T spectrum are implemented in numerical codes **HqT** [Bozzi,Catani,de Florian,Grazzini('03,'06,'08)], [de Florian,G.F.,Grazzini,Tommasini('11)] and **DYqT** [Bozzi,Catani,de Florian,G.F.,Grazzini('08,'10)] (public versions of both codes are available).

HqT/DYqT: q_T -resummation at NNLL

- We have performed the resummation up to **NNLL(NNLO)+NLO**. It means that our complete formula includes:
 - **NNLL** logarithmic contributions to all orders (i.e. $\mathcal{O}(\alpha_S^n L^{n-1})$ in the exponent);
 - **NNLO** corrections (i.e. $\mathcal{O}(\alpha_S^2)$) at small q_T ;
 - **NLO** corrections (i.e. $\mathcal{O}(\alpha_S^2)$) at large q_T ;
 - **NNLO** result (i.e. $\mathcal{O}(\alpha_S^2)$) for the total cross section (upon integration over q_T).
- NLO+PS generators (MC@NLO/POWHEG) reach NLL+LO accuracy (NLO for total cross section).
- The calculation of the resummed q_T spectrum are implemented in numerical codes **HqT** [Bozzi,Catani,de Florian,Grazzini('03,'06,'08)], [de Florian,G.F.,Grazzini,Tommasini('11)] and **DYqT** [Bozzi,Catani,de Florian,G.F.,Grazzini('08,'10)] (public versions of both codes are available).

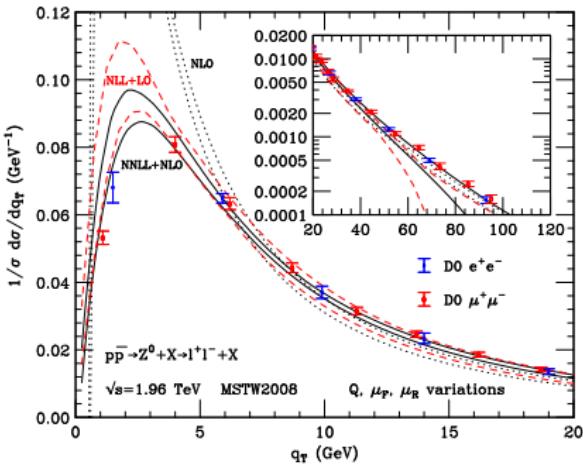
HqT/DYqT: q_T -resummation at NNLL

- We have performed the resummation up to **NNLL(NNLO)+NLO**. It means that our complete formula includes:
 - **NNLL** logarithmic contributions to all orders (i.e. $\mathcal{O}(\alpha_S^n L^{n-1})$ in the exponent);
 - **NNLO** corrections (i.e. $\mathcal{O}(\alpha_S^2)$) at small q_T ;
 - **NLO** corrections (i.e. $\mathcal{O}(\alpha_S^2)$) at large q_T ;
 - **NNLO** result (i.e. $\mathcal{O}(\alpha_S^2)$) for the total cross section (upon integration over q_T).
- NLO+PS generators (MC@NLO/POWHEG) reach NLL+LO accuracy (NLO for total cross section).
- The calculation of the resummed q_T spectrum are implemented in numerical codes **HqT** [Bozzi,Catani,de Florian,Grazzini('03,'06,'08)],
[deFlorian,G.F.,Grazzini,Tommasini('11)] and **DYqT** [Bozzi,Catani,de
Florian,G.F.,Grazzini('08,'10)] (public versions of both codes are available).

HqT/DYqT: q_T -resummation at NNLL

- We have performed the resummation up to **NNLL(NNLO)+NLO**. It means that our complete formula includes:
 - **NNLL** logarithmic contributions to all orders (i.e. $\mathcal{O}(\alpha_S^n L^{n-1})$ in the exponent);
 - **NNLO** corrections (i.e. $\mathcal{O}(\alpha_S^2)$) at small q_T ;
 - **NLO** corrections (i.e. $\mathcal{O}(\alpha_S^2)$) at large q_T ;
 - **NNLO** result (i.e. $\mathcal{O}(\alpha_S^2)$) for the total cross section (upon integration over q_T).
- NLO+PS generators (MC@NLO/POWHEG) reach NLL+LO accuracy (NLO for total cross section).
- The calculation of the resummed q_T spectrum are implemented in numerical codes **HqT** [Bozzi,Catani,de Florian,Grazzini('03,'06,'08)],
[de Florian,G.F.,Grazzini,Tommasini('11)] and **DYqT** [Bozzi,Catani,de Florian,G.F.,Grazzini('08,'10)] (public versions of both codes are available).

