TMD Evolution Results

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Outline

- TMD factorization & evolution: general aspects
- TMD evolution: Sivers and Collins asymmetries
- Higgs transverse momentum distribution
- Higher twist
- Small x

TMD factorization

“Evolution” of TMD Factorization

- Collins & Soper, 1981: $e^+e^- \to h_1\ h_2\ X$ [NPB 193 (1981) 381]
- X. Ji, J.-P. Ma & F. Yuan, 2004/5: SIDIS & Drell-Yan (DY) [PRD 71 (2005) 034005 & PLB 597 (2004) 299]
- Collins (JCC), 2011: “Foundations of perturbative QCD” [Cambridge Univ. Press]
- P. Sun, B.-W. Xiao & F. Yuan, 2011: Higgs prod. (gluon TMDs)[PRD 84 (2011) 094005]
- Echevarria, Idilbi & Scimemi (EIS), 2012/4: DY & SIDIS (SCET)[JHEP 1207 (2012) 002 & PRD 90 (2014) 014003]
- J.P. Ma, J.X. Wang & S. Zhao, 2012: quarkonium prod. 1-loop [PRD 88 (2013) 014027]
- J.P. Ma, J.X. Wang & S. Zhao, 2014: breakdown of factorization in P-wave quarkonium production beyond 1-loop [PLB 737 (2014) 103]

Main differences among the various approaches:
- treatment of rapidity/LC divergences, in order to make each factor well-defined
- redistribution of terms to avoid large logarithms

TMD factorization

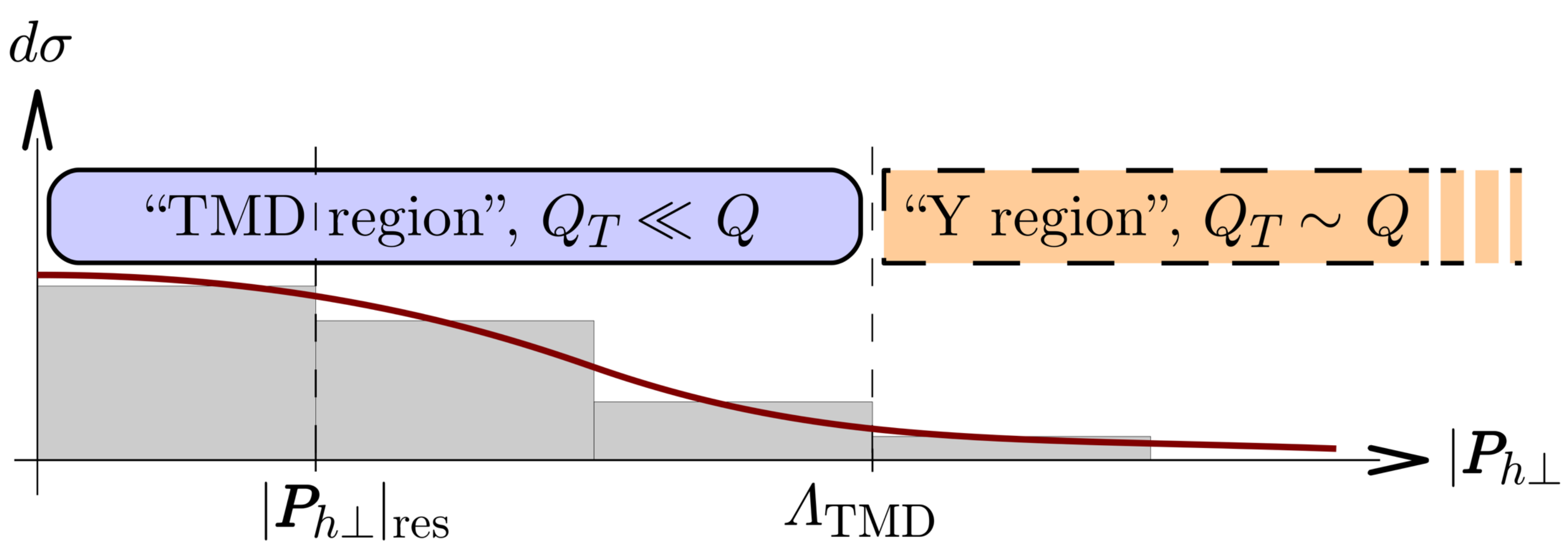
TMD factorization for SIDIS, $e^+e^- \to h_1\ h_2\ X$ and Drell-Yan (DY)

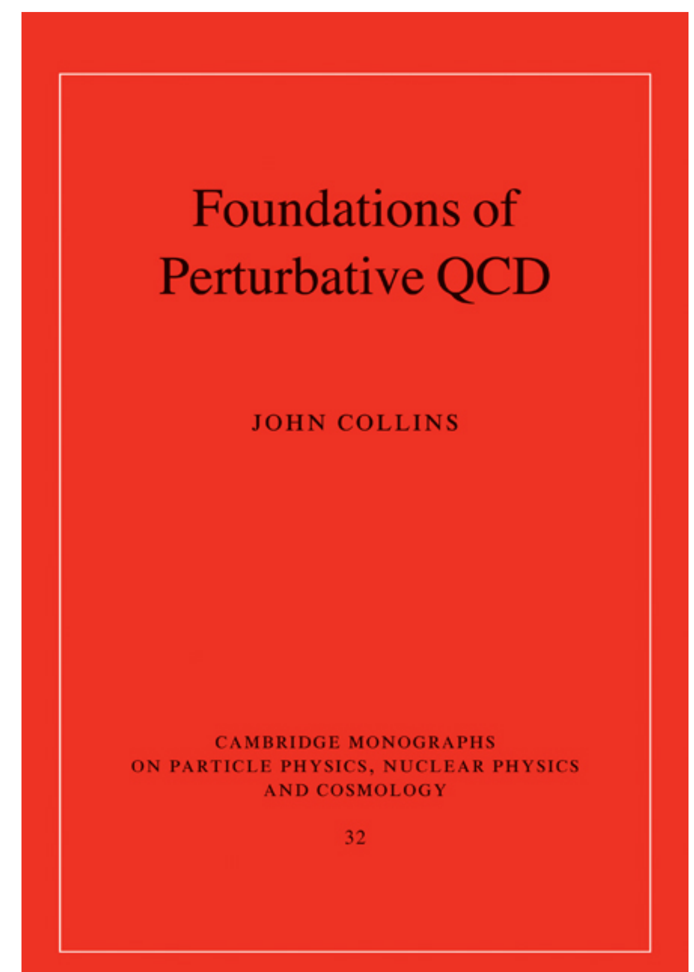
Schematic form of (new) TMD factorization "JCC" [Collins 2011]:

$$d\sigma = H \times \text{convolution of } A\,B + \text{high-}q_T \text{ correction } (Y) + \text{power-suppressed}$$

A & B are TMD pdfs or FFs
(a soft factor has been absorbed in them)

Details in book by J.C. Collins
Summarized in arXiv:1107.4123





Convolution in terms of A and B best deconvoluted by Fourier transform

New TMD factorization expressions

$$\frac{d\sigma}{d\Omega d^4 q} = \int d^2 b \, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T} \tilde{W}(\boldsymbol{b}, Q; x, y, z) + \mathcal{O}\left(Q_T^2/Q^2\right)$$

$$\tilde{W}(\boldsymbol{b}, Q; x, y, z) = \sum_a \tilde{f}_1^a(x, \boldsymbol{b}^2; \zeta_F, \mu) \tilde{D}_1^a(z, \boldsymbol{b}^2; \zeta_D, \mu) H(y, Q; \mu)$$

Fourier transforms of the TMDs are functions of the momentum fraction *x* (or *z*), the transverse coordinate b, a rapidity variable ζ, and the renormalization scale μ

$$\zeta_F = M^2 x^2 e^{2(y_P - y_s)} \qquad \zeta_D = M_h^2 e^{2(y_s - y_h)}/z^2$$

y_s is an arbitrary rapidity that drops out of the final answer

$$\zeta_F \zeta_D \approx Q^4 \qquad \zeta_F \approx \zeta_D \approx Q^2$$

The TMDs in principle also depend on the Wilson line U

$$\tilde{f}^{[\mathcal{U}]}(x, b_T^2; \zeta, \mu)$$

New TMD factorization expressions

Y term

$$\frac{d\sigma}{d\Omega d^4q} = \int d^2b\, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T}\,\tilde{W}(\boldsymbol{b}, Q; x, y, z) + \mathcal{O}\left(Q_T^2/Q^2\right)$$

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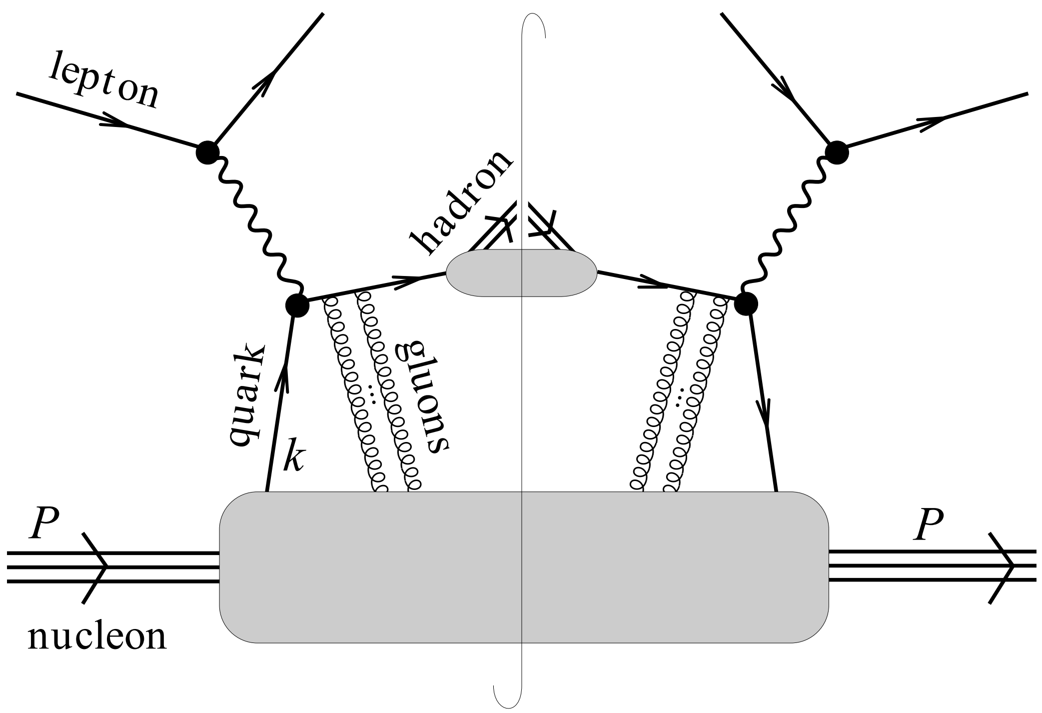
Gauge invariance of TMD correlators



summation of all gluon insertions leads to path-ordered exponentials in the correlators

$$\mathcal{L}_{\mathcal{C}}[0,\xi] = \mathcal{P}\exp\left(-ig\int_{\mathcal{C}[0,\xi]} ds_\mu\, A^\mu(s)\right)$$

$$\Phi \propto \langle P|\overline{\psi}(0)\mathcal{L}_{\mathcal{C}}[0,\xi]\psi(\xi)|P\rangle$$

Efremov & Radyushkin, Theor. Math. Phys. 44 ('81) 774

Resulting Wilson lines depend on whether the color is incoming or outgoing

[Collins & Soper, 1983; DB & Mulders, 2000; Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, X. Ji & F. Yuan, 2003; DB, Mulders & Pijlman, 2003]

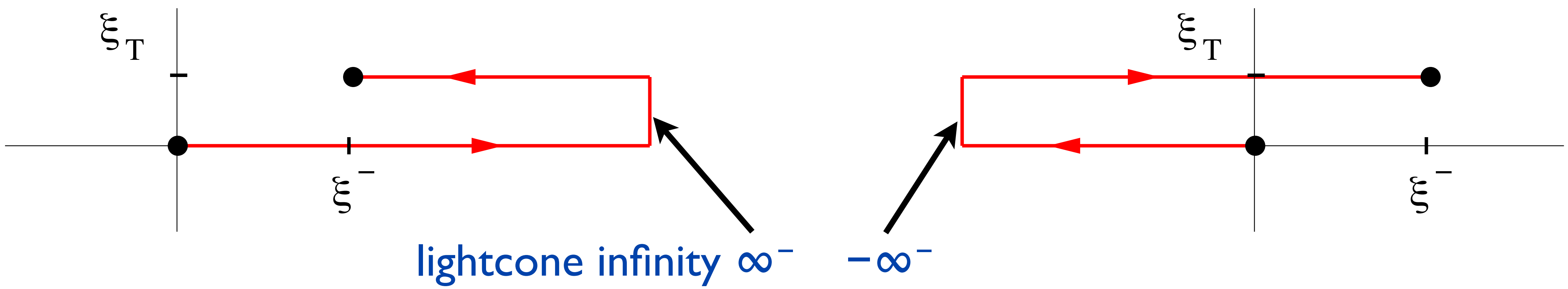
This does not automatically imply that this affects observables, but it turns out that it does in certain cases, for example, Sivers asymmetries

[Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, Ji & Yuan, 2003]

Process dependence of Sivers TMD

Gauge invariant definition of TMDs in semi-inclusive DIS contains a future pointing Wilson line, whereas in Drell-Yan (DY) it is past pointing
[Belitsky, X. Ji & F. Yuan '03]

$\gamma^* p \to h\, X$ (SIDIS) $\qquad\qquad pp \to \gamma^* X$ (Drell-Yan)



One can use parity and time reversal invariance to relate the Sivers functions:

$$f_{1T}^{\perp[\mathrm{SIDIS}]} = -f_{1T}^{\perp[\mathrm{DY}]}$$

[Collins '02]

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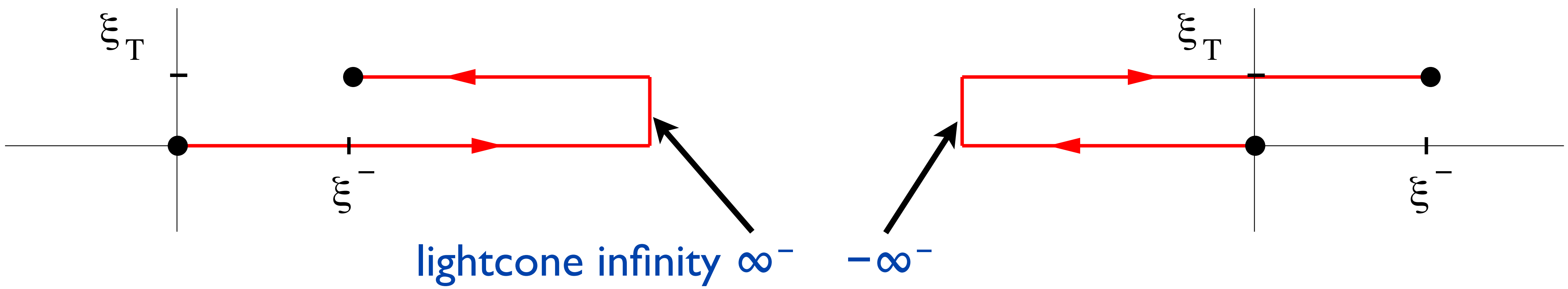

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The more hadrons are observed in a process, the more complicated the end result: more complicated N_c-dependent prefactors
[Bomhof, Mulders & Pijlman '04; Buffing, Mulders '14]

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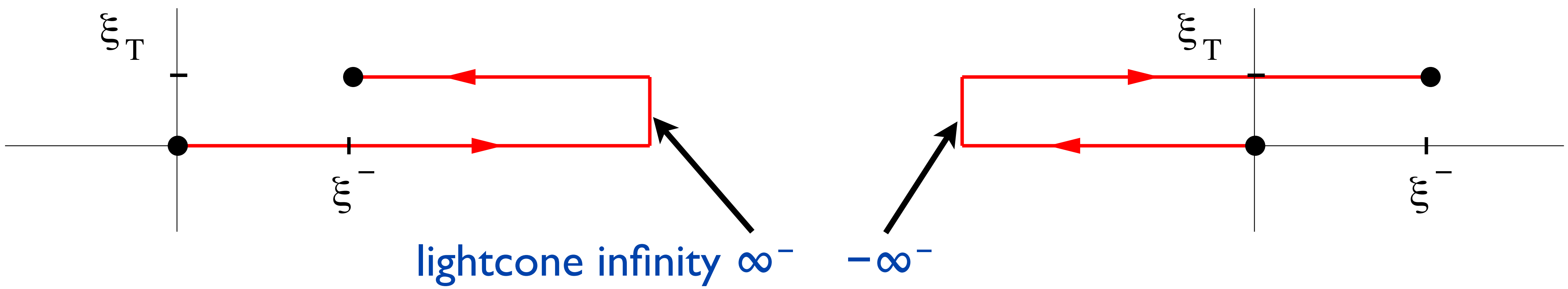

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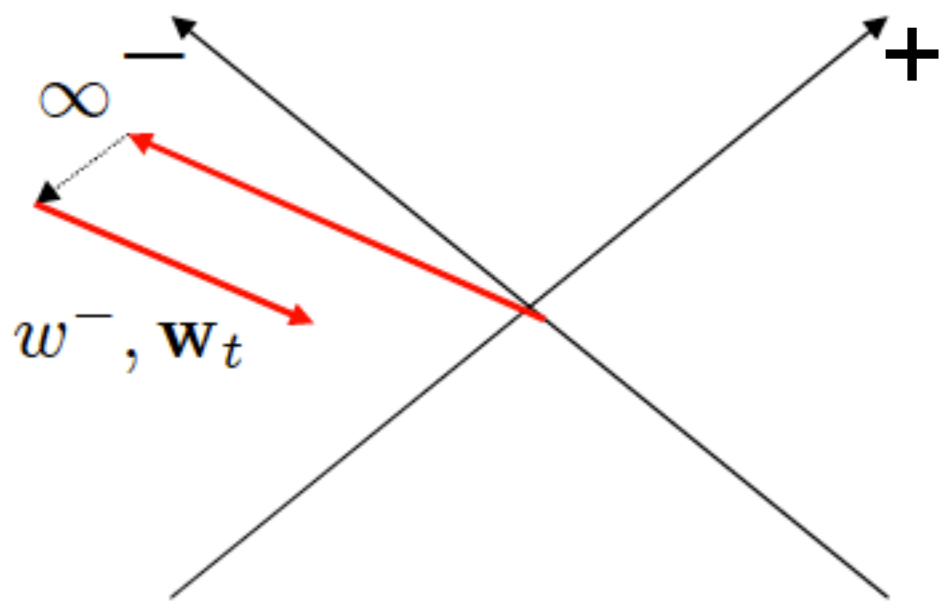
When color flow is in too many directions: *factorization breaking*
[Collins & J. Qiu '07; Collins '07; Rogers & Mulders '10]

Scale dependence of TMDs

QCD corrections will also attach to the Wilson line, which needs renormalization
This determines the change with renormalization scale μ

Wilson lines not smooth → cusp anomalous dimension
[Polyakov '80; Dotsenko & Vergeles '80; Brandt, Neri, Sato '81; Korchemsky, Radyushkin '87]

As a regularization of rapidity/LC divergences of a *lightlike* Wilson line, in JCC's TMD factorization the path is taken off the lightfront, the variation in rapidity determines the change with ζ



$$\tilde{f}^{[\mathcal{U}]}(x, b_T^2; \zeta, \mu)$$

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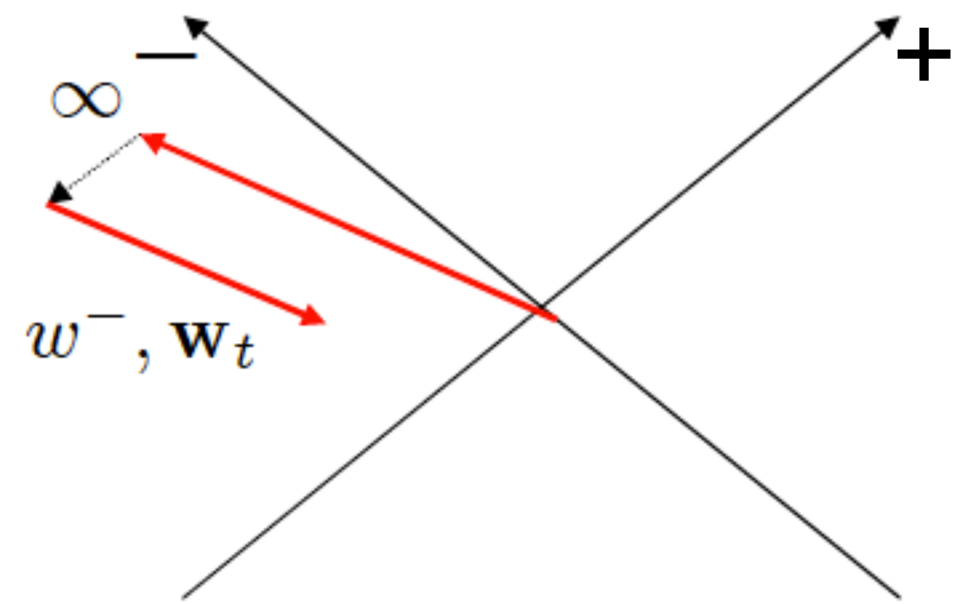


$$\tilde{f}^{[\mathcal{U}]}(x, b_T^2; \zeta, \mu)$$

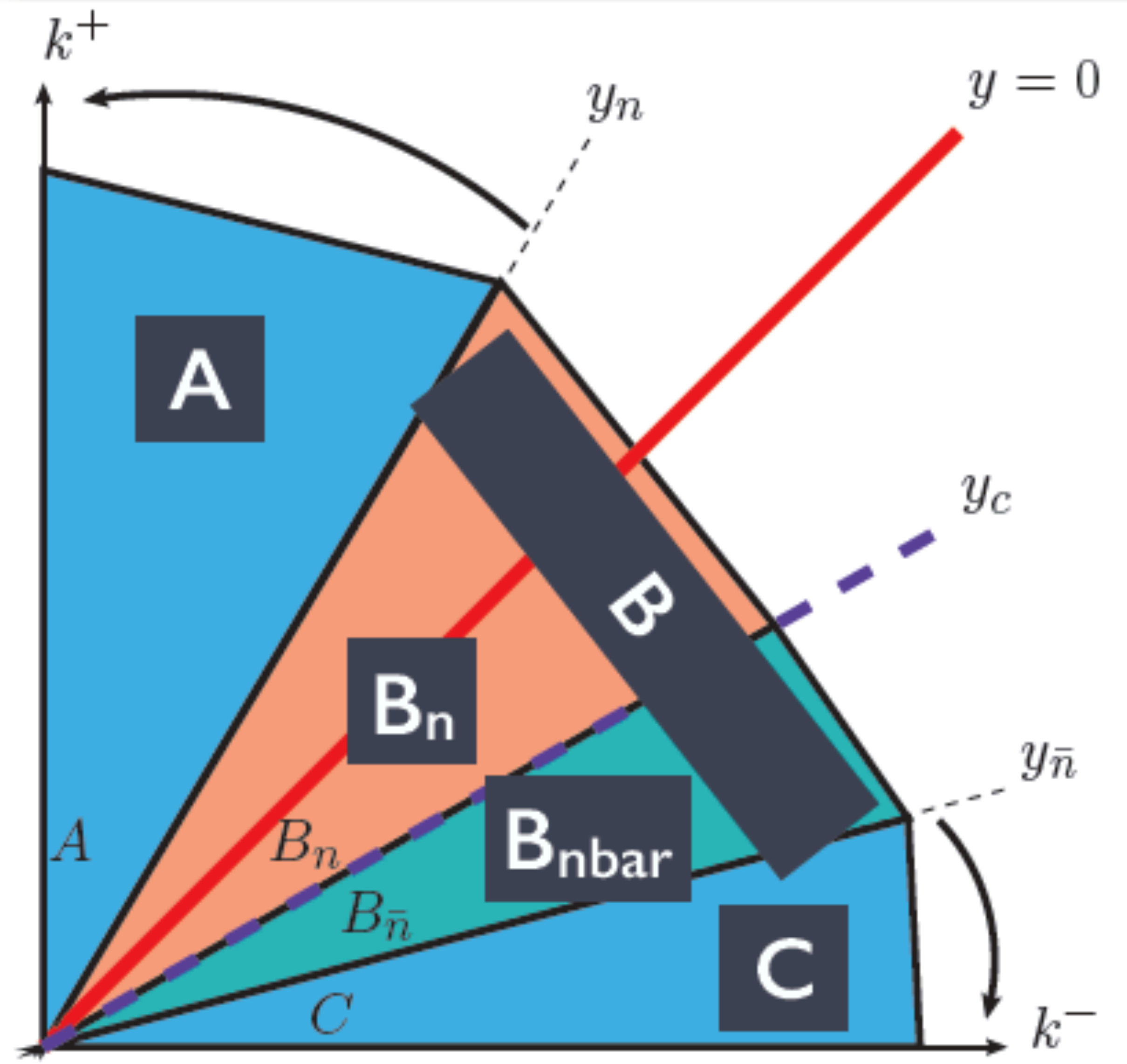
Two important consequences:

- yields energy evolution of TMD observables
- allows for calculation of the Sivers and Boer-Mulders effect on the lattice
  Musch, Hägler, Engelhardt, Negele & Schäfer, 2012

Definition of TMDPDFs: Cancellation of RDs

MGE, Idilbi, Scimemi JHEP'12, PLB'13

- Pictorially, the relevant (anti-)collinear and soft modes are represented as:



$$k_n \sim (1, \lambda^2, \lambda)$$
$$k_{\bar{n}} \sim (\lambda^2, 1, \lambda)$$
$$k_s \sim (\lambda, \lambda, \lambda)$$

$$y = \frac{1}{2} \ln \left| \frac{k^+}{k^-} \right|$$

- **Naive collinear = A+B**
- **Soft = B**
- **Naive anticollinear = C+B**
- (Pure collinear = A)
- (Pure anticollinear = C)
- Each piece is boost invariant and depends on the difference of rapidities at the borders.
- **x-section = (A+B) + (C+B) - B = A+B+C**

- **Divergences at yn and ynbar as spurius…**
- **(Anti-)Collinear and Soft are ill-defined!!!**

So in order to cancel rapidity divergences, we define the TMDPDFs as:

$$G^{\mu\nu}_{g/A}(x_A, \boldsymbol{k}_{n\perp}, S_A; \zeta_A, \mu^2) = A + B_n$$
$$G^{\mu\nu}_{g/B}(x_B, \boldsymbol{k}_{\bar{n}\perp}, S_B; \zeta_B, \mu^2) = C + B_{\bar{n}}$$

Definition of TMDPDFs: Cancellation of RDs

The goal is to <u>cancel Rapidity Divergences</u>. The particular regulator is irrelevant!!

MGE, Idilbi, Scimemi JHEP'12, PLB'13

- Rapidity regulator I: Δ-regulator (MGE, Idilbi, Scimemi JHEP'12)

$$\tilde{G}^{\mu\nu}_{g/A}(x_A, b_T, S_A; \zeta_A, \mu^2) = \tilde{J}^{\mu\nu}_n(x_A, b_T, S_A; Q^2, \mu^2; \Delta^+)\, \tilde{S}_+^{-1}(b_T; \zeta_B, \mu^2; \Delta^+)$$

$\zeta_A = Q^2/\alpha$
$\zeta_B = Q^2\alpha$

- Rapidity regulator II: rapidity-regulator (eta) (Chiu, Jain, Neill, Rothstein PRL'12)

$$\tilde{G}^{\mu\nu}_{g/A}(x_A, b_T, S_A; \zeta_A, \mu^2) = \tilde{J}^{\mu\nu\,(0)}_n(x_A, b_T, S_A; Q^2, \mu^2; \nu_-; \eta)\, \tilde{S}_-(b_T; \mu^2; \alpha\nu_-; \eta)$$

$\zeta_A = Q^2/\alpha$
$\zeta_B = Q^2\alpha$

- Rapidity regulator III: "combining integrands" (Collins'11)

$$\tilde{G}^{\mu\nu}_{g/A}(x_A, b_T, S_A; \zeta_A, \mu^2) = \lim_{\substack{y_n \to +\infty \\ y_{\bar{n}} \to -\infty}} \tilde{J}^{\mu\nu}_n(x_A, b_T, S_A; \mu^2; y_{\bar{n}}) \sqrt{\frac{\tilde{S}(y_n, y_c)}{\tilde{S}(y_c, y_{\bar{n}})\, \tilde{S}(y_n, y_{\bar{n}})}}$$

$\zeta_A = (p^+)^2 e^{-2y_c}$
$\zeta_B = (\bar{p}^-)^2 e^{+2y_c}$

- **One could also use off-shellnesses, masses, "real Δ's", analytic regulator, etc... Yet they all mean (pictorially):**

$$\tilde{G}^{\mu\nu}_{g/A}(x_A, b_T, S_A; \zeta_A, \mu^2) = A + B_n$$

Previous slide!

