Applications of the High-Energy QCD Effective Action

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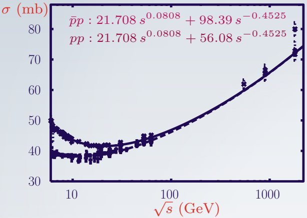
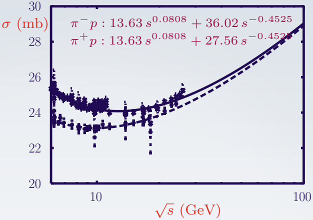
† Based on work in collaboration with G. Chachamis, M. Hentschinski, B. Murdaca and A. Sabio Vera [NPB**861** (201) 133, NPB**876** (2013) 453, PRD**87** (2013) 076009, Phys.Part.Nucl.**45** (2014) 788, PLB**735** (2014) 168, NPB**887** (2014) 309, and NPB**889** (2014) 549]

Multi-Regge Factorization & Lipatov's Action

Why the Multi-Regge Limit?

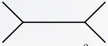
◇ Total hadronic cross-sections rise with energy $\sqrt{s}$





◇ Leading asymptotic behavior ($s \to \infty$, t fixed) given by ladder diagrams with steps strongly ordered in rapidity $y_k = \frac{1}{2} \ln \frac{k^+}{k^-}$

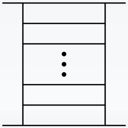
$\sim$ means *as* $s \to \infty$



$A^{(1)} \sim \frac{g^2}{s}$

$A^{(2)} \sim \frac{g^2}{s} K(t) \ln s$

$K(t) \sim g^2 \int \frac{d^2\mathbf{k}_\perp}{(\mathbf{k}_\perp^2 + m^2)[(\mathbf{k}_\perp + \mathbf{q}_\perp)^2 + m^2]}$



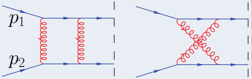
$A^{(n)} \sim \frac{g^2}{s} \frac{(K(t) \ln s)^{n-1}}{(n-1)!}$

$$A(s,t) = \sum_{n=1}^{\infty} A^{(n)} \sim \sum_{n=1}^{\infty} \frac{g^2}{s} \frac{(K(t) \ln s)^{n-1}}{(n-1)!} \simeq \frac{g^2}{s} e^{K(t) \ln s} \simeq g^2 s^{\alpha(t)}$$

Reggeization in QCD & Leading-$\ln s$ Resummation

Corrections to Born Scattering

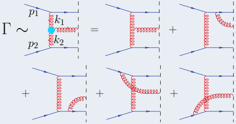
- **Virtual** ($\mathbf{8}_a$ Projected)



$\simeq$ Born $\times\, \omega(\boldsymbol{q}^2) \ln \frac{s}{s_0}$

$$\omega(\boldsymbol{q}^2) = -\frac{g^2 N_c}{8\pi^2} \ln \frac{\boldsymbol{q}^2}{\mu^2}$$

- **Real** (Lipatov's Vertex)



$$\int d\Pi\, \Gamma\Gamma^* \sim \ln \frac{s}{s_0}$$

IR singularities cancel

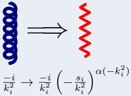
High-Energy Factorization

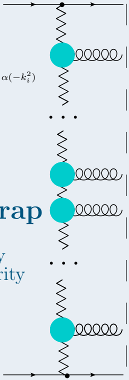
$$A^{\mathrm{MRK}}_{2\to 2+n} = A^{\mathrm{tree}}_{2\to 2+n} \prod_{i=1}^{n+1} s_i^{\omega(t_i)}, \qquad A^{\mathrm{tree}}_{2\to 2+n} = 2gsT^{c_1}_{A'A}$$

$$\times \Gamma_1 \frac{1}{t_1} gT^{d_1}_{c_2 c_1} \Gamma^1_{2,1} \frac{1}{t_2} \cdots gT^{d_n}_{c_{n+1} c_n} \Gamma^n_{n+1,n} \frac{1}{t_{n+1}} gT^{c_{n+1}}_{B'B} \Gamma_2$$

Lipatov's Ansatz

[Lipatov'76]





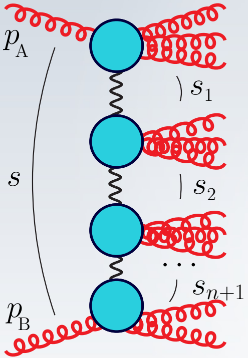
The ansatz satisfies

- ☐ **Bootstrap**
- ☐ Consistency with Unitarity

Leading $\ln s$ terms captured by strong ordering in rapidity

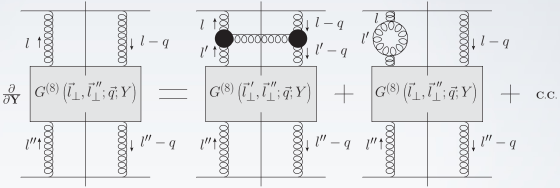
Quasi-Multi-Regge Factorization

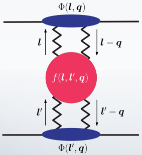
Effective expansion parameter in high-energy limit: $\alpha_s \ln s$



Clusters strongly ordered in rapidity: $y_0 \gg y_1 \gg \cdots \gg y_{n+1}$

- We lose one $\ln s$ factor each time we allow two emissions close in rapidity
- One can get elastic amplitude from production amplitude using unitarity: BFKL approach [Fadin, Kuraev & Lipatov '75,76,77; Lipatov'76; Balitsky & Lipatov '78]





Amplitudes are convolutions of *universal* BFKL Green's function and impact factors

BFKL Observables at NLO

☐ BFKL Green's function known at NLO, both in forward [Fadin & Lipatov '98; Ciafaloni & Camici '98; Kotikov & Lipatov '00] and non-forward [Fadin & Fiore '05; Fadin, Fiore & Papa '12] cases

- NLO corrections are rather large, although stabilized through collinear resummation [Salam '98; Ciafaloni, Colferai, Salam & Staśto '02,'03,'04]

Also a number of impact factors known at NLO:

- Colliding partons [Fadin, Fiore, Kotsky & Papa '00]
- Forward jet production [Bartels, Colferai & Vacca '02,'03; Caporale *et al.*'12]
- Forward vector meson production [Ivanov, Kotsky & Papa '04]
- $\gamma^* \to \gamma^*$ transition [Bartels *et al.* '02,'03; Balitsky & Chirilli '11,'13]

(However, none of them proves non-forward BFKL!)