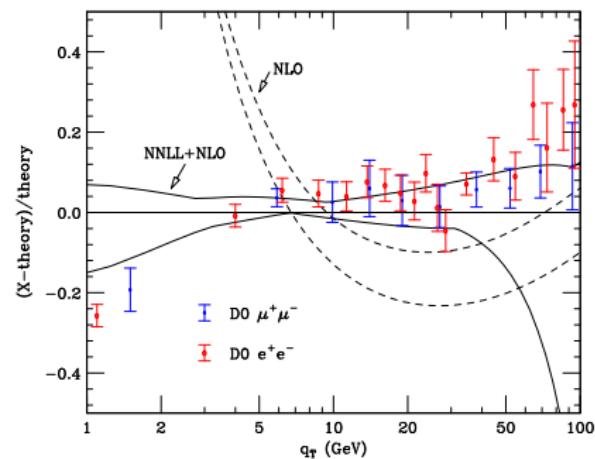
DY q_T results: q_T spectrum of Z boson at the Tevatron



D0 data for the Z q_T spectrum compared with perturbative results.

- Uncertainty bands obtained varying μ_R , μ_F , Q independently:
 $\frac{1}{2} \leq \{\mu_F/m_Z, \mu_R/m_Z, 2Q/m_Z, \mu_F/\mu_R, Q/\mu_R\} \leq 2$
- Significant reduction of scale dependence from NLL to NNLL for all q_T .
- Good convergence of resummed results: NNLL and NLL bands overlap (contrary to the fixed-order case).
- Good agreement between data and resummed predictions (without any model for non-perturbative effects).
The perturbative uncertainty of the NNLL results is comparable with the experimental errors.

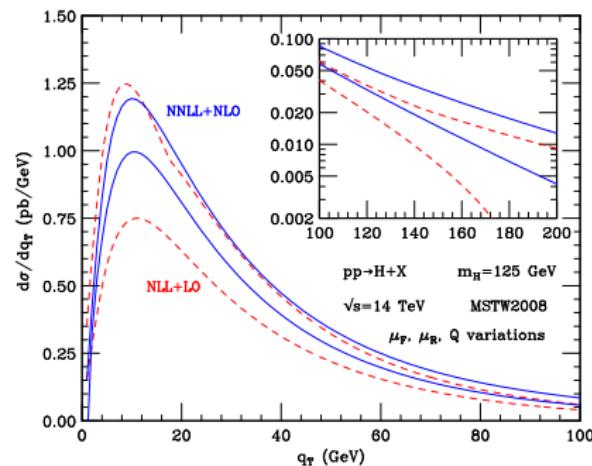
DY q_T results: q_T spectrum of Z boson at the Tevatron



D0 data for the Z q_T spectrum: Fractional difference with respect to the reference result: NNLL, $\mu_R = \mu_F = 2Q = m_Z$.

- NNLL scale dependence is $\pm 6\%$ at the peak, $\pm 5\%$ at $q_T = 10$ GeV and $\pm 12\%$ at $q_T = 50$ GeV. For $q_T \geq 60$ GeV the resummed result loses predictivity.
- At large values of q_T , the NLO and NNLL bands overlap.
At intermediate values of transverse momenta the scale variation bands do not overlap.
- The resummation improves the agreement of the NLO results with the data.
In the small- q_T region, the NLO result is theoretically unreliable and the NLO band deviates from the NNLL band.

