Results on TMD evolution

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(M.C.Escher, Circle Limit III)

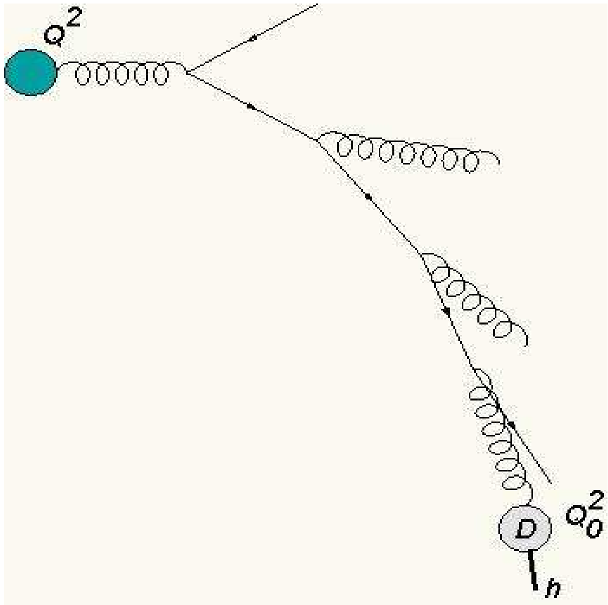
Based on F.A.Ceccopieri and L.Trentadue, arXiv:1407.7972

Outline

- TMD DGLAP equations
- Soft gluon resummation at small transverse momenta
- Analytical solutions
- Comparison with other results in the literature
- Conclusions

Overview

- TMD FF DGLAP equations originally derived by Bassetto, Ciafaloni, Marchesini, '80
- In the collinear limit, at each branching, the active parton acquires a small p_t
- Collinear emissions give however leading logarithmic corrections
- Such contributions are resummed by k_t-evolution equations
- The radiative process in the collinear limit generates therefore an appreciable p_t which adds to the non-perturbative fragmentation one



- Notably even logs due to multiple soft gluon emissions (dominant at small p_t) can be taken into account in the formalism

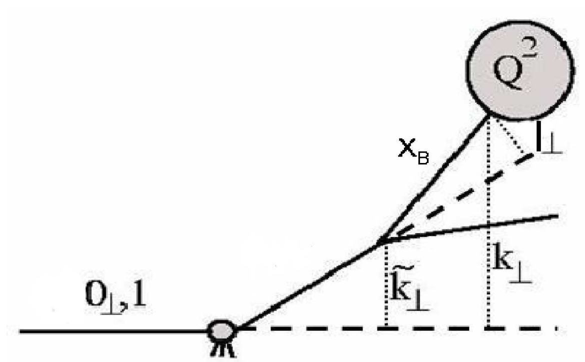
Unpolarised evolution

- Spacelike version FC, Trentadue '06

$$\mu^2 \frac{\partial}{\partial \mu^2} f_{i/P}(x, \boldsymbol{k}_\perp, \mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \int_x^1 \frac{du}{u^3} P_{ji}(u) \int \frac{d^2 \boldsymbol{l}_\perp}{\pi} \cdot$$

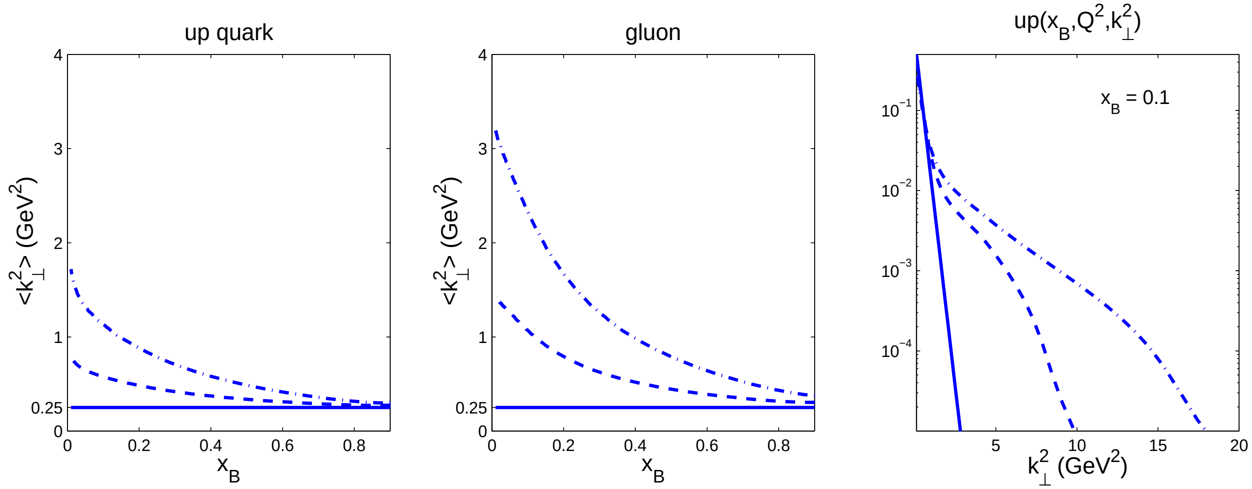
$$\cdot\, \delta[l_\perp^2 - (1-u)\mu^2] f_{j/P}\Big(\frac{x}{u}, \frac{\boldsymbol{k}_\perp - \boldsymbol{l}_\perp}{u}, \mu^2\Big)$$