In general,

- NLO corrections turn out to be also large for impact factors (see, e.g. [Colferai, Schwennsen, Szymanowski & Wallon'02,'03,'04])
- Uncertainties in renormalization, factorization and reggeization scales reduced
- Realistic jets (containing more than one parton)

The Effective Action Approach

Why an Effective Theory for High-Energy QCD?

- Provides a unified formalism for different phenomena in high-energy QCD
- Incorporates all requirements of unitarity
- Sets a connection with Gribov's reggeon field theory
- Simplifies the computation of scattering amplitudes in the Regge limit
- In leading log approximation the theory may be integrable
  [Lipatov'91; Kirschner, Lipatov & Szymanowski'93,'94]

Such an action exists... [Lipatov'95,'97]

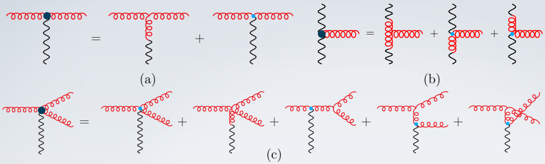
High-Energy (QMRK) Factorization + Gauge Invariance

= Lipatov's Action

Gauge Invariance & Lipatov's Action

[Lipatov'95,'97]

Consider local-in-rapidity production scattering amplitudes



In order for these amplitudes to be **gauge invariant**, *a new induced vertex has to be introduced at each order in perturbation theory*
[Ward Identities ⟹ Recurrence Relations]

$$\Delta^{\nu_0\nu_1\cdots\nu_r +}_{d_0 d_1 \cdots d_n c}(k_0^+, \cdots, k_r^+) = \frac{(n^+)^{\nu_r}}{k_r^+} \sum_{i=0}^{r-1} t^a_{a_r a_i} \Delta^{\nu_0\nu_1\cdots\nu_{r-1}+}_{d_0 d_1\cdots d_n c}(k_0^+, \cdots k_{r-1}^+), \quad (n > 2)$$

- For an action linear in reggeon fields, these identities build a current coupling to the reggeon of the form

$$W_\pm[v(x)] = -g^{-1}\partial_\pm \mathcal{P} \exp\left(-\frac{g}{2}\int_{-\infty}^{x^\mp} dz^\pm v_\pm(z)\right)$$

Feynman Rules for the Effective Action

[Antonov, Cherednikov, Kuraev & Lipatov'05]

$$S_{\text{eff}} = S_{\text{QCD}} + S_{\text{ind}};$$

$$S_{\text{ind}} = \int d^4x \,\text{Tr}\left[\left(W_+[v(x)] - \mathscr{A}_+(x)\right)\partial_\perp^2 \mathscr{A}_-(x)\right] + \int d^4x\,\text{Tr}\left[\left(W_-[v(x)] - \mathscr{A}_-(x)\right)\partial_\perp^2 \mathscr{A}_+(x)\right];$$

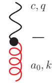
$$W_\pm[v] = v_\pm \frac{1}{D_\pm}\partial_\pm = v_\pm - g v_\pm \frac{1}{\partial_\pm} v_\pm + \cdots$$

$\mathscr{A}_\pm$: reggeons, v_μ: gluons

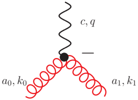
Kinematical Constraints

$$\partial_\pm \mathscr{A}_\mp(x) = 0, \quad \sum_{i=0}^{r} k_i^\pm = 0$$

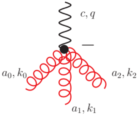
Reggeon fields invariant under *local* gauge transformations



$$= \Delta^{\nu_0 -}_{a_0 c} = -i\boldsymbol{q}^2 \delta^{a_0 c}(n^-)^{\nu_0},$$



$$= g\boldsymbol{q}^2 f^{a_0 a_1 c}\frac{1}{k_0^-}(n^-)^{\nu_0}(n^-)^{\nu_1},$$



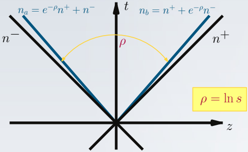
$$= \Delta^{\nu_0\nu_1\nu_2 -}_{a_0 a_1 a_2 c} = ig^2\boldsymbol{q}^2\left(\frac{f^{a_2 a_1 a} f^{a_0 a c}}{k_2^- k_0^-} + \frac{f^{a_2 a_0 a} f^{a_1 a c}}{k_2^- k_1^-}\right)(n^-)^{\nu_0}(n^-)^{\nu_1}(n^-)^{\nu_2},$$

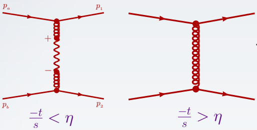
$$= \frac{i}{2\boldsymbol{q}^2}.$$

Lipatov's Action Beyond Tree Level

☐ When dealing with loops, it is needed to regularize new rapidity divergences and avoid overcounting of diagrams

☐ This can be achieved in a manifestly gauge-invariant way [Hentschinski & Sabio Vera '11; Chachamis, Hentschinski, JDM & Sabio Vera '13]



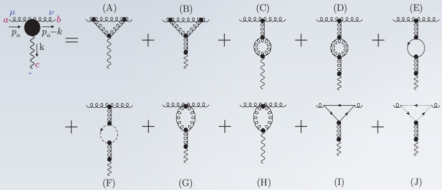


• This procedure has already been checked successfully for 1-loop corrections to forward jet vertex [Hentschinski & Sabio Vera '11; Chachamis, Hentschinski, JDM & Sabio Vera '13] and 2-loop gluon Regge trajectory [Chachamis, Hentschinski, JDM & Sabio Vera '12,'13]

1-Loop Forward Jet Vertex

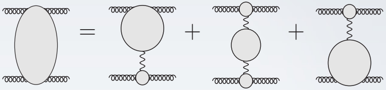
[Hentschinski & Sabio Vera'11; Chachamis, Hentshcinski, JDM & Sabio Vera'12]