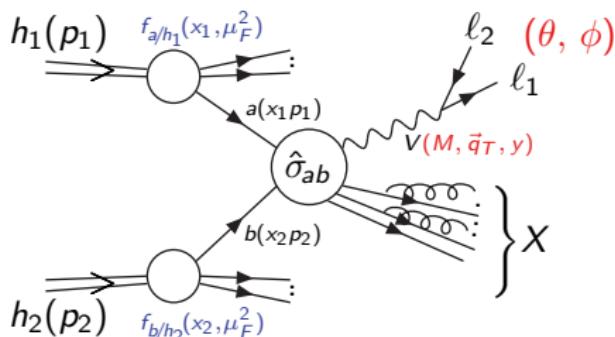
HqT results: q_T spectrum of H boson at the LHC $\sqrt{s} = 14 \text{ TeV}$



- Uncertainty bands obtained as before: $1/2 \leq \{\mu_F/m_Z, \mu_R/m_Z, 2Q/m_Z, \mu_F/\mu_R, Q/\mu_R\} \leq 2$
- Significant reduction of scale dependence from NLL+LO to NNLL+NLO for all q_T .
- Good convergence of resummed results: NNLL+NLO and NLL+LO bands overlap (contrary to the fixed-order case).

Higgs q_T spectrum for $m_H = 125$ GeV at LHC.

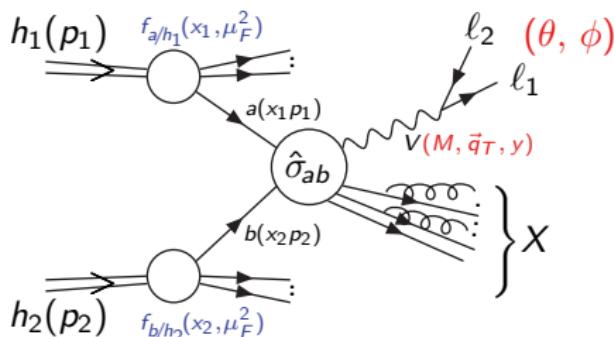
q_T -resummation with decay variables dependence



- Experiments have finite acceptance: important to provide exclusive theoretical predictions.
- Analytic resummation formalism inclusive over soft-gluon emission: not possible to apply selection cuts on final state partons.

- We have included the full dependence on the decay variables: possible to apply cuts on vector/Higgs boson and decay products.
- To construct the “finite” part we rely on the fully-differential NNLO result from the codes `HNNLO/DYNNNLO` [Catani,Cieri,de Florian,GF,Grazzini(’09)], [Catani,Grazzini(’09)].
- Calculation implemented in numerical codes (`HRES/DYRES`) which include spin correlations, finite-width effects and compute distributions in form of bin histograms.

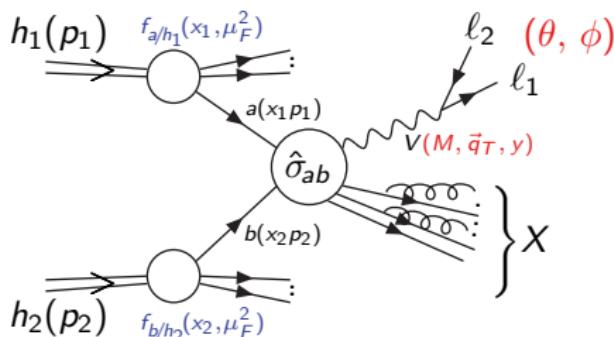
q_T -resummation with decay variables dependence



- Experiments have finite acceptance: important to provide exclusive theoretical predictions.
- Analytic resummation formalism inclusive over soft-gluon emission: not possible to apply selection cuts on final state partons.

- We have included the full dependence on the decay variables: possible to apply cuts on vector/Higgs boson and decay products.
- To construct the “finite” part we rely on the fully-differential NNLO result from the codes `HNNLO/DYNNNLO` [Catani,Cieri,de Florian,GF,Grazzini(’09)], [Catani,Grazzini(’09)].
- Calculation implemented in numerical codes (`HRES/DYRES`) which include spin correlations, finite-width effects and compute distributions in form of bin histograms.

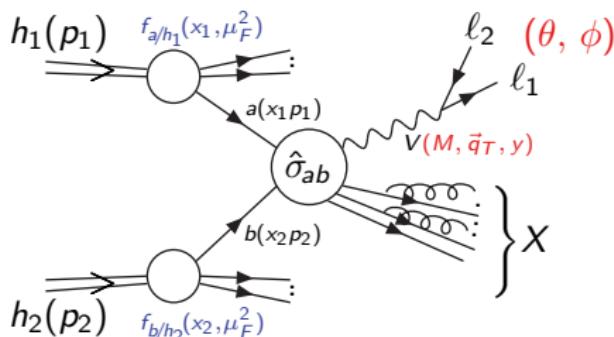
q_T -resummation with decay variables dependence



- Experiments have finite acceptance: important to provide exclusive theoretical predictions.
- Analytic resummation formalism inclusive over soft-gluon emission: not possible to apply selection cuts on final state partons.

