Resummation, Evolution, Factorization

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Perturbative QCD, CSS/TMD resummation and non-perturbative aspects in SIDIS processes

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Outline

- **Resummation in SIDIS**
  - *The resummed W-term*
  - *The regular Y-term*
- **Matching prescriptions**
  - *Non-perturbative contributions to the Sudakov factor*
  - *Dependence of the total cross section on the b_{max} parameter*
  - *Y-term matching*
  - *Matching with the inclusion of non-perturbative contributions*
- **Practical implementations for 3 kinematical configurations**
  - *high energy and large Q^2*
  - *HERA-like*
  - *COMPASS-like*
- **Conclusions and outlook**

Resummation of large logarithms

- *Calculating a cross section which describes a hadronic process over the whole q_T range is a highly non-trivial task*
- *It requires a proper treatment of the non-perturbative regime and the resummation of large logarithms, in the limit $q_T << Q$, arising from emission of soft and collinear gluons*
- ***Collins – Soper – Sterman (CSS) resummation (⇒ TMD formalism)***

Nucl. Phys. B250, 199 (1985) *Phys. Rev. D83, 114042 (2011)*

$$\boxed{\frac{1}{\sigma_0}\frac{d\sigma}{dQ^2 dy dq_T^2} = X_{div}(q_T) + Y_{reg}(q_T)}$$

Divergent part for $q_T \to 0$
contains terms proportional to (Q^2/q_T^2) and $(Q^2/q_T^2)\ln(Q^2/q_T^2)$
MUST BE RESUMMED

Regular part for $q_T \to 0$
contains terms less singular than (Q^2/q_T^2), i.e. terms that are at most logaritmically divergent as $q_T \to 0$

CSS in SIDIS

$$\frac{d\sigma^{total}}{dx\,dy\,dz\,dq_T^2} = \pi\sigma_0^{DIS}\int\frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}W^{SIDIS}(x,z,b_T,Q) + Y^{SIDIS}(x,z,q_T,Q)$$

Resummed part

Regular part

$$\boxed{W^{SIDIS}(x,z,b_T,Q) = \exp\left[S_{pert}(b_T,Q)\right]\sum_j e_j^2\sum_{i,k} C_{ji}^{\rm in}\otimes f_i(x,\mu_b^2)\,C_{kj}^{\rm out}\otimes D_k(z,\mu_b^2),}$$

PDFs convoluted with Wilson Coefficients

$$C\otimes f(x) \equiv \int_x^1\frac{d\hat{x}}{\hat{x}}C\left(\frac{x}{\hat{x}}\right)f(\hat{x})$$

$$C(x,\alpha_s(\mu_b)) = \sum_{n=0}^{\infty}\left(\frac{\alpha_s(\mu_b)}{\pi}\right)^n C^{(n)}(x)$$

CSS in SIDIS

$$\frac{d\sigma^{total}}{dx\,dy\,dz\,dq_T^2} = \pi\sigma_0^{DIS}\int\frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}W^{SIDIS}(x,z,b_T,Q) + Y^{SIDIS}(x,z,q_T,Q)$$

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Sudakov factor

$$S_{pert}(b_T,Q) = -\int_{\mu_b^2}^{Q^2}\frac{d\mu^2}{\mu^2}\left[A(\alpha_s(\mu))\ln\left(\frac{Q^2}{\mu^2}\right) + B(\alpha_s(\mu))\right]$$

$$A(\alpha_s(\mu)) = \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n A^{(n)}$$

$$B(\alpha_s(\mu)) = \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n B^{(n)}$$

Leading Log (LL)	:	$A^{(1)}$;
Next to LL (NLL)	:	$A^{(2)}, B^{(1)}, C^{(1)}$;
Next to NLL (NNLL)	:	$A^{(3)}, B^{(2)}, C^{(2)}$;
Fixed order $\alpha_s(FXO)$	:	$A^{(1)}, B^{(1)}, C^{(1)}$;

CSS in SIDIS

$$\frac{d\sigma^{total}}{dx\,dy\,dz\,dq_T^2} = \pi\sigma_0^{DIS}\int\frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2} W^{SIDIS}(x,z,b_T,Q) + Y^{SIDIS}(x,z,q_T,Q)$$

$$\boxed{W^{SIDIS}(x,z,b_T,Q) = \exp\left[S_{pert}(b_T,Q)\right]\sum_j e_j^2 \sum_{i,k} C_{ji}^{\rm in}\otimes f_i(x,\mu_b^2)\, C_{kj}^{\rm out}\otimes D_k(z,\mu_b^2),}$$

- The resummed cross section, W, does not describe the whole q_T range.
  It sums all known logarithmic terms dominating the low q_T region, but does not take into account the full fixed order (NLO) corrections, which are important at large q_T values.

- Because of the oscillatory nature of the Fourier integrand,
  W may become **negative** (i.e. unphysical) at large q_T

Warning: here NLO means first order in α_s of the collinear QCD cross section

- For a consistent description over the whole q_T range we need to **MATCH** the resummed cross section with the NLO (fixed order) cross section

The Y factor and the asymptotic part

$$\frac{d\sigma}{dx dz dQ^2 d^2 q_T} = \sigma_0^{SIDIS} \left\{ \int \frac{d^2 \boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot \boldsymbol{b}_T}}{(2\pi)^2} \sum_j e_j^2 W_j^{SIDIS}(x, z, b_T, Q) + Y^{SIDIS} \right\}$$

$$\frac{d\sigma^{NLO}}{dx\, dy\, dz\, dq_T^2} = \frac{d\sigma^{ASY}}{dx\, dy\, dz\, dq_T^2} + Y$$