- light-like gauge
- transverse boost: $\widetilde{\boldsymbol{k}}_\perp = (\boldsymbol{k}_\perp - \boldsymbol{l}_\perp)/u$
- P_{ji} : AP splitting functions
- Normalisation : $\int d^2 \mathbf{k}_\perp f_P^i(x, \mu^2, \mathbf{k}_\perp) = f_P^i(x, \mu^2)$
- δ-function : energy-momentum conservation
- $\boldsymbol{l}_\perp$: relative transverse momentum of the emitted gluon



Numerical solutions

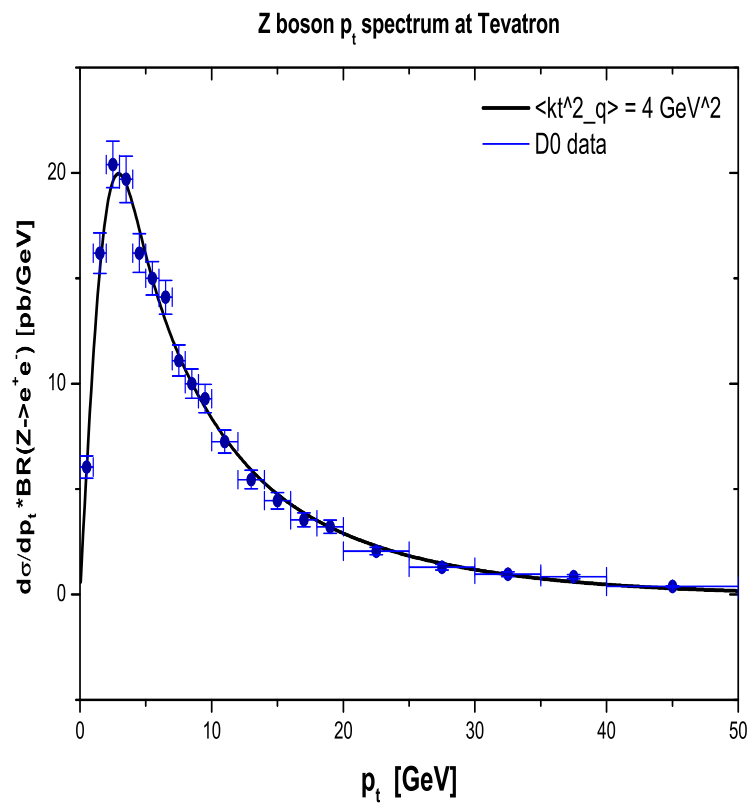
- $\langle \boldsymbol{k}_\perp^2 \rangle$ vs Q^2 at x=0.1
- Gaussian initial condition, $\langle \boldsymbol{k}^2_{\perp(0)q,g} \rangle = 0.25$ GeV^2 at $Q_0^2 = 5$ GeV^2.
- GRV94 PDF set, $Q^2 = 50$ GeV^2



- For $x \to 0$: $\langle \boldsymbol{k}_\perp^2(x) \rangle = \langle \boldsymbol{k}^2_{\perp(0)} \rangle x^\gamma, \gamma < 0$
- For $x \to 1$: $\langle \boldsymbol{k}_\perp^2 \rangle \to \langle \boldsymbol{k}^2_{\perp(0)} \rangle\ \forall Q^2$.
- Broadening of the k_t-distribution as Q^2 increases

Some phenomenology

- Almost insensitive to gluon intrinsic transverse momentum
- $\langle k^2_{\perp,0,q}\rangle$=4 Ge$V^2$ $\langle k^2_{\perp,0,g}\rangle$=1 Ge$V^2$
- Large values of the LL running coupling $\alpha_s(M_Z^2) = 0.150$
- Changes in $\langle k^2_{\perp,0,q}\rangle$ shift the position of the maximum
- Changes in $\alpha_s(M_Z^2)$ influence the height of the maximum
- What about matching?