- We have included the full dependence on the decay variables: possible to apply cuts on vector/Higgs boson and decay products.
- To construct the “finite” part we rely on the fully-differential NNLO result from the codes `HNNLO/DYNNNLO` [Catani,Cieri,de Florian,GF,Grazzini(’09)], [Catani,Grazzini(’09)].
- Calculation implemented in numerical codes (`HRES/DYRES`) which include spin correlations, finite-width effects and compute distributions in form of bin histograms.

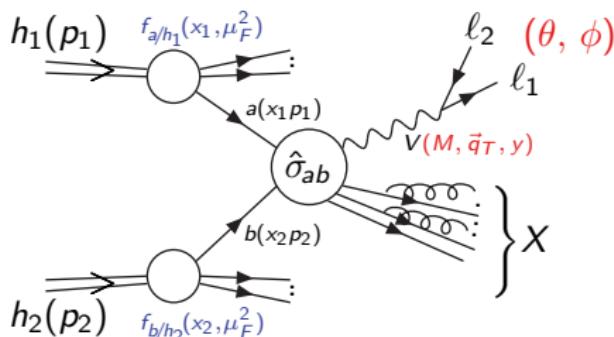
q_T -resummation with decay variables dependence



- Experiments have finite acceptance: important to provide exclusive theoretical predictions.
- Analytic resummation formalism inclusive over soft-gluon emission: not possible to apply selection cuts on final state partons.

- We have included the full dependence on the decay variables: possible to apply cuts on vector/Higgs boson and decay products.
- To construct the “finite” part we rely on the fully-differential NNLO result from the codes HNNLO/DYNNNLO [Catani,Cieri,de Florian,GF,Grazzini(’09)], [Catani,Grazzini(’09)].
- Calculation implemented in numerical codes (HRES/DYRES) which include spin correlations, finite-width effects and compute distributions in form of bin histograms.

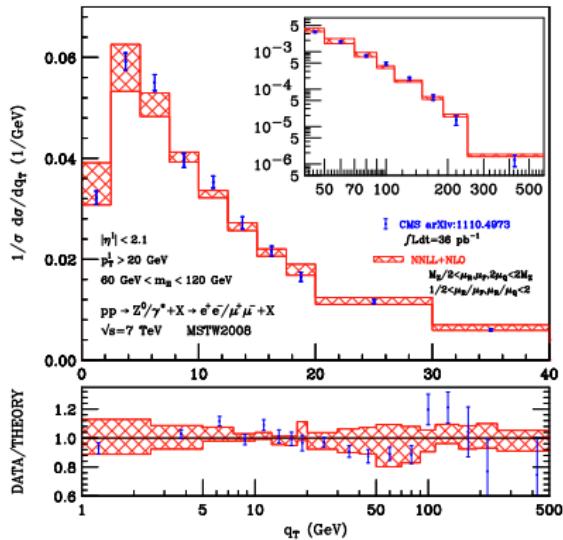
q_T -resummation with decay variables dependence



- Experiments have finite acceptance: important to provide exclusive theoretical predictions.
- Analytic resummation formalism inclusive over soft-gluon emission: not possible to apply selection cuts on final state partons.