New TMD factorization expressions

$$\frac{d\sigma}{d\Omega d^4q} = \int d^2b\, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T} \tilde{W}(\boldsymbol{b}, Q; x, y, z) + \mathcal{O}\left(Q_T^2/Q^2\right)$$

$$\tilde{W}(\boldsymbol{b}, Q; x, y, z) = \sum_a \tilde{f}_1^a(x, \boldsymbol{b}^2; \zeta_F, \mu)\tilde{D}_1^a(z, \boldsymbol{b}^2; \zeta_D, \mu) H\left(y, Q; \mu\right)$$

Take $\mu = Q$

$$H\left(Q; \alpha_s(Q)\right) \propto e_a^2 \left(1 + \alpha_s(Q^2) F_1 + \mathcal{O}(\alpha_s^2)\right)$$

This choice avoids large logarithms in H, but now they will appear in the TMDs

Use renormalization group equations to evolve the TMDs to the scale:

$$\mu_b = C_1/b = 2e^{-\gamma_E}/b \quad (C_1 \approx 1.123)$$

Or to a fixed low (but still perturbative) scale Q_0, although that only works for not too large Q

RG and CS equations

$$\frac{d\ln\tilde{f}(x,b;\zeta,\mu)}{d\ln\sqrt{\zeta}} = \tilde{K}(b;\mu)$$ Collins-Soper equation

$$\frac{d\ln\tilde{f}(x,b;\zeta,\mu)}{d\ln\mu} = \gamma_F(g(\mu);\zeta/\mu^2)$$ RG equation

$$d\tilde{K}/d\ln\mu = -\gamma_K(g(\mu))$$

$$\gamma_F(g(\mu);\zeta/\mu^2) = \gamma_F(g(\mu);1) - \frac{1}{2}\gamma_K(g(\mu))\ln(\zeta/\mu^2)$$

Using these equations one can evolve the TMDs to the scale μ_b

$$\tilde{f}_1^a(x,b^2;\zeta_F,\mu)\,\tilde{D}_1^b(z,b^2;\zeta_D,\mu) = e^{-S(b,Q)}\,\tilde{f}_1^a(x,b^2;\mu_b^2,\mu_b)\,\tilde{D}_1^b(z,b^2;\mu_b^2,\mu_b)$$

with Sudakov factor

$$S(b,Q) = -\ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b,\mu_b) - \int_{\mu_b^2}^{Q^2}\frac{d\mu^2}{\mu^2}\left[\gamma_F(g(\mu);1) - \frac{1}{2}\ln\left(\frac{Q^2}{\mu^2}\right)\gamma_K(g(\mu))\right]$$

Perturbative expressions

At leading order in α_s

$$\tilde{K}(b,\mu) = -\alpha_s(\mu)\frac{C_F}{\pi}\ln(\mu^2 b^2/C_1^2) + \mathcal{O}(\alpha_s^2)$$

$$\gamma_K(g(\mu)) = 2\alpha_s(\mu)\frac{C_F}{\pi} + \mathcal{O}(\alpha_s^2)$$

$$\gamma_F(g(\mu),\zeta/\mu^2) = \alpha_s(\mu)\frac{C_F}{\pi}\left(\frac{3}{2} - \ln\left(\zeta/\mu^2\right)\right) + \mathcal{O}(\alpha_s^2)$$

Such that the perturbative expression for the Sudakov factor becomes:

$$S_p(b,Q) = \frac{C_F}{\pi}\int_{\mu_b^2}^{Q^2}\frac{d\mu^2}{\mu^2}\alpha_s(\mu)\left(\ln\frac{Q^2}{\mu^2} - \frac{3}{2}\right) + \mathcal{O}(\alpha_s^2)$$

It can be used whenever the restriction $b^2 << 1/\Lambda^2$ is justified (e.g. at very large Q^2)

If also larger b contributions are important, at moderate Q and small Q_T for instance, then one needs to include a *nonperturbative* Sudakov factor

Nonperturbative Sudakov factor

$$\tilde{W}(b) \equiv \tilde{W}(b_*)\, e^{-S_{NP}(b)} \qquad b_* = b/\sqrt{1+b^2/b_{\max}^2} \leq b_{\max}$$

$$b_{\max} = 1.5\ \mathrm{GeV}^{-1} \Rightarrow \alpha_s(b_0/b_{\max}) = 0.62$$

such that W(b_*) can be calculated within perturbation theory
In general the nonperturbative Sudakov factor is Q dependent and of the form:

$$S_{NP}(b,Q) = \ln(Q^2/Q_0^2)g_1(b) + g_A(x_A,b) + g_B(x_B,b) \qquad Q_0 = \frac{1}{b_{\max}}$$

Collins, Soper & Sterman, NPB 250 (1985) 199

The $g_{..}$ functions need to be fitted to data

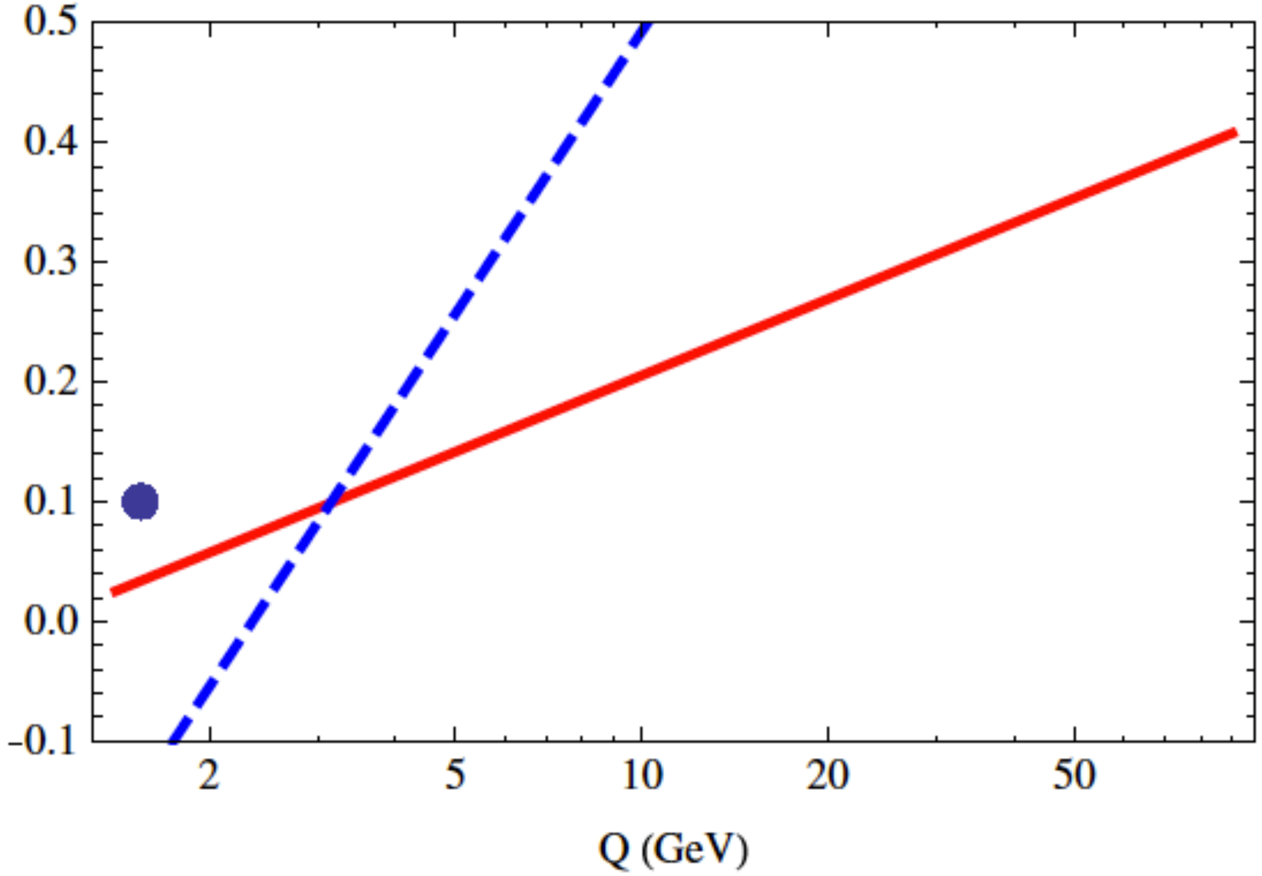
Until recently S_{NP} typically chosen as a Gaussian, e.g. Aybat & Rogers (x=0.1):

$$S_{NP}(b,Q,Q_0) = \left[0.184 \ln \frac{Q}{2Q_0} + 0.332\right] b^2$$

Recently alternatives considered in: P. Sun & F. Yuan, PRD 88 (2013) 034016
P. Sun, Isaacson, C.-P. Yuan & F. Yuan, arXiv:1406.3073

Form suggested by Collins at QCD evolution workshop 2013: $e^{-m\left(\sqrt{b^2+b_0^2}-b_0\right)}$

S_{NP}



Problem is to find one single universal S_{NP} that describes both SIDIS and DY/Z data

Figure 6. Coefficient of $-b_T^2$ in the exponent in Eq. (6), from Sun and Yuan [13], as a function of Q at $x = 0.1$. The blue dashed line is for the BLNY fit, and the red solid line for a KN fit with $b_{\max} = 1.5\,\text{GeV}^{-1}$. The dot represents the value needed for SIDIS at HERMES.

From Collins, 1409.5408 based on P. Sun & F. Yuan, PRD 88 (2013) 034016

BLNY = Brock, Landry, Nadolsky, C.-P. Yuan, PRD67 (2003) 073016
KN = Konychev & Nadolsky, PLB 633 (2006) 710

Further resummations

$$\tilde{F}(x, b_T; \zeta_f, \mu_f) = \tilde{R}(b_T; \zeta_i, \mu_i, \zeta_f, \mu_f)\tilde{F}(x, b_T; \zeta_i, \mu_i)$$

Evolutor:

$$\tilde{R}(b_T; \zeta_i, \mu_i, \zeta_f, \mu_f) = \exp\left\{\int_{\mu_i}^{\mu_f} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_F\left(\alpha_s(\bar{\mu}), \ln\frac{\zeta_f}{\bar{\mu}^2}\right)\right\}\left(\frac{\zeta_f}{\zeta_i}\right)^{-D(b_T;\mu_i)}$$

$$\boxed{D(b_T, \mu) = -\frac{1}{2}\tilde{K}(b_T, \mu)} \qquad \frac{dD(b_T, \mu)}{d\ln\mu} = \Gamma_{\mathrm{cusp}} = \frac{1}{2}\gamma_K$$

Echevarria, Idilbi, Schäfer, Scimemi, EPJC 73 (2013) 2636:

$$\begin{aligned} D^R(b_T;\mu) = &-\frac{\Gamma_0}{2\beta_0}\ln(1-X) + \frac{1}{2}\left(\frac{a_s}{1-X}\right)\left[-\frac{\beta_1\Gamma_0}{\beta_0^2}(X+\ln(1-X)) + \frac{\Gamma_1}{\beta_0}X\right] \\ &+ \frac{1}{2}\left(\frac{a_s}{1-X}\right)^2\left[2d_2(0) + \frac{\Gamma_2}{2\beta_0}(X(2-X)) + \frac{\beta_1\Gamma_1}{2\beta_0^2}\left(X(X-2) - 2\ln(1-X)\right) + \frac{\beta_2\Gamma_0}{2\beta_0^2}X^2\right. \\ &\left. + \frac{\beta_1^2\Gamma_0}{2\beta_0^3}(\ln^2(1-X) - X^2)\right], \end{aligned}$$

where we have used the notation

$$a_s = \frac{\alpha_s(\mu)}{4\pi}, \qquad X = a_s\beta_0 L_T, \qquad L_T = \ln\frac{\mu^2 b_T^2}{4e^{-2\gamma_E}} = \ln\frac{\mu^2}{\mu_b^2}.$$

Convergence fails as b approaches b_X which to leading order is $b_X = \frac{C_1}{\mu_i} \exp\left(\frac{2\pi}{\beta_0 \alpha_s(\mu_i)}\right)$

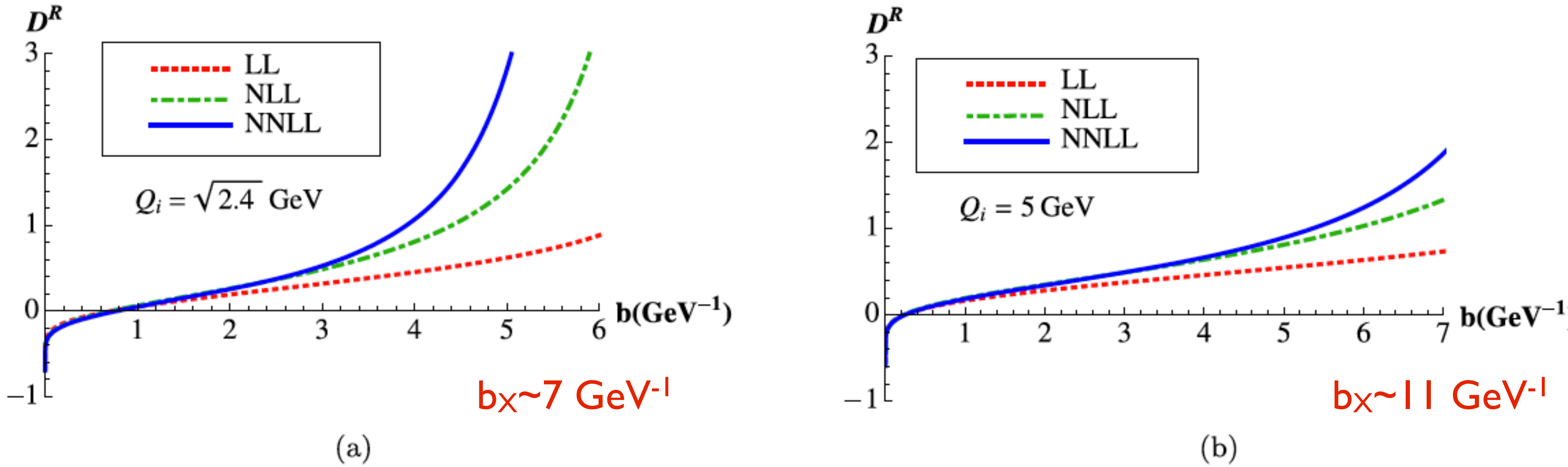


Fig. 1 Resummed D at $Q_i = \sqrt{2.4}$ GeV with $n_f = 4$ (**a**) and $Q_i = 5$ GeV with $n_f = 5$ (**b**)

Echevarria, Idilbi, Schäfer, Scimemi, EPJC 73 (2013) 2636

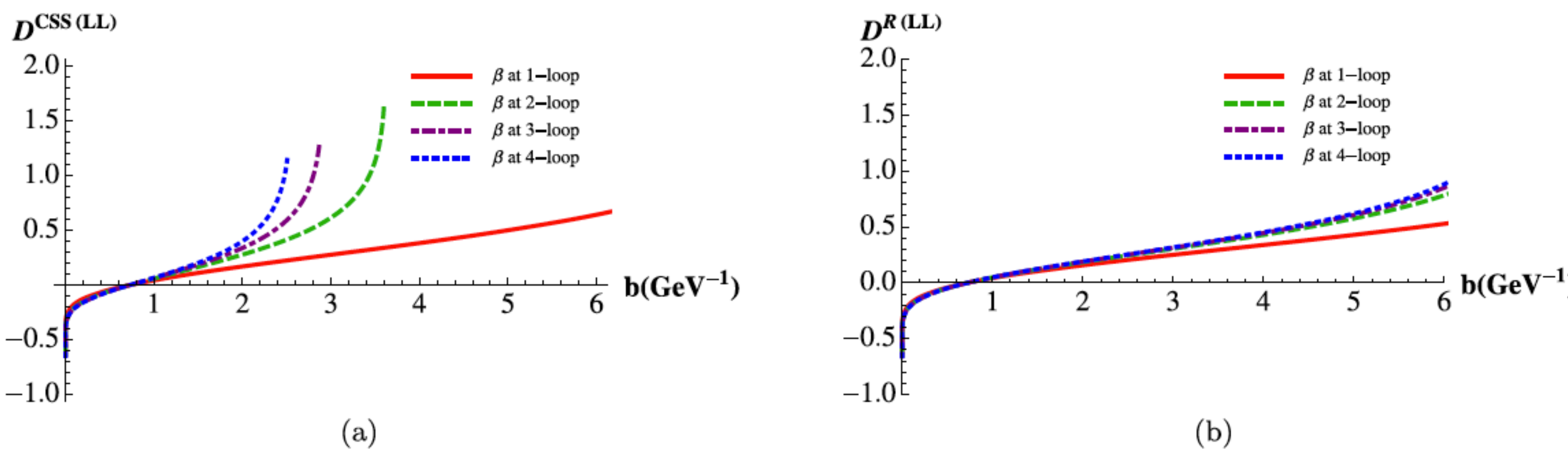


Fig. 3 Resummed $D(b; Q_i = \sqrt{2.4})$ at LL of Eqs. (25), (**a**), and (26), (**b**), with the running of the strong coupling at various orders and decoupling coefficients included

Evolutor R vanishes well before $b \sim b_X$ if $Q_f \gg Q_i$, reduces sensitivity to large b region

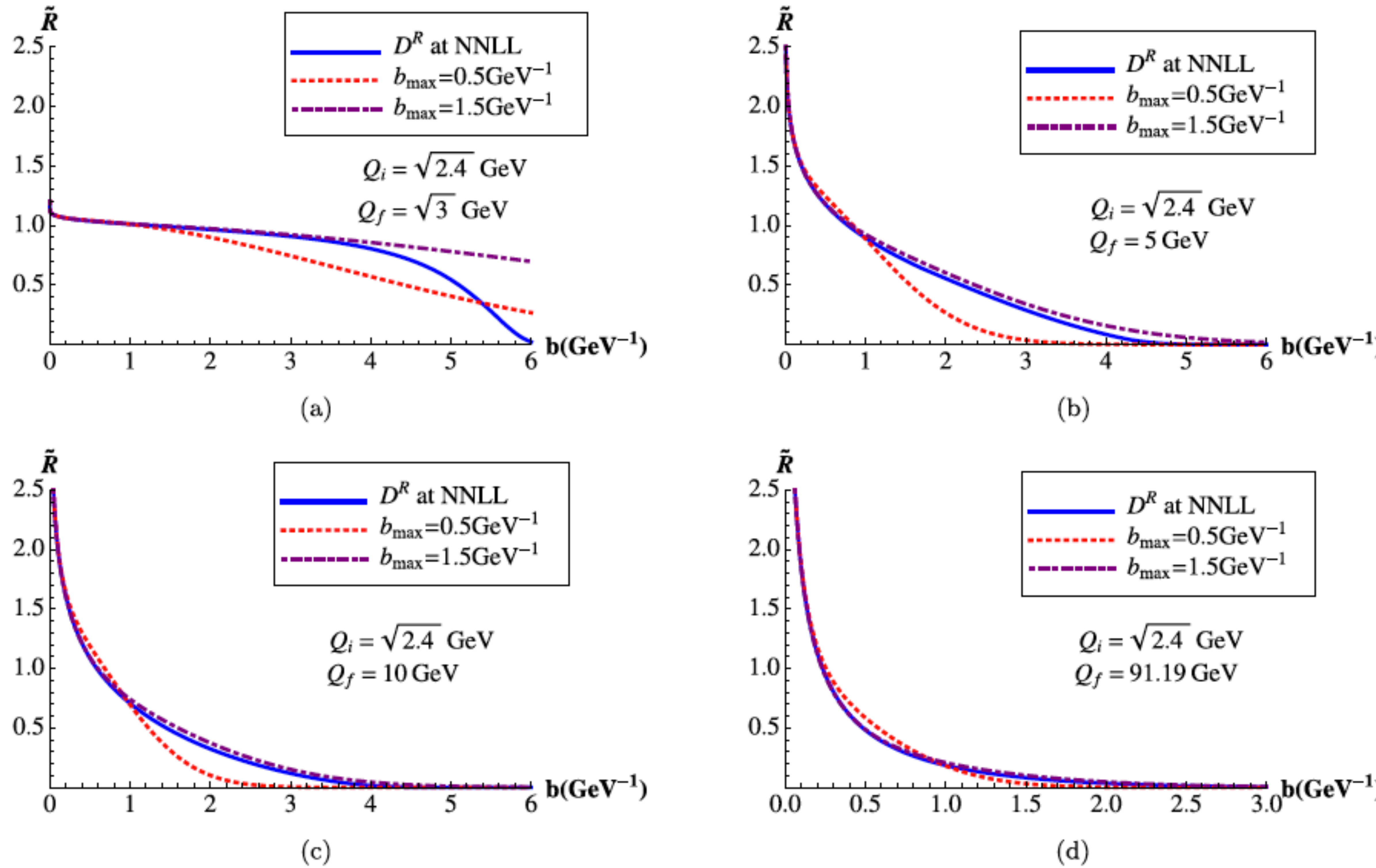


Fig. 4 Evolution kernel from $Q_i = \sqrt{2.4}$ GeV up to $Q_f = \{\sqrt{3}, 5, 10, 91.19\}$ GeV using ours and CSS approaches, both at NNLL

Echevarria, Idilbi, Schäfer, Scimemi, EPJC 73 (2013) 2636

This approach favors b_{max} = 1.5 GeV^{-1}

Further resummations

For the TMD at small b one often considers the perturbative tail, which is calculable

$$\tilde{f}_{g/P}(x, b^2; \mu, \zeta) = \sum_{i=g,q} \int_x^1 \frac{d\hat{x}}{\hat{x}} C_{i/g}(x/\hat{x}, b^2; g(\mu), \mu, \zeta) f_{i/P}(\hat{x}; \mu) + \mathcal{O}((\Lambda_{\rm QCD} b)^a)$$

To extend it to be valid at larger b values one can perform further resummation:

$$\tilde{F}^{\rm pert}_{q/N}(x, b_T; \zeta, \mu) = \left(\frac{\zeta b_T^2}{4e^{-2\gamma_E}}\right)^{-D^R(b_T;\mu)} e^{h_\Gamma^R(b_T;\mu) - h_\gamma^R(b_T;\mu)} \sum_j \int_x^1 \frac{dz}{z} \hat{C}_{q\leftarrow j}(x/z, b_T; \mu)\, f_{j/N}(z; \mu)$$

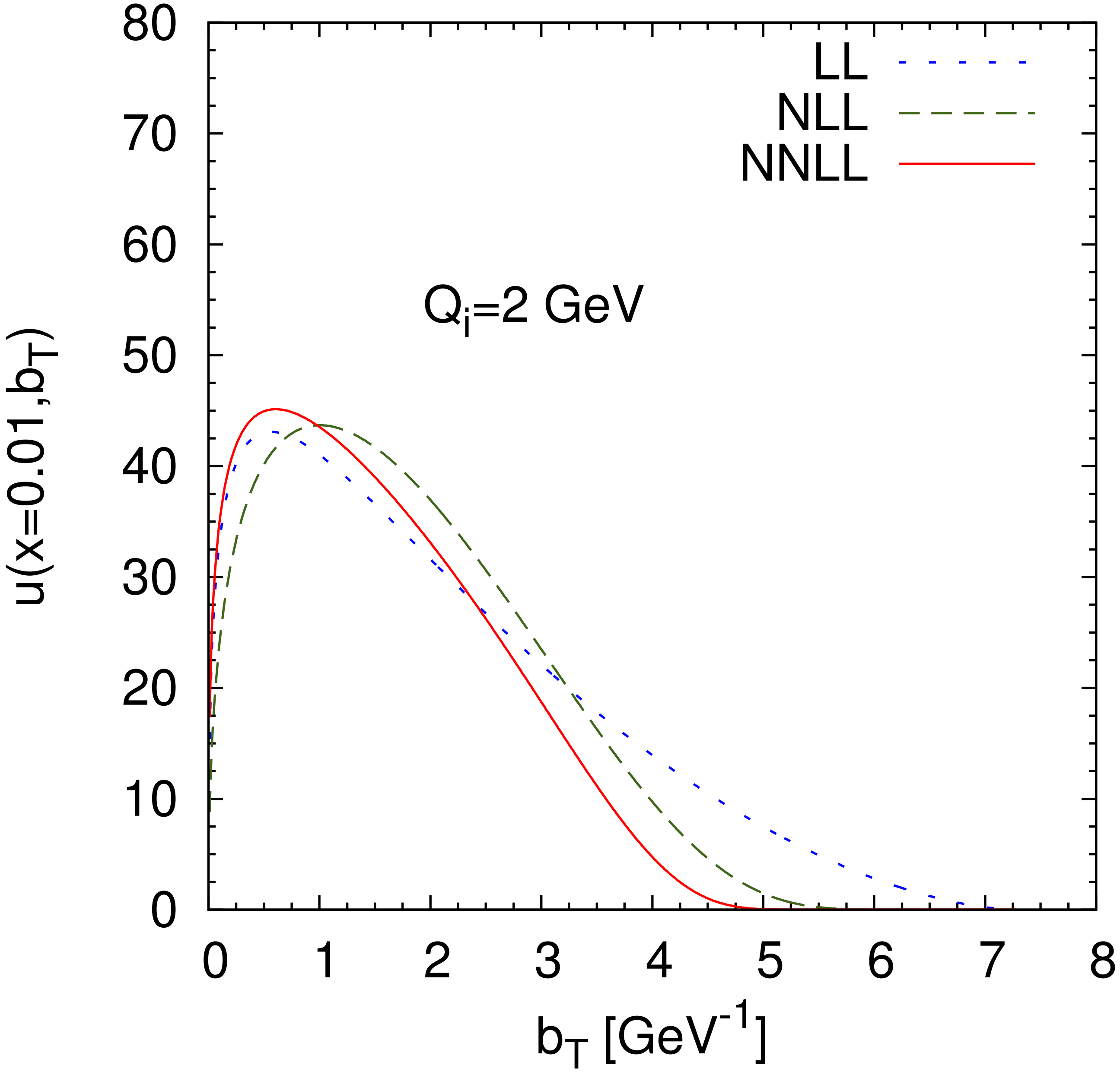
$$\tilde{F}_{q/N}(x, b_T; Q_i^2, \mu_i) = \tilde{F}^{\rm pert}_{q/N}(x, b_T; Q_i^2, \mu_i)\, \tilde{F}^{\rm NP}_{q/N}(x, b_T; Q_i)$$

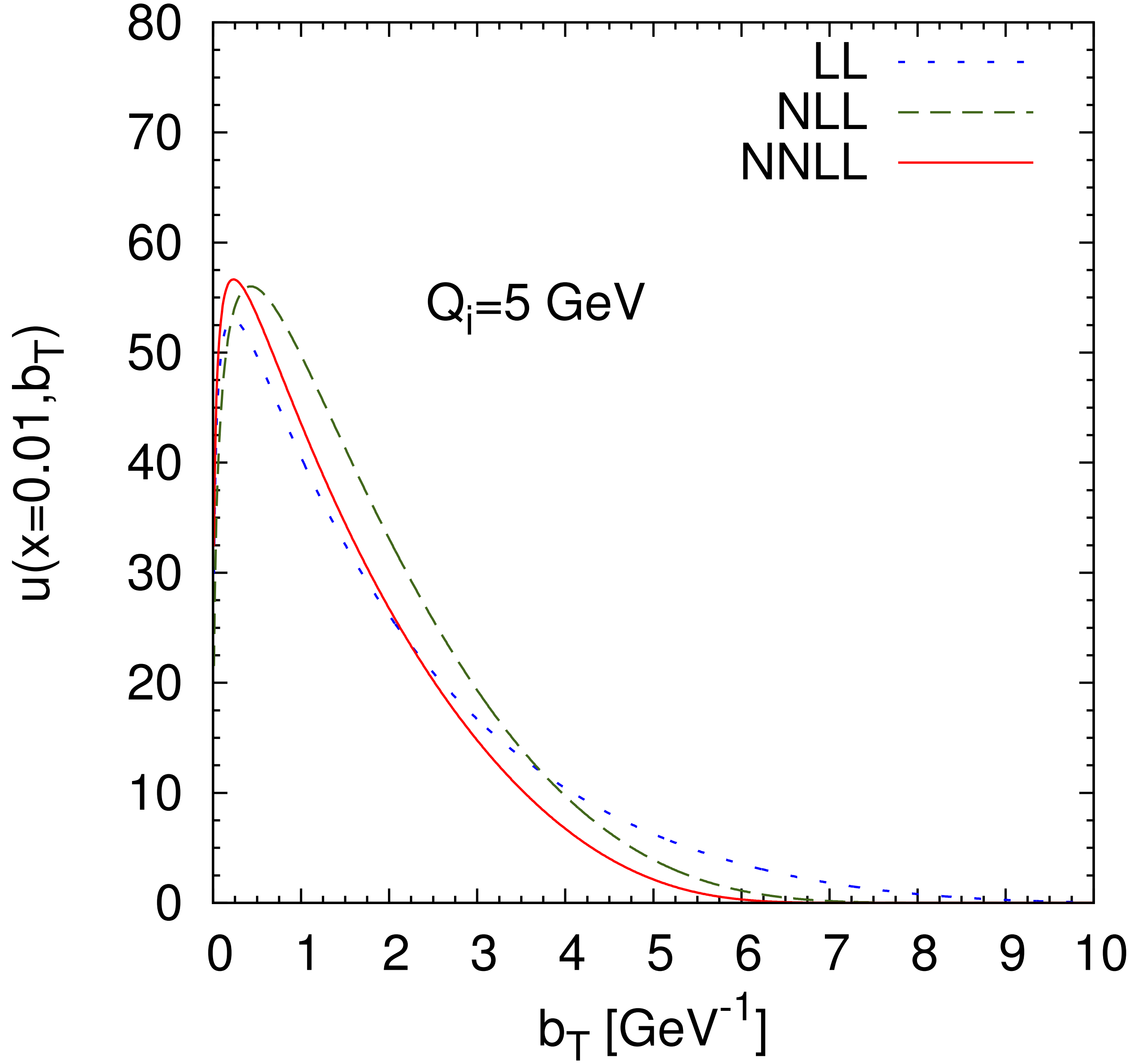
$$\tilde{F}^{\rm NP}_{q/N}(x, b_T; Q_i) \equiv \tilde{F}^{\rm NP}_{q/N}(x, b_T) \left(\frac{Q_i^2}{Q_0^2}\right)^{-D^{\rm NP}(b_T)}$$

D'Alesio, Echevarria, Melis, Scimemi, arXiv:1407.3311

Further resummations

$$\tilde{F}^{\text{pert}}_{q/N}(x, b_T; \zeta, \mu) = \left(\frac{\zeta b_T^2}{4e^{-2\gamma_E}}\right)^{-D^R(b_T;\mu)} e^{h^R_\Gamma(b_T;\mu) - h^R_\gamma(b_T;\mu)} \sum_j \int_x^1 \frac{dz}{z} \hat{C}_{q\leftarrow j}(x/z, b_T; \mu)\, f_{j/N}(z; \mu)$$

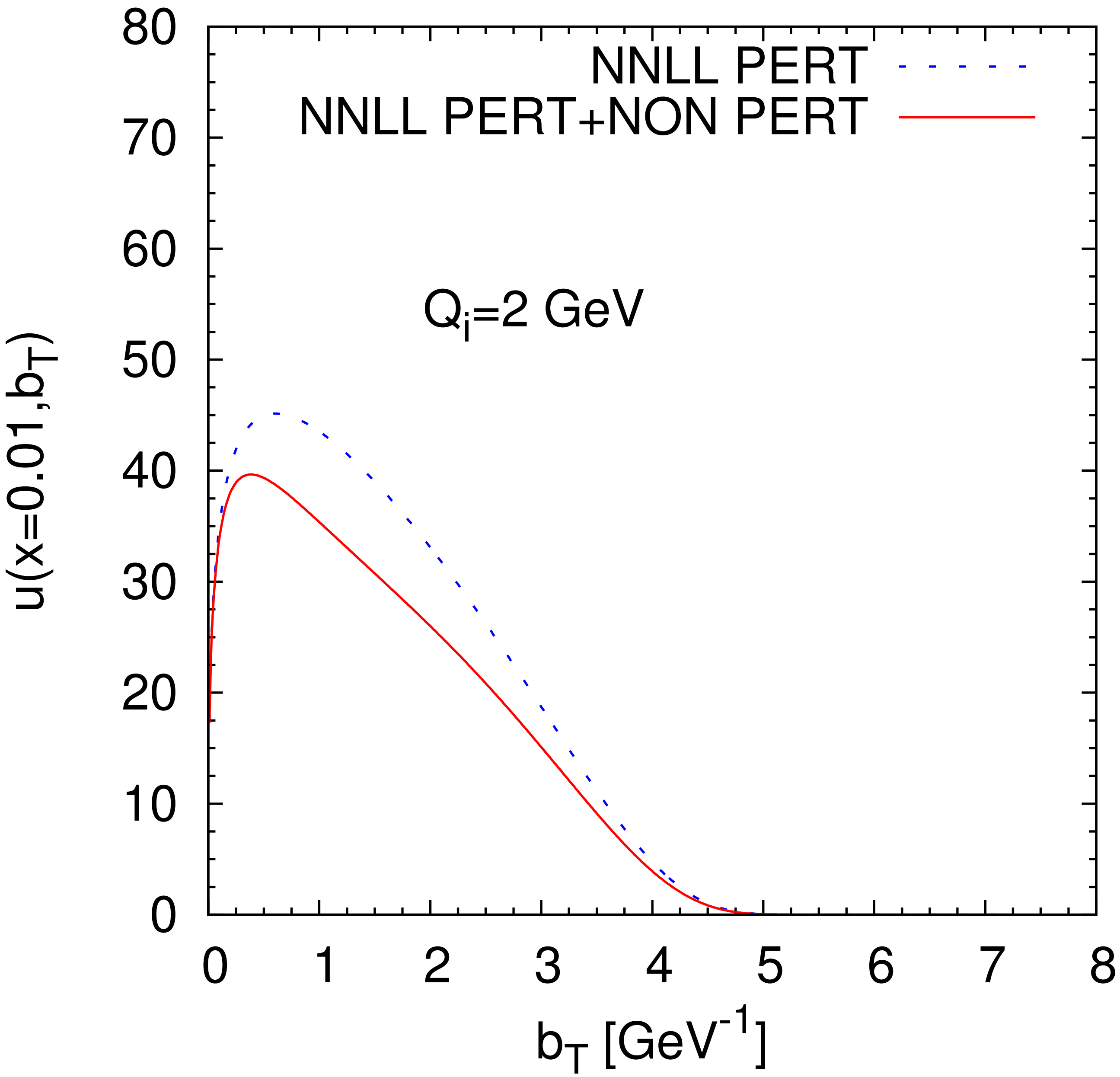




D'Alesio, Echevarria, Melis, Scimemi, arXiv:1407.3311

Resummed TMD at low scales is reduced at large b_T where $\alpha_s(\mu_b)$ is very large

New approach to Landau pole problem



Sensitivity to Landau pole minimized by using $Q_i = Q_0 + q_T$ rather than μ_b

Correspondingly a new F^{NP} form is considered

High Q data (DY/Z) need only λ_1 & λ_2
Low Q (SIDIS) needs modification (λ_3)

$$\tilde{F}^{\rm NP}_{q/N}(x, b_T; Q) = e^{-\lambda_1 b_T}\,(1+\lambda_2 b_T^2)\left(\frac{Q^2}{Q_0^2}\right)^{-\frac{\lambda_3}{2} b_T^2}$$

D'Alesio, Echevarria, Melis, Scimemi, arXiv:1407.3311

Comparison

Formalisms used: They don't all appear compatible

Parton model:	QCD complications ignored
Original CSS:	non-light-like axial gauge; soft factor
Ji–Ma–Yuan:	non-light-like Wilson lines; soft factor; parameter ρ
New CSS:	clean up, Wilson lines mostly light-like;
	absorb (square roots of) soft factor in TMD pdfs
Becher–Neubert:	SCET, but without actual finite TMD pdfs
Echevarría–Idilbi–Scimemi:	SCET
Mantry–Petriello:	SCET
Boer, Sun-Yuan:	Approximations on CSS

Disagreement on non-perturbative contribution to evolution ($\tilde{K}(b_{\rm T})$ at large $b_{\rm T}$), or even whether it exists.