D0 Collab. '99

Evolution at small transverse momenta

- These TMD equations do not capture the leading (double) logarithmic corrections at small k_t (known from fixed order calculations)
- change the argument of the running coupling to $l_\perp^2 = (1-u)\mu^2$ Parisi, Petronzio, '79

$$\overline{P}_{qq}(u,\mu^2) = \left[C_F \frac{1+u^2}{1-u} \frac{\alpha_s((1-u)\mu^2)}{2\pi} \right]_+$$

- Upon expansion of the strong coupling around $u \sim 0$

$$\overline{P}_{qq}(u,\mu^2) = C_F \frac{\alpha_s(\mu^2)}{2\pi} \left[\frac{1+u^2}{1-u} \right]_+ - \beta_0 C_F \frac{\alpha_s^2(\mu^2)}{2\pi} \left[\frac{1+u^2}{1-u} \ln(1-u) \right]_+ + \cdots$$

- one gets a whole tower of (soft) large logarithms (*c.f.r.* DIS coefficient functions)
- easy way to perform SGR

Evolution at small transverse momenta

- Consider the non-singlet channel (enhanced soft gluon emission $\longrightarrow$ small p_t)
- Perform 2D Fourier-trasform w.r.t to $\boldsymbol{k}_\perp$

$$\mathcal{F}_{ns}(x, \boldsymbol{b}_\perp, \mu^2) = \int d^2\boldsymbol{k}_\perp e^{-i\boldsymbol{b}_\perp \cdot \boldsymbol{k}_\perp} f_{ns}(x, \boldsymbol{k}_\perp, \mu^2)$$

- Assuming that $\mathcal{F}_{ns}$ does not depend upon the azimuthal angle between $\boldsymbol{b}_\perp$ and $\boldsymbol{l}_\perp$

$$\mu^2 \frac{\partial}{\partial \mu^2} \mathcal{F}_{ns}(x, b, \mu^2) = \int_x^1 \frac{du}{u} \overline{P}_{qq}(u, \mu^2) \int dl^2 \delta[l^2 - (1-u)\mu^2] J_0(bl) \mathcal{F}_{ns}\Big(\frac{x}{u}, ub, \mu^2\Big)$$

- where $|\boldsymbol{b}_\perp| \equiv b$ and $|\boldsymbol{l}_\perp| \equiv l$
- leading log corrections arise in the soft gluon emission limit
- Set $u \longrightarrow 1$ in slowly varying functions to get

$$\mathcal{F}_{ns}(x, b, Q^2) = \mathcal{F}_{ns}(x, b, Q_0^2) \exp[T(Q_0^2, Q^2, b)]$$

The quark form factor (1)

- The exponent of the quark form factor is given by

$$T(Q_0^2, Q^2, b) = \int_{Q_0^2}^{Q^2} \frac{d\mu^2}{\mu^2} \int_x^1 du \left[\frac{\alpha_s((1-u)\mu^2)}{2\pi} \widehat{P}_{qq}(u) \right]_+ J_0(b\sqrt{(1-u)\mu^2})$$

- hierarchy : $Q_0^2 < b_0^2/b^2 < Q^2$
- Apply $+$-prescription
- change to $q^2 = (1-u)\mu^2$ and set $Q_0^2 = b_0^2/b^2$ (improve convergence)
- LL : $J_0(bq) \sim \theta(bq - b_0)$
- for the special value $b_0 = 2e^{-\gamma_E}$ we get (LL accuracy)

$$T_q^{KT}(b_0^2/b^2, Q^2) = -\frac{C_F}{\pi} \int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} \alpha_s(q^2) \left[1 \cdot \ln \frac{Q^2}{q^2} - \frac{3}{4} \right]$$

Kodaira, Trentadue ’82

The quark form factor (2)

- Origin of numerical factors in the form factor

$$\hat{P}_{qq}^{(0)}(u) = C_F\Big(\frac{2}{1-u} - 1 - u\Big)$$

$$T_q^{KT}(b_0^2/b^2, Q^2) = -\frac{C_F}{\pi}\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2}\alpha_s(q^2)\Big[\ln\frac{Q^2}{q^2} - \frac{3}{4}\Big]$$

- Accuracy: LL
- to go to NLL, include the residue of two loop $P_{qq}^{(1)}(u)$ for $u \to 1$ KT'82

$$K = \Big[C_F C_A\Big(\frac{67}{9} - \frac{\pi^2}{3}\Big) + 2C_F n_f T_R\Big(-\frac{10}{9}\Big)\Big]$$

AR calculation (1)

- perturbative form factor, term B Aybat, Rogers '11, Collins '11

$$T^{AR}(\mu_b, \mu, \sqrt{\zeta_F}) = \ln \frac{\sqrt{\zeta_F}}{\mu_b} \tilde{K}(b; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right]$$

