Introduction to QCD Evolution of Parton Distributions: Comparative View

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Outline:

- **Introduction:** from $1D$- to $3D$-structure of the nucleon in QCD
- $3D$-structure in the momentum representation
- Beyond the tree-approximation: **gauge invariance and Wilson lines**
- Beyond the tree-approximation: **singularity issues**
- **Evolution and resummation** approaches for unintegrated PDFs in different energy-rapidity regions
- **Outlook**

QCD analysis of DIS: Collinear factorization

$1D$-structure of hadrons is captured within the **well-defined QCD-based framework**

Hadronic tensor

$$W_{\mu\nu} = \frac{1}{2\pi} \Im m \left[i \int d^4\xi \ e^{iq\xi} \langle P | T\{J_\mu(\xi) J_\nu(0)\} | P \rangle \right]$$

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) F_1(x_B, Q^2) +$$

$$+ \frac{1}{P \cdot q} \left(P_\mu - q_\mu \frac{P \cdot q}{q^2} \right) \left(P_\nu - q_\nu \frac{P \cdot q}{q^2} \right) F_2(x_B, Q^2)$$

Collinear factorization: Structure functions

$$F(x, Q^2) = H(x, Q^2/\mu^2) \otimes \mathcal{F}(x, \mu^2)$$
$$= \sum_i \int_x^1 \frac{d\xi}{\xi} \, C_i\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}\right) \, \mathcal{F}_i(\xi, \mu^2)$$

Renormalization properties: DGLAP

$$\mu \frac{d}{d\mu} \mathcal{F}_i(x, \mu^2) = \sum_j \int_x^1 \frac{dz}{z} \, P_{ij}\left(\frac{x}{z}\right) \mathcal{F}_j(x, \mu^2)$$

Collinear (integrated) PDF: Operator definition

Longitudinal **momentum fraction:**

$$x k^+ = P^+$$

$$\mathcal{F}(x, \mu^2) = \frac{1}{2} \int \frac{dz^-}{2\pi} \, \mathrm{e}^{-ik^+z^-} \langle h | \bar{\psi}(z^-, \mathbf{0}_\perp) \gamma^+ \psi(0^-, \mathbf{0}_\perp) | h \rangle$$

Gauge invariance:

$$\psi(x) \to U(x)\psi(x)$$
$$\bar{\psi}(x) \to \bar{\psi}(x) U^\dagger(x)$$

Gauge invariance of bi-local operator products

Generic bi-local product:

$$\Delta(y,x) = \bar{\psi}(y)\psi(x)$$

$$\Delta(y,x) \to \bar{\psi}(y)U^\dagger(y)U(x)\psi(x)$$

Problem: to find a 'transporter'

$$T_{[y,x]}\psi(x) \to U(y)[T_{[y,x]}\psi(x)]$$

Bi-local product supplied with the **transporter** is gauge invariant:

$$\bar{\psi}(y)\,T_{[y,x]}\psi(x) \to$$

$$\bar{\psi}(y)U^\dagger(y)U(y)[T_{[y,x]}\psi(x)] = \bar{\psi}(y)\,T_{[y,x]}\psi(x)$$

Parallel transport equation

$$\frac{d}{dt}\ T_{[y,x]} = \pm ig\mathcal{A}_\gamma(t)\, T_{[y,x]}$$

Path-dependence:

$$z \in \gamma$$

$$dz_\mu = \dot{\gamma}_\mu(t) dt,\ z(0) = x,\ z(t) = y$$

$$\mathcal{A}_\gamma(t) = A_\mu[z(t)]\ \dot{\gamma}_\mu(t)$$

Parallel transport equation: Wilson line

$$T^{(0)} = T_{[x,x]} = 1$$

$$T_{[y,x]} = \mathcal{P}\exp\left[\pm ig\int_x^y A_\mu[z]dz_\mu\right]_\gamma$$

Parallel transporter is a Wilson line:

$$T_{[y,x]} = U_\gamma[y,x]$$

Gauge-invariant correlation functions: Main issues

$$\mathcal{F}(k)_\gamma = \text{F.T.} \ \langle h| \ \bar{\Psi}(z) \ \mathcal{W}_\gamma[z,0] \ \Psi(0) \ |h\rangle$$