Virtual Corrections to RGG Vertex

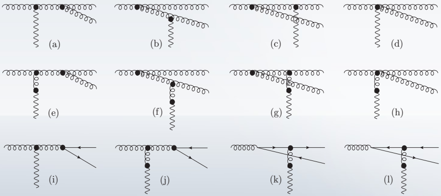


Subtracted 1-loop RGG Vertex

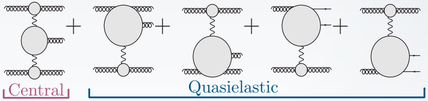
Full 1-loop $gg \to gg$ Amplitude

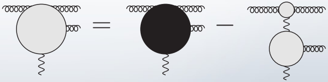


Real Emission Corrections



Contributions to the 1-loop Jet Vertex

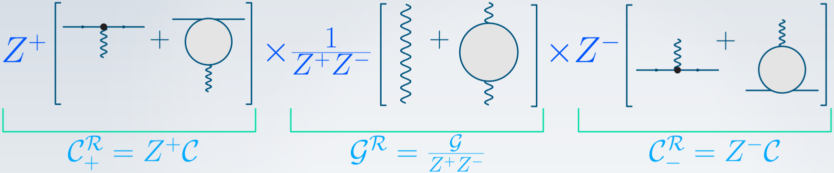




Renormalization Interpretation

- Exact cancellation of ρ-divergences between impact factors and reggeon self-energy explicitly checked

  [Hentschinski & Sabio Vera'11; Chachamis, Hentshcinski, JDM & Sabio Vera'12]
- Enables definition of reggeon vertex and wavefunction renormalization



- Reggeon is initially a background field, renormalized by perturbative corrections ~ Matching

1-loop quark- and gluon initiated jet vertices found in agreement with previous results

[Bartels, Colferai & Vacca'01,'02]

$$\mathcal{C}^{\mathrm{R}}_{gr^*\to g}\left(1;\epsilon,\frac{\boldsymbol{q}^2}{\mu^2}\right) = 2gf_{abc}\cdot\left[\Gamma_a^{(+)}\delta_{\lambda_a,\lambda_1} + \Gamma_a^{(-)}\delta_{\lambda_a,-\lambda_1}\right],$$

$$\Gamma_a^{(+)} = -\frac{1}{2}\omega^{(1)}\left[-\psi(1)+2\psi(2\epsilon)-\psi(1-\epsilon)+\frac{1}{4(1+2\epsilon)(3+2\epsilon)}+\frac{7}{4(1+2\epsilon)}-\frac{n_f}{N_c}\frac{1+\epsilon}{(1+2\epsilon)(3+2\epsilon)}\right]$$

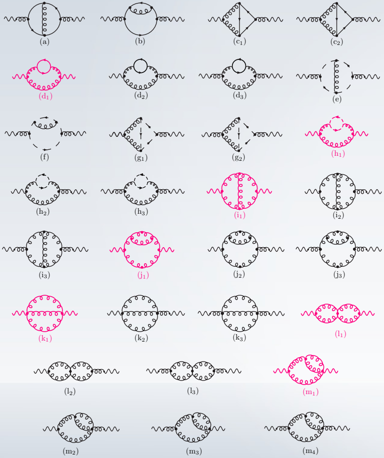
$$= \frac{\alpha_s N_c}{4\pi}\left(\frac{\boldsymbol{q}^2}{\mu^2}\right)^{\epsilon}\left[-\frac{1}{\epsilon^2}+\frac{\beta_0}{2\epsilon}-\frac{(67-\pi^2)N_c-10n_f}{18}\right]+\mathcal{O}(\epsilon), \qquad \beta_0 = \frac{11}{3}N_c-\frac{2}{3}n_f;$$

$$\Gamma_a^{(-)} = -\frac{1}{2}\omega^{(1)}\left[\frac{\epsilon}{(1+\epsilon)(1+2\epsilon)(3+2\epsilon)}\left(1+\epsilon-\frac{n_f}{N_c}\right)\right] = \frac{\alpha_s}{12\pi}(N_c-n_f)+\mathcal{O}(\epsilon).$$

2-Loop Gluon Regge Trajectory

[Chachamis, Hentschinski, JDM & Sabio Vera'12,'13]

Computation of Reggeon Self-Energy



Subtractions ~ Iteration of 1-loop trajectory

- We get trajectory from subtracted self-energy requiring ρ-independence of renormalized gluon propagator
- Loop integrals can be adressed with usual covariant techniques (e.g. Mellin-Barnes)
- Ambiguities from mixed divergences fixed [Chachamis, Hentschinski, JDM & Sabio Vera'13]

- Exact agreement with literature [Fadin, Fiore & Kotsky'95,'96; Fadin, Fiore & Quartarolo'96; Del Duca & Glover'01]

$$\omega^{(2)}(\boldsymbol{q}^2) = \frac{(\omega^{(1)}(\boldsymbol{q}^2))^2}{4}\left[\frac{11}{3} - \frac{2n_f}{3N_c} + \left(\frac{\pi^2}{3} - \frac{67}{9}\right)\epsilon + \left(\frac{404}{27} - 2\zeta(3)\right)\epsilon^2\right]$$

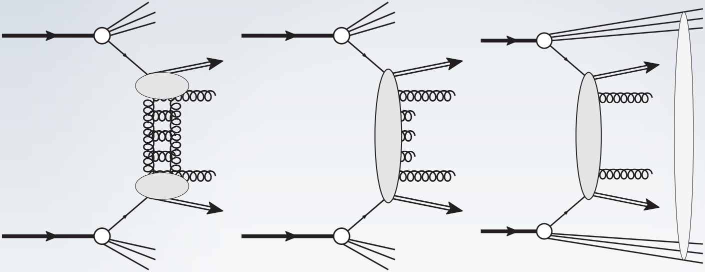
- Easy extraction of cusp anomalous dimension from this expression [Korchemskaya & Korchemsky'96]
- Universality explicitly revealed in our formalism
- Imaginary parts under control: important role of symmetric prescription

NLO Mueller-Tang Vertex

The Mueller-Tang Cross-Section

- **Singlet Exchange $\Longrightarrow$ Rapidity Gap $\Longrightarrow$ Diffraction**

Gap is never empty... $\Longrightarrow$ Need to introduce resolution scale E_{gap}



Singlet	Octet	Residual Interaction
(α_s suppressed)	(Sudakov suppressed)	(from soft rescattering)
		Rapidity Gap Survival Probability

The Challenge of Dijets with Rapidity Gap

- The high-p_T of the tagged jets ensures one can control the pomeron-proton coupling perturbatively

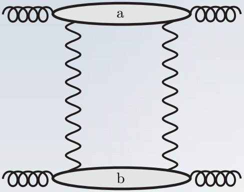
  In this respect, is the Mueller-Tang prescription for Green's function valid? [Mueller & Tang '92; Bartels *et al.* '95]