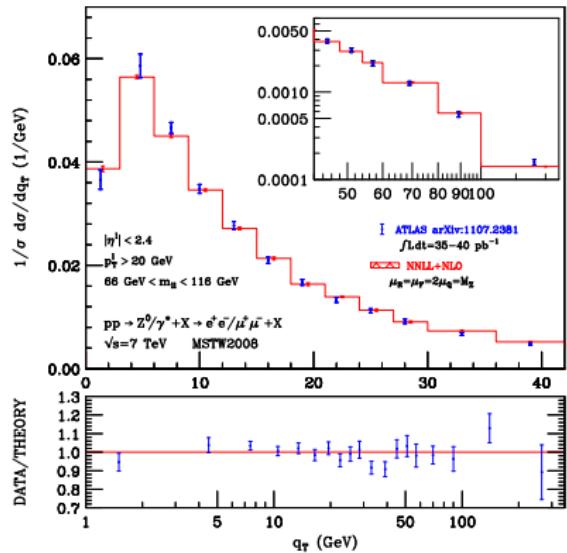
- We have included the full dependence on the decay variables: possible to apply cuts on vector/Higgs boson and decay products.
- To construct the “finite” part we rely on the fully-differential NNLO result from the codes HNNLO/DYNNNLO [Catani,Cieri,de Florian,GF,Grazzini(’09)], [Catani,Grazzini(’09)].
- Calculation implemented in numerical codes (HRES/DYRES) which include spin correlations, finite-width effects and compute distributions in form of bin histograms.

DYRES results: q_T spectrum of Z boson at the LHC

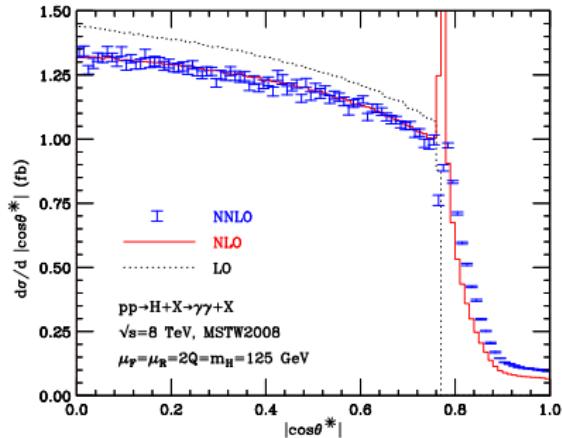


CMS data for the Z q_T spectrum compared with NNLL result. Scale variation:

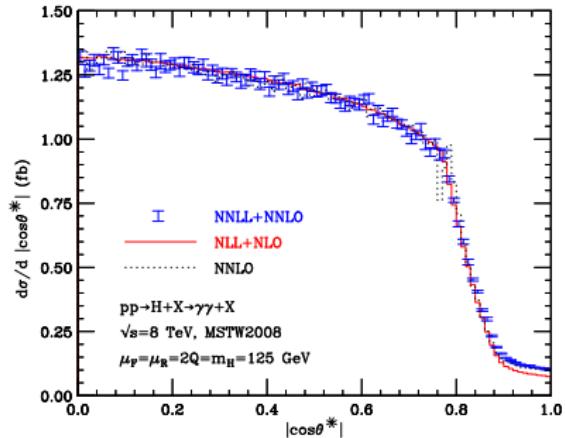
$$1/2 \leq \{\mu_F/m_Z, \mu_R/m_Z, \mu_F/\mu_R, 2Q/m_Z, Q/\mu_R\} \leq 2$$



HRES results: q_T -resummation with H boson decay

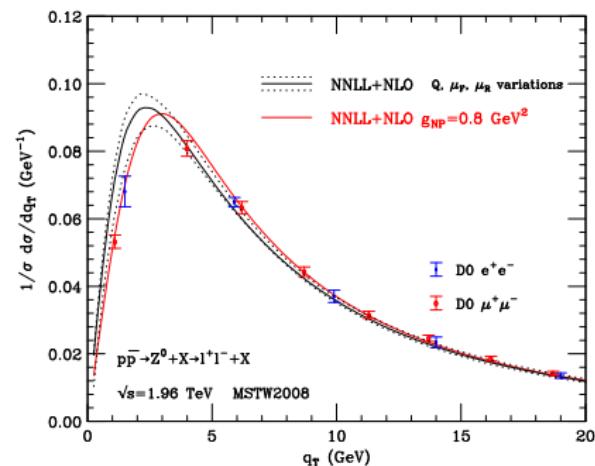


Fixed order results for
 $|\cos \theta^*| = \sqrt{1 - 4p_{T,\gamma}^2/m_H^2}$ distribution
 at the LHC.



Resummed results for
 $|\cos \theta^*| = \sqrt{1 - 4p_{T,\gamma}^2/m_H^2}$ distribution
 at the LHC.