Gauge invariance is guaranteed by the **Wilson line**

$$\mathcal{W}_\gamma = \mathcal{P} \ \exp\left[\pm ig \int_0^z d\zeta^\mu \mathcal{A}_\mu(\zeta)\right]_\gamma$$

Issues:

- Gauge invariance $\to$ complicated **structure of the Wilson lines**
- Path dependence $\to$ **universality** is geopardized
- Singularities $\to$ problems with **renormalization**
- Factorization $\to$ **evolution**

3D Hadronic Correlators: Structure of Nucleon beyond the Collinear Approximation

@ [Belitsky, Ji, Yuan (2003); Boer, Mulders, Pijlman (2003)]

Generic 3D hadronic correlator with the light-like and transverse gauge links

$$\mathcal{F}(k^+, k_\perp; \text{scales}) \sim$$
$$\text{F.T. } \langle h|\bar{\Psi}(z)\ \mathcal{W}_\gamma[z^-, z_\perp; 0^-, 0_\perp]\Psi(0)|h\rangle$$

$$\gamma \to \{n \cup l_\perp\}$$

Tree-level:

$$\mathcal{F}^{(0)}(k^+, k_\perp) = \delta(k^+ - p^+)\delta^{(2)}(k_\perp)$$

$$\int d^2k_\perp \mathcal{F}(k^+, k_\perp) = \mathcal{F}(k^+) = \text{collinear limit}$$

$$\mathcal{F}(k^+, \mu) = \int dz^-\ \mathrm{e}^{-ik^+z^-} \langle h|\bar{\Psi}(z)\ \mathcal{W}_n[z^-, 0^-]\Psi(0)|h\rangle$$

Quantum corrections: $\to$ emergent (light-cone/rapidity/overlapping) singularities $\to$ problems with renormalization and evolution

Beyond the tree approximation: Why divergences?

$$\langle h|_H \bar{\Psi}_H(z)\ \mathcal{W}_\gamma[z^-, z_\perp; 0^-, 0_\perp]\Psi_H(0)|h\rangle_H$$

$$\to$$

$$\langle h|\bar{\Psi}(z)\ \mathcal{W}_\gamma[z^-, z_\perp; 0^-, 0_\perp]\Psi(0)\ \mathbf{S}_{\rm int}|h\rangle$$

$$\mathbf{S}_{\rm int} = \int d^4x \mathcal{L}^{\rm QCD}_{\rm int}(x)$$

$\to$ Perturbative expansion, Feynman graphs etc.

Classification of Singularities in the leading $\mathcal{O}(\alpha_s)$ order

- **Ultraviolet poles**
- **Overlapping divergences**: contain the UV and rapidity poles simultaneously
- Pure **rapidity divergences**
- Specific **self-energy** divergences: stem from the gauge links, treated by modifications of the soft factors

@ [ICh, Stefanis (2008, 2009, 2010); Collins (2003, 2008, 2011, 2012, 2014 etc.); Chiu, Jain, Neill, Rothstein (2011, 2012); Avsar (2012) Echevarría, Idilbi, Scimemi (2011, 2012, 2014)]

- Penetration of the extra singularities in the anomalous dimensions of the TMDs

@ [ICh, Stefanis (2008, 2009, 2010)]

- **Collinear case**: cancellation in the interplay of the virtual and real gluon contributions

@ [Furmanski, Curci, Petronzio (1980); Fleming, Zhang (2012)]

(Unintegrated) Parton Distributions: Evolution Methods

1. **Fully inclusive processes:** collinear ($\mathbf{k}_\perp$-integrated) PDFs
   - DGLAP:
     - strong ordering in the transverse momenta of emitted patrons;
     - hard matrix elements are on-shell
     - sums up $\ln \mu^2/\Lambda^2$ logs
2. **Semi-Inclusive processes:** unintegrated PDFs
   - BFKL
   - CCFM
   - SCET
   - TMD

(Unintegrated) Parton Distributions: Evolution Methods

1. **Fully inclusive processes:** collinear ($\mathbf{k}_\perp$-integrated) PDFs
   - DGLAP

2. **Semi-Inclusive processes:** unintegrated PDFs
   - BFKL:
     - high energy = small-x domain;
     - ordering in the rapidities of emitted partons;
     - hard matrix elements are off-shell
     - sums up $\ln 1/x$ logs
     - nonlinear extensions
   - CCFM
   - SCET
   - TMD