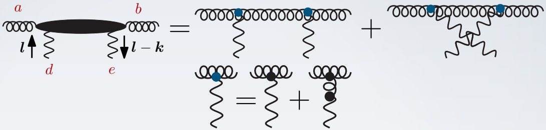
- The exclusive character of the observable may preclude applicability of collinear factorization

The Leading-Order Cross-Section

Impact factors determined from parton (quark/gluon)-pomeron coupling

Pomeron $\sim$ Two Reggeons in color singlet





- Rapidity factorization suggests including integral over light-cone component of loop momentum in impact factors

$$i\mathcal{M}^{(0)}_{g_a g_b \to g_1 g_2} = \int \frac{\mathrm{d}^{2+2\epsilon}\boldsymbol{l}}{(2\pi)^{2+2\epsilon}} \phi_{gg,\mathrm{a}} \phi_{gg,\mathrm{b}} \frac{1}{\boldsymbol{l}^2 (\boldsymbol{k}-\boldsymbol{l})^2},$$

$$i\phi_{gg,\mathrm{a}} = \int \frac{\mathrm{d}l^-}{8\pi} i\tilde{\mathcal{M}}^{abde}_{gr^*r^* \to g} P^{de}, \quad P^{de} = \frac{\delta^{de}}{\sqrt{N_c^2 - 1}}$$

The Leading-Order Cross-Section

$$\frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}^2\boldsymbol{k}} = \int \frac{\mathrm{d}^2\boldsymbol{l}_1 \mathrm{d}^2\boldsymbol{l}_1'}{\pi} \frac{\mathrm{d}^2\boldsymbol{l}_2 \mathrm{d}^2\boldsymbol{l}_2'}{\pi} h_{i,\mathrm{a}}^{(0)} h_{j,\mathrm{b}}^{(0)} G\left(\boldsymbol{l}_1, \boldsymbol{l}_1', \boldsymbol{k}, \frac{s}{s_0}\right) G\left(\boldsymbol{l}_2, \boldsymbol{l}_2', \boldsymbol{k}, \frac{s}{s_0}\right),$$

$$h_q^{(0)} = C_f^2 h^{(0)}, \quad h_g^{(0)} = C_a^2(1+\epsilon)h^{(0)}; \quad h^{(0)} = \frac{\alpha_{s,\epsilon}^2 2^\epsilon}{\mu^{4\epsilon}\Gamma^2(1-\epsilon)(N_c^2-1)}$$

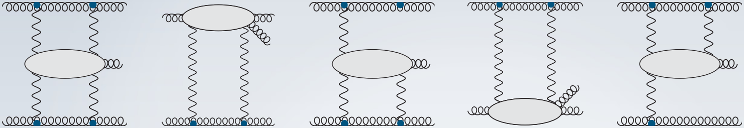
G is the non-forward BFKL Green's function

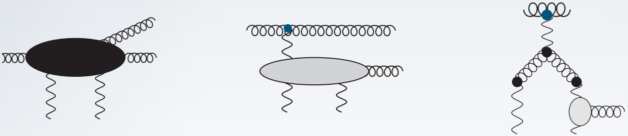
- We expect that, for $s \to \infty$, Green's function avoids singularities in transverse momentum integral as it occurs at LO

  [Motyka, Martin & Ryskin '02]

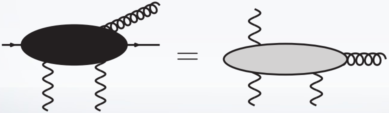
Types of NLO Corrections

- Virtual corrections already computed in [Fadin, Fiore, Kotsky & Papa '00]



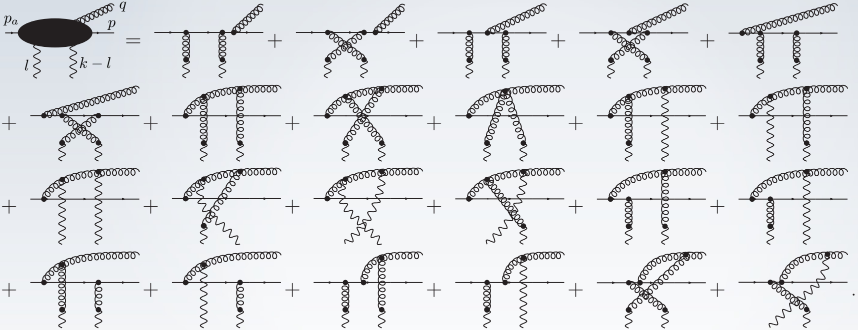


$\lim_{s_{qg}\to\infty}$



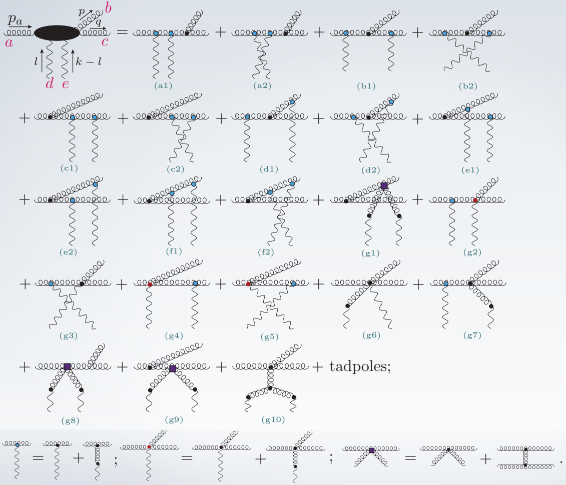
Diagrams for Quasielastic Corrections

$$q(\bar{q}) \to q(\bar{q})g$$



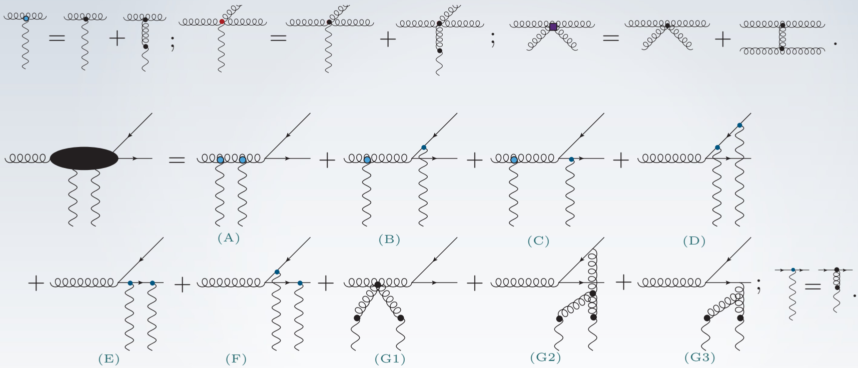
Diagrams for Quasielastic Corrections

$g \to gg$



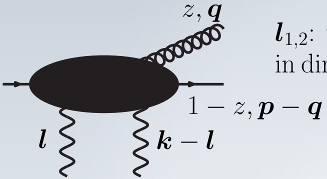
Diagrams for Quasielastic Corrections

$g \to q\bar{q}$



$+$ crossing counterparts of diagrams (A)-(F);