Collins, QCD Evolution workshop, May 12, 2014

Tool to compare different methods: The L function

(JCC & Rogers, in preparation)

- Shape change of transverse momentum distribution comes only from b_T-dependence of $\tilde{K}$
- So define scheme independent

$$L(b_T) = -\frac{\partial}{\partial \ln b_T^2} \frac{\partial}{\partial \ln Q^2} \ln \tilde{W}(b_T, Q, x_A, x_B) \stackrel{\text{CSS}}{=} -\frac{\partial}{\partial \ln b_T^2} \tilde{K}(b_T, \mu)$$

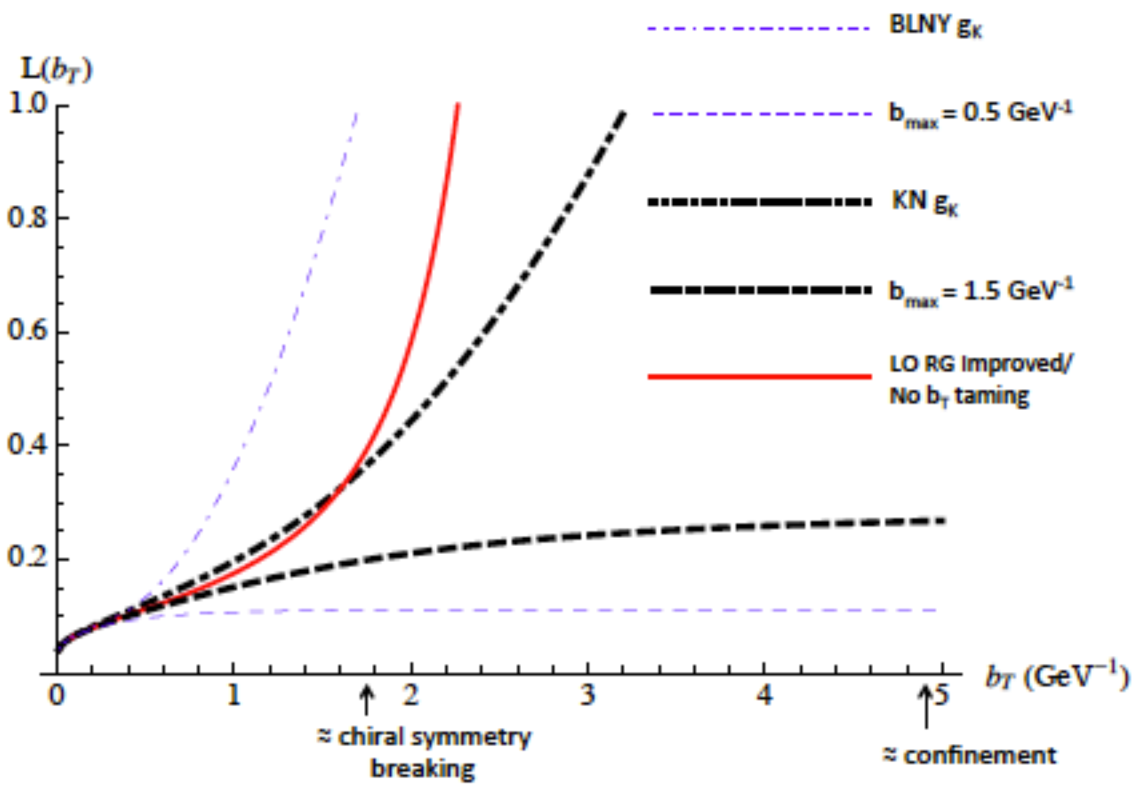
- QCD predicts it is
  - independent of Q, x_A, x_B
  - independent of light-quark flavor
  - RG invariant
  - perturbatively calculable at small b_T
  - non-perturbative at large b_T

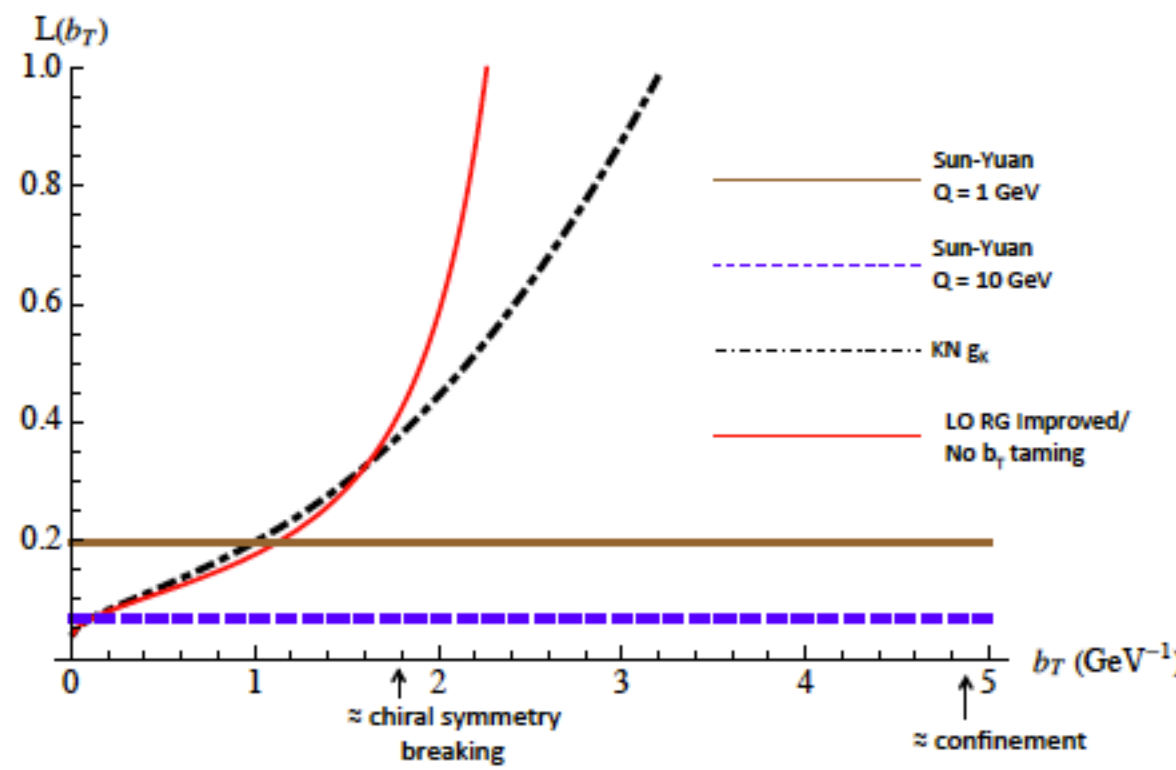
Collins, QCD Evolution workshop, May 12, 2014

L is called A in Collins, 1409.5408

Comparing different results using the L function

(Preliminary)





Q	Typical b_T
$2\,\mathrm{GeV}$	$3\,\mathrm{GeV}^{-1}$
$10\,\mathrm{GeV}$	$1.2\,\mathrm{GeV}^{-1}$
m_Z	$0.5\,\mathrm{GeV}^{-1}$

SY = Sun & Yuan (PRD 88, 114012 (2013)):

$$L_{\mathrm{SY}} = C_F \frac{\alpha_s(Q)}{\pi}$$

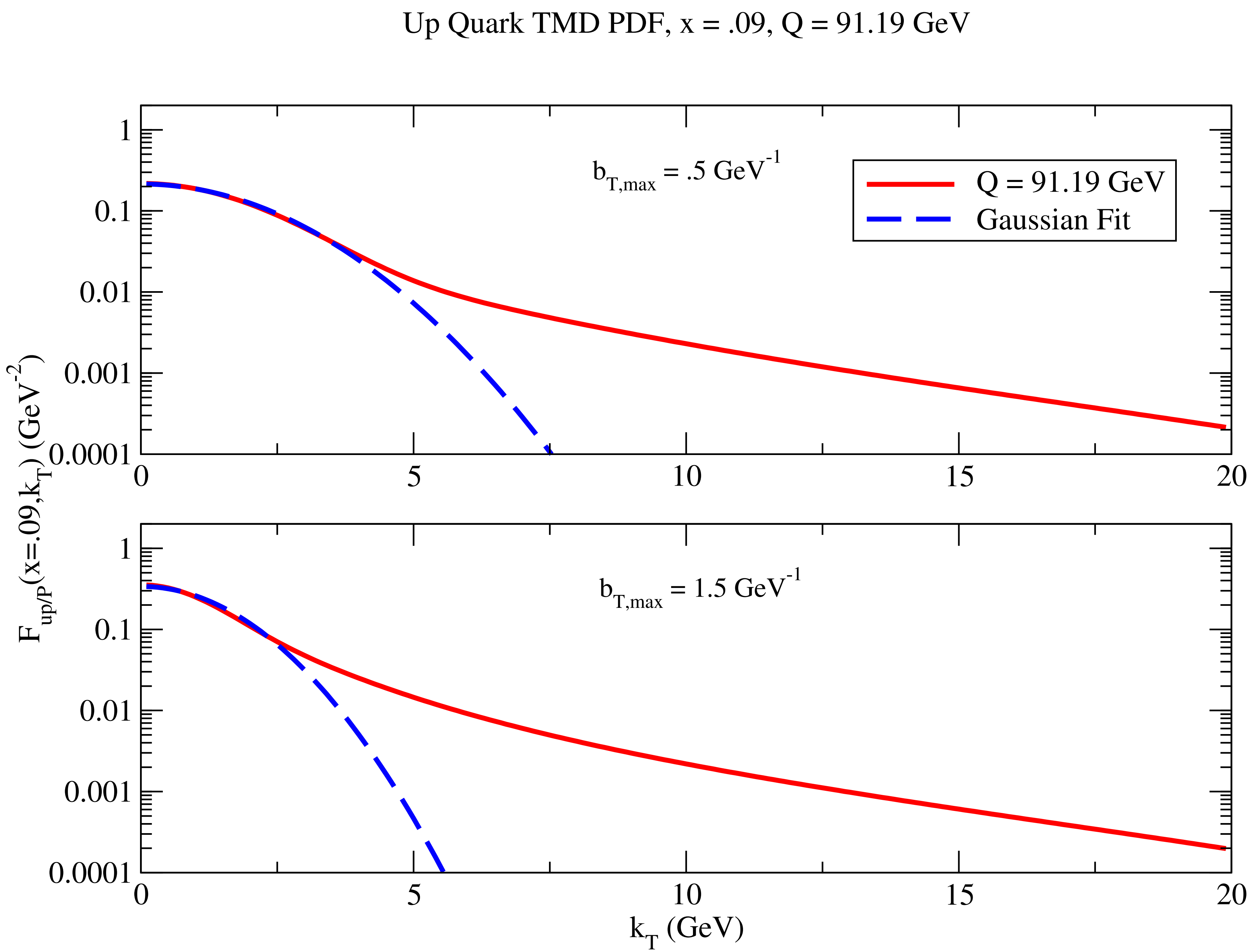
Depends on Q: contrary to QCD

TMD evolution

Large p_T tail

Factorization dictates the evolution:

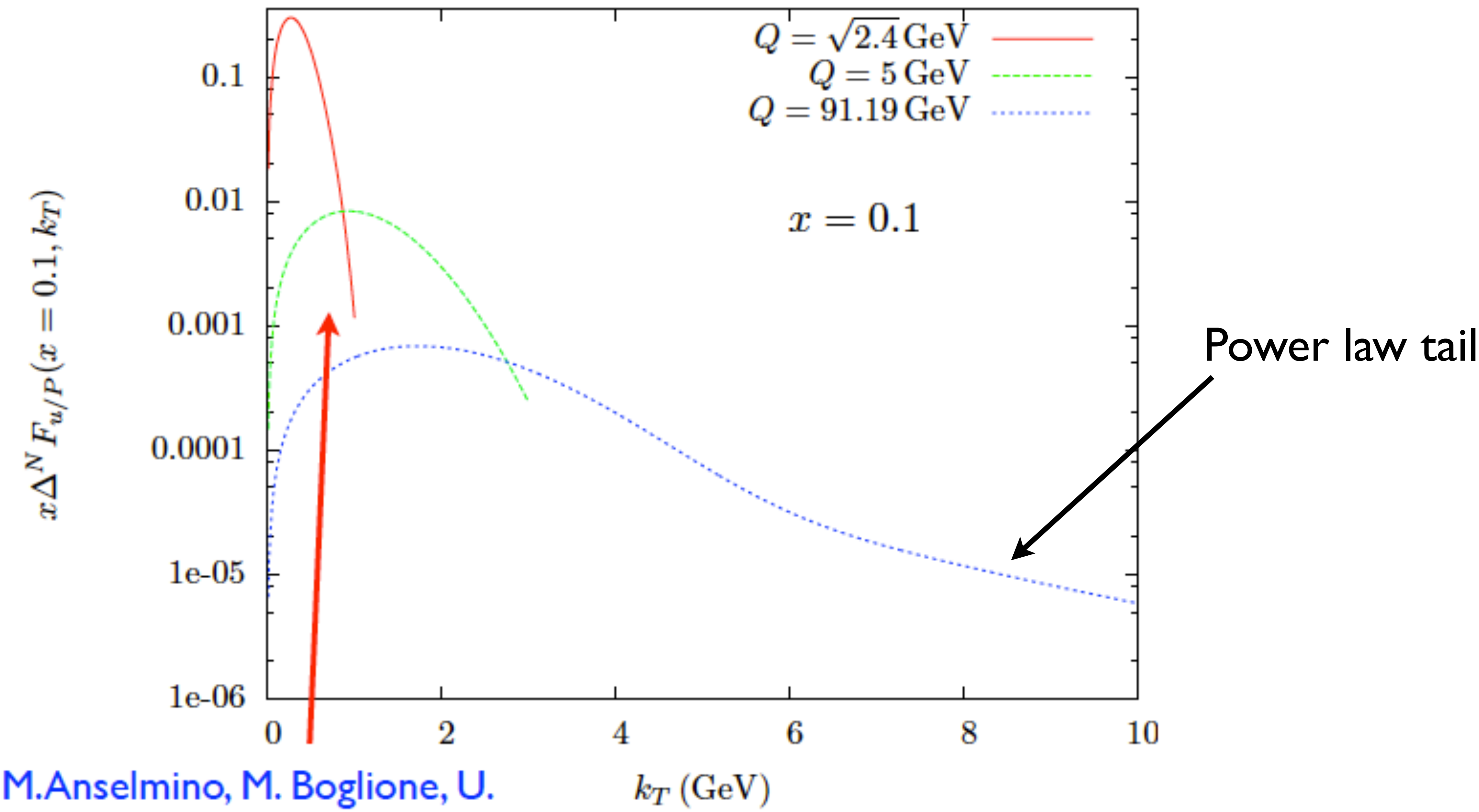
Under evolution TMDs develop a power law tail



Aybat & Rogers, PRD 83 (2011) 114042

Evolution of Sivers function

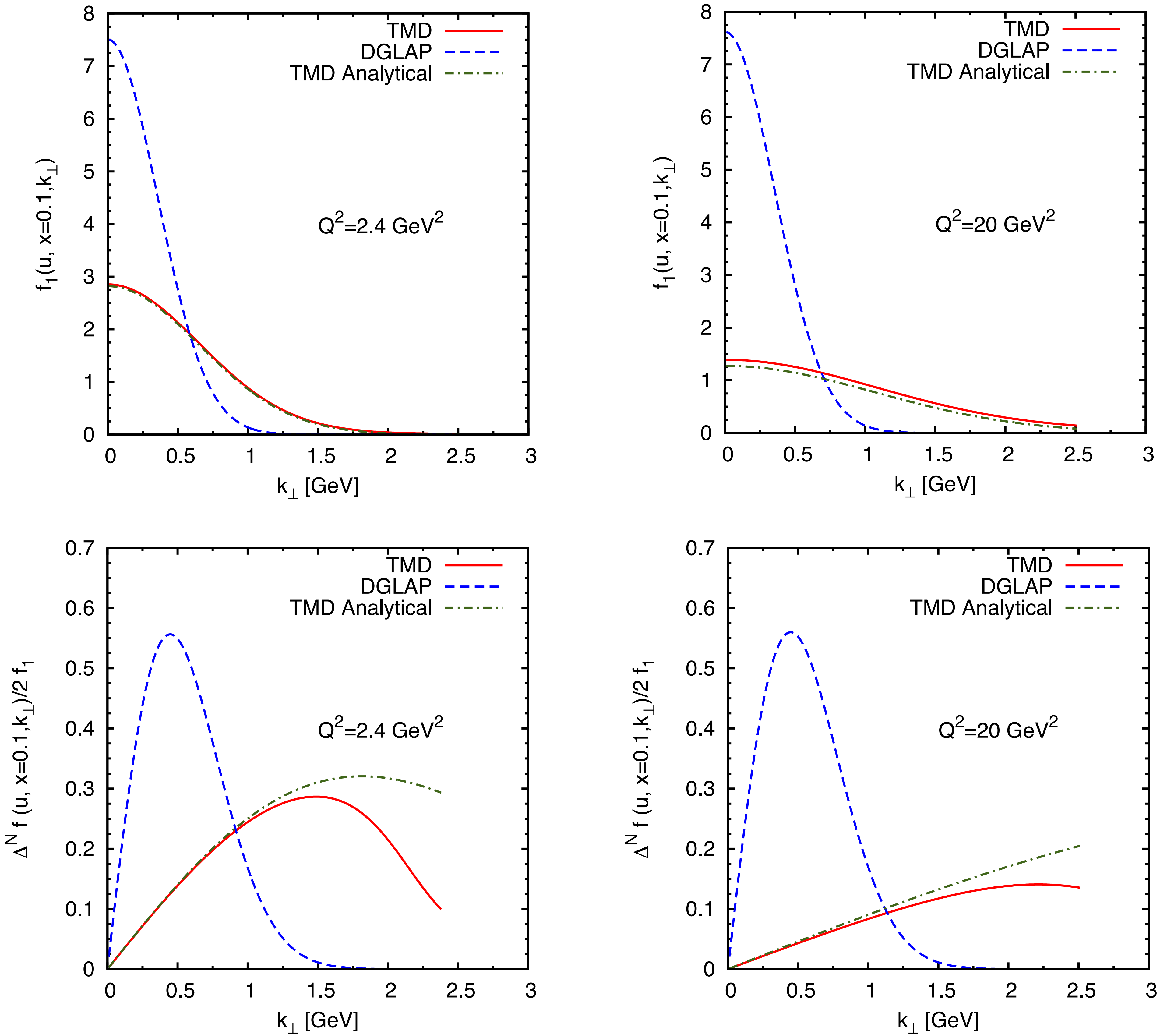
TMDs and their asymmetries become broader and smaller with increasing energy



M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, S. Melis, F. Murgia, A. Prokudin, C. Turk; 2009

Aybat & Rogers, PRD 83 (2011) 114042
Aybat, Collins, Qiu, Rogers, PRD 85 (2012) 034043

Comparing TMD and DGLAP evolution



Anselmino, Boglione, Melis
PRD 86 (2012) 014028

All curves evolved from
$Q^2 = 1\ \mathrm{GeV}^2$

Makes quite a difference in this limited range of Q: from 1.5 to 4.5 GeV

S_{NP} dominates evolution

TMD evolution of azimuthal asymmetries

- Sivers effect in SIDIS and DY

  [Idilbi, Ji, Ma & Yuan, 2004; Aybat, Prokudin & Rogers, 2012; Anselmino, Boglione, Melis, 2012; Sun & Yuan, 2013; D.B., 2013; Echevarria, Idilbi, Kang & Vitev, 2014]

- Collins effect in e^+e^- and SIDIS

  [D.B., 2001 & 2009; Echevarria, Idilbi, Scimemi, 2014]

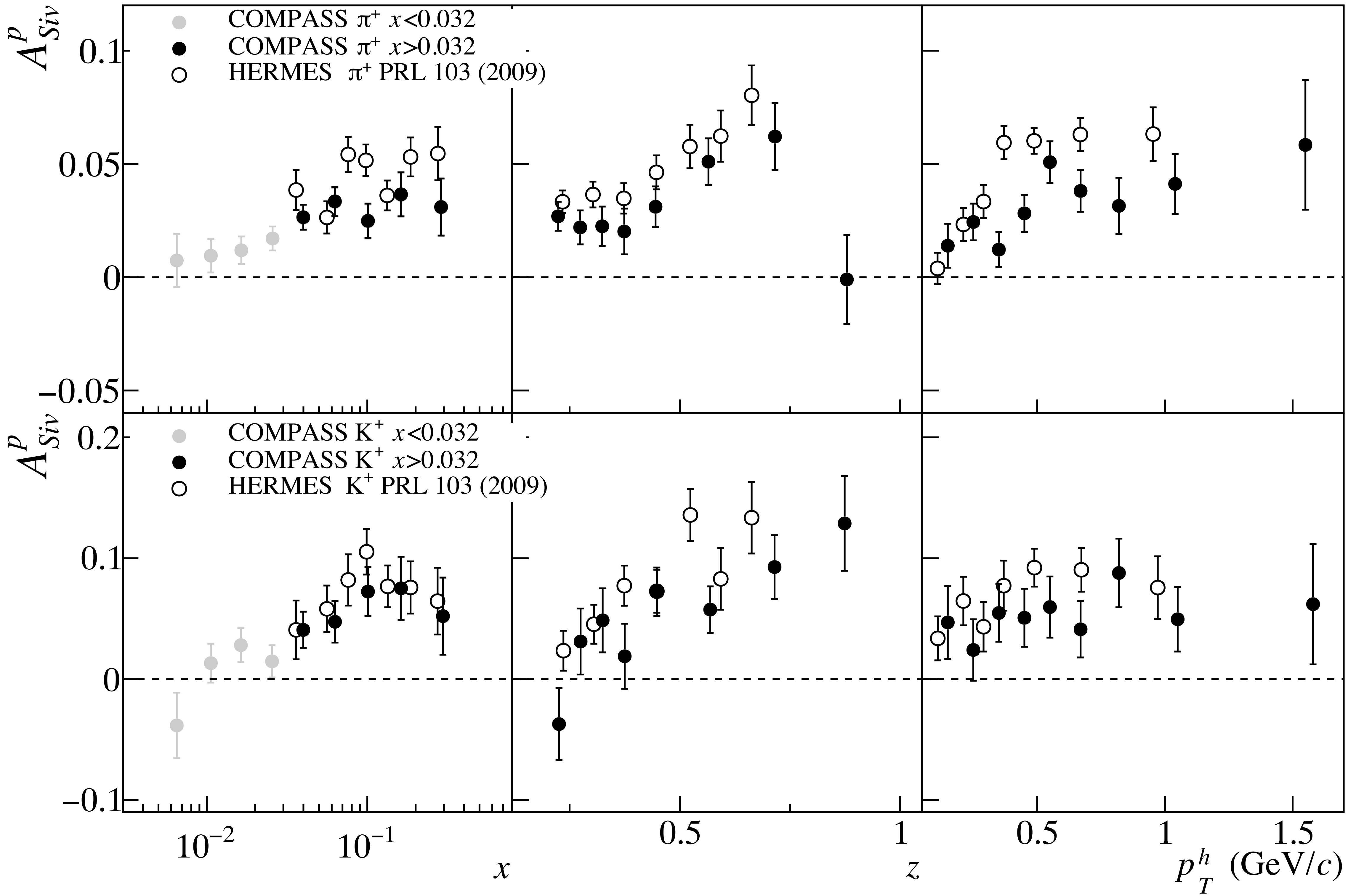
- Sivers effect in J/ψ production

  [Godbole, Misra, Mukherjee, Rawoot, 2013; Godbole, Kaushik, Misra, Rawoot, 2014]

Main differences among the various approaches:
- treatment of nonperturbative Sudakov factor
- treatment of leading logarithms, i.e. the level of perturbative accuracy

TMD evolution of the Sivers asymmetry

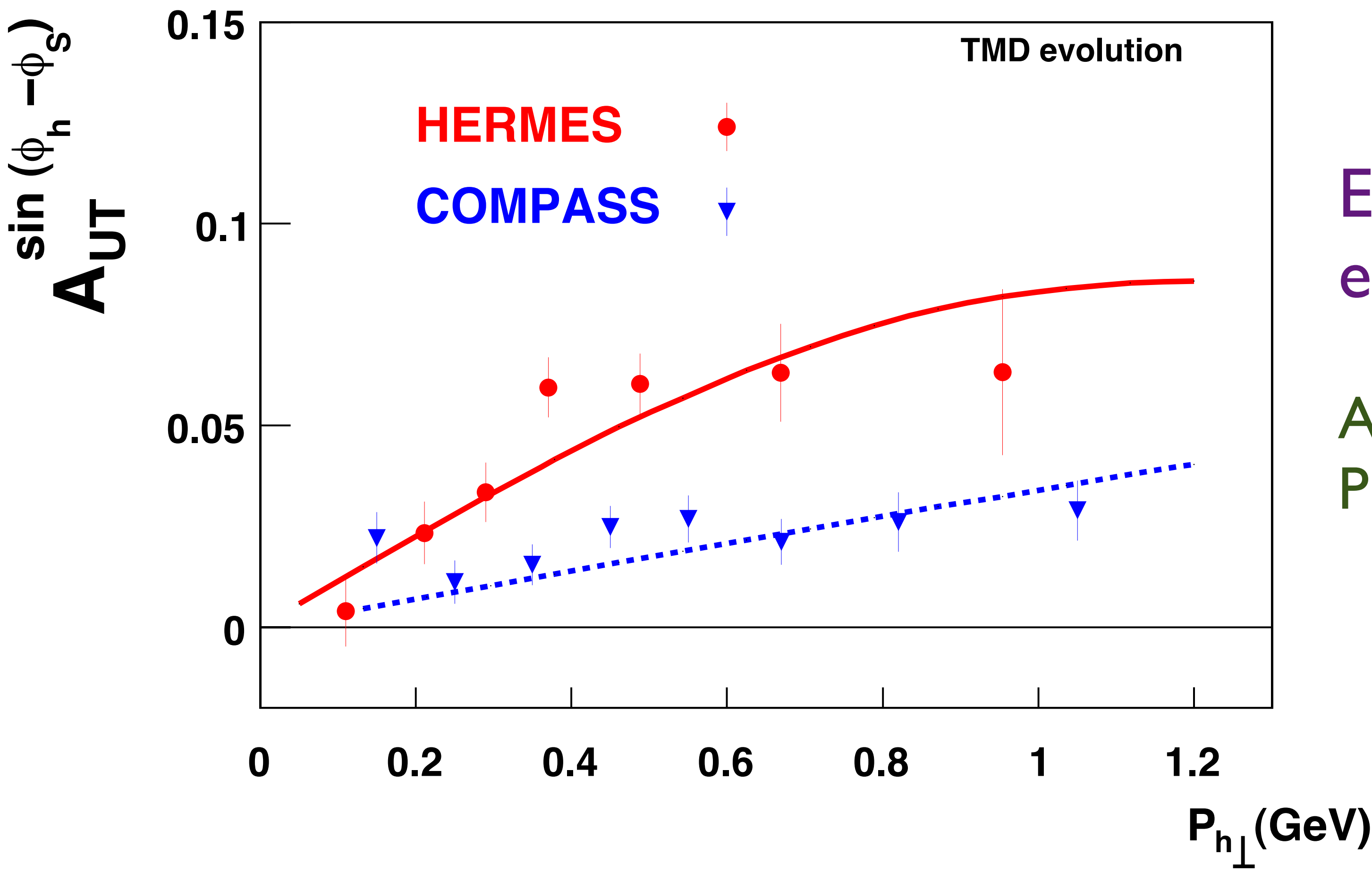
Sivers Asymmetry



[COMPASS, arXiv:1408.4405]

HERMES data ($\langle Q^2 \rangle \sim 2.4$ GeV^2) mostly above COMPASS data ($\langle Q^2 \rangle \sim 3.8$ GeV^2)

Evolution of the Sivers Asymmetry



Evolution from HERMES to COMPASS energy scale seems to work well

Aybat, Prokudin & Rogers,
PRL 108 (2012) 242003

This is obtained using the 2011 TMD factorization, including some approximations that should be applicable at small Q:

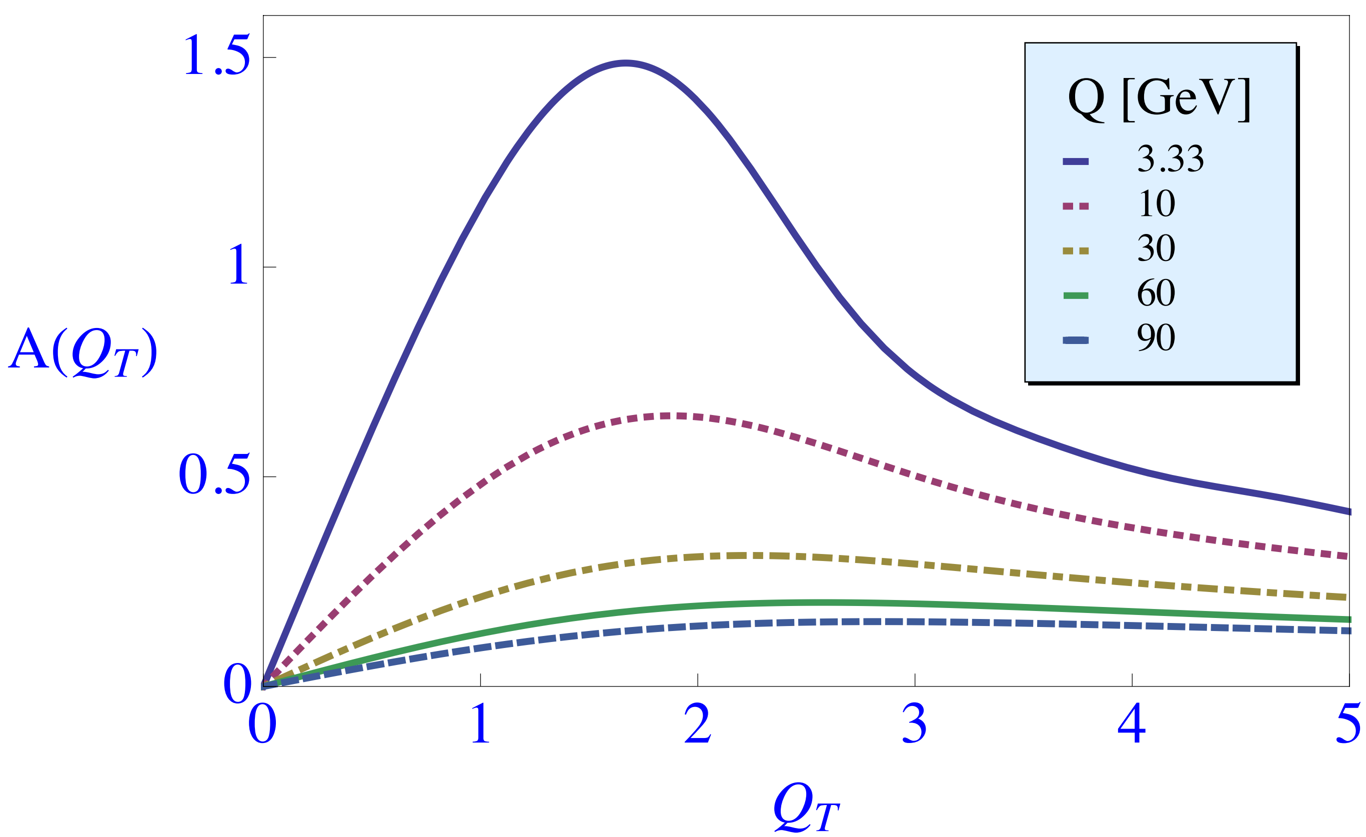
- Y term is dropped (or equivalently the perturbative tail)
- evolution from a fixed starting Q_0 rather than μ_b
- TMDs at starting scale Q_0 Gaussian

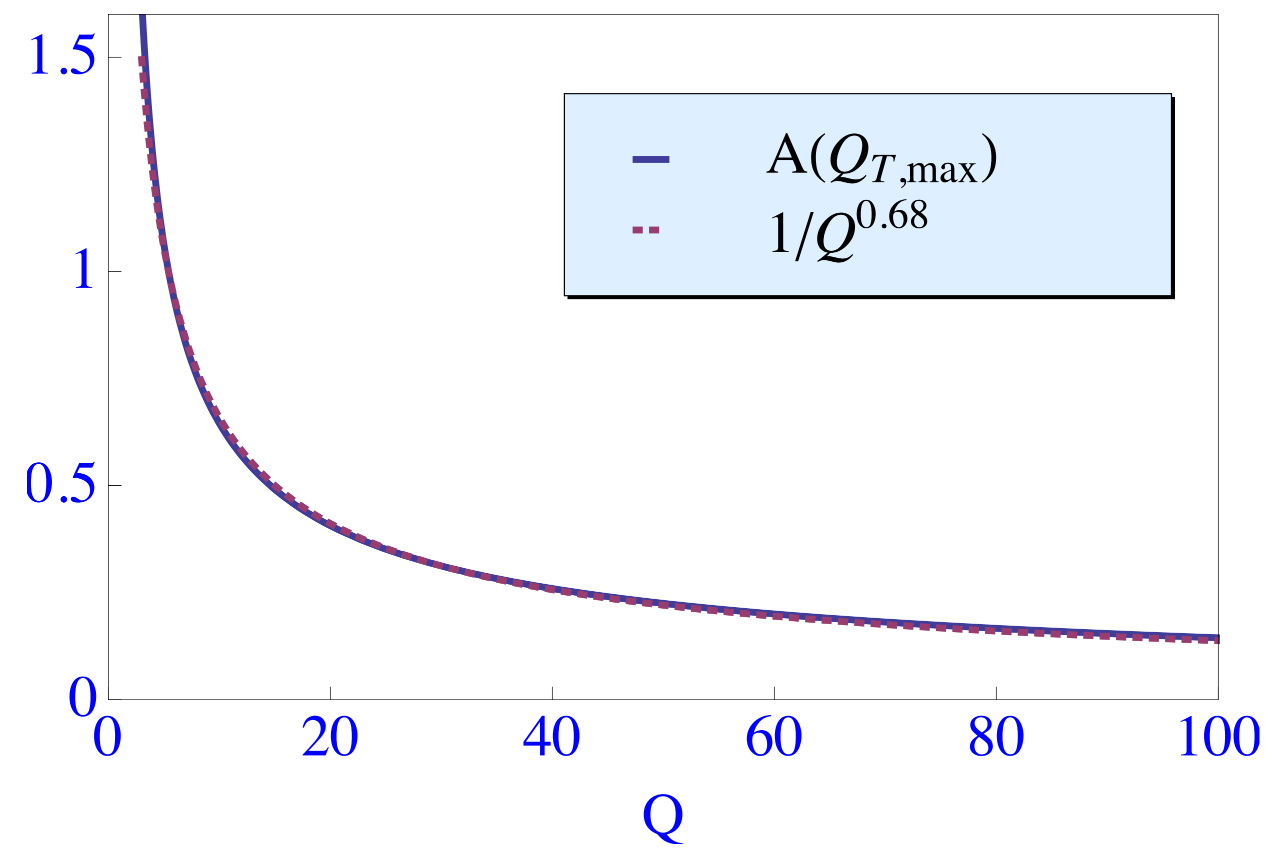
TMD evolution of the Sivers asymmetry

If in addition one assumes that the TMDs of b* are slowly varying functions of b in the dominant b region ($b \sim 1/Q_T >> 1/Q$, hence $b_* \approx b_{max} = 1/Q_0$): $\Phi(x,b_*) \approx \Phi(x,1/Q_0)$, then the Q dependence of the Sivers asymmetry resides in an overall factor:

$$A_{UT}^{\sin(\phi_h-\phi_S)} \propto \mathcal{A}(Q_T, Q)$$

[D.B., NPB 874 (2013) 217]



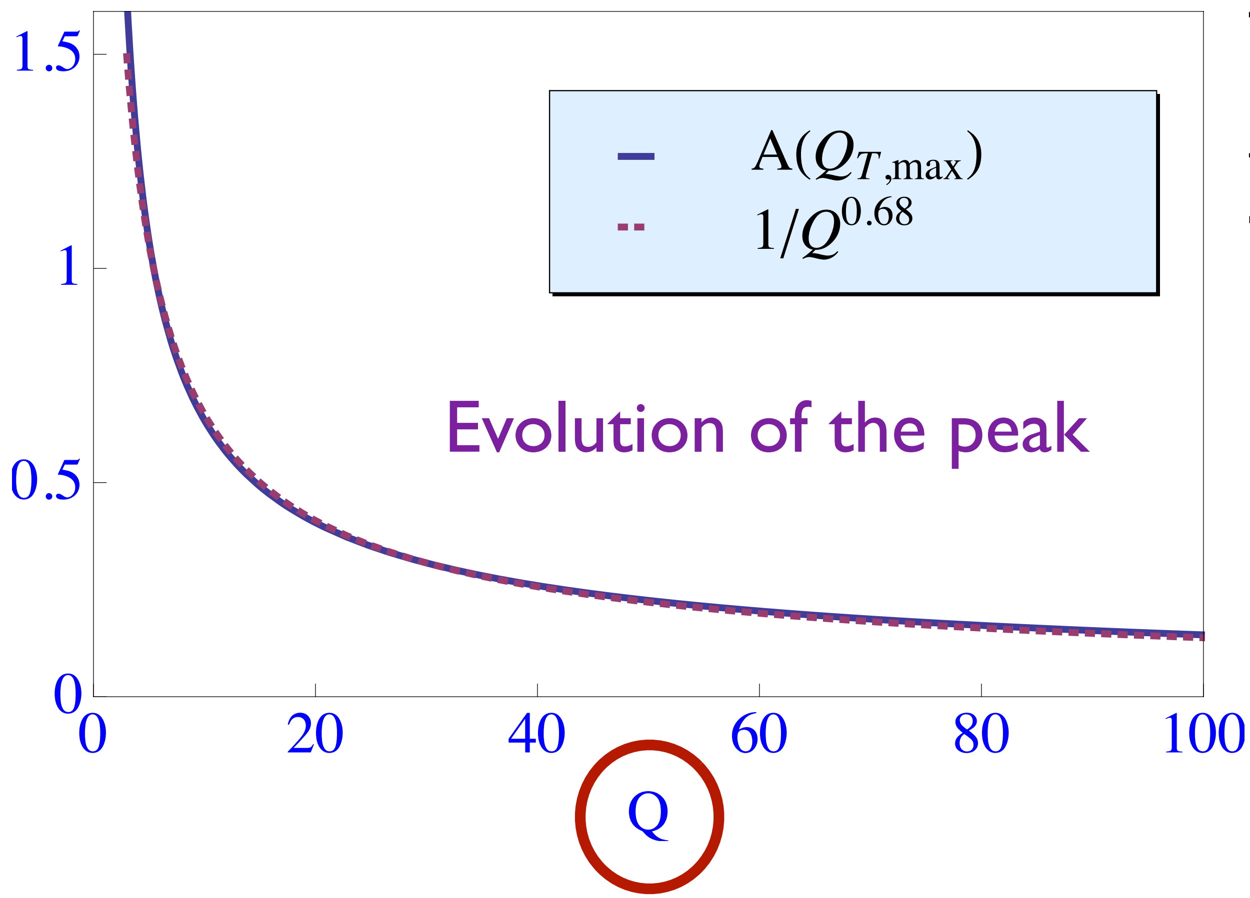


Observations:

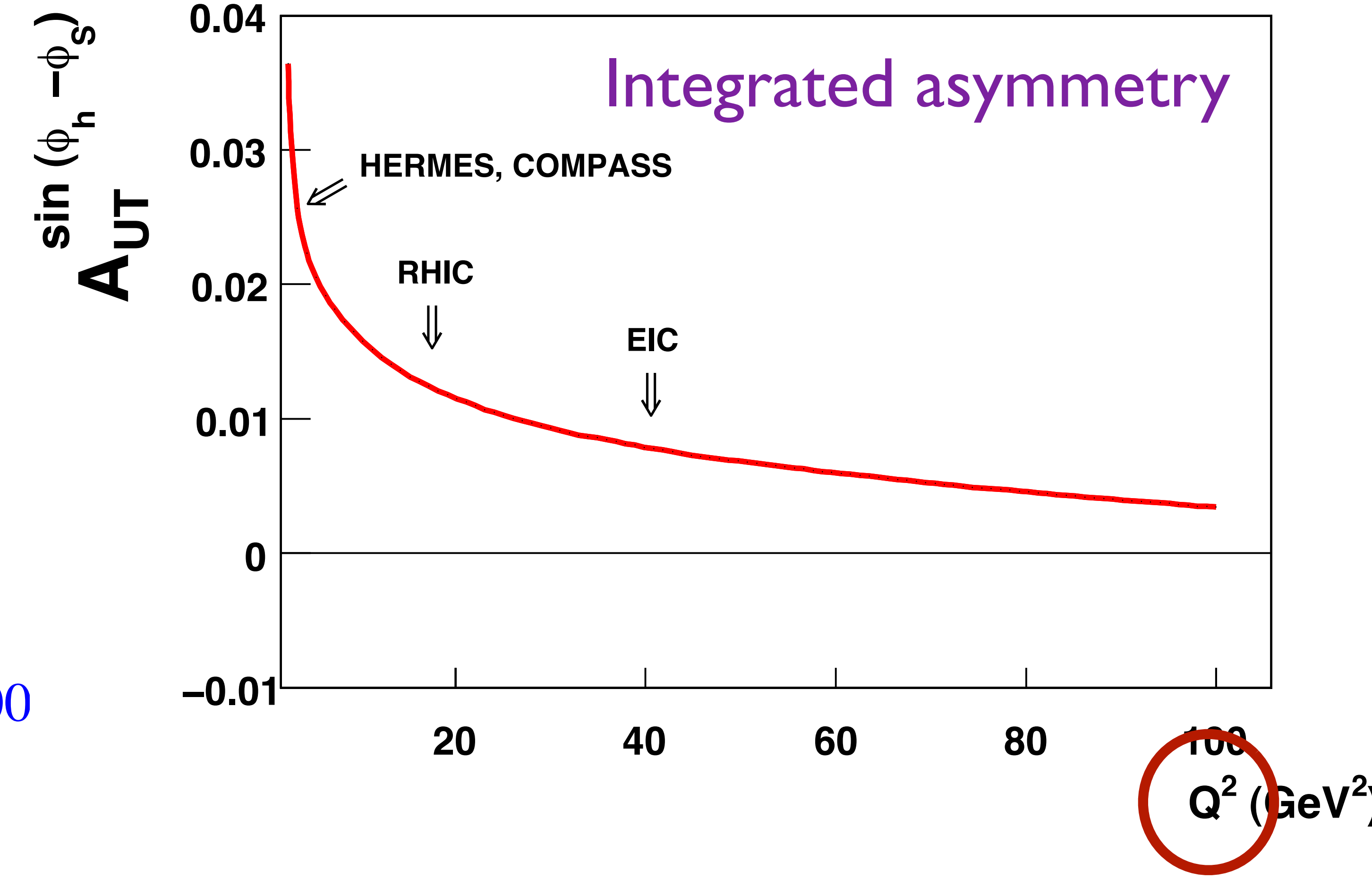
- the peak of the Sivers asymmetry decreases as $1/Q^{0.7\pm0.1}$ ("Sudakov suppression")
- the peak of the asymmetry shifts slowly towards higher Q_T

Testing these features needs a larger Q range, requiring a high-energy EIC

TMD evolution of the Sivers asymmetry



D.B., NPB 874 (2013) 217



Aybat, Prokudin & Rogers, PRL 108 (2012) 242003

Both approaches use the same formalism (2011 TMD factorization), very similar approximations and ingredients, the key difference is in the integration over x, z, $P_{h\perp}$

The two results are not necessarily in contradiction with each other

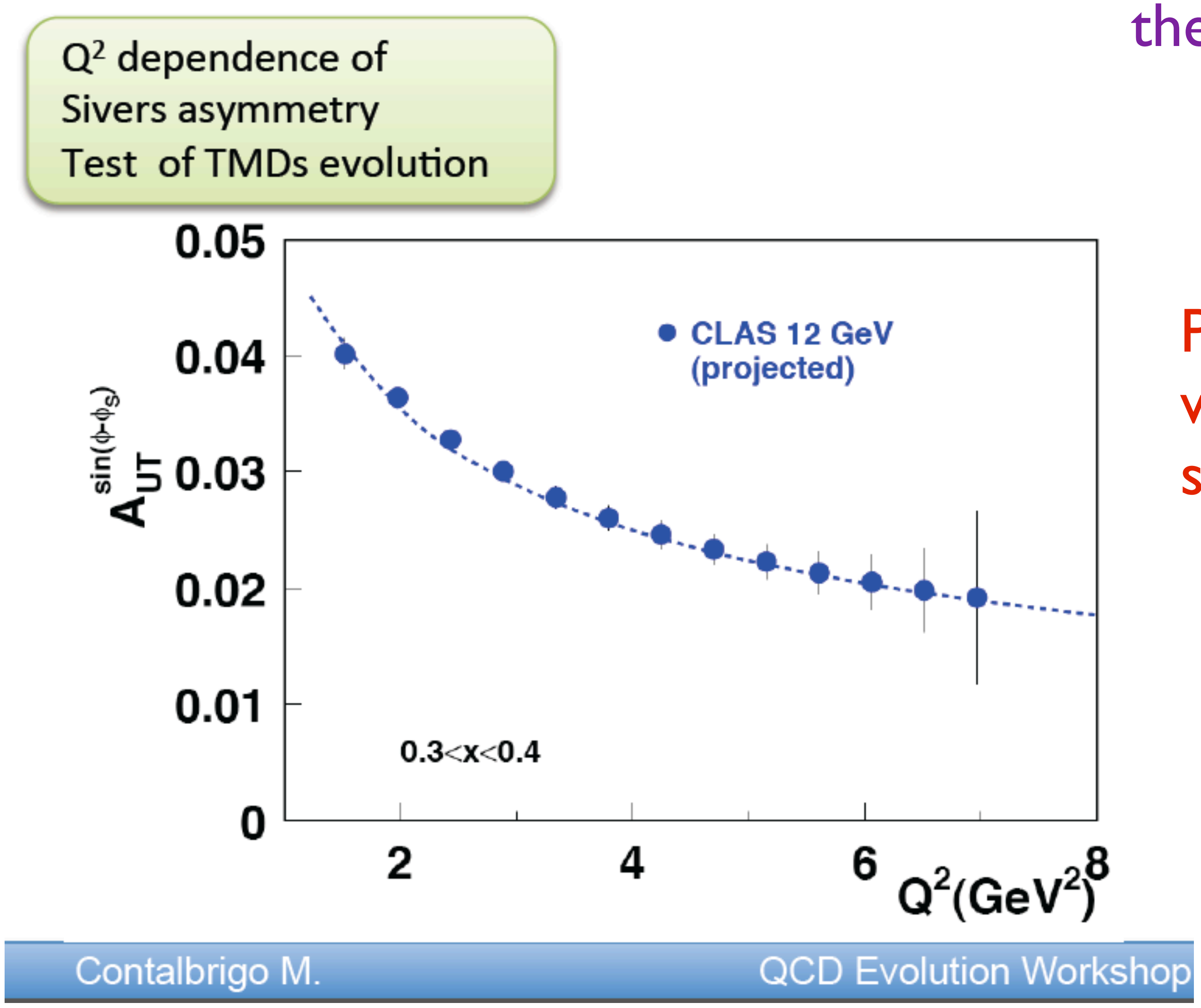
The *integrated* asymmetry falls off fast, not of form $1/Q^{\alpha}$, but in the considered range it falls off faster than $1/Q$ but slower than $1/Q^2$

TMD evolution of the Sivers asymmetry

At low Q^2 (up to ~20 GeV^2), the Q^2 evolution is dominated by S_{NP}

[Anselmino, Boglione, Melis,PRD 86 (2012) 014028]

Uncertainty in S_{NP} determines the ±0.1 in $1/Q^{0.7\pm0.1}$



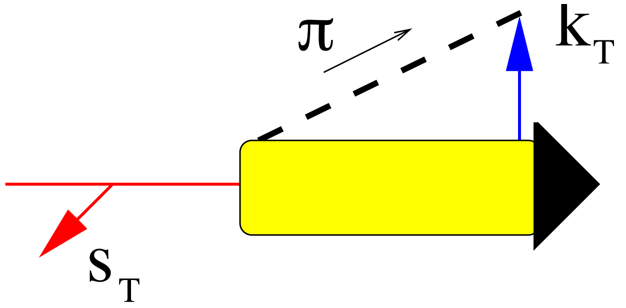
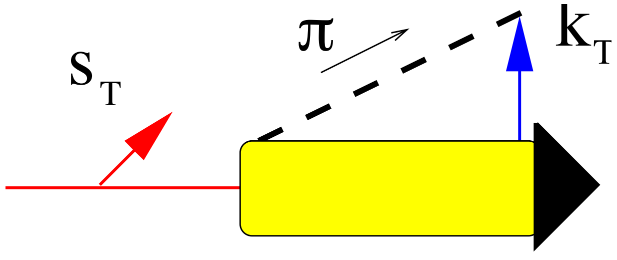
Precise low Q^2 data from JLab 12 GeV will help to determine the form and size of S_{NP}, incl. its x and z-dependence

TMD evolution of Collins asymmetries

Collins Effect

Collins effect is described by a TMD fragmentation function:

[NPB 396 (1993) 161]

$H_1^{\perp} =$  - 

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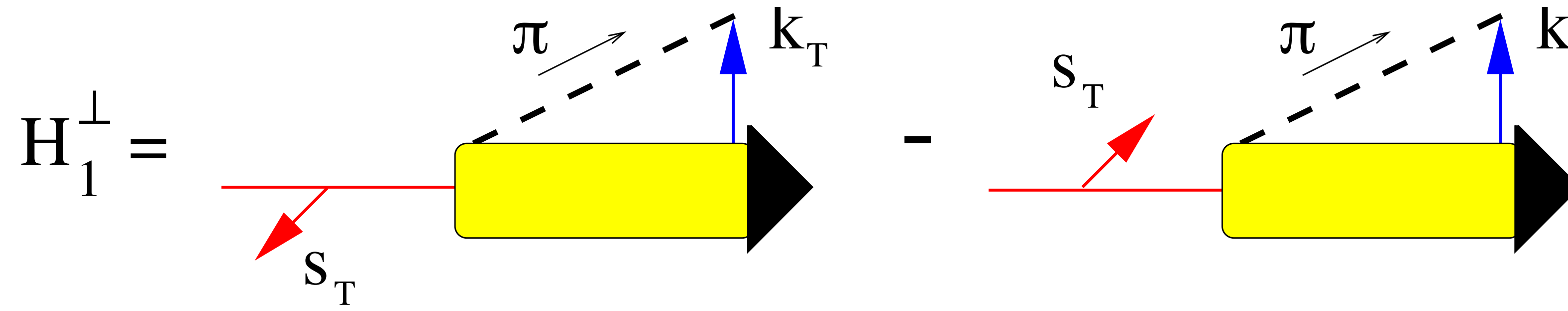

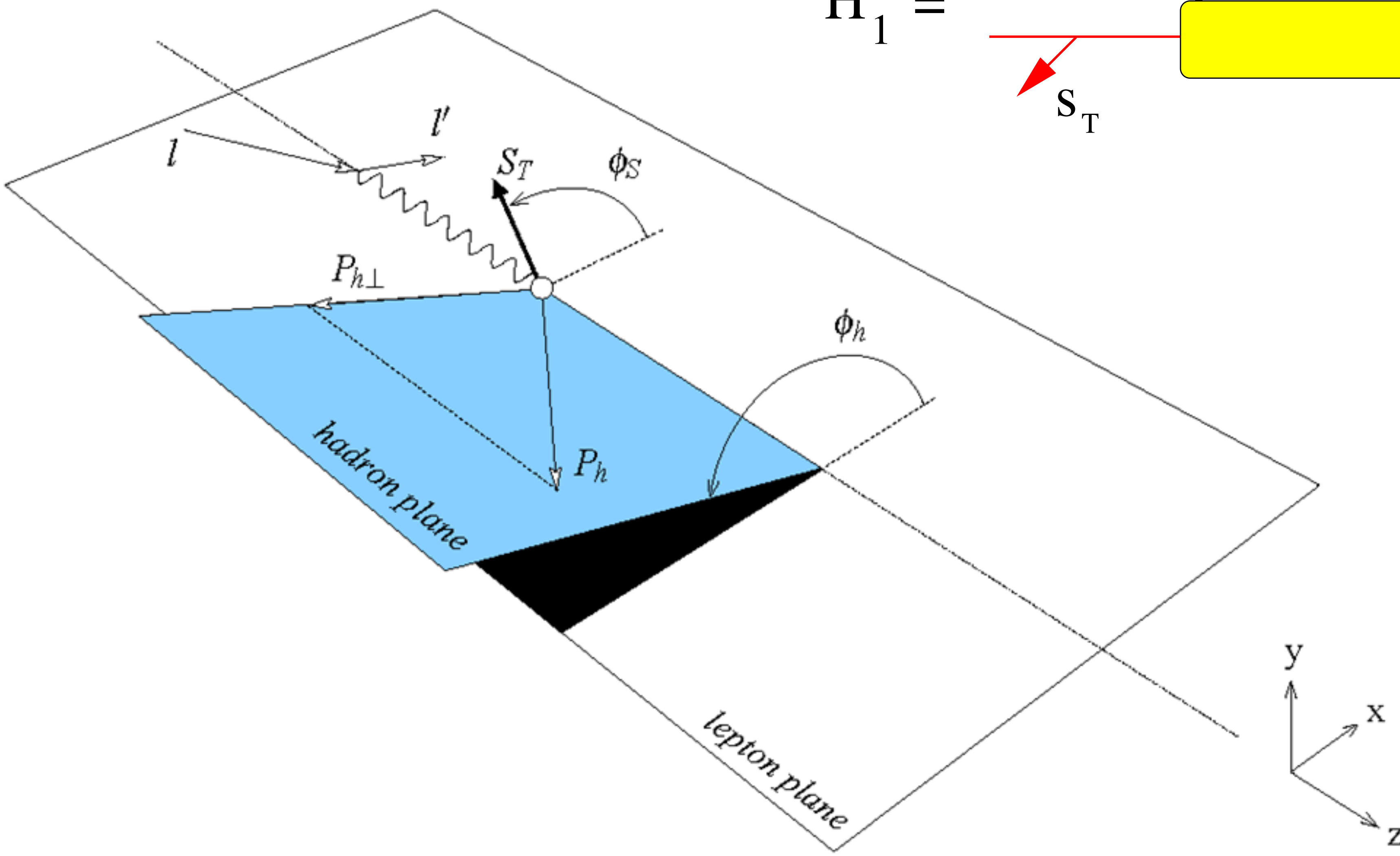

It gives rise to a sin(φ_h+φ_S) asymmetry in SIDIS:

$$\frac{d\sigma(e\,p^{\uparrow} \to e'\pi X)}{d\phi_\pi^e\, d|\boldsymbol{P}_\perp^\pi|^2} \propto \left\{1 + |\boldsymbol{S}_T|\ \sin(\phi_\pi^e - \phi_S^e)\ f_{1T}^\perp D_1 + |\boldsymbol{S}_T|\ \sin(\phi_\pi^e + \phi_S^e)\ h_1 H_1^\perp\right\}$$

Collins Effect

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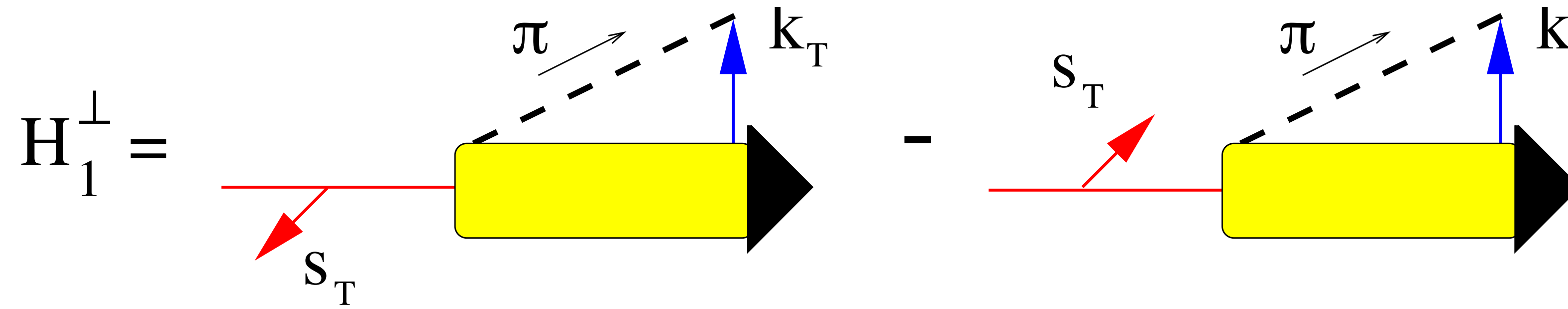
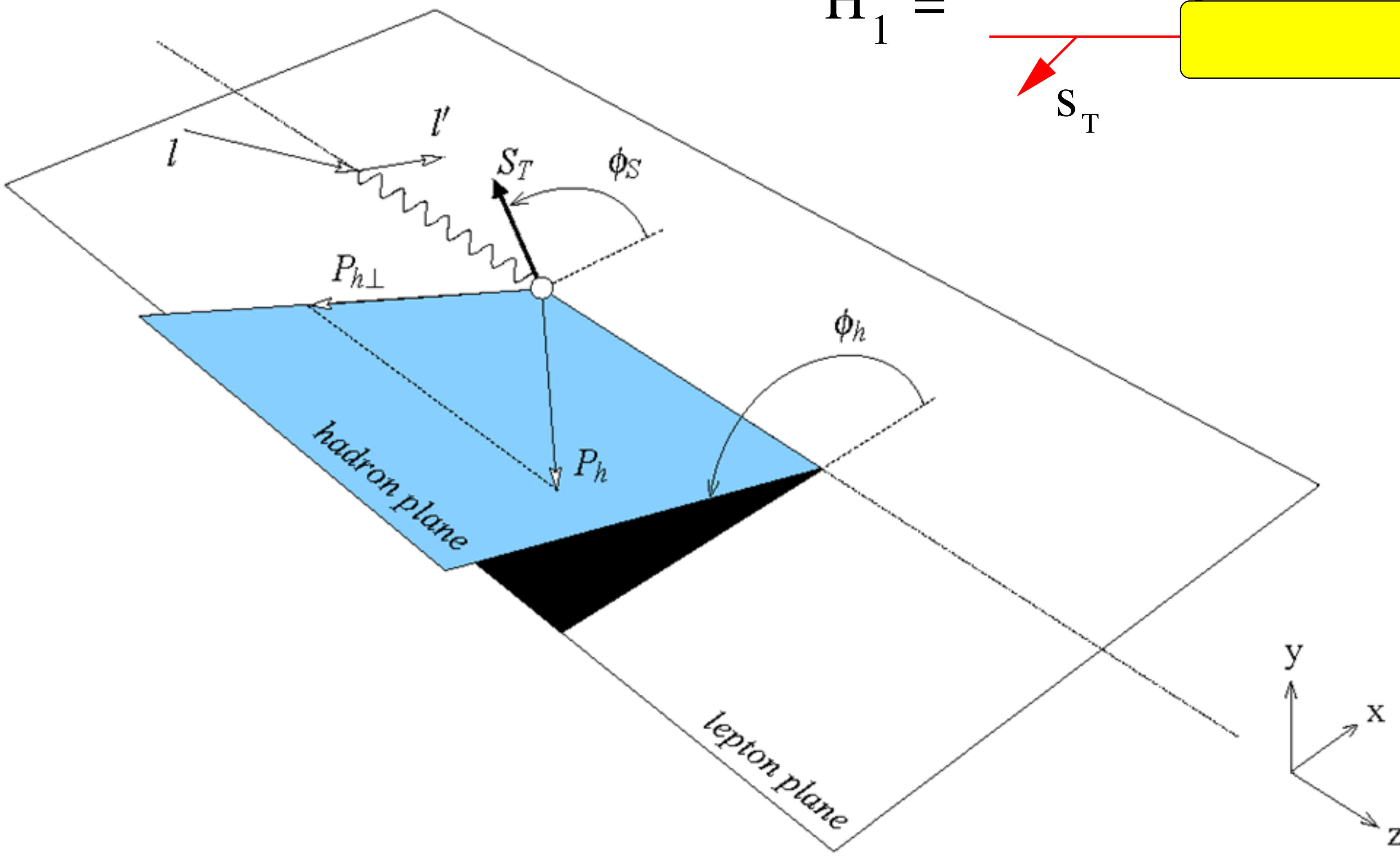