(Unintegrated) Parton Distributions: Evolution Methods

1. **Fully inclusive processes:** collinear ($\mathbf{k}_\perp$-integrated) PDFs
   - DGLAP

2. **Semi-Inclusive processes:** unintegrated PDFs
   - BFKL
   - CCFM
     - angular ordering of the emitted patrons;
     - hard matrix elements are off-shell;
     - dependence on extra scale (maximumal angle);
     - valid for low as well as for high x
     - sums up $\ln 1/x$ a well as $\ln 1/(1-x)$ logs
   - SCET
   - TMD

(Unintegrated) Parton Distributions: Evolution Methods

1. **Fully inclusive processes:** collinear ($\mathbf{k}_\perp$-integrated) PDFs
   - DGLAP

2. **Semi-Inclusive processes:** unintegrated PDFs
   - BFKL
   - CCFM:
     - scale separation and effective Lagrangian
     - finally equivalent to the TMD approach?
   - SCET
   - TMD

(Unintegrated) Parton Distributions: Evolution Methods

1. **Fully inclusive processes:** collinear ($\mathbf{k}_\perp$-integrated) PDFs
   - DGLAP

2. **Semi-Inclusive processes:** unintegrated PDFs
   - BFKL
   - CCFM
   - SCET
   - TMD:
     - direct generalisation of the collinear picture;
     - valid (formally?) in the whole range of x;
     - extra rapidity scale;
     - combined evolution

(Unintegrated) Parton Distributions: Operator definitions?

1. **Fully inclusive processes:** collinear ($\mathbf{k}_\perp$-integrated) PDFs
   - DGLAP: defined non-perturbatively — YES
2. **Semi-Inclusive processes:** unintegrated PDFs
   - BFKL: defined via perturbative evolution — NO
   - CCFM: defined via perturbative evolution — NO
   - SCET: defined non-perturbatively — YES
   - TMD: defined non-perturbatively — YES

All Unintegrated Parton Distributions from Wigner Function?

Universal Winger function allows for **fully non-perturbative analysis**

Quantum mechanical **Wigner function**

$$\mathbf{W}(x,p) = \int d\zeta \; \mathrm{e}^{i\zeta p} \; \psi^*(x-\zeta/2)\psi(x+\zeta/2)$$

Standard distributions from the **Wigner function**

$$\int dx \; \mathbf{W}(x,p) = \tilde{\psi}^*(p)\tilde{\psi}(p) \quad , \quad \psi(x) = \int dp \; \mathrm{e}^{-ixp} \; \tilde{\psi}(p)$$

$$\int dp \; \mathbf{W}(x,p) = \psi^*(x)\psi(x)$$

All Unintegrated Parton Distributions from the Wigner function?

Quark **Wigner function**

$$\mathbf{W}(\vec{r},\vec{k})_{\mathrm{r}^+=0} = \int \frac{dk^-}{2\pi} \int d^4\zeta \ \mathrm{e}^{i\zeta k} \ \bar{\Psi}(r-\zeta/2)\Psi(r+\zeta/2)$$

Reductions of the **Wigner function**

$$\int d^2k \ \mathbf{W}(\vec{r},\vec{k}) \to \mathrm{GPD}$$

$$\int d^2r \ \mathbf{W}(\vec{r},k^+,\mathbf{k}_\perp)_{r^+=0} \to \mathrm{TMD}$$

$$\int dk^+ \int d^2k \ \mathbf{W}(\vec{r},\vec{k}) \to \mathrm{form-factors}$$

$$\int d^2k \int d^2r \ \mathbf{W}(\vec{r},k^+,\mathbf{k}_\perp)_{r^+=0} \to \mathrm{collinear\ PDF}$$

Outlook:

- Comparison of the TMD evolution with DGLAP, BFKL, nonlinear small-x, CCFM for specific processes and in general: key-talks at the REF-2014
- Is it possible to contract a consistent QCD picture of the partonic structure of the nucleon starting from universal Wigner function?
- uPDFs: a unique theoretical scheme for small- and large-x experiments (at EIC, JLab, RHIC, AFTER@LHC)