Differential Partonic Impact Factor



$\boldsymbol{l}_{1,2}$: transverse momenta of pomeron loop in direct and complex conjugate amplitude

$$h^{(1)}_{r,ij}\mathrm{d}\Gamma^{(2)} = \frac{h^{(0)}(1+\epsilon)}{\mu^{2\epsilon}\Gamma(1-\epsilon)}\frac{\alpha_{s,\epsilon}}{2\pi}P_{ij}(z,\epsilon)$$

$$\left[A^{(1)}_{ij}\frac{\boldsymbol{\Delta}}{\boldsymbol{\Delta}^2} - A^{(2)}_{ij}\frac{\boldsymbol{q}}{\boldsymbol{q}^2} - A^{(3)}_{ij}\frac{\boldsymbol{p}}{\boldsymbol{p}^2} - \frac{1}{2}A^{(4)}_{ij}\left(\frac{\boldsymbol{q}-\boldsymbol{l}_1}{(\boldsymbol{q}-\boldsymbol{l}_1)^2} + \frac{\boldsymbol{l}_1-\boldsymbol{p}}{(\boldsymbol{l}_1-\boldsymbol{p})^2}\right)\right]\cdot\left[\{\boldsymbol{l}_1 \leftrightarrow \boldsymbol{l}_2\}\right]\mathrm{d}\Gamma^{(2)},$$

$$ij = gq, gg, qg$$

$$P_{gq}(z,\epsilon) = C_f\frac{1+(1-z)^2+\epsilon z^2}{z} \qquad P_{gg}(z,\epsilon) = 2C_a\frac{(1-z(1-z))^2}{z(1-z)} \qquad P_{qg}(z,\epsilon) = \frac{1}{2}\left(1-\frac{2z(1-z)}{1+\epsilon}\right)$$

$$A^{(k)}_{gq} = \frac{1}{1+\epsilon}(C_f, C_f, C_a, C_a) \qquad A^{(k)}_{gg} = \frac{1}{2!}(C_a, C_a, C_a, C_a) \qquad A^{(k)}_{gq} = (C_a, C_f, C_f, 2(C_f - C_a))$$

$$\mathrm{d}\Gamma^{(2)} = \mathrm{d}z\,\mathrm{d}^{2+2\epsilon}\boldsymbol{q}/\pi^{1+\epsilon} \qquad \boldsymbol{\Delta} = \boldsymbol{q} - z\boldsymbol{k}$$

Including the Jet Function

- Collinear and infrared singularities manifest as poles in dimensional regularization parameter ϵ
- In order to define an infrared and collinear safe NLO cross section, we need to convolute the partonic cross section with a jet function S_J:

$$\frac{\mathrm{d}\hat{\sigma}_J}{\mathrm{d}J_1\mathrm{d}J_2\mathrm{d}^2\boldsymbol{k}} = \mathrm{d}\hat{\sigma} \otimes S_{J_1} S_{J_2}, \quad \mathrm{d}J_i = \mathrm{d}^{2+2\epsilon}\boldsymbol{k}_{J_i}\mathrm{d}y_{J_i}, \; i = 1, 2.$$

⋆ At LO, jet = parton: $S_J^{(2)}(\boldsymbol{p}, x) = x\,\delta\left(x - \frac{|\boldsymbol{k}_J|e^{y_J}}{\sqrt{s}}\right)\delta^{2+2\epsilon}(\boldsymbol{p} - \boldsymbol{k}_J)$

⋆ At NLO, collinear and IR safe definition of jet function must satisfy

$$S_J^{(3)}(\boldsymbol{p}, \boldsymbol{q}, zx, x) \xrightarrow{\boldsymbol{p}\to 0} S_J^{(2)}(\boldsymbol{k}, zx); \quad S_J^{(3)}(\boldsymbol{p}, \boldsymbol{q}, zx, x) \xrightarrow{\frac{\boldsymbol{q}}{z} \to \frac{\boldsymbol{p}}{1-z}} S_J^{(2)}(\boldsymbol{k}, x).$$

Final Result for NLO Jet Vertex

- After renormalization of the coupling and parton densities (UV and collinear counterterms) and including virtual corrections, finite jet vertex in $d = 4$
- To see explicitly the cancellation, we isolate the poles with a phase slicing parameter $\lambda^2 \to 0$

  ⋆ Remanent dependence of jet vertex on λ satisfies $\frac{\mathrm{d}}{\mathrm{d}\ln\lambda^2}\frac{\mathrm{d}\hat{V}^{(1)}}{\mathrm{d}J} \to 0$ for $\lambda^2 \ll \boldsymbol{k}^2$
- Within collinear factorization

$$\frac{\mathrm{d}\sigma_{J,H_1H_2}}{\mathrm{d}J_1\mathrm{d}J_2\mathrm{d}^2\boldsymbol{k}} = \frac{1}{\pi^2}\int \mathrm{d}\boldsymbol{l}_1\mathrm{d}\boldsymbol{l}_1'\mathrm{d}\boldsymbol{l}_2\mathrm{d}\boldsymbol{l}_2'\frac{\mathrm{d}V(\boldsymbol{l}_1,\boldsymbol{l}_2,\boldsymbol{k},\boldsymbol{p}_{J,1},y_1,s_0)}{\mathrm{d}J_1}$$
$$\times G\left(\boldsymbol{l}_1,\boldsymbol{l}_1',\boldsymbol{k},\frac{\hat{s}}{s_0}\right)G\left(\boldsymbol{l}_2,\boldsymbol{l}_2',\boldsymbol{k},\frac{\hat{s}}{s_0}\right)\frac{\mathrm{d}V(\boldsymbol{l}_1',\boldsymbol{l}_2',\boldsymbol{k},\boldsymbol{p}_{J,2},y_2,s_0)}{\mathrm{d}J_2},$$