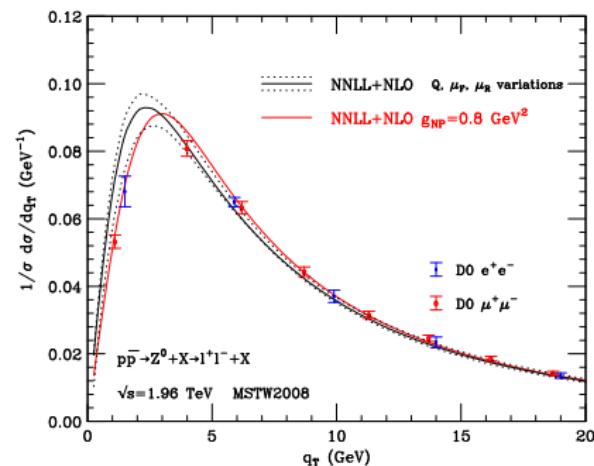
Non perturbative intrinsic k_T effects



D0 data for the Z q_T spectrum.

- Up to now result in a complete perturbative framework (plus PDFs).
- Non perturbative *intrinsic* k_T effects can be parametrized by a NP form factor $S_{NP} = \exp\{-g_{NP}b^2\}$:
$$S_c(\alpha_S, \tilde{L}) \rightarrow S_c(\alpha_S, \tilde{L}) S_{NP}$$
$$g_{NP} \simeq 0.8 \text{ GeV}^2 \quad [\text{Kulesza et al. ('02)}]$$
- With NP effects the q_T spectrum is harder. Quantitative impact of intrinsic k_T effects is comparable with perturbative uncertainties and with non perturbative effects from PDFs.

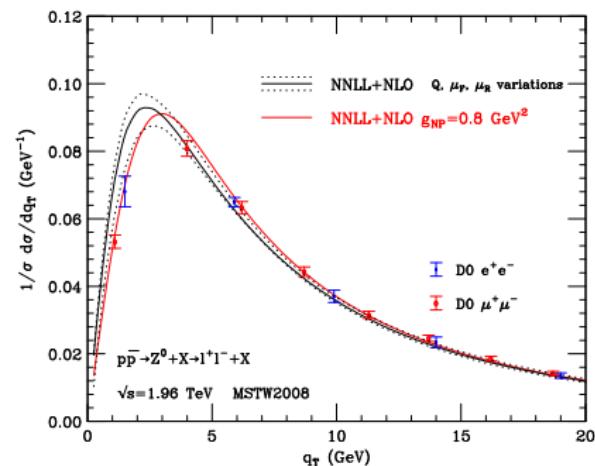
Non perturbative intrinsic k_T effects



D0 data for the Z q_T spectrum.

- Up to now result in a complete perturbative framework (plus PDFs).
- Non perturbative *intrinsic* k_T effects can be parametrized by a NP form factor $S_{NP} = \exp\{-g_{NP}b^2\}$:
$$S_c(\alpha_S, \tilde{L}) \rightarrow S_c(\alpha_S, \tilde{L}) S_{NP}$$
$$g_{NP} \simeq 0.8 \text{ GeV}^2 \quad [\text{Kulesza et al. ('02)}]$$
- With NP effects the q_T spectrum is harder. Quantitative impact of intrinsic k_T effects is comparable with perturbative uncertainties and with non perturbative effects from PDFs.

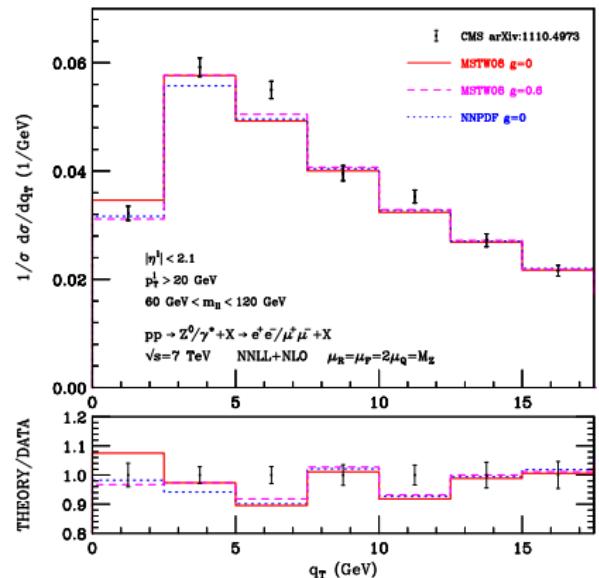
Non perturbative intrinsic k_T effects



D0 data for the Z q_T spectrum.