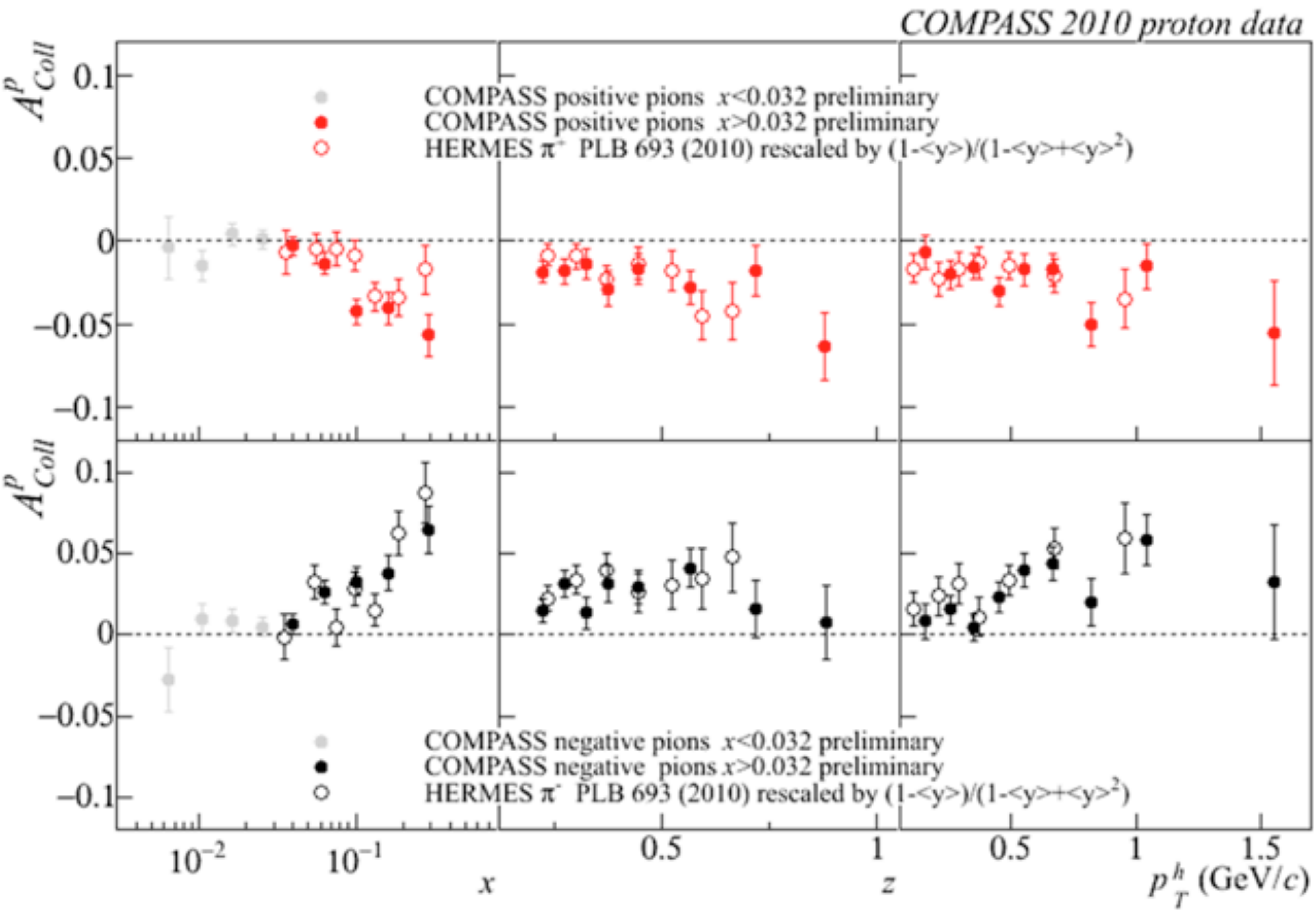


It gives rise to a $\sin(\varphi_h+\varphi_S)$ asymmetry in SIDIS:

transversity ⊗ Collins function

$$\frac{d\sigma(e\,p^\uparrow \to e'\pi X)}{d\phi_\pi^e\, d|\boldsymbol{P}_\perp^\pi|^2} \propto \{1 + |\boldsymbol{S}_T|\ \sin(\phi_\pi^e - \phi_S^e)\ f_{1T}^\perp D_1 + |\boldsymbol{S}_T|\ \sin(\phi_\pi^e + \phi_S^e)\, h_1 H_1^\perp\}$$

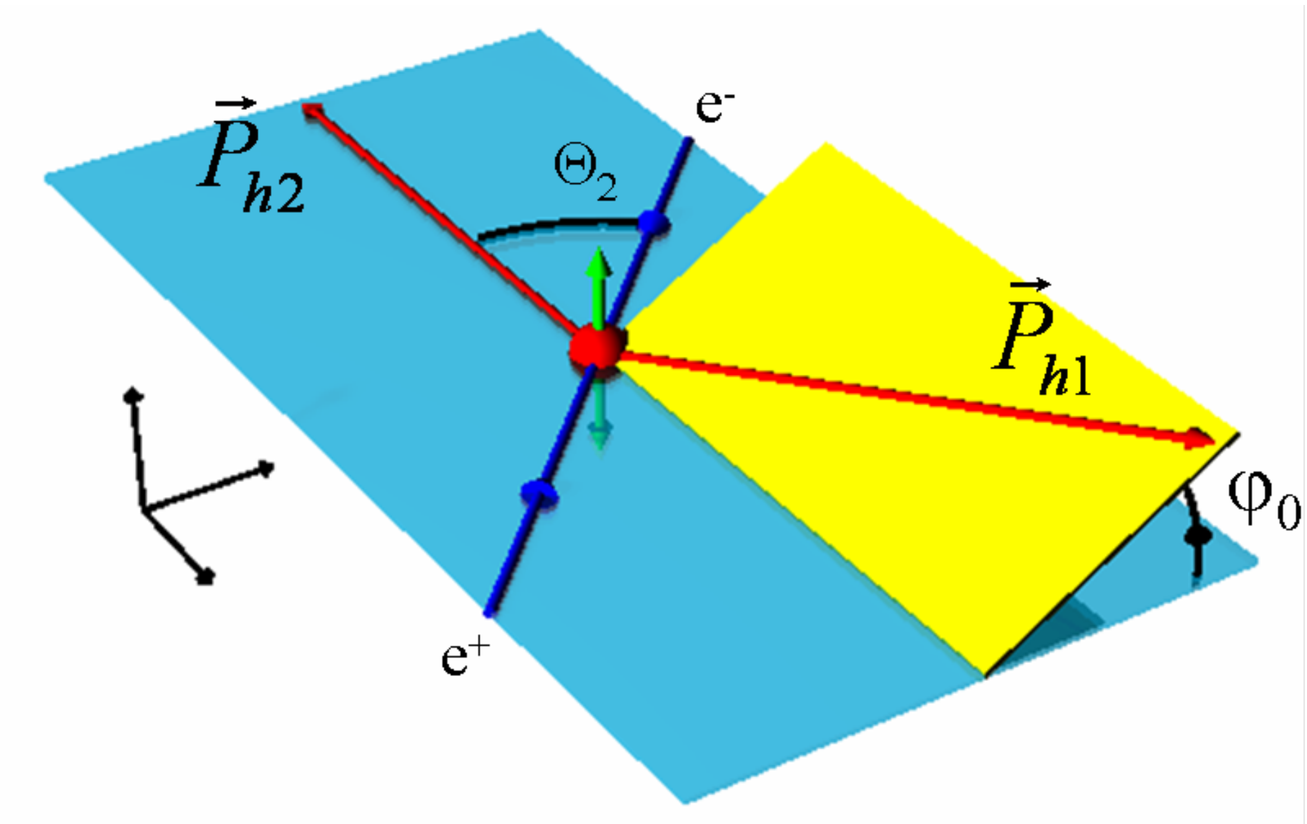
Collins Asymmetry in SIDIS



No clear need for TMD evolution from HERMES to COMPASS

Double Collins Effect

The Collins fragmentation function provides a way to probe transversity (h_1), if measured independently in another process

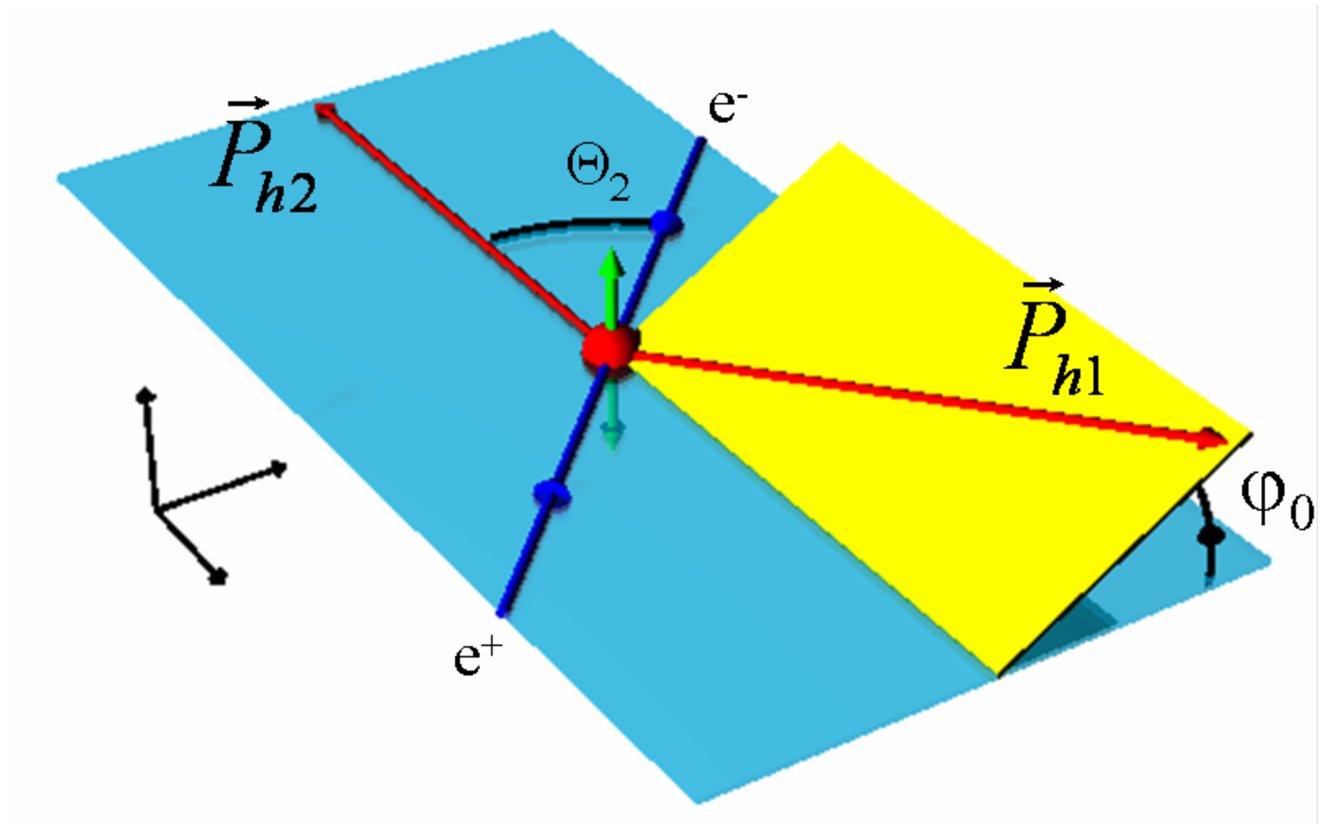


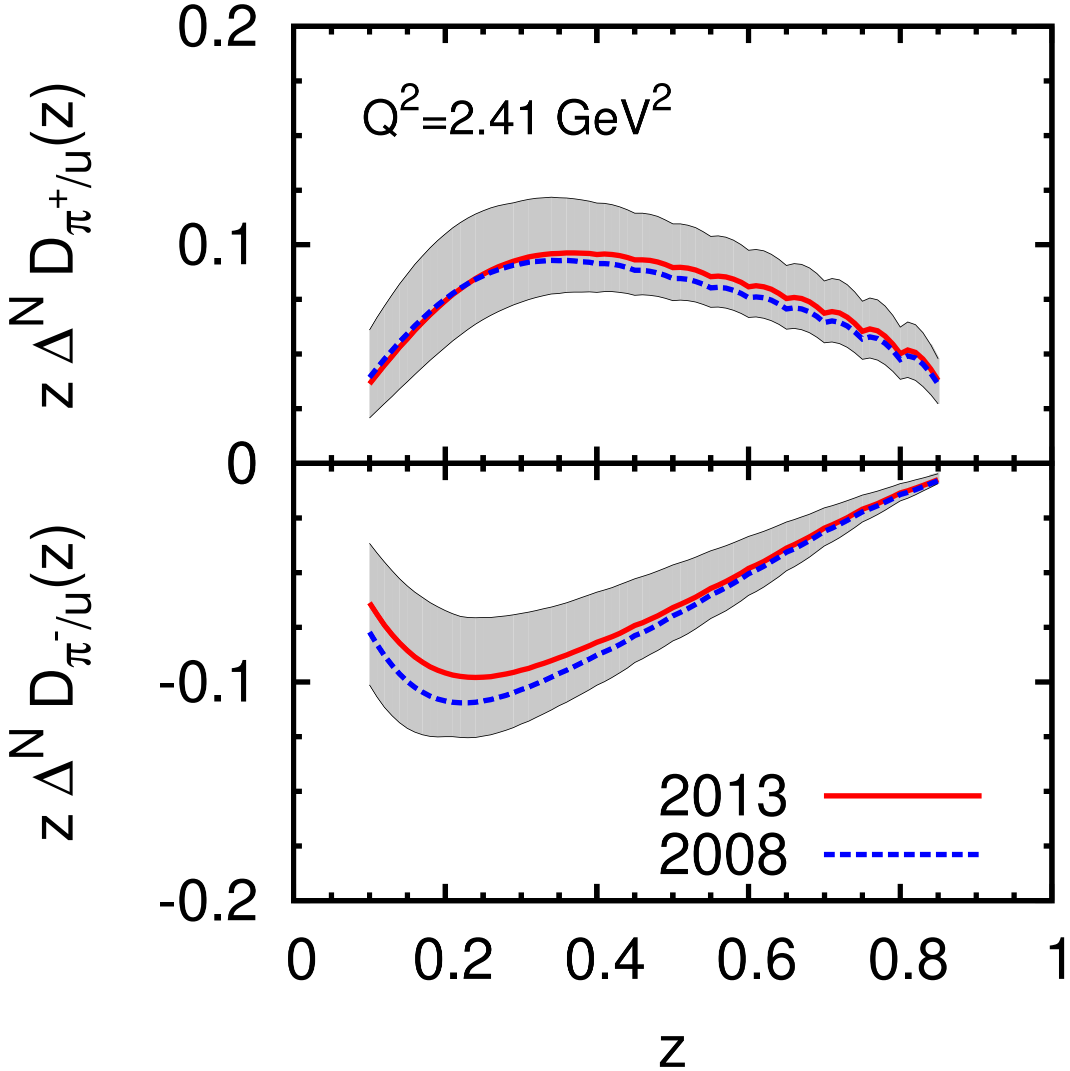
Double Collins effect gives rise to a cos 2φ asymmetry in $e^+e^- \rightarrow h_1\ h_2\ X$
[D.B., Jakob, Mulders, NPB 504 (1997) 345]

Clearly observed in experiment by BELLE (R. Seidl *et al.*, PRL '06; PRD '08) and BaBar (I. Garzia at Transversity 2011 & J.P. Lees et al., arXiv:1309.527)

Double Collins Effect

The Collins fragmentation function provides a way to probe transversity (*h_1*), if measured independently in another process





Anselmino *et al.*, PRD 87 (2013) 094019

Double Collins effect gives rise to a cos 2φ asymmetry in $e^+e^- \to h_1\, h_2\, X$

[D.B., Jakob, Mulders, NPB 504 (1997) 345]

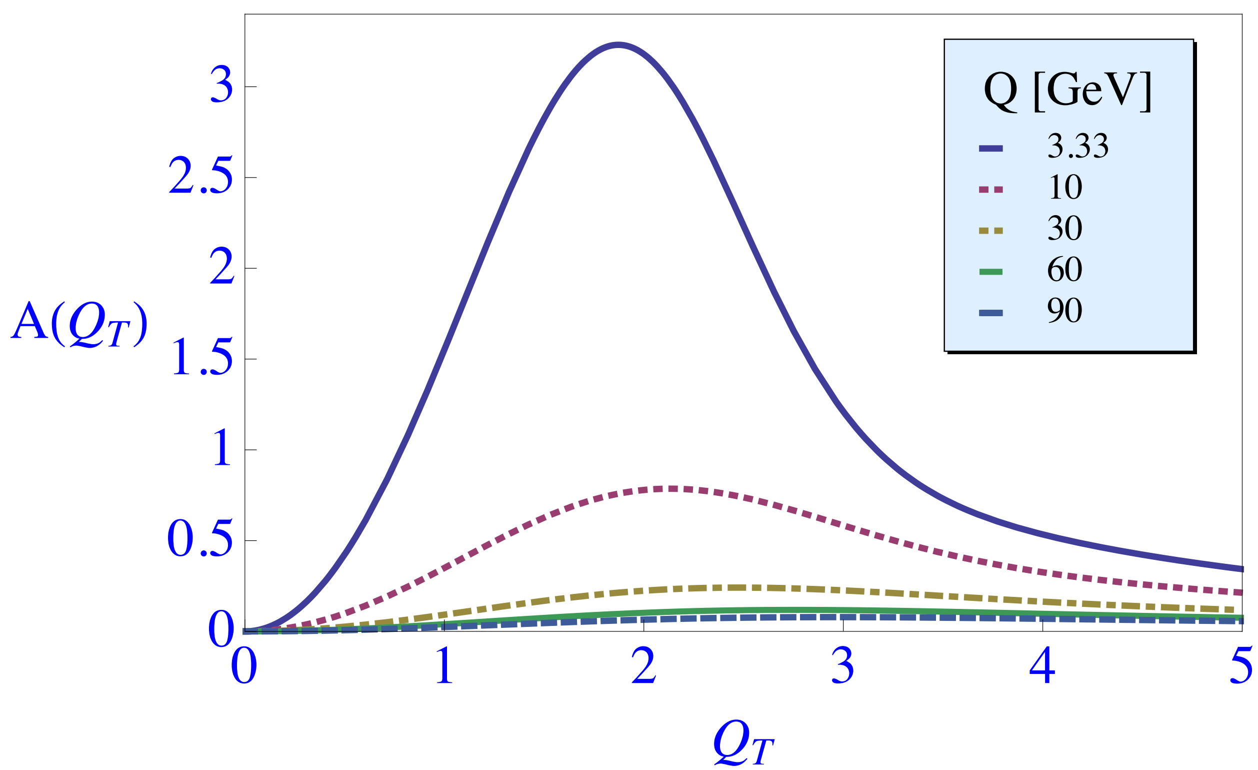
Clearly observed in experiment by BELLE (R. Seidl *et al.*, PRL '06; PRD '08) and BaBar (I. Garzia at Transversity 2011 & J.P. Lees et al., arXiv:1309.527)

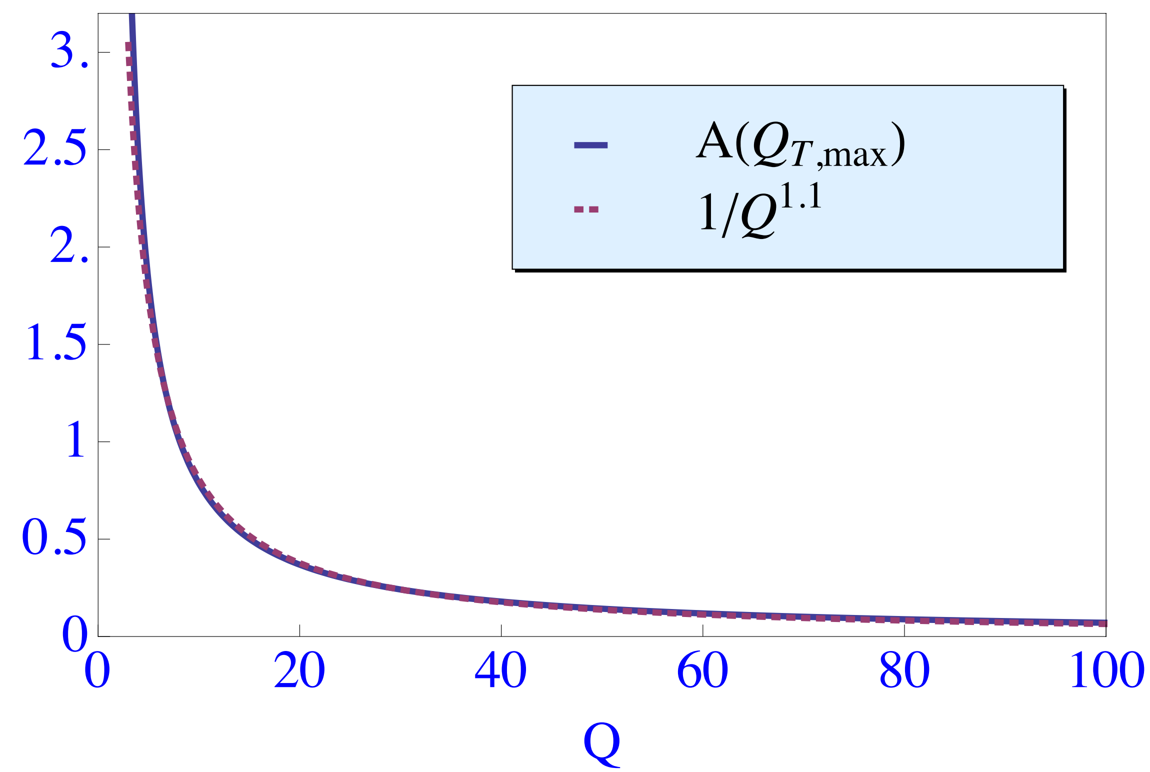
Double Collins Asymmetry

$$\frac{d\sigma(e^+e^- \to h_1 h_2 X)}{dz_1 dz_2 d\Omega d^2\boldsymbol{q}_T} \propto \{1 + \cos 2\phi_1 A(\boldsymbol{q}_T)\}$$

Under similar assumptions as for the Sivers asymmetry:

$$A(Q_T) = \frac{\sum_a e_a^2 \sin^2\theta\; H_1^{\perp(1)a}(z_1;Q_0)\; \overline{H}_1^{\perp(1)a}(z_2;Q_0)}{\sum_b e_b^2(1+\cos^2\theta)\; D_1^b(z_1;Q_0)\; \overline{D}_1^b(z_2;Q_0)}\; \mathcal{A}(Q_T)$$



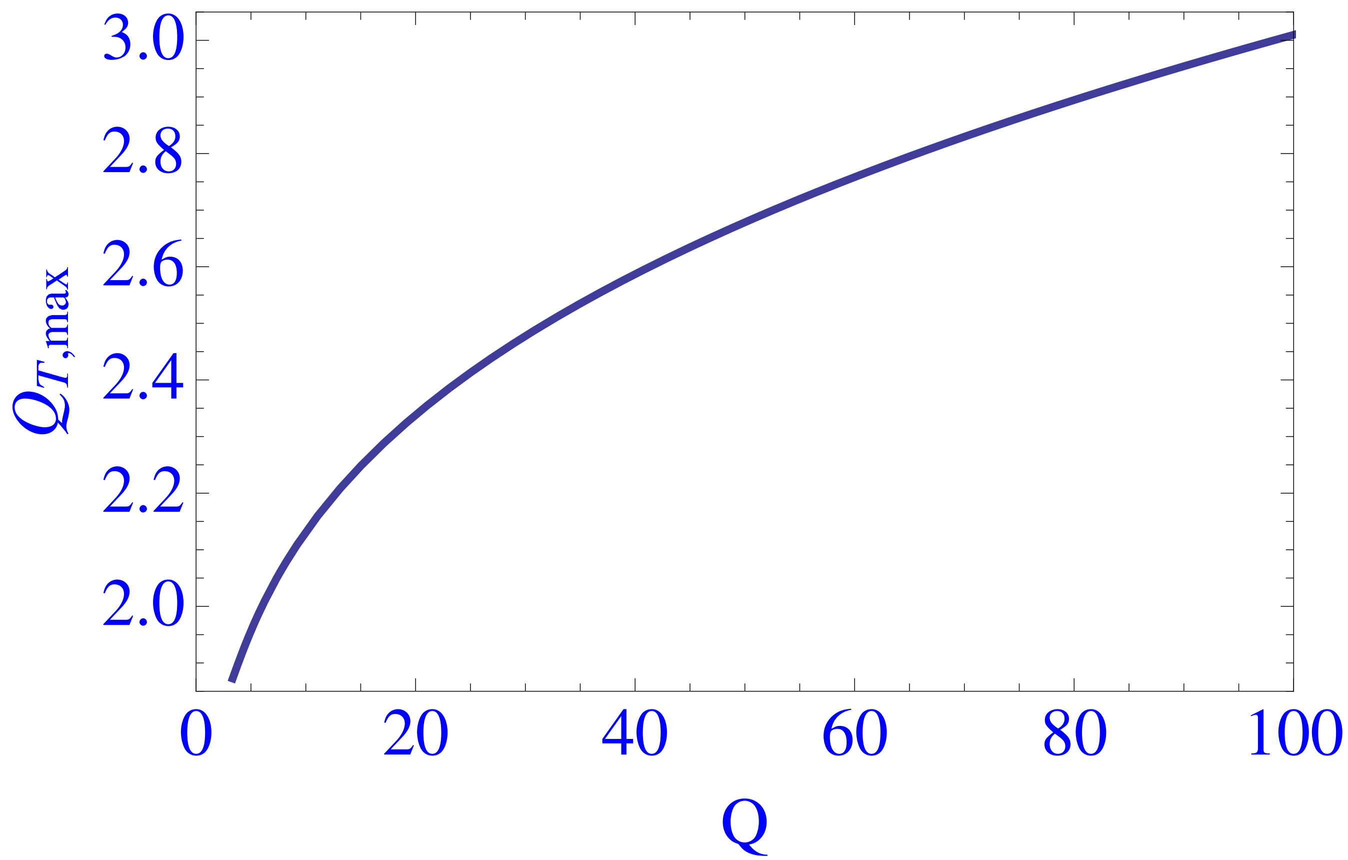


Considerable Sudakov suppression ~1/Q (effectively twist-3)

D.B., NPB 603 (2001) 195 & NPB 806 (2009) 23 & NPB 874 (2013) 217 & arXiv:1308.4262

Next steps

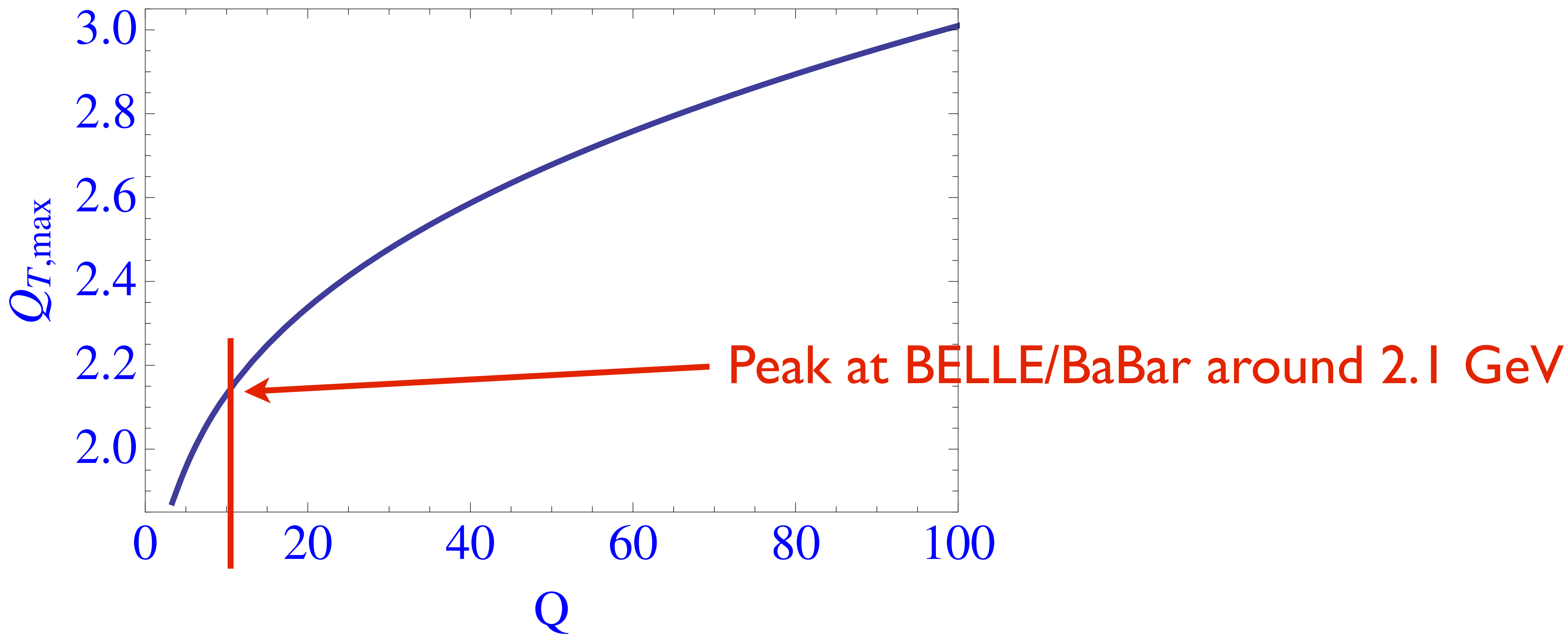
Peak of the asymmetry shifts slowly towards higher Q_T, offers a test