Final Result for NLO Jet Vertex

$$\frac{\mathrm{d}V}{\mathrm{d}J} = \sum_{j=\{q,\bar{q},g\}} \int_{x_0}^{1} \mathrm{d}x\, f_{j/H}(x,\mu_F^2) \left(\frac{\mathrm{d}\hat{V}_j^{(0)}}{\mathrm{d}J} + \frac{\mathrm{d}\hat{V}_j^{(1)}}{\mathrm{d}J} \right), \qquad x_0 = \frac{-t}{M_{x,\max}^2 - t}$$

$$\frac{\mathrm{d}\hat{V}_j^{(0)}}{\mathrm{d}J} = \frac{\alpha_s^2 C_j^2}{N_c^2 - 1} S_J^{(2)}(\boldsymbol{k}, x), \qquad C_{q,\bar{q}} = C_f, \quad C_g = C_a$$

$$\frac{\mathrm{d}\hat{V}_j^{(1)}}{\mathrm{d}J} = \left(\frac{\mathrm{d}\hat{V}_{j,v}^{(1)}}{\mathrm{d}J} + \frac{\mathrm{d}\hat{V}_{j,r}^{(1)}}{\mathrm{d}J} + \frac{\mathrm{d}\hat{V}_{j,\mathrm{UV\,ct.}}^{(1)}}{\mathrm{d}J} + \frac{\mathrm{d}\hat{V}_{j,\mathrm{col.\,ct.}}^{(1)}}{\mathrm{d}J} \right),$$

$$\frac{\mathrm{d}\hat{V}_{j,v}^{(1)}}{\mathrm{d}J} = h_{v,j}\; S_J^{(2)}(\boldsymbol{k}, x),$$

$$\frac{\mathrm{d}\hat{V}_{j,r}^{(1)}}{\mathrm{d}J} = \int d\Gamma^{(2)} \sum_i h_{r,ij}^{(1)}\; S_J^{(3)}(\boldsymbol{p}, \boldsymbol{q}, zx, x)\ .$$

Final Result for NLO Jet Vertex

$$\alpha_s = \alpha_s(\mu^2), \qquad \phi_i = \arccos \frac{\boldsymbol{l}_i \cdot (\boldsymbol{k} - \boldsymbol{l}_i)}{|\boldsymbol{l}_i||\boldsymbol{k} - \boldsymbol{l}_i|},$$

$$P_0(z) = C_a\left[\frac{2(1-z)}{z} + z(1-z)\right], \quad P_1(z) = C_a\Big[\frac{2z}{[1-z]_+} + z(1-z)\Big], \quad P_{qq}^{(0)}(z) = C_f\left(\frac{1+z^2}{1-z}\right)_+,$$

$$P_{qg}^{(0)}(z) = \frac{z^2 + (1-z)^2}{2} \quad P_{gq}^{(0)}(z) = C_f\frac{1+(1-z)^2}{z}, \quad P_{gg}^{(0)}(z) = P_0(z) + P_1(z) + \frac{\beta_0}{2}\delta(1-z)\ ,$$

$$J_1(\boldsymbol{q},\boldsymbol{k},\boldsymbol{l}_i,z) = \frac{1}{4}\left[2\frac{\boldsymbol{k}^2}{\boldsymbol{p}^2}\left(\frac{(1-z)^2}{\boldsymbol{\Delta}^2} - \frac{1}{\boldsymbol{q}^2}\right) - \left(\frac{(\boldsymbol{l}_i - z\boldsymbol{k})^2}{\boldsymbol{\Delta}^2(\boldsymbol{q}-\boldsymbol{l}_i)^2} - \frac{\boldsymbol{l}_i^2}{\boldsymbol{q}^2(\boldsymbol{q}-\boldsymbol{l}_i)^2}\right) - \left(\frac{(\boldsymbol{l}_i - (1-z)\boldsymbol{k})^2}{\boldsymbol{\Delta}^2(\boldsymbol{p}-\boldsymbol{l}_i)^2} - \frac{(\boldsymbol{l}_i-\boldsymbol{k})^2}{\boldsymbol{q}^2(\boldsymbol{p}-\boldsymbol{l}_i)^2}\right)\right],$$

$$\begin{aligned} J_2(\boldsymbol{q},\boldsymbol{k},\boldsymbol{l}_1,\boldsymbol{l}_2) = \frac{1}{4}\Bigg[&\frac{\boldsymbol{l}_1^2}{\boldsymbol{p}^2(\boldsymbol{p}-\boldsymbol{l}_1)^2} + \frac{(\boldsymbol{k}-\boldsymbol{l}_1)^2}{\boldsymbol{p}^2(\boldsymbol{q}-\boldsymbol{l}_1)^2} + \frac{\boldsymbol{l}_2^2}{\boldsymbol{p}^2(\boldsymbol{p}-\boldsymbol{l}_2)^2} + \frac{(\boldsymbol{k}-\boldsymbol{l}_2)^2}{\boldsymbol{p}^2(\boldsymbol{q}-\boldsymbol{l}_2)^2} \\ &- \frac{1}{2}\left(\frac{(\boldsymbol{l}_1-\boldsymbol{l}_2)^2}{(\boldsymbol{q}-\boldsymbol{l}_1)^2(\boldsymbol{q}-\boldsymbol{l}_2)^2} + \frac{(\boldsymbol{k}-\boldsymbol{l}_1-\boldsymbol{l}_2)^2}{(\boldsymbol{p}-\boldsymbol{l}_1)^2(\boldsymbol{q}-\boldsymbol{l}_2)^2} + \frac{(\boldsymbol{k}-\boldsymbol{l}_1-\boldsymbol{l}_2)^2}{(\boldsymbol{q}-\boldsymbol{l}_1)^2(\boldsymbol{p}-\boldsymbol{l}_2)^2} + \frac{(\boldsymbol{l}_1-\boldsymbol{l}_2)^2}{(\boldsymbol{p}-\boldsymbol{l}_1)^2(\boldsymbol{p}-\boldsymbol{l}_2)^2}\right)\Bigg], \end{aligned}$$