- Up to now result in a complete perturbative framework (plus PDFs).
- Non perturbative *intrinsic k_T* effects can be parametrized by a NP form factor
 $S_{NP} = \exp\{-g_{NP}b^2\}$:
 $S_c(\alpha_S, \tilde{L}) \rightarrow S_c(\alpha_S, \tilde{L}) S_{NP}$
 $g_{NP} \simeq 0.8 \text{ GeV}^2$ [Kulesza et al. ('02)]
- With NP effects the q_T spectrum is harder.
Quantitative impact of intrinsic k_T effects is comparable with perturbative uncertainties and with non perturbative effects from PDFs.

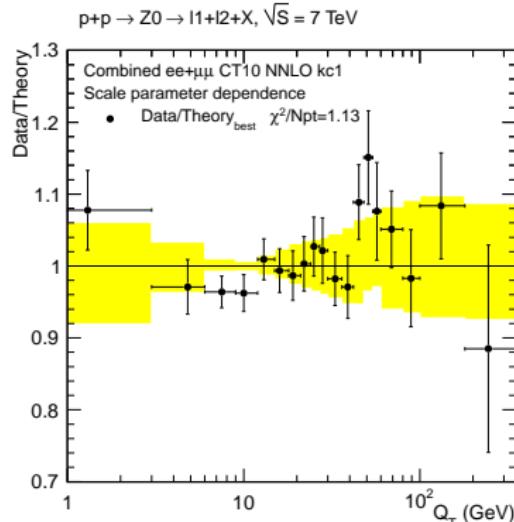
Non perturbative intrinsic k_T effects



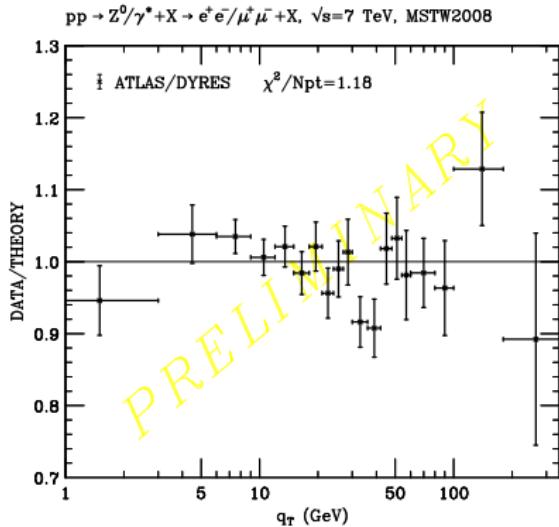
CMS data for the Z q_T spectrum.

- Up to now result in a complete perturbative framework (plus PDFs).
- Non perturbative *intrinsic k_T* effects can be parametrized by a NP form factor $S_{NP} = \exp\{-g_{NP}b^2\}$:
$$S_c(\alpha_S, \tilde{L}) \rightarrow S_c(\alpha_S, \tilde{L}) S_{NP}$$
$$g_{NP} \simeq 0.8 \text{ GeV}^2 \quad [\text{Kulesza et al. ('02)}]$$
- With NP effects the q_T spectrum is harder. Quantitative impact of intrinsic k_T effects is comparable with perturbative uncertainties and with non perturbative effects from PDFs.

Non perturbative intrinsic k_T effects

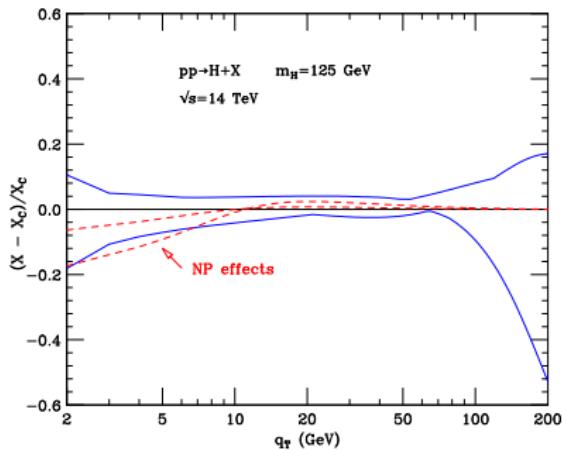


ATLAS ('11) data for the Z q_T spectrum compared with **ResBos** predictions with a Non Perturbative smearing parameter $g_{NP} = 1.1 \text{ GeV}^2$ [Guzzi, Nadolsky, Wang ('13)].

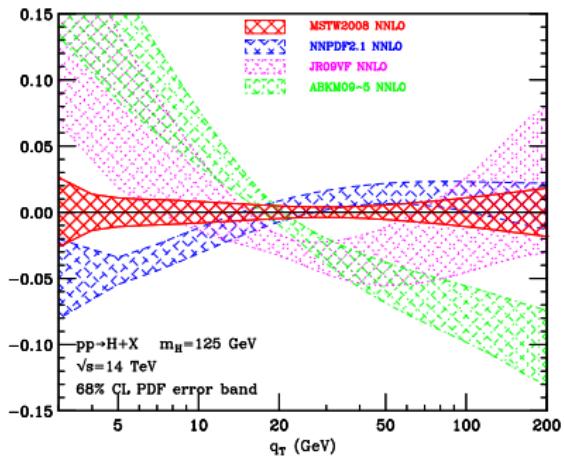


ATLAS ('11) data for the Z q_T spectrum compared with **DYRES** predictions without Non Perturbative smearing ($g_{NP} = 0$).

Non perturbative intrinsic k_T effects



Uncertainties in the normalized q_T spectrum of the Higgs boson at the LHC. NNLL+NLO uncertainty bands (solid) compared to an estimate of NP effects with smearing parameter $g_{NP} = 1.67 - 5.64 \text{ GeV}^2$ (dashed).



The q_T spectrum has a strong sensitivity from collinear PDFs (especially from the gluon density).

q_T resummation for heavy-quark hadroproduction

[Catani, Grazzini, Torre ('14)]

$$\frac{d\sigma^{(res)}}{d^2 q_T dM^2 dy d\Omega} = \frac{M^2}{s} \sum_{c=q,\bar{q},g} \left[d\sigma_{c\bar{c}}^{(0)} \right] \int \frac{d^2 \mathbf{b}}{(2\pi)^2} e^{i\mathbf{b} \cdot \mathbf{q}_T}$$

$$x S_c(M, b) \sum_{a_1, a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} [(\mathbf{H}\Delta) C_1 C_2]_{c\bar{c}; a_1 a_2}$$

$$x f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) f_{a_2/h_2}(x_2/z_2, b_0^2/b^2),$$

- Main difference with colourless case: soft factor (colour matrix) $\Delta(\mathbf{b}, M; \Omega)$ which embodies soft (wide-angle) emissions from $Q\bar{Q}$ and from initial/final-state interferences (no collinear emission from heavy-quarks). Its contribution starts at NLL.
- Soft radiation produce colour-dependent azimuthal correlations at small- q_T entangled with the azimuthal dependence due to gluonic collinear radiation.
- Explicit results for coefficients obtained up NLO and NNLL accuracy.
- Soft-factor $\Delta(\mathbf{b}, M; \Omega)$ consistent with breakdown (in weak form) of TMD factorization (additional process-dependent non-perturbative factor needed) [Collins, Qiu ('07)].

Conclusions

- Overview on q_T resummation formalism: difference between $q\bar{q}$ annihilation and gluon fusion process, hard-collinear factors and universality.
- **NNLL(NNLO)+NLO q_T -resummation** for Drell-Yan and Higgs production in gluon fusion.
Reduction of scale uncertainties from NLL+LO to NNLL+NLO accuracy. The NNLL+NLO results for Drell-Yan consistent with the experimental data in a wide region of q_T .
- Added full kinematical dependence on the vector/Higgs boson and on the final state leptons/photons.
- Intrinsic k_T effects: good agreement between data and NNLL+NLO results and LHC/Tevatron data without any model for non perturbative effects.
- Public version of the numerical codes **HqT/DYqT** and **HRES/DYRES** available.