Data from charm factory (BEPC) important by providing data around Q≈4 GeV

Next steps

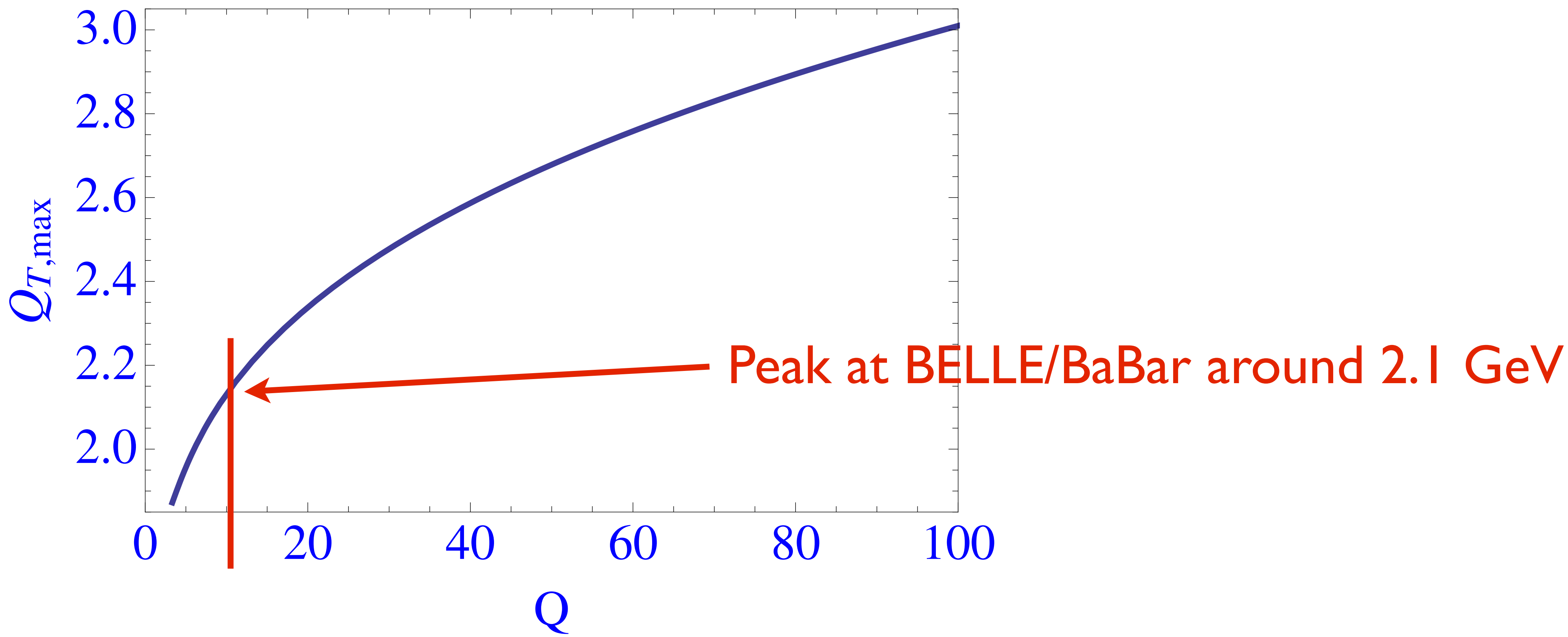
Peak of the asymmetry shifts slowly towards higher Q_T, offers a test



Data from charm factory (BEPC) important by providing data around Q≈4 GeV

Next steps

Peak of the asymmetry shifts slowly towards higher Q_T, offers a test



Data from charm factory (BEPC) important by providing data around Q≈4 GeV

The 1/Q behavior should modify the transversity extraction using Collins effect, full TMD evolution still to be implemented (for Q ~ 10 GeV S_{pert} is important)

Need to check the TMD evolution of the Collins asymmetry in SIDIS, which is slower than that of the double Collins asymmetry (Jefferson Lab & possibly EIC)

Double Collins Asymmetry

Data from BES important by providing data at lower Q

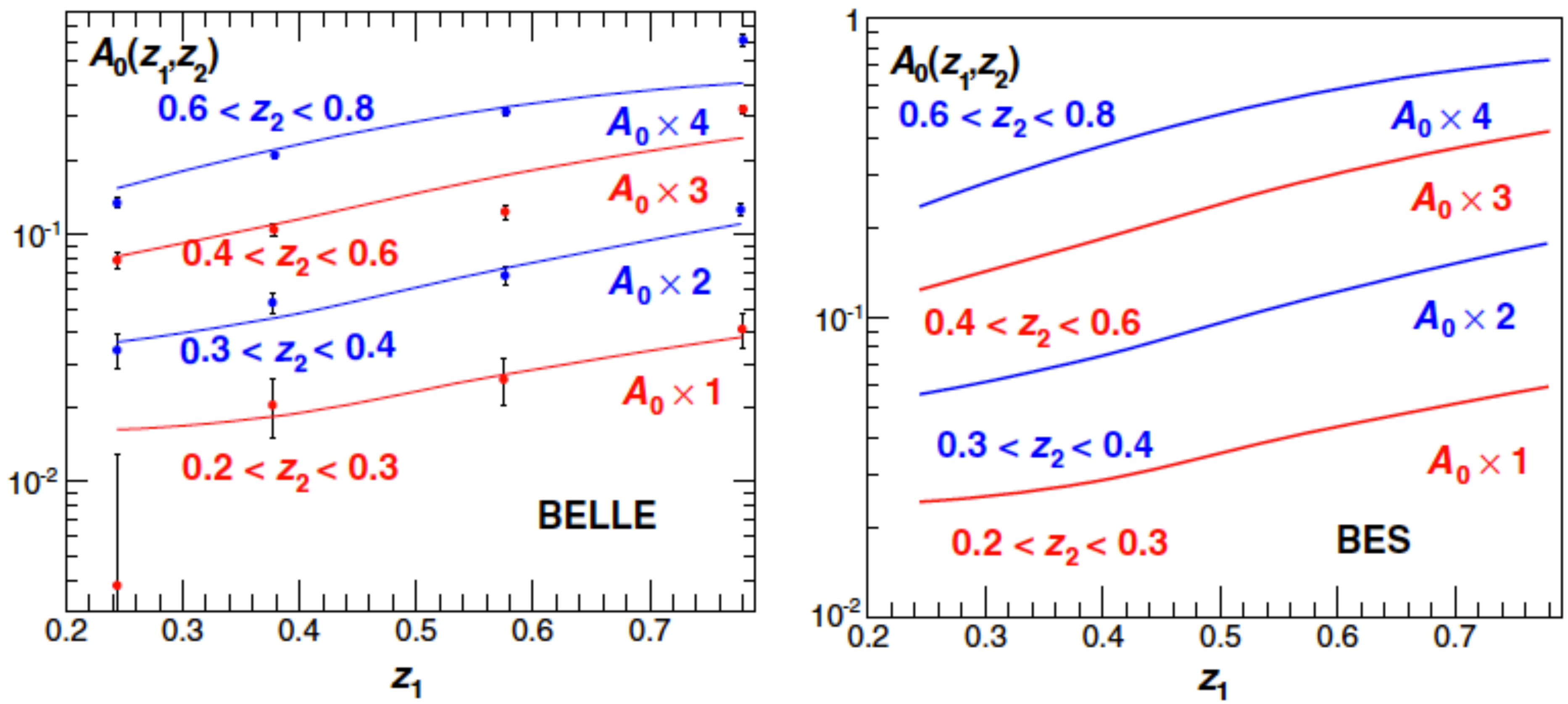


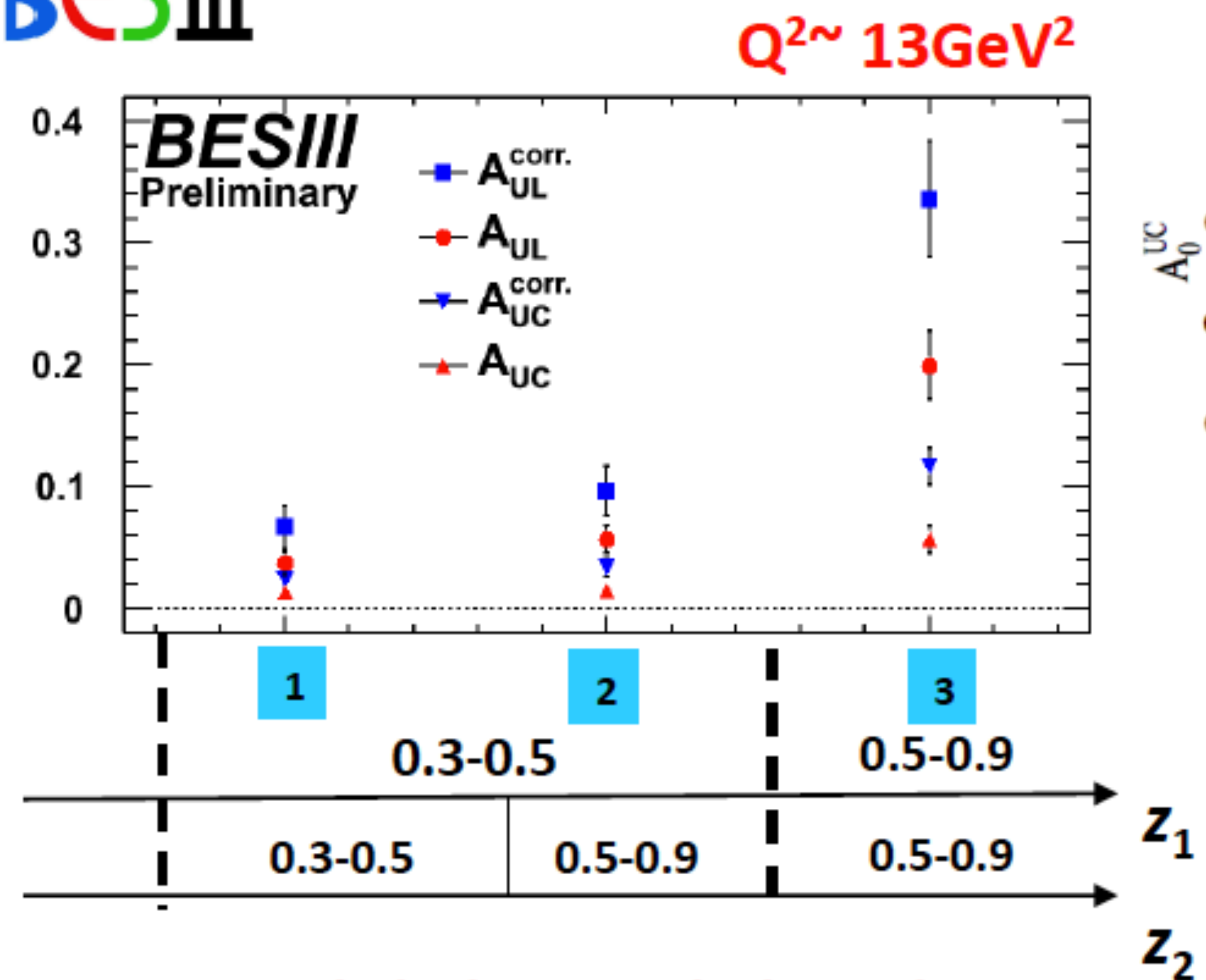
FIG. 4 (color online). The Collins asymmetries in di-hadron azimuthal angular distributions in e^+e^- annihilation processes: fit to the BELLE experiment at $\sqrt{S} = 10.6$ GeV Ref. [8], and predictions for the experiment at BEPC at $\sqrt{S} = 4.6$ GeV.

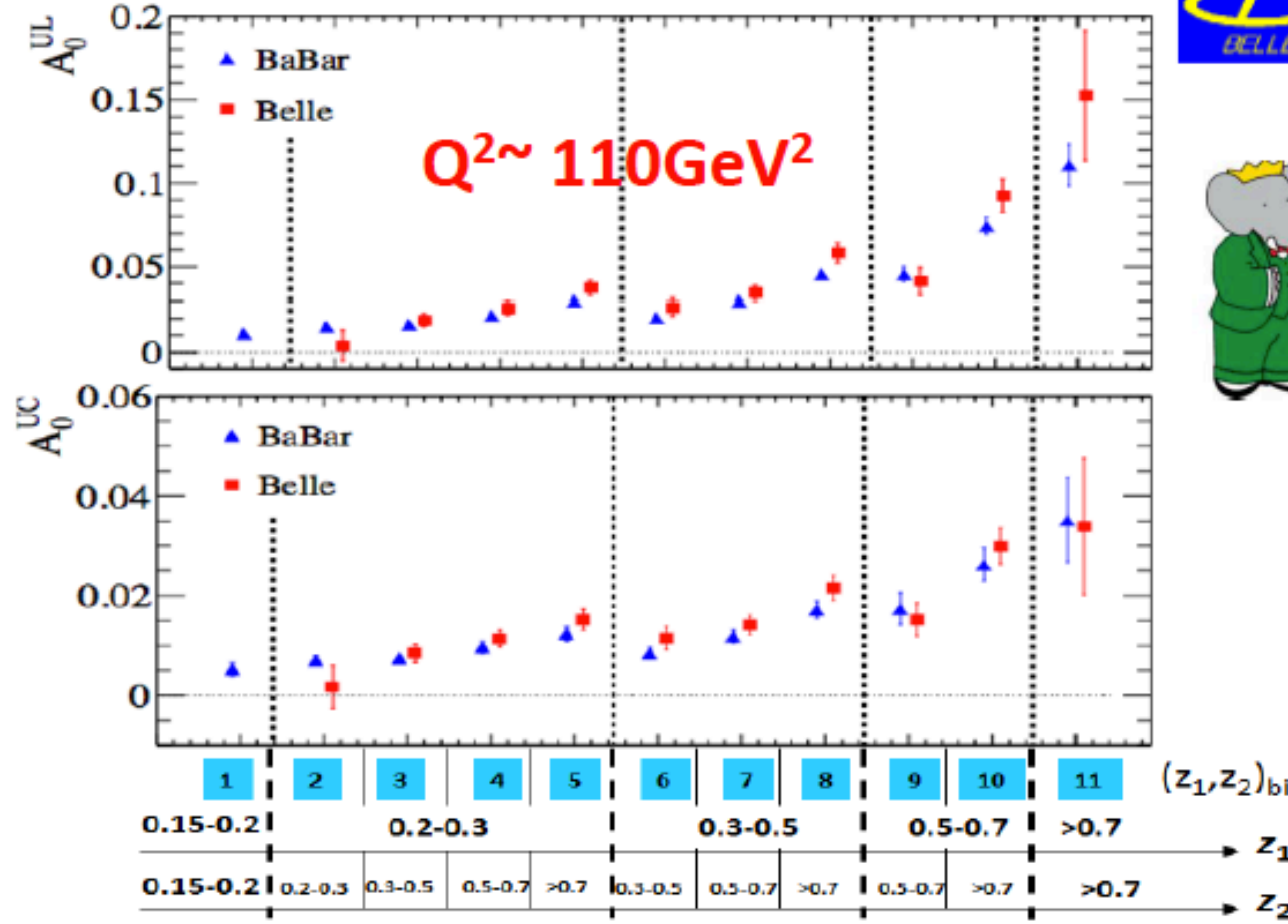
P. Sun & F. Yuan, PRD 88 (2013) 034016

One does have to worry about $1/Q^2$ corrections (analogue of the Cahn effect), which can be bounded by study simultaneously the $1/Q$ $\cos\varphi$ asymmetry

E.L. Berger, ZPC 4 (1980) 289; Brandenburg, Brodsky, Khoze & D. Mueller, PRL 73 (1994) 939

Yinghui Guan for BESIII at SPIN2014, Beijing





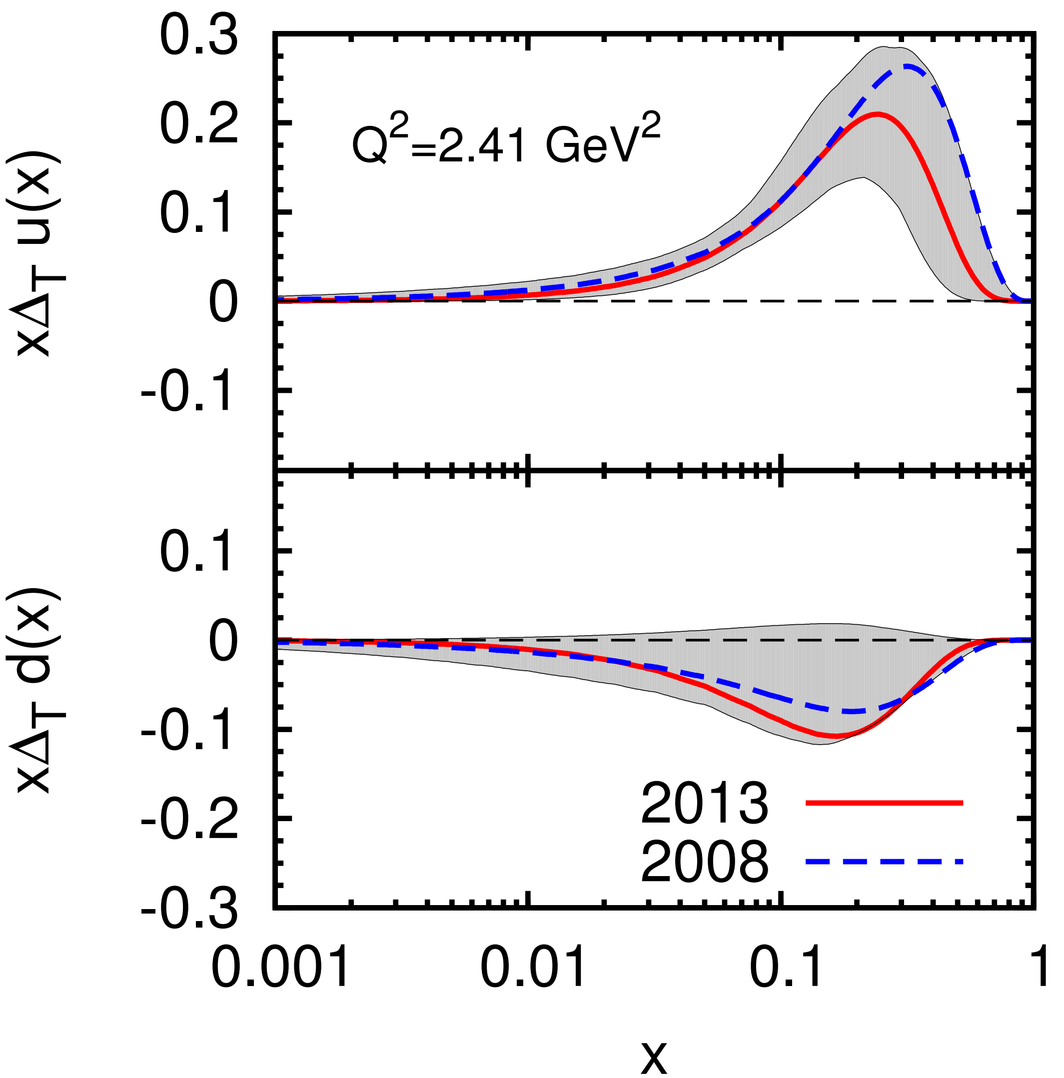
- Statistical uncertainties only.

Compatible with a 1/Q type of evolution!
(which also applies to double ratios)

- **The measured Collins asymmetries at BESIII is larger than those at higher Q^2 at B factories.**
- **This trend accords with predictions in PRD 88. 034016 (2013).** = Sun & Yuan

Transversity extraction using Collins effect

$$\frac{d\sigma(e\,p^{\uparrow}\to e'\pi X)}{d\phi_\pi^e d|\boldsymbol{P}_\perp^\pi|^2} \propto \{1+|\boldsymbol{S}_T|\ \sin(\phi_\pi^e-\phi_S^e)\ f_{1T}^\perp D_1 + |\boldsymbol{S}_T|\ \sin(\phi_\pi^e+\phi_S^e)\ h_1 H_1^\perp\}$$



Extraction of $h_1^q(x)=\Delta_T q(x)$ at $Q^2=2.4$ GeV^2 from HERMES, COMPASS & BELLE data

Anselmino *et al.*, PRD 75 (2007) 054032 & PRD 87 (2013) 094019

It shows: $h_1^q(x) \approx f_1^q(x)/3$

About half its maximally allowed value

Similar in size as $\Delta q(x)$

This extraction uses that the Collins function is universal

Metz '02; Collins & Metz '04; Yuan '08; Gamberg, Mukherjee & Mulders' 08; Meissner & Metz '09

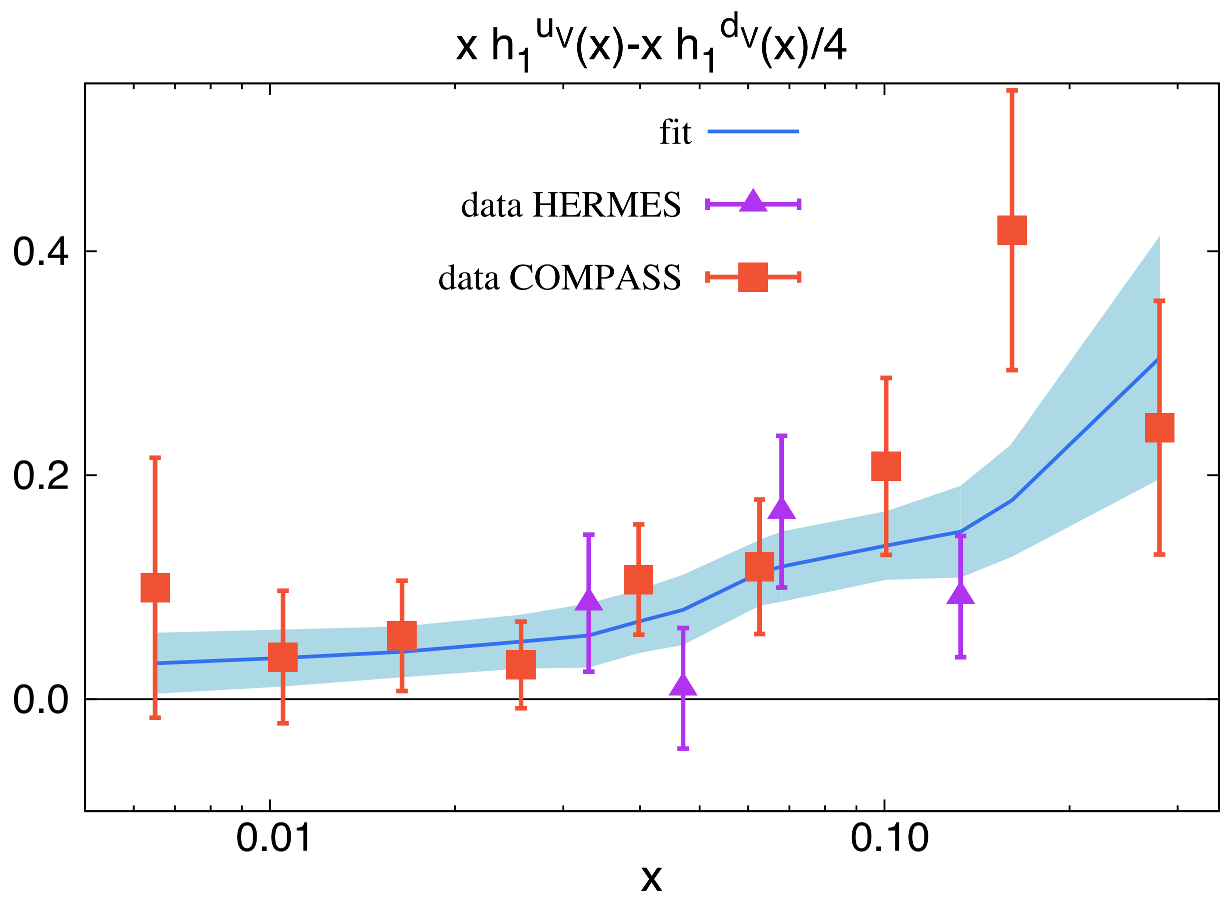
Transversity extraction using DiFF

Dihadron or Interference Fragmentation Functions (DiFF or IFF) also allow for transversity extraction using SIDIS and e^+e^- data

$$e\,p^{\uparrow} \to e'\,(\pi^+\,\pi^-)\,X \qquad h_1 \otimes H_1^{\sphericalangle} \qquad H_1^{\sphericalangle}(z, M_{\pi\pi}^2)$$

Ji '94; Collins, Heppelmann, Ladinsky '94; Jaffe, Jin, Tang '98; ...

not a TMD!



Bacchetta, Courtoy, Radici
JHEP 1303 (2013) 119

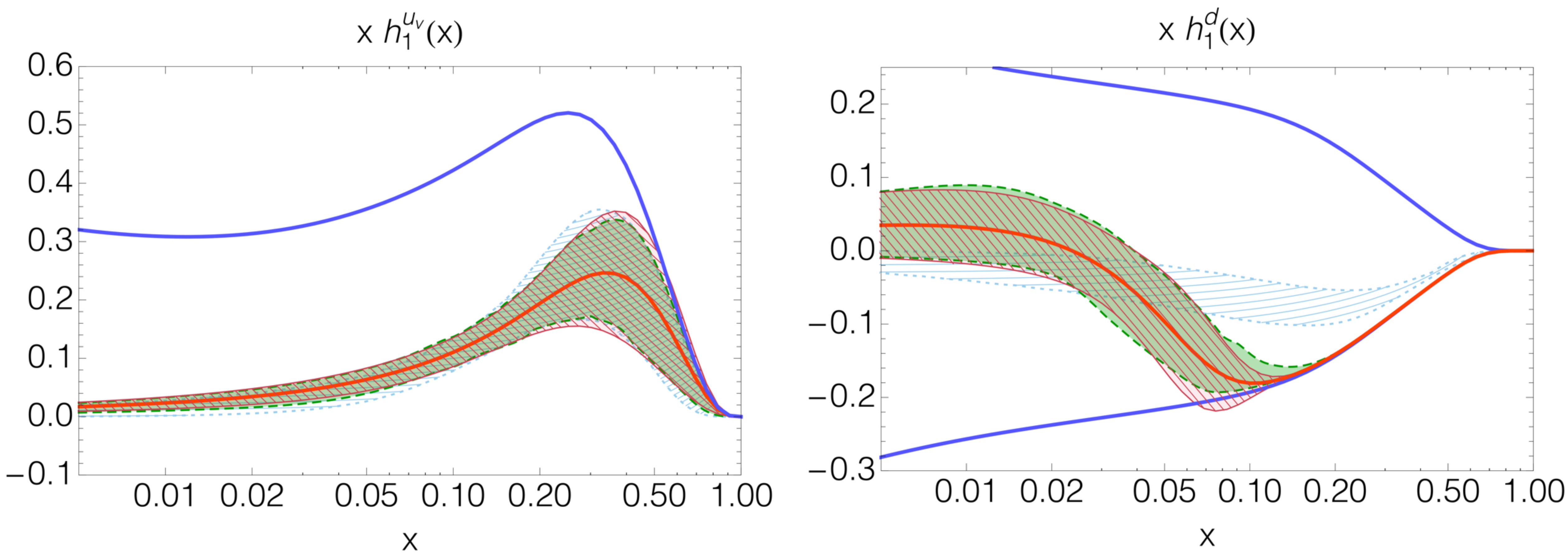
Vossen et al., BELLE Collaboration
PRL 107 (2011) 072004

From a theoretical point of view very clean: collinear factorization & universal
Currently offers the safest and easiest way to extract transversity

Transversity extraction using DiFF

Allows a transversity extraction from COMPASS & HERMES and BELLE data using different data selections

$$e\, p^{\uparrow} \to e'\, (\pi^{+}\, \pi^{-})\, X \qquad h_1 \otimes H_1^{\sphericalangle} \qquad H_1^{\sphericalangle}(z, M_{\pi\pi}^2)$$



[Bacchetta, Courtoy, Radici, JHEP 1303 (2013) 119]

The two extractions (Collins and DiFF methods) are compatible with each other

BELLE vs SIDIS data

Both transversity extractions are compatible with each other
But should they be?