Final Result for NLO Jet Vertex

$$\frac{\mathrm{d}\hat{V}_q^{(1)}(x,\boldsymbol{k},\boldsymbol{l}_1,\boldsymbol{l}_2;x_J,\boldsymbol{k}_J;M_{X,\max},s_0)}{\mathrm{d}J} = v^{(0)}\frac{\alpha_s}{2\pi}(Q_1+Q_2+Q_3)$$

$$Q_1 = S_J^{(2)}(\boldsymbol{k},x)C_f^2\left[-\frac{\beta_0}{4}\left\{\left[\ln\left(\frac{\boldsymbol{l}_1^2}{\mu^2}\right)+\ln\left(\frac{(\boldsymbol{l}_1-\boldsymbol{k})^2}{\mu^2}\right)+\{1\leftrightarrow 2\}\right]-\frac{20}{3}\right\}-4C_f\right.$$

$$+\frac{C_a}{2}\left(\left\{\frac{3}{2\boldsymbol{k}^2}\left[\boldsymbol{l}_1^2\ln\left(\frac{(\boldsymbol{l}_1-\boldsymbol{k})^2}{\boldsymbol{l}_1^2}\right)+(\boldsymbol{l}_1-\boldsymbol{k})^2\cdot\ln\left(\frac{\boldsymbol{l}_1^2}{(\boldsymbol{l}_1-\boldsymbol{k})^2}\right)-4|\boldsymbol{l}_1||\boldsymbol{l}_1-\boldsymbol{k}|\phi_1\sin\phi_1\right]-\frac{3}{2}\left[\ln\left(\frac{\boldsymbol{l}_1^2}{\boldsymbol{k}^2}\right)\right.\right.\right.$$

$$\left.\left.\left.+\ln\left(\frac{(\boldsymbol{l}_1-\boldsymbol{k})^2}{\boldsymbol{k}^2}\right)\right]-\ln\left(\frac{\boldsymbol{l}_1^2}{\boldsymbol{k}^2}\right)\ln\left(\frac{(\boldsymbol{l}_1-\boldsymbol{k})^2}{s_0}\right)-\ln\left(\frac{(\boldsymbol{l}_1-\boldsymbol{k})^2}{\boldsymbol{k}^2}\right)\cdot\ln\left(\frac{\boldsymbol{l}_1^2}{s_0}\right)-2\phi_1^2+\{1\leftrightarrow 2\}\right\}+2\pi^2+\frac{14}{3}\right)\right],$$

$$Q_2 = \int_{z_0}^1 \mathrm{d}z\; S_J^{(2)}(\boldsymbol{k},zx)\left[\ln\frac{\lambda^2}{\mu_F^2}\left(C_f^2P_{qq}^{(0)}(z)+C_a^2P_{gq}^{(0)}(z)\right)+C_f(1-z)\left(C_f^2-\frac{2}{z}C_a^2\right)+2C_f(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+\right],$$

$$Q_3 = \int_0^1 \mathrm{d}z\int\frac{\mathrm{d}^2\boldsymbol{q}}{\pi}\left[\Theta\left(\hat{M}_{X,\max}^2-\frac{(\boldsymbol{p}-z\boldsymbol{k})^2}{z(1-z)}\right)S_J^{(3)}(\boldsymbol{p},\boldsymbol{q},(1-z)x,x)C_f^2P_{qq}^{(0)}(z)\Theta\left(\frac{|\boldsymbol{q}|}{1-z}-\lambda^2\right)\frac{\boldsymbol{k}^2}{\boldsymbol{q}^2(\boldsymbol{p}-z\boldsymbol{k})^2}\right.$$

$$\left.+\Theta\left(\hat{M}_{X,\max}^2-\frac{\boldsymbol{\Delta}^2}{z(1-z)}\right)S_J^{(3)}(\boldsymbol{p},\boldsymbol{q},zx,x)P_{gq}^{(0)}(z)\left\{C_fC_a[J_1(\boldsymbol{q},\boldsymbol{k},\boldsymbol{l}_1)+J_1(\boldsymbol{q},\boldsymbol{k},\boldsymbol{l}_2)]+C_a^2J_2(\boldsymbol{q},\boldsymbol{k},\boldsymbol{l}_1,\boldsymbol{l}_2)\Theta(\boldsymbol{p}^2-\lambda^2)\right\}\right].$$

Final Result for NLO Jet Vertex

$$\frac{\mathrm{d}\hat{V}^{(1)}(x,\boldsymbol{k},\boldsymbol{l}_1,\boldsymbol{l}_2;x_J,\boldsymbol{k}_J;M_{X,\max},s_0)}{\mathrm{d}J} = v^{(0)}\frac{\alpha_s}{2\pi}\big(G_1+G_2+G_3\big)$$

$$G_1 = C_a^2\, S_J^{(2)}(\boldsymbol{k},x)\Bigg[C_a\left(\pi^2-\frac{5}{6}\right)-\beta_0\left(\ln\frac{\lambda^2}{\mu^2}-\frac{4}{3}\right)+\left(\frac{\beta_0}{4}+\frac{11C_a}{12}+\frac{n_f}{6C_a^2}\right)\left(\ln\frac{\boldsymbol{k}^4}{\boldsymbol{l}_1^2(\boldsymbol{k}-\boldsymbol{l}_1^2)}+\ln\frac{\boldsymbol{k}^4}{\boldsymbol{l}_2^2(\boldsymbol{k}-\boldsymbol{l}_2)^2}\right)$$
$$+\frac{1}{2}\Bigg\{C_a\left(\ln^2\frac{\boldsymbol{l}_1^2}{(\boldsymbol{k}-\boldsymbol{l}_1)^2}+\ln\frac{\boldsymbol{k}^2}{\boldsymbol{l}_1^2}\ln\frac{\boldsymbol{l}_1^2}{s_0}+\ln\frac{\boldsymbol{k}^2}{(\boldsymbol{k}-\boldsymbol{l}_1)^2}\ln\frac{(\boldsymbol{k}-\boldsymbol{l}_1)^2}{s_0}\right)-\left(\frac{n_f}{3C_a^2}+\frac{11C_a}{6}\right)\frac{\boldsymbol{l}_1^2-(\boldsymbol{k}-\boldsymbol{l}_1)^2}{\boldsymbol{k}^2}\ln\frac{\boldsymbol{l}_1^2}{(\boldsymbol{k}-\boldsymbol{l}_1)^2}$$
$$-2\left(\frac{n_f}{C_a^2}+4C_a\right)\frac{(\boldsymbol{l}_1^2(\boldsymbol{k}-\boldsymbol{l}_1)^2)^{\frac{1}{2}}}{\boldsymbol{k}^2}\phi_1\sin\phi_1+\frac{1}{3}\left(C_a+\frac{n_f}{C_a^2}\right)\Bigg[16\frac{(\boldsymbol{l}_1^2(\boldsymbol{k}-\boldsymbol{l}_1)^2)^{\frac{3}{2}}}{(\boldsymbol{k}^2)^3}\phi_1\sin^3\phi_1$$
$$-4\frac{\boldsymbol{l}_1^2(\boldsymbol{k}-\boldsymbol{l}_1)^2}{(\boldsymbol{k}^2)^2}\left(2-\frac{\boldsymbol{l}_1^2-(\boldsymbol{k}-\boldsymbol{l}_1)^2}{\boldsymbol{k}^2}\ln\frac{\boldsymbol{l}_1^2}{(\boldsymbol{k}-\boldsymbol{l}_1)^2}\right)\sin^2\phi_1+\frac{(\boldsymbol{l}_1^2(\boldsymbol{k}-\boldsymbol{l}_1)^2)^{\frac{1}{2}}}{(\boldsymbol{k}^2)^2}\cos\phi_1$$
$$\left(4\boldsymbol{k}^2-12(\boldsymbol{l}_1^2(\boldsymbol{k}-\boldsymbol{l}_1)^2)^{\frac{1}{2}}\phi_1\sin\phi_1-(\boldsymbol{l}_1^2-(\boldsymbol{k}-\boldsymbol{l}_1)^2)\ln\frac{\boldsymbol{l}_1^2}{(\boldsymbol{k}-\boldsymbol{l}_1)^2}\right)\Bigg]-2C_a\phi_1^2+\{\boldsymbol{l}_1\leftrightarrow\boldsymbol{l}_2,\phi_1\leftrightarrow\phi_2\}\Bigg\}\Bigg]$$