- with $\mu_b = C_1/b$ and $C_1 = 2e^{-\gamma_E} \equiv b_0$
- the relevant kernels at one loop are given by

$$\begin{aligned} \gamma_F(g(\mu); \zeta_F/\mu^2) &= \alpha_s \frac{C_F}{\pi} \left(\frac{3}{2} - \ln \frac{\zeta_F}{\mu^2} \right) + \mathcal{O}(\alpha_s^2) , \\ \gamma_K(g(\mu)) &= 2 \frac{\alpha_s C_F}{\pi} + \mathcal{O}(\alpha_s^2) , \\ \tilde{K}(b, \mu) &= -\frac{\alpha_s C_F}{\pi} [\ln(\mu^2 b^2) - \ln 4 + 2\gamma_E] + \mathcal{O}(\alpha_s^2) \end{aligned}$$

AR calculation (2)

- $\tilde{K}(b, \mu_b) = 0$

$$T^{AR}(\mu_b, \mu, \sqrt{\zeta_F}) = \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\alpha_s(\mu') \frac{C_F}{\pi} \frac{3}{2} - \ln \frac{\sqrt{\zeta_F}}{\mu'} 2 \frac{\alpha_s(\mu') C_F}{\pi} \right]$$

- we set $q^2 = \mu'^2$ with $\mu^2 = Q^2$ and $\mu_b = b_0/b$

$$T^{AR}(b_0/b, Q^2, \zeta_F) = -\frac{C_F}{\pi} \int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} \alpha_s(q^2) \left[\frac{1}{2} \ln \frac{\zeta_F}{q^2} - \frac{3}{4} \right]$$

- T^{AR} closely resembles T^{KT} with $\zeta_F = Q^2$
- single logarithms OK (3/4 term)
- coefficients in front of double logarithms are different: $1/2$ for T^{AR} and 1 for T^{KT}

DY as a benchmark

- consider $q + \bar{q} \to \gamma^* + X$
- AR: $\zeta_q = \zeta_{\bar{q}} = Q^2$, $1 = 1/2 + 1/2$
  (preferred for TMD PDFs evolution, symmetric evolution)
- KT: $\zeta_q = Q^4/q^2$ and $\zeta_{\bar{q}} = q^2$, $1 = 1 + 0$ (or explicit calculation)
  (double logs completely associated to the quark line, TMD PDFs asymmetric evolution)
- Tension resolved once gauge independent quantities are evaluated, *e.g.* the p_t spectrum of gauge bosons

$$T^{DY,LL}(b_0^2/b^2, Q^2) = T_q(b_0^2/b^2, Q^2) + T_{\bar{q}}(b_0^2/b^2, Q^2) = -\frac{C_F}{\pi}\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2}\alpha_s(q^2)\left[1 \cdot \ln\frac{Q^2}{q^2} - \frac{3}{2}\right]$$

EIS calculation

- perturbative form factor, Echevarria, Idilbi and Scimemi '12

$$R(b, Q_i, \mu_i, Q_f, \mu_f) = \exp\left\{\int_{\mu_i}^{\mu_f} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_F\Big(\alpha_s, \ln\frac{Q_f^2}{\bar{\mu}^2}\Big)\right\}\left(\frac{Q_f^2}{Q_i^2}\right)^{-D(b,\mu_i)}$$

- We set $\mu_f = Q_f$ and $\mu_i = Q_i$. Taking logs and substituing relevant expressions for anomalous dimensions

$$\ln R(b, Q_i, Q_f) = -\frac{C_F}{\pi}\int_{Q_i^2}^{Q_f^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\alpha_s(\bar{\mu}^2)\left[\frac{1}{2}\ln\frac{Q_f^2}{\bar{\mu}^2} - \frac{3}{4}\right] - D(b, Q_i)\ln\frac{Q_f^2}{Q_i^2}$$

- setting $Q_i^2 = b_0^2/b^2$, $D = 0$

$$\ln R(b, b_0/b, Q_f) = -\frac{C_F}{\pi}\int_{b_0^2/b^2}^{Q_f^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\alpha_s(\bar{\mu}^2)\left[\frac{1}{2}\ln\frac{Q_f^2}{\bar{\mu}^2} - \frac{3}{4}\right]$$

Conclusions

- TMD evolution eqns can be recast (in the soft limit) in a form analogous to the ones from two indipendent calculations in the literature
- KT has double logs shared asymmetrically on parton legs
- AR & EIS has double logs shared symmetrically on parton legs
- All formulations give consistent results, as far as physical observables are considered
- Convergence of the formalisms is a good point for phenomenology
- What about all other approach in the same limit (KMR, CCFM, etc?)
- Import in this field all the knowledge already accumulated on SGR by other communities