BELLE and SIDIS data are obtained at quite different scales:
$Q^2=110$ GeV^2 vs $\langle Q^2 \rangle = 2.4$ GeV^2

Collins effect method requires TMD evolution
DiFF method requires DGLAP evolution, which is much slower

Extraction of $h_1^q(x)=\Delta_T q(x)$ using Collins effect method used DGLAP-like evolution (the one of D_1 not H_1)

$$H_1^{\perp}(z,\boldsymbol{k}_T^2;Q) \equiv D_1(z;Q)F(z,\boldsymbol{k}_T^2)$$

Anselmino *et al.*, PRD 75 (2007) 054032 & PRD 87 (2013) 094019

This type of DGLAP-like evolution for the Sivers function is quite different from the TMD evolution, especially at low energies

Anselmino, Boglione, Melis, PRD 86 (2012) 014028

The h_1 extraction "conundrum"

For small b (in W(b*)) one can consider the perturbative tail, which is calculable

$$\tilde{f}_{g/P}(x, b^2; \mu, \zeta) = \sum_{i=g,q} \int_x^1 \frac{d\hat{x}}{\hat{x}} C_{i/g}(x/\hat{x}, b^2; g(\mu), \mu, \zeta) f_{i/P}(\hat{x}; \mu) + \mathcal{O}((\Lambda_{\rm QCD} b)^a)$$

TMD factorized expression with TMD perturbative tails only = CSS expression

For the Collins asymmetry: [Kang, Prokudin, Sun & Yuan, arXiv:1410.4877]

$$F_{UT} = -\frac{1}{2z_h^3} \int \frac{db\, b^2}{(2\pi)} J_1\left(\frac{P_{h\perp} b}{z_h}\right) e^{-S_{\rm PT}(Q,b_*) - S^{\rm (SIDIS)}_{\rm NP\ coll}(Q,b)}$$
$$\times\ \delta C_{q\leftarrow i} \otimes h_1^i(x_B, \mu_b)\ \delta \hat{C}^{\rm (SIDIS)}_{j\leftarrow q} \otimes \hat{H}^{(3)}_{h/j}(z_h, \mu_b), (2)$$

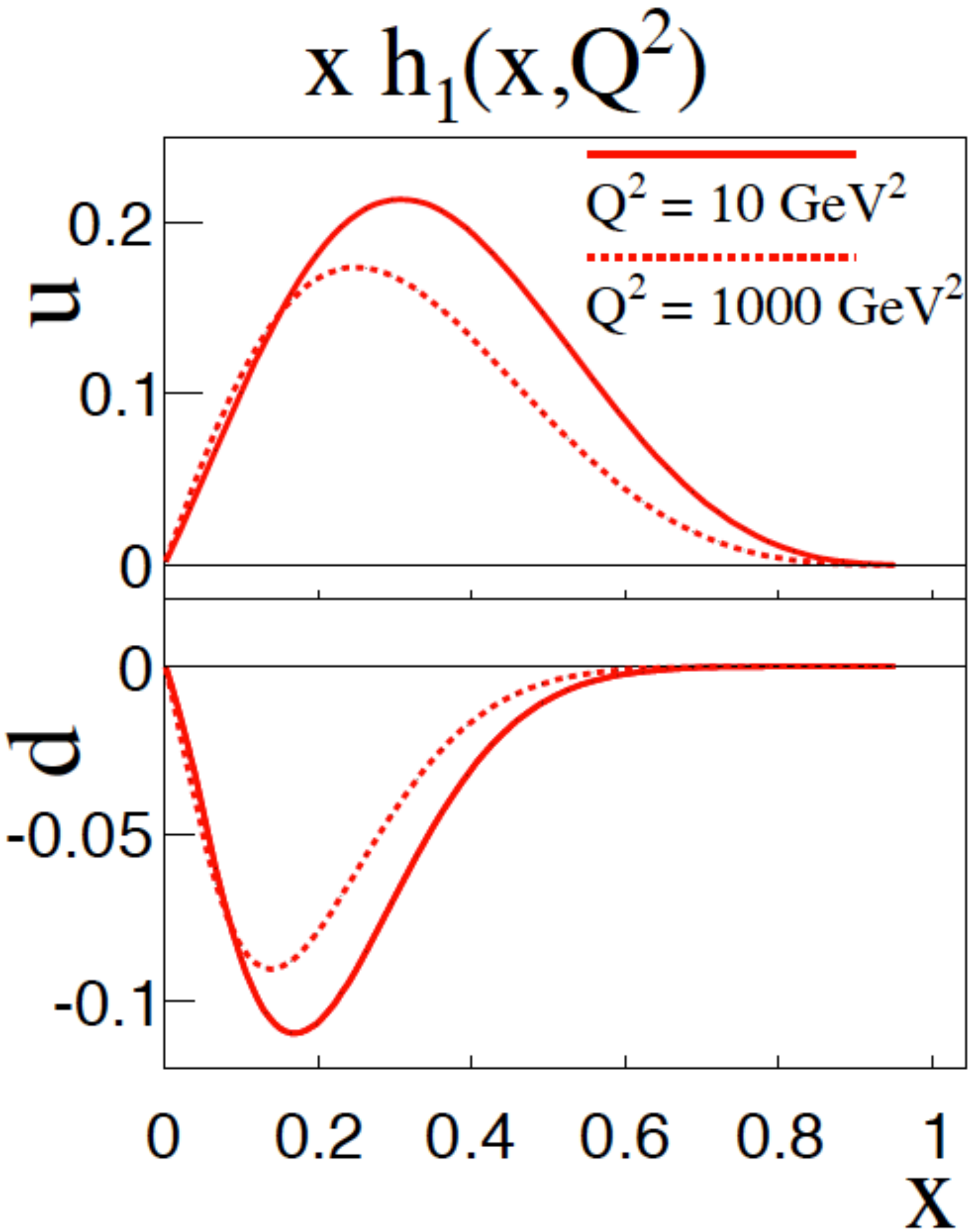
Include evolution of the tail, but only the homogeneous part:

hand, the evolution equation for $\hat{H}^{(3)}_{h/q}$ is more complicated [26, 27, 43]. However, if we keep only the homogeneous term, it reduces to a simpler form as

$$\frac{\partial}{\partial \ln \mu^2} \hat{H}^{(3)}_{h/q}(z, \mu) = \frac{\alpha_s}{2\pi} P^{\rm coll}_{q\leftarrow q} \otimes \hat{H}^{(3)}_{h/q}(z, \mu)\ , \qquad (5)$$

Evolution kernel same as of transversity

The h_1 extraction "conundrum"



$$\delta q^{[x_{\min},x_{\max}]}\left(Q^2\right) \equiv \int_{x_{\min}}^{x_{\max}} dx\, h_1^q(x,Q^2)\ . \qquad (16)$$

In Fig. 3, we plot the χ^2 Monte Carlo scanning of SIDIS data for the contribution to the tensor charge from such a region, and find

$$\delta u^{[0.0065,0.35]} = +0.30^{+0.12}_{-0.11}\ , \qquad (17)$$

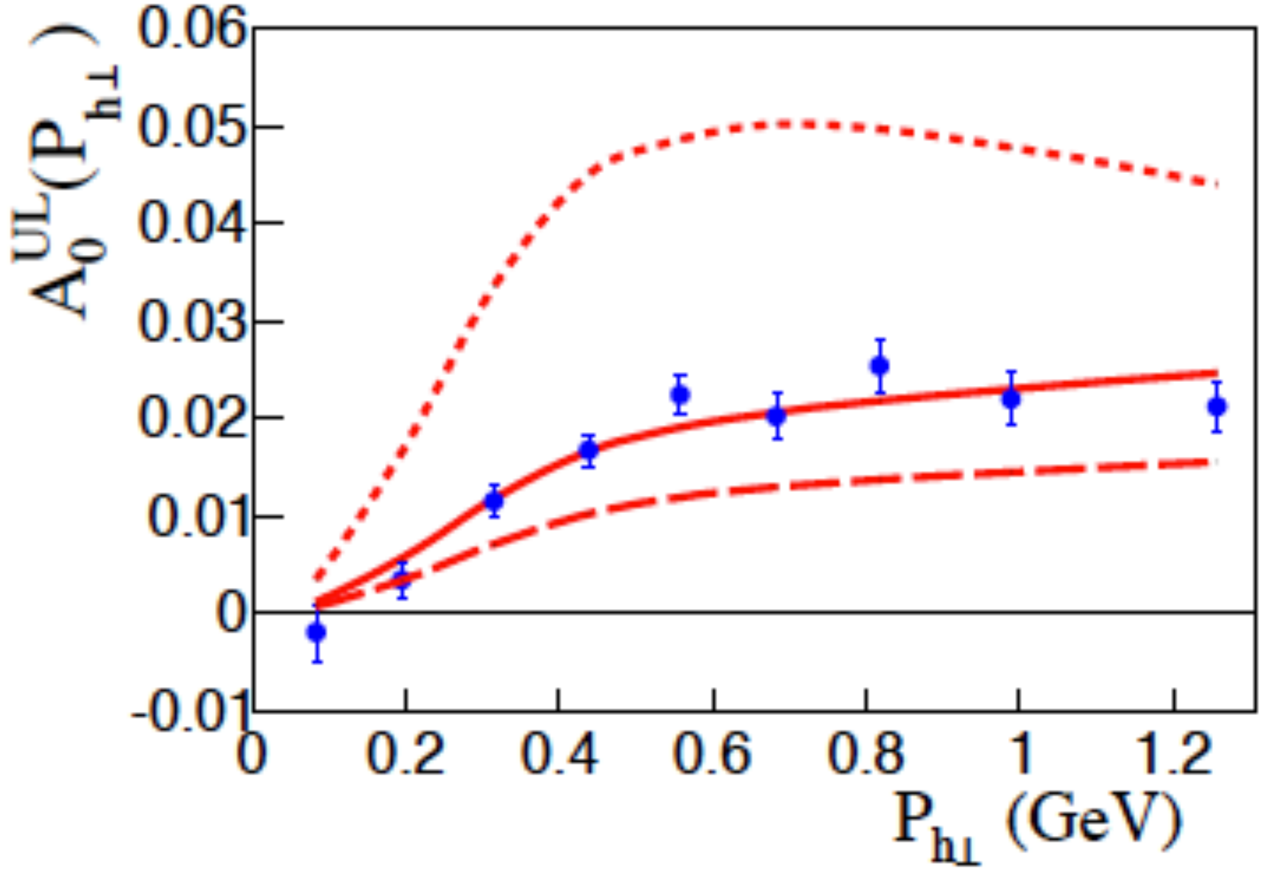
$$\delta d^{[0.0065,0.35]} = -0.20^{+0.36}_{-0.13}\ , \qquad (18)$$

at 90% C.L. at $Q^2 = 10$ GeV^2. We notice that this result is comparable with previous TMD extractions without evolution [19–21] and di-hadron method [35, 36].

Kang, Prokudin, Sun & Yuan, arXiv:1410.4877

This leads to very similar h_1 as other methods and gives very similar tensor charge, but why? Is the evolution too slow to matter?

The h_1 extraction “conundrum”



$P_{h\perp}$ distribution very sensitive to evolution however

FIG. 2. Collins asymmetries measured by BABAR [17] collaboration as a function of $P_{h\perp}$ in production of unlike sign “U” over like sign “L” pion pairs at $Q^2 = 110$ GeV^2. The solid line corresponds to the full NLL$'$ calculation, the dashed line to the LL calculation, and the dotted to the calculation without TMD evolution. Calculations are performed with parameters from Table I.

Kang, Prokudin, Sun & Yuan, arXiv:1410.4877

Higgs transverse momentum distribution

Higgs transverse momentum

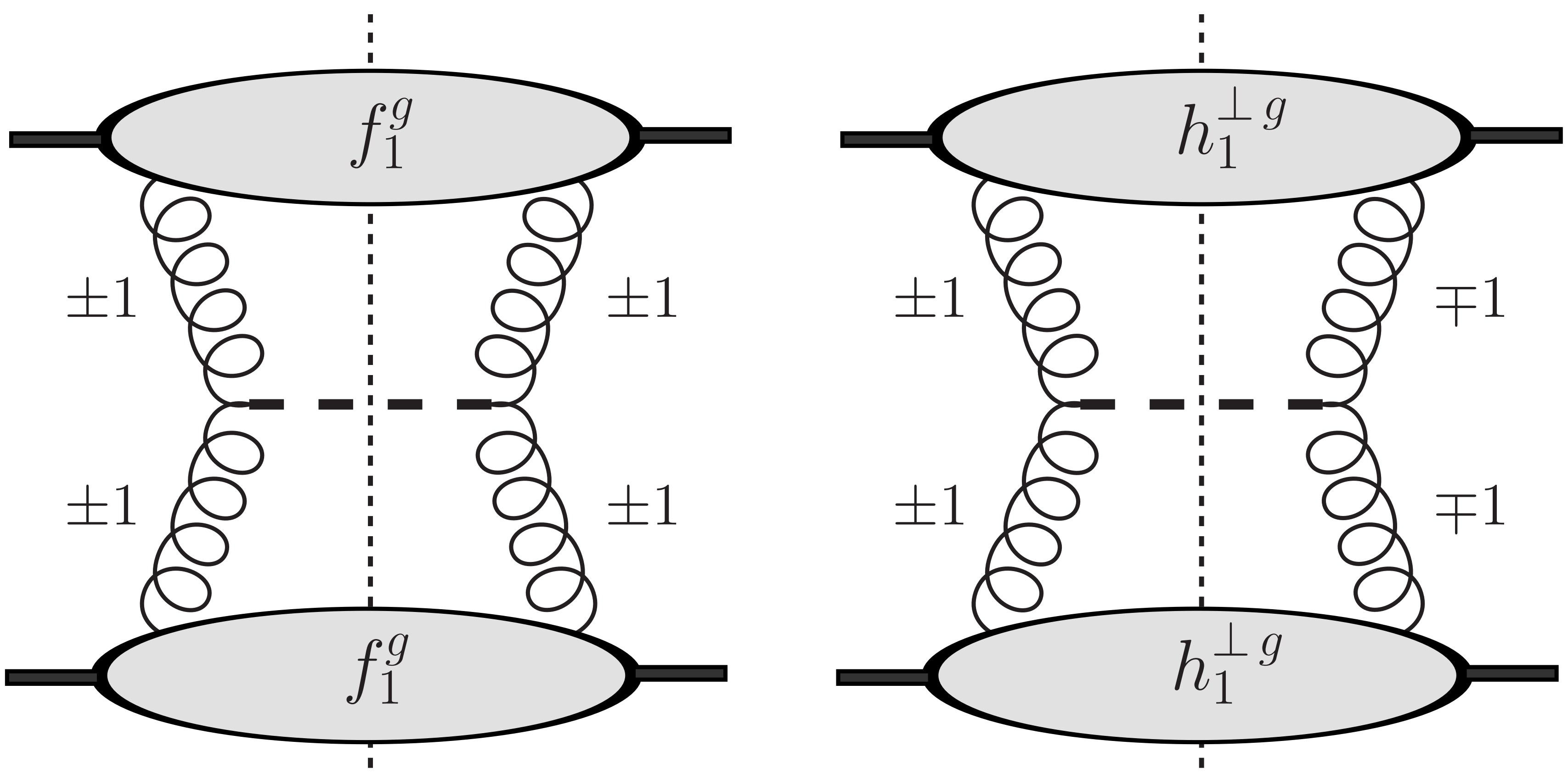
The transverse momentum distribution in Higgs production at LHC is also a TMD factorizing process

P. Sun, B.-W. Xiao & F. Yuan, PRD 84 (2011) 094005

In this case starting the evolution from a fixed scale Q_0 is not appropriate due to the large Q/Q_0 ratio

The linear polarization of gluons inside the unpolarized protons plays a role

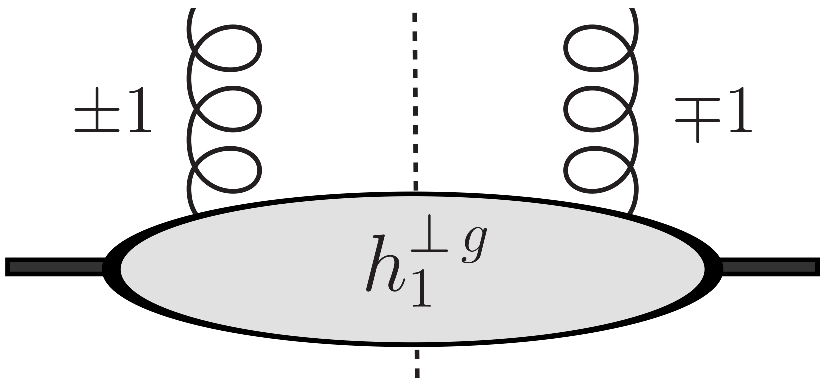
Catani & Grazzini, 2010; Sun, Xiao, Yuan, 2011; D.B., Den Dunnen, Pisano, Schlegel, Vogelsang, 2012



Gluon polarization inside unpolarized protons

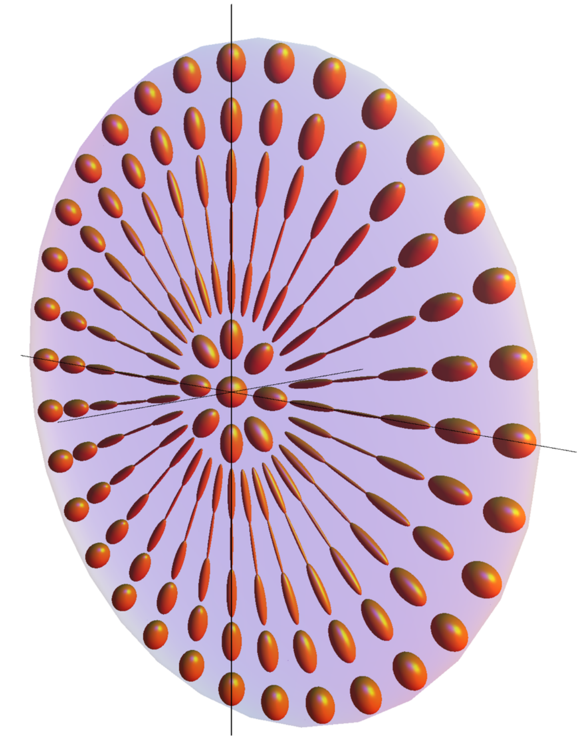
Linearly polarized gluons exist in unpolarized hadrons

Mulders, Rodrigues, 2001



an interference between
±1 helicity gluon states

For $h_1^{\perp\, g} > 0$ gluons prefer to be polarized along k_T,
with a $\cos 2\phi$ distribution of linear polarization
around it, where $\phi = \angle(k_T, \varepsilon_T)$



It affects the transverse momentum distribution in $pp \to HX$ (Higgs production)

Catani & Grazzini, 2010; Sun, Xiao, Yuan, 2011; D.B., Den Dunnen, Pisano, Schlegel, Vogelsang, 2012

TMD factorization expressions

$$\frac{d\sigma}{dx_A dx_B d\Omega d^2\boldsymbol{q}_T} = \int d^2b\, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T}\,\tilde{W}(\boldsymbol{b}, Q; x_A, x_B) + \mathcal{O}\left(\frac{Q_T^2}{Q^2}\right)$$

$$\tilde{W}(\boldsymbol{b}, Q; x_A, x_B) = \tilde{f}_1^g(x_A, \boldsymbol{b}^2; \zeta_A, \mu)\, \tilde{f}_1^g(x_B, \boldsymbol{b}^2; \zeta_B, \mu) H\left(Q; \mu\right)$$

TMD factorization expressions

$$\frac{d\sigma}{dx_A dx_B d\Omega d^2\boldsymbol{q}_T} = \int d^2b\, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T} \tilde{W}(\boldsymbol{b}, Q; x_A, x_B) + \mathcal{O}\left(\frac{Q_T^2}{Q^2}\right)$$

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This is a naive expression, since gluons can be polarized inside unpolarized protons

[Mulders, Rodrigues '01]

$$\begin{aligned}\Phi_g^{\mu\nu}(x, \boldsymbol{p}_T) &= \frac{n_\rho\, n_\sigma}{(p{\cdot}n)^2} \int \frac{d(\xi{\cdot}P)\, d^2\xi_T}{(2\pi)^3}\, e^{ip\cdot\xi}\, \langle P|\, \mathrm{Tr}\big[\, F^{\mu\rho}(0)\, F^{\nu\sigma}(\xi)\, \big]\, |P\rangle \,\big]_{\rm LF} \\ &= -\frac{1}{2x}\left\{ g_T^{\mu\nu}\, f_1^g - \left(\frac{p_T^\mu p_T^\nu}{M^2} + g_T^{\mu\nu}\frac{\boldsymbol{p}_T^2}{2M^2}\right) h_1^{\perp\, g}\right\}\end{aligned}$$

Second term requires nonzero k_T, but is k_T even, chiral even and T even

$$\tilde{\Phi}_g^{ij}(x, \boldsymbol{b}) = \frac{1}{2x}\left\{ \delta^{ij}\, \tilde{f}_1^g(x, b^2) - \left(\frac{2b^i b^j}{b^2} - \delta^{ij}\right) \tilde{h}_1^{\perp\, g}(x, b^2)\right\}$$

Cross section

$$\frac{E\, d\sigma^{pp\to HX}}{d^3\vec{q}}\bigg|_{q_T\ll m_H} = \frac{\pi\sqrt{2}G_F}{128 m_H^2 s}\left(\frac{\alpha_s}{4\pi}\right)^2 |\mathcal{A}_H(\tau)|^2$$

$$\times\left(\mathcal{C}\left[f_1^g\, f_1^g\right] + \mathcal{C}\left[w_H\, h_1^{\perp g}\, h_1^{\perp g}\right]\right) + \mathcal{O}\left(\frac{q_T}{m_H}\right)$$

$$w_H = \frac{(\boldsymbol{p}_T\cdot\boldsymbol{k}_T)^2 - \frac{1}{2}\boldsymbol{p}_T^2\boldsymbol{k}_T^2}{2M^4} \qquad\qquad \tau = m_H^2/(4m_t^2)$$

The relative effect of linearly polarized gluons:

$$\mathcal{R}(Q_T) \equiv \frac{\mathcal{C}[w_H\, h_1^{\perp g}\, h_1^{\perp g}]}{\mathcal{C}[f_1^g\, f_1^g]}$$

$$\mathcal{R}(Q_T) = \frac{\int d^2\boldsymbol{b}\, e^{i\boldsymbol{b}\cdot\boldsymbol{q}_T}\, e^{-S_A(b_*,Q)-S_{NP}(b,Q)}\ \tilde{h}_1^{\perp g}(x_A, b_*^2; \mu_{b_*}^2, \mu_{b_*})\ \tilde{h}_1^{\perp g}(x_B, b_*^2; \mu_{b_*}^2, \mu_{b_*})}{\int d^2\boldsymbol{b}\, e^{i\boldsymbol{b}\cdot\boldsymbol{q}_T}\ e^{-S_A(b_*,Q)-S_{NP}(b,Q)}\, \tilde{f}_1^g(x_A, b_*^2; \mu_{b_*}^2, \mu_{b_*})\, \tilde{f}_1^g(x_B, b_*^2; \mu_{b_*}^2, \mu_{b_*})}$$

CSS approach

Consider now only the perturbative tails:

$$\tilde{f}_1^g(x, b^2; \mu_b^2, \mu_b) = f_{g/P}(x; \mu_b) + \mathcal{O}(\alpha_s)$$

$$\tilde{h}_1^{\perp g}(x, b^2; \mu_b^2, \mu_b) = \frac{\alpha_s(\mu_b) C_A}{2\pi} \int_x^1 \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1\right) f_{g/P}(\hat{x}; \mu_b) + \mathcal{O}(\alpha_s^2)$$

This coincides with the CSS approach

[Nadolsky, Balazs, Berger, C.-P. Yuan, '07; Catani, Grazzini, '10; P. Sun, B.-W. Xiao, F. Yuan, '11]

PHYSICAL REVIEW D **86,** 094026 (2012)

Improved resummation prediction on Higgs boson production at hadron colliders

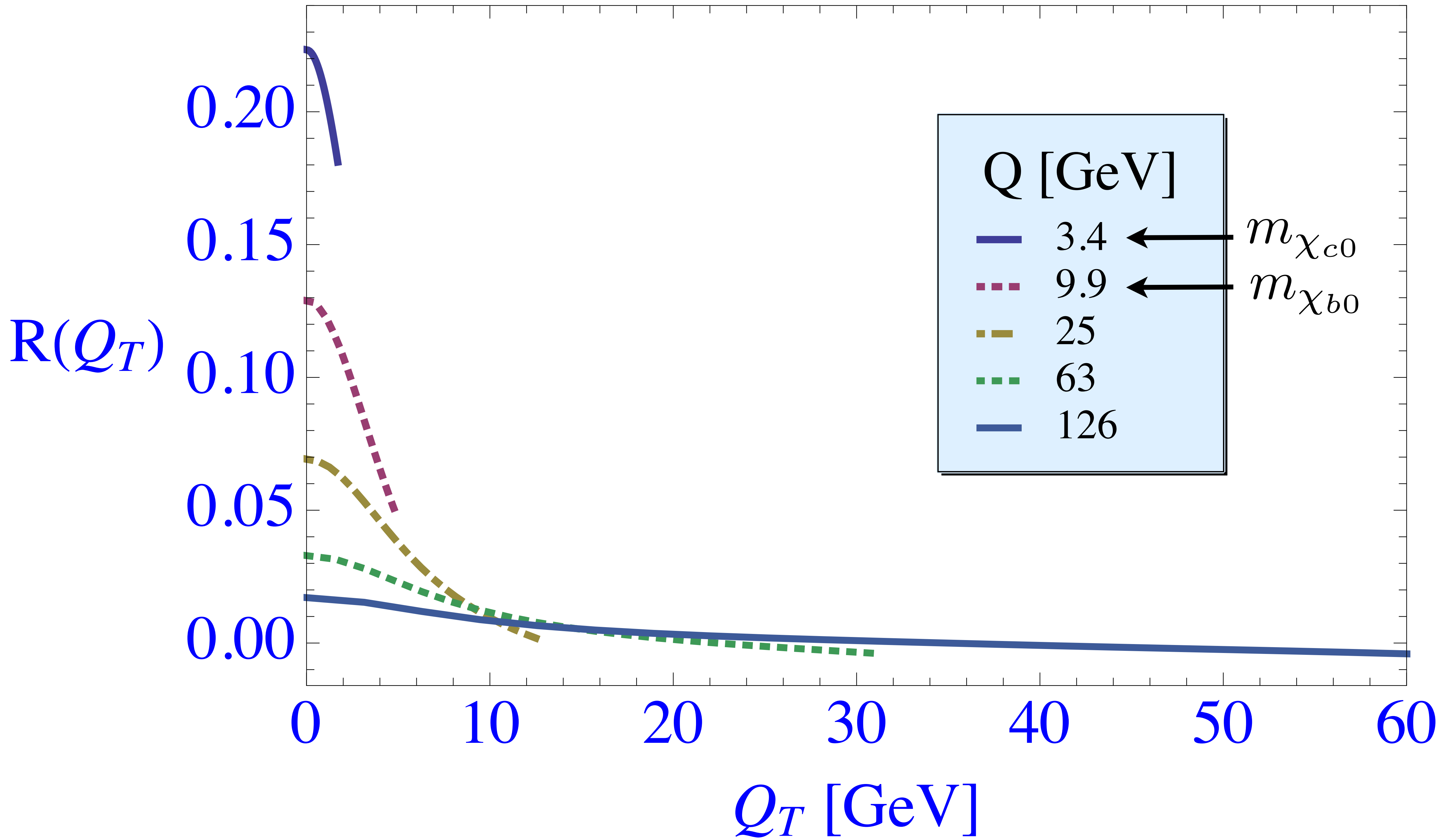
Jian Wang,[1] Chong Sheng Li,[1,2,*] Hai Tao Li,[1] Zhao Li,[3,†] and C.-P. Yuan[2,3,‡]

They find permille level effects at the Higgs scale, but using the TMD approach at the LL level yields percent level effects

D.B. & den Dunnen, NPB 886 (2014) 421

Wang et al. include α_s^2 terms, but in denominator only, and also use a different pdf set and S_{NP}

TMD / CSS evolution effects



$x_A = x_B = Q/(8\text{TeV})$

MSTW08 LO gluon distribution

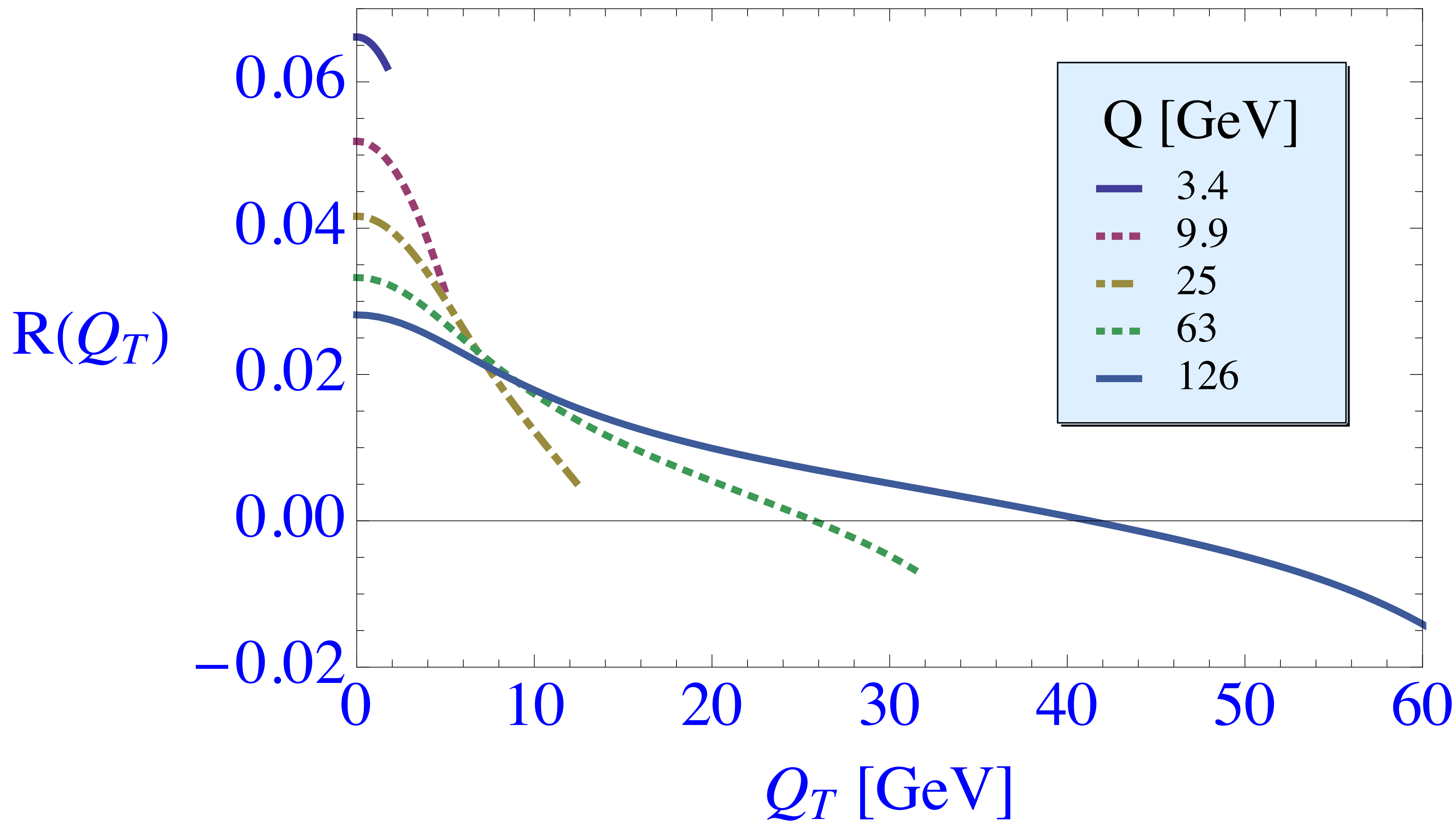
D.B. & den Dunnen, NPB 886 (2014) 421

Beyond CSS

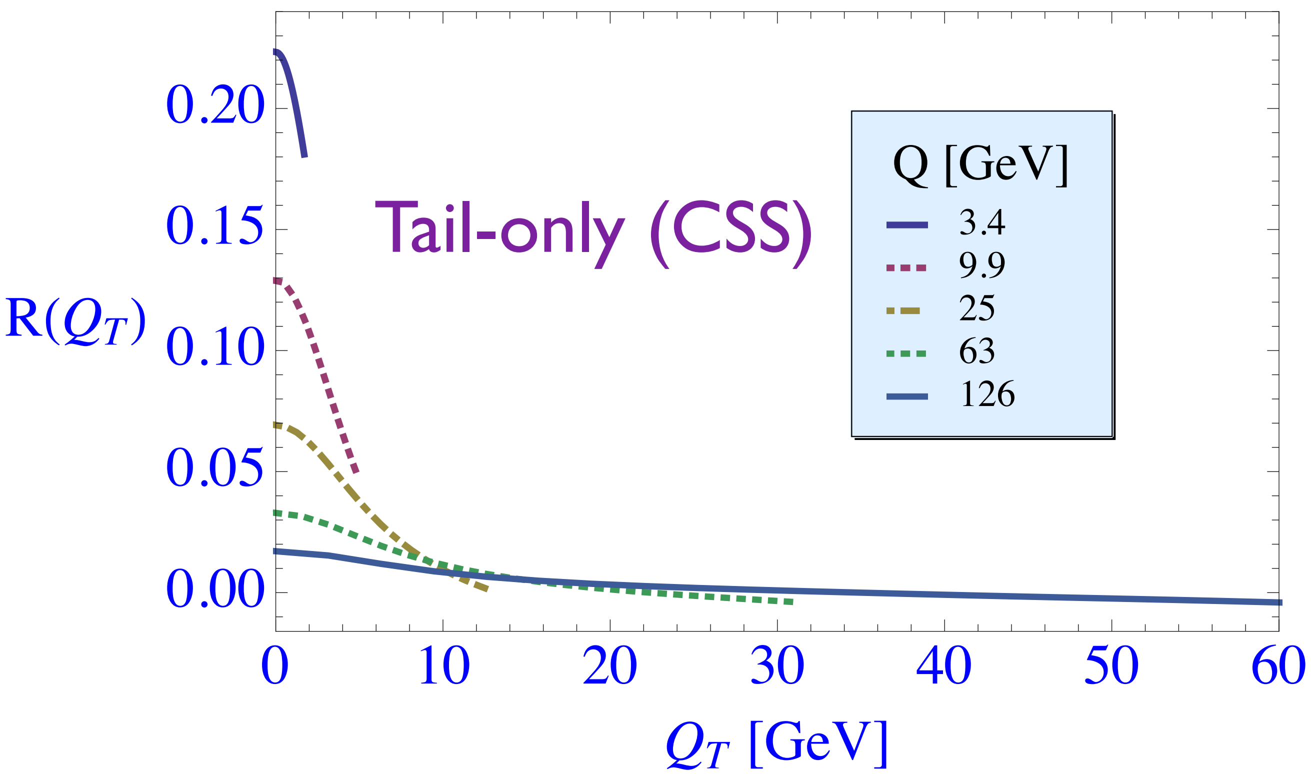
In the TMD factorized expression there may be nonperturbative contributions from small p_T which mainly affect large b