$$G_2=\int_{z_0}^1\mathrm{d}z\, S_J^{(2)}(\boldsymbol{k},zx)\Bigg\{2n_f P_{qg}^{(0)}(z)\left(C_f^2\ln\frac{\lambda^2}{\mu_F^2}+C_a^2\ln(1-z)\right)$$
$$+C_a^2P_{gg}^{(0)}(z)\ln\frac{\lambda^2}{\mu_F^2}+C_f^2n_f+2C_a^3z\left((1-z)\ln(1-z)+2\left[\frac{\ln(1-z)}{1-z}\right]_+\right)$$

Final Result for NLO Jet Vertex

$$
\begin{aligned}
G_3 = & \int_0^1 \mathrm{d}z \int \frac{\mathrm{d}^2\boldsymbol{q}}{\pi} \Bigg\{ n_f P_{qg}^{(0)}(z) \Bigg[C_a^2 \Theta\left(\hat{M}_{X,\max}^2 - \frac{z\boldsymbol{p}^2}{(1-z)} \right) S_J^{(3)}(\boldsymbol{k} - z\boldsymbol{q}, z\boldsymbol{q}, zx, x) \\
& \left[\frac{\Theta(\boldsymbol{p}^2 - \lambda^2)\boldsymbol{k}^2}{(\boldsymbol{p}^2 + \boldsymbol{q}^2)\boldsymbol{p}^2} + \frac{\boldsymbol{k}^2}{(\boldsymbol{p}^2 + \boldsymbol{q}^2)\boldsymbol{q}^2} \right] - \Theta\left(\hat{M}_{X,\max}^2 - \frac{\boldsymbol{\Delta}^2}{z(1-z)} \right) S_J^{(3)}(\boldsymbol{p}, \boldsymbol{q}, zx, x) \left(C_a^2 \frac{\boldsymbol{k}^2}{(\boldsymbol{p}^2 + \boldsymbol{q}^2)\boldsymbol{q}^2} - 2C_f^2 \frac{\boldsymbol{k}^2 \Theta(\boldsymbol{q}^2 - \lambda^2)}{(\boldsymbol{p}^2 + \boldsymbol{q}^2)\boldsymbol{q}^2} \right) \Bigg] \\
& + P_1(z) \Theta\left(\hat{M}_{X,\max}^2 - \frac{(\boldsymbol{p} - z\boldsymbol{k})^2}{z(1-z)} \right) S_J^{(3)}(\boldsymbol{p}, \boldsymbol{q}, (1-z)x, x) \frac{(1-z)^2 \boldsymbol{k}^2}{(1-z)^2(\boldsymbol{p} - z\boldsymbol{k})^2 + \boldsymbol{q}^2} \Bigg[\Theta\left(\frac{|\boldsymbol{q}|}{1-z} - \lambda \right) \frac{1}{\boldsymbol{q}^2} \\
& + \Theta\left(\frac{|\boldsymbol{p} - z\boldsymbol{k}|}{1-z} - \lambda \right) \frac{1}{(\boldsymbol{p} - z\boldsymbol{k})^2} + \Theta\left(\hat{M}_{X,\max}^2 - \frac{\boldsymbol{\Delta}^2}{z(1-z)} \right) S_J^{(3)}(\boldsymbol{p}, \boldsymbol{q}, zx, x) \Bigg[\frac{n_f}{C_a^2} P_{qg}^{(0)} \left(J_2(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{l}_1, \boldsymbol{l}_2) - \frac{\boldsymbol{k}^2}{\boldsymbol{p}^2(\boldsymbol{q}^2 + \boldsymbol{p}^2)} \right) \\
& - n_f P_{qg}^{(0)} \Big(J_1(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{l}_1, z) + J_1(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{l}_2, z) \Big) + P_0(z) \Big(J_1(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{l}_1) + J_1(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{l}_2) + J_2(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{l}_1, \boldsymbol{l}_2) \Theta(\boldsymbol{p}^2 - \lambda^2) \Big) \Bigg] \Bigg] \Bigg\}.
\end{aligned}
$$

Conclusions & Outlook

❶
- Lipatov's effective action is a powerful tool to implement computation of amplitudes in multi-Regge limit
- Several nontrivial checks performed (1-loop Mueller-Navelet jet vertex, 2-loop gluon Regge trajectory)

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- Jet-gap-jet events ideal to study perturbative pomeron in QCD
- NLO jet vertex for pomeron-forward jet coupling computed with the help of Lipatov's action

  ⋆ Finite result for jet vertex within collinear factorization
- Open door to compute diffractive jet production in pQCD at NLO (reduced scale uncertainties, realistic jet)

  ⋆ Possible Monte-Carlo implementation (solution of NLO nonforward BFKL directly in $\boldsymbol{k}_T$-space) allowing to obtain exclusive distributions

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