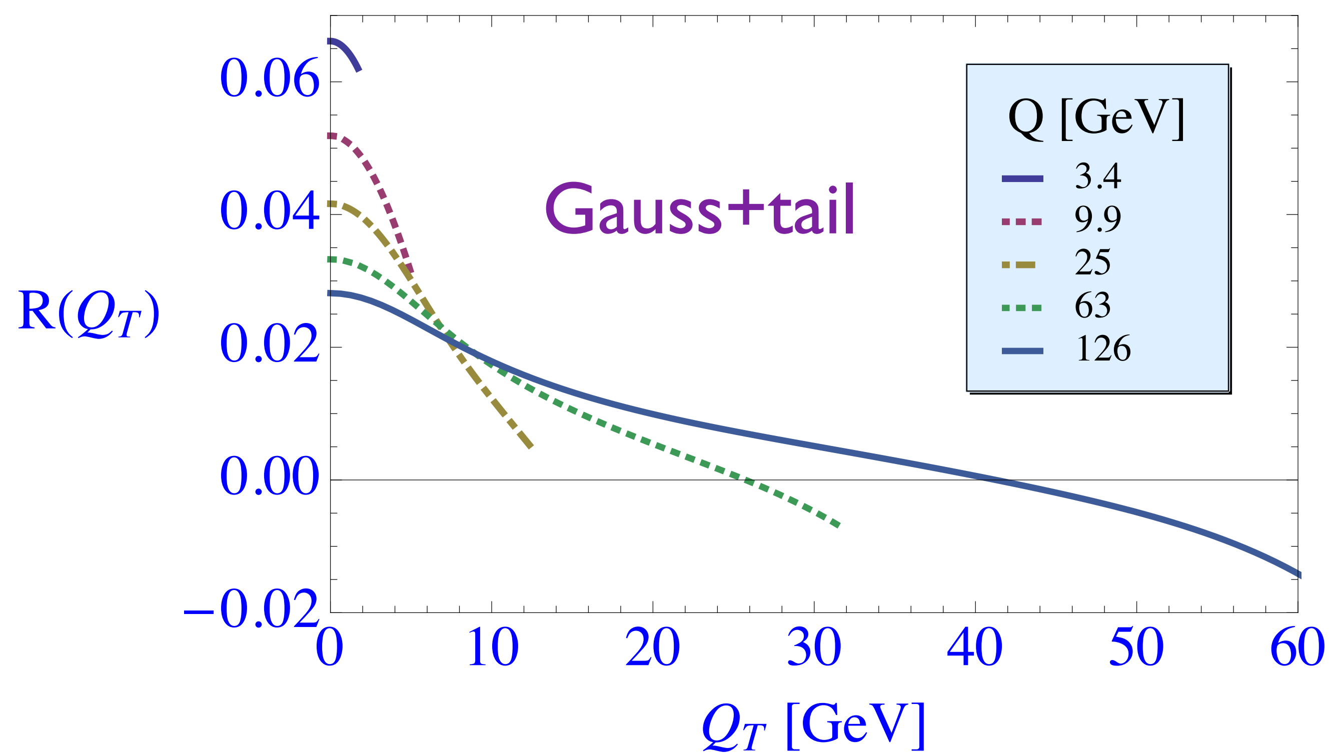
CSS only allows NP contribution via S_{NP} and does not allow all possibilities of the TMD approach

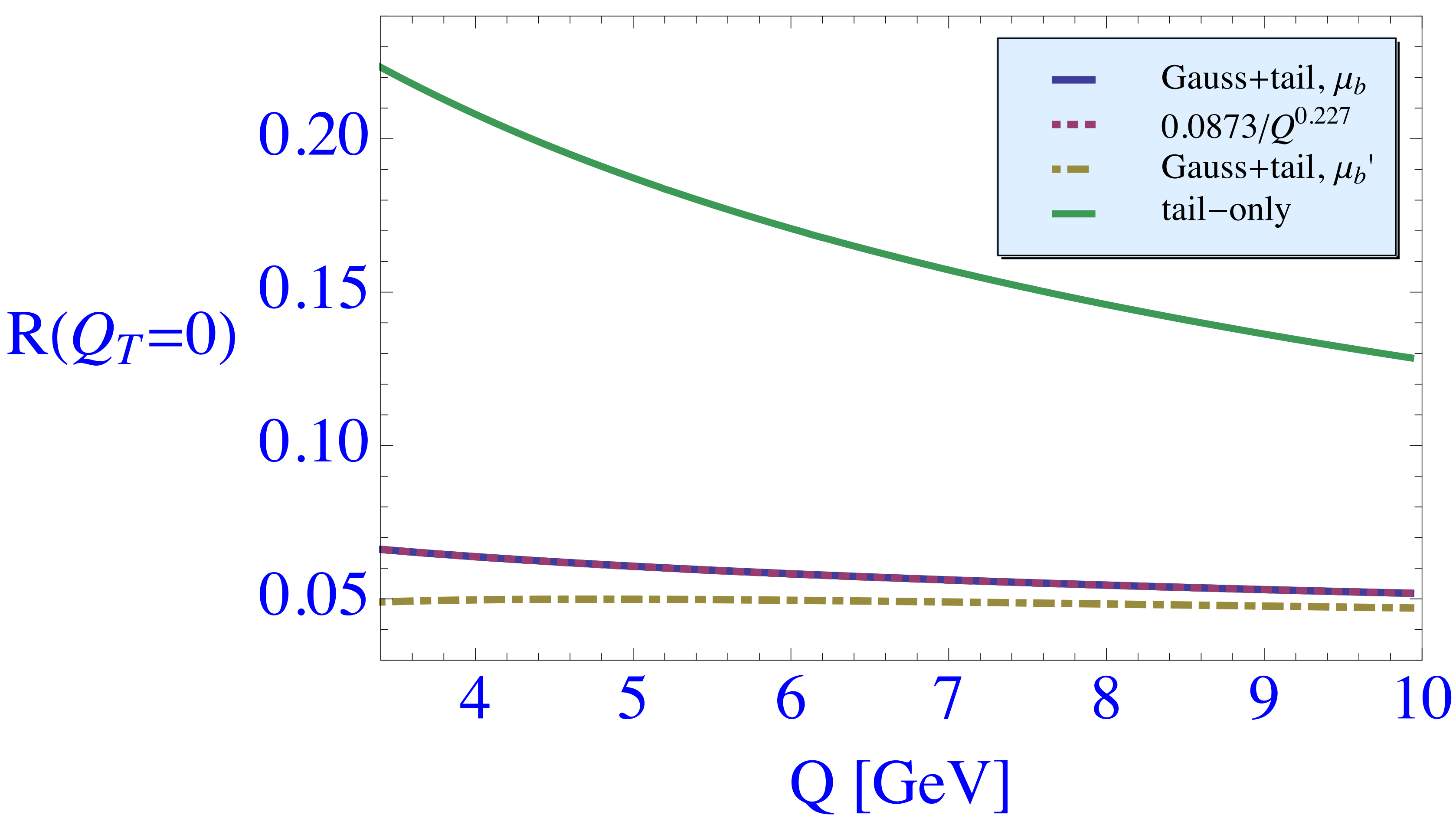
To illustrate this we consider a model which is approximately Gaussian at low p_T and has the correct tail at high p_T or small b



Comparison







Gaussian+tail evolves much more slowly than tail-only (CSS) expression

Very small b region

For very small b region ($b << 1/Q$) the perturbative expressions for S_A are all incorrect

$$S_A(b,Q) = \frac{C_A}{\pi}\int_{\mu_b^2}^{Q^2}\frac{d\mu^2}{\mu^2}\alpha_s(\mu)\,[\ldots] \overset{b\ll 1/Q}{\longrightarrow} -\frac{C_A}{\pi}\int_{Q^2}^{\mu_b^2}\frac{d\mu^2}{\mu^2}\alpha_s(\mu)\,[\ldots]$$

As a consequence e^{-0} becomes $e^{-\infty}$, in other words, F.T. [W(b)] < 0 at larger q_T

See e.g. Boglione, Gonzalez Hernandez, Melis, Prokudin, 1412.1383

Standard regularization:

$$Q^2/\mu_b^2 = b^2 Q^2/b_0^2 \to Q^2/{\mu_b'}^2 \equiv (bQ/b_0 + 1)^2$$

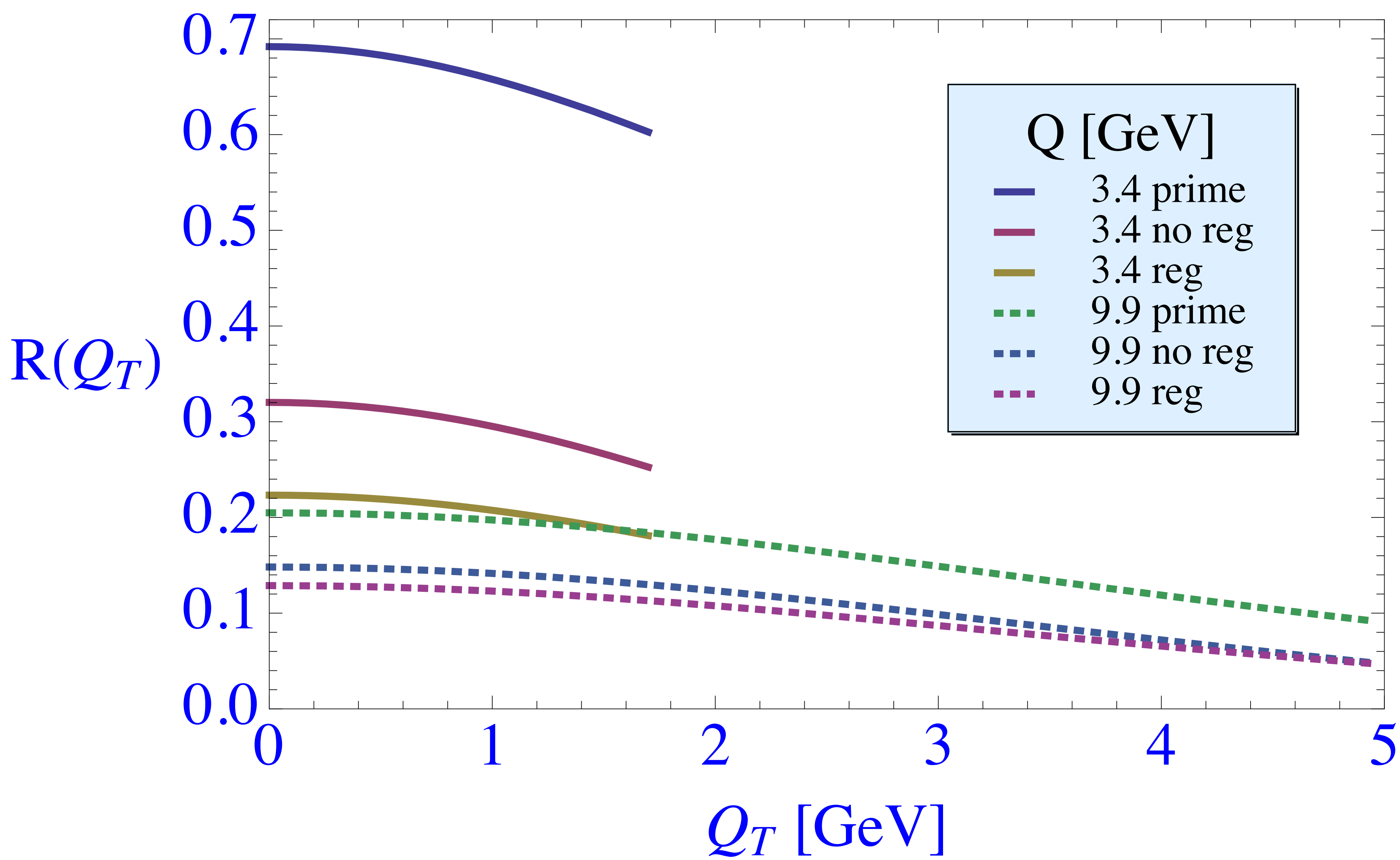
Parisi, Petronzio, 1985

Although $b << 1/Q$ is perturbative, it is not clear what is the right expression to take in TMD factorization

Precise form of Parisi-Petronzio regularization usually irrelevant since matching to Y-term is needed anyway, *but not so in the Higgs case where the problem already arises at q_T=0*!

Very small b region

At low Q there is quite some uncertainty from the very small b region (b << 1/Q) where the perturbative expressions for S_A are all incorrect (don't satisfy S(0)=0)



reg=standard regularization:

$$Q^2/\mu_b^2 = b^2 Q^2/b_0^2 \rightarrow Q^2/\mu_b'^2 \equiv (bQ/b_0 + 1)^2$$

Parisi, Petronzio, 1985

prime=evolve everything to scale μ_b'

Very small b region

Altarelli, Ellis, Martinelli, 1985:

$$\frac{d\sigma}{dq_T^2} = Y(q_T^2) + \int \frac{d^2\mathbf{b}}{4\pi} e^{-i\mathbf{q}_T\cdot\mathbf{b}} \sigma_0(1+A) \exp S(b)$$

where

$$S(b) = \int_0^{A_T^2} \frac{dk^2}{k^2} (J_0(bk) - 1) \left(B \ln \frac{Q^2}{k^2} + C \right).$$

$$A_T^2 = A_T^2(y) = \frac{(S+Q^2)^2}{4S\cosh^2 y} - Q^2.$$

$$\exp S = \exp \int_0^{A_T^2} \approx \left(1 + \int_{Q^2}^{A_T^2}\right) \exp \int_0^{Q^2}$$

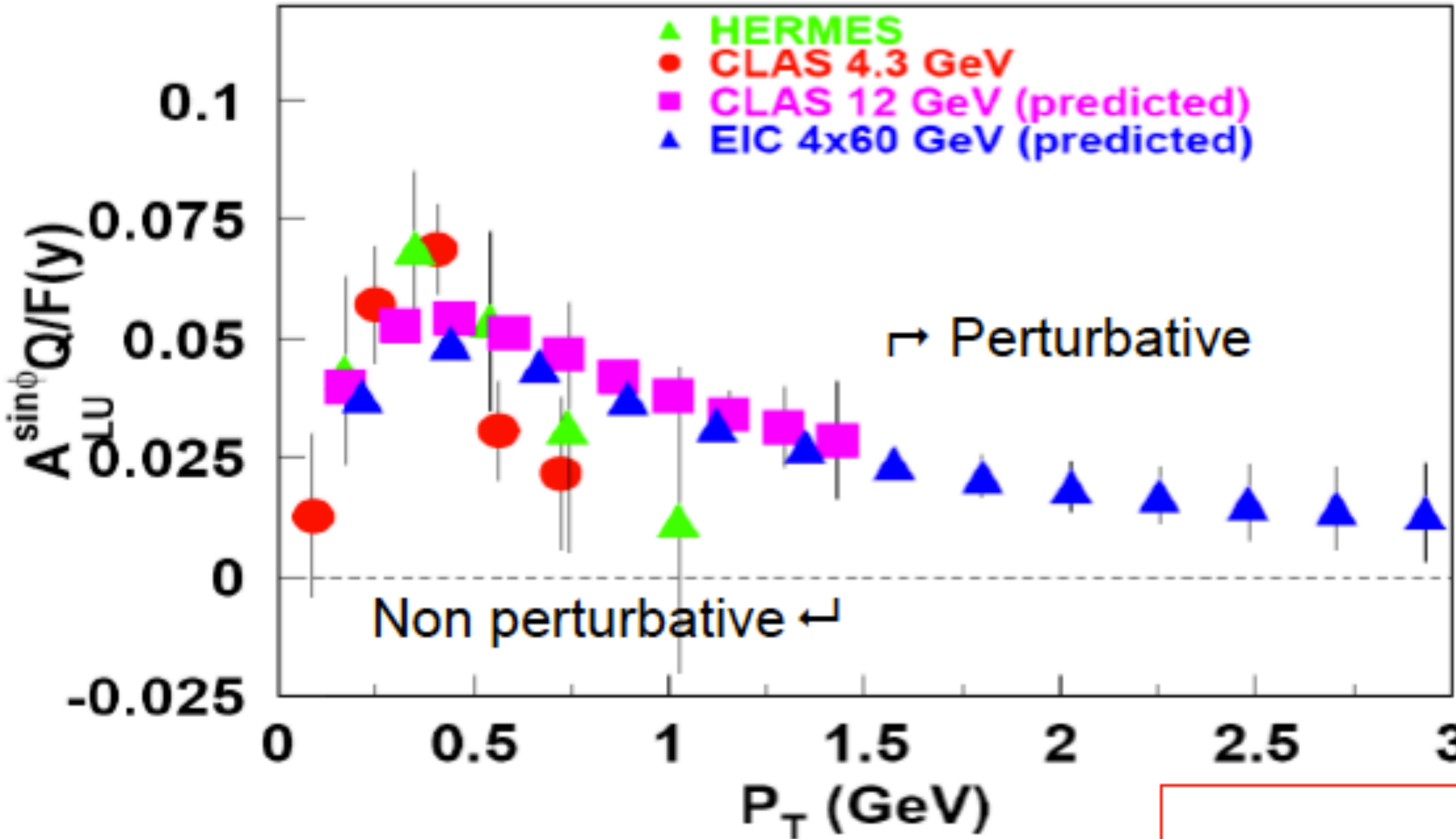
This expression does satisfy S(0)=0

Additional resummations by Echevarria *et al.* may reduce this very small b problem?

(and perhaps the ones in Boglione, Gonzalez Hernandez, Melis, Prokudin, 1412.1383)

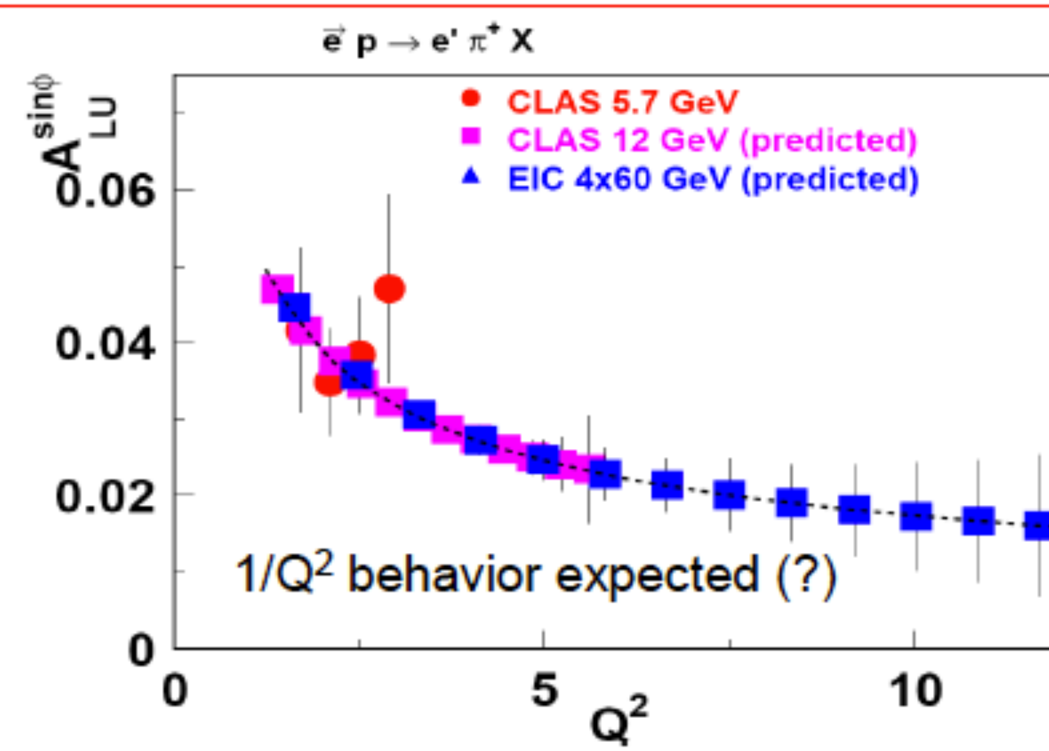
Higher twist

P_T and Q^2-dep Higher Twist $A_{LU}^{\sin\phi}$



Study for SSA transition from non-perturbative to perturbative regime.

E12-06-112



Study for Q^2 dependence of beam SSA allows to check the higher twist nature and access quark-gluon correlations

TMDs beyond leading twist

Subleading twist asymmetries are relevant for HERMES, COMPASS, JLab, J-Parc

There is no TMD factorization established yet for subleading twist

Promising hints regarding TMD factorization beyond leading twist are found in:

Boer & Vogelsang, PRD 74 (2006) 014004

Bacchetta, Boer, Diehl & Mulders, JHEP 0808 (2008) 023

Processes involving higher twist TMD $f^{\perp}$ (which enters the Cahn effect)

Small x

Small x

TMD factorization breaking processes can be TMD factorizing in small-x limit

Factorization breaking contributions may become suppressed

Small x

TMD factorization breaking processes can be TMD factorizing in small-x limit

Factorization breaking contributions may become suppressed

for nearly back-to-back di-jets ($q_\perp \equiv |k_\perp + k'_\perp| \ll |k_\perp|, |k'_\perp| \equiv P_\perp$) :

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{d^2k_\perp d^2k'_\perp} = \sum_a x_p f_{a/p}(x_p) \sum_i F_{g/A}^{(a,i)}(x_A, q_\perp)\; H_{ag}^{(i)} \;\exp\left[-b_i \ln^2\left(\frac{P_\perp}{q_\perp}\right)\right]$$

$F^{(a,i)}$: obtained from two-independent unintegrated gluons $G^{(1)}$ and $G^{(2)}$ (with different operator definitions)

Dominguez, Marquet, Xiao and Yuan (2011)

hard matrix elements

CSS-like Sudakov factors

Mueller, Xiao and Yuan (2013)

Small x

TMD factorization breaking processes can be TMD factorizing in small-x limit

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Mueller, Xiao and Yuan (2013)

$$xG^{(1)}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\text{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}\right]|P\rangle$$

$$xG^{(2)}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\text{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}\right]|P\rangle$$

Small x

Involvement of the two "universal" gluon distributions in other processes:

	DIS and DY	SIDIS	hadron in pA	photon-jet in pA	Dijet in DIS	Dijet in pA
$G^{(1)}$ (WW)	×	×	×	×	√	√
$G^{(2)}$ (dipole)	√	√	√	√	×	√

Dominguez *et al.*: "The large N_c limit is essential in order to eliminate other non-universal distributions or correlators in other different dijet channels, i.e., $qg \to qg$, $gg \to q\bar{q}$ and $gg \to gg$ in pA collisions"

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"One-Loop Factorization for Inclusive Hadron Production in p-A Collisions in the Saturation Formalism", Chirilli, Xiao, Yuan, PRL 108 (2012) 122301

$$\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} = \sum_a \int \frac{dz}{z^2}\frac{dx}{x}\xi x f_a(x,\mu) D_{h/c}(z,\mu) \times \int [dx_\perp] S^Y_{a,c}([x_\perp]) \mathcal{H}_{a\to c}(\alpha_s, \xi, [x_\perp]\mu),$$

$$\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} = \int \frac{dz}{z^2}\frac{dx}{x}\xi x q(x,\mu) D_{h/q}(z,\mu) \times \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2}\Big\{ S^{(2)}(x_\perp, y_\perp) \times \Big[\mathcal{H}^{(0)}_{2qq} + \frac{\alpha_s}{2\pi}\mathcal{H}^{(1)}_{2qq}\Big] + \int \frac{d^2b_\perp}{(2\pi)^2} S^{(4)}(x_\perp, b_\perp, y_\perp)\frac{\alpha_s}{2\pi}\mathcal{H}^{(1)}_{4qq}\Big\}$$

Gluon polarization at small x

Often it is said that polarization does not matter at small-x
More specifically this refers to $\Delta g(x)$ which at small x is suppressed w.r.t. g(x)

Evolution kernel does not have 1/x behavior, see e.g. Maul's CCFM study, 2002

$$\Delta P_{gg}(z) = \frac{2C_A(2-z)}{1-z}$$

This is relevant for the spin sum rule, where one integrates over all x values

Δg corresponds to circularly polarized gluons

Linearly polarized gluon distribution inside unpolarized protons does grow with 1/x, it can even become maximal

At small x the k_T-factorization approach implies maximum polarization:

$$\Phi_g^{\mu\nu}(x, \boldsymbol{p}_T)_{\mathrm{max\ pol}} = \frac{2}{x}\, \frac{p_T^\mu p_T^\nu}{\boldsymbol{p}_T^2}\, f_1^g$$

Catani, Ciafaloni, Hautmann, 1991

Applied to Higgs production by Lipatov, Malyshev, Zotov in 1402.6481

Gluon polarization at small x

At small x the WW (or CGC) gluon field and the dipole distribution have been studied:

$$h_{1,WW}^{\perp g} \ll f_{1,WW}^{\perp g} \quad \text{for } k_\perp \ll Q_s, \qquad h_{1,WW}^{\perp g} = 2 f_{1,WW}^{\perp g} \quad \text{for } k_\perp \gg Q_s$$

$$x h_{1,DP}^{\perp g}(x, k_\perp) = 2 x f_{1,DP}^{g}(x, k_\perp)$$

Metz, Zhou, 2011

	DIS and DY	SIDIS	hadron in pA	photon-jet in pA	Dijet in DIS	Dijet in pA
$G^{(1)}$ (WW)	×	×	×	×	√	√
$G^{(2)}$ (dipole)	√	√	√	√	×	√

DIS, DY, SIDIS, hadron and photon+jet in pA are not sensitive to $h_1^{\perp g}$

It would thus be very interesting to study dijet DIS at a high-energy EIC (small x in and outside the saturation region)

Pisano, Boer, Brodsky, Buffing & Mulders, JHEP 10 (2013) 024

Note: for dijet in DIS the result does not require large N_c

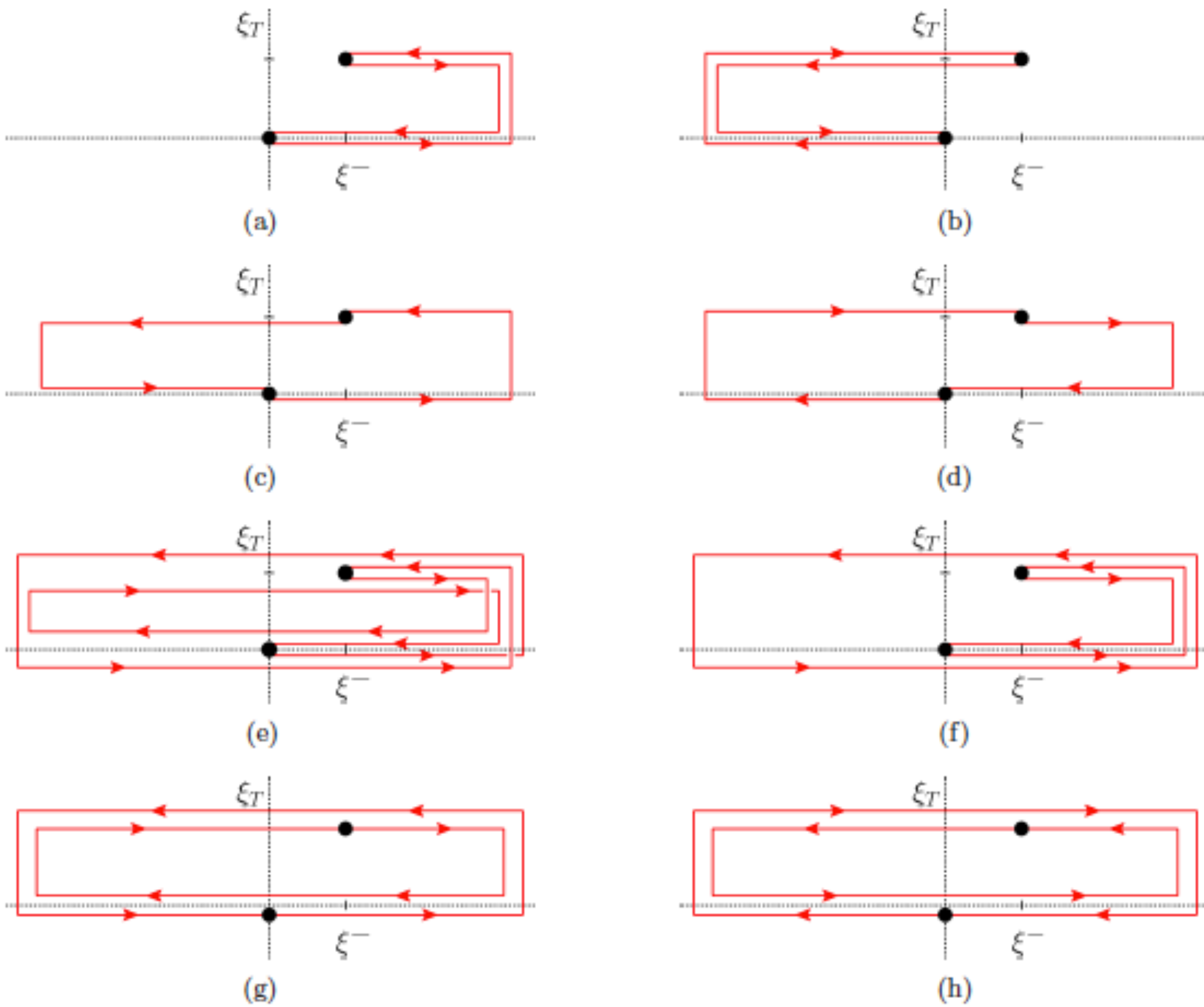
Nonuniversality

For dijet in pA the result does require large N_c. More generally there are 5 TMDs:

$$h_1^{\perp g[U]}(x, p_T^2) \;=\; h_1^{\perp g(A)}(x, p_T^2) + \sum_{c=1}^{4} C_{GG,c}^{[U]}\, h_1^{\perp g(Bc)}(x, p_T^2)$$

Note: without ISI/FSI it can still be nonzero

Buffing, Mukherjee, Mulders, 2013



Conclusions

Conclusions

- Significant recent developments on TMD factorization and evolution:
  - New TMD factorization expressions by JCC (2011) & EIS (2012)
  - Improvements through additional resummations (Echevarria *et al.*) lifts analyses to the NNLL level (2013/4)
  - Progress towards describing SIDIS, DY & Z production data by a universal non-perturbative function (2013/4)

- Consequences of TMD evolution studied (in varying levels of accuracy) for:
  - Sivers & (single and double) Collins effect asymmetries
  - Higgs production including the effect of linear gluon polarization

- Future data from JLab12 and BES and perhaps a high-energy EIC can help to map out the Q dependence of Sivers and Collins asymmetries in greater detail

- Future data from LHC on Higgs and $\chi_{c/b0}$ production and from dijet DIS at a high-energy EIC can shed light on $h_1^{\perp g}$ effects and gluon dominated TMD processes

- TMD (non-)factorization at next-to-leading twist remains entirely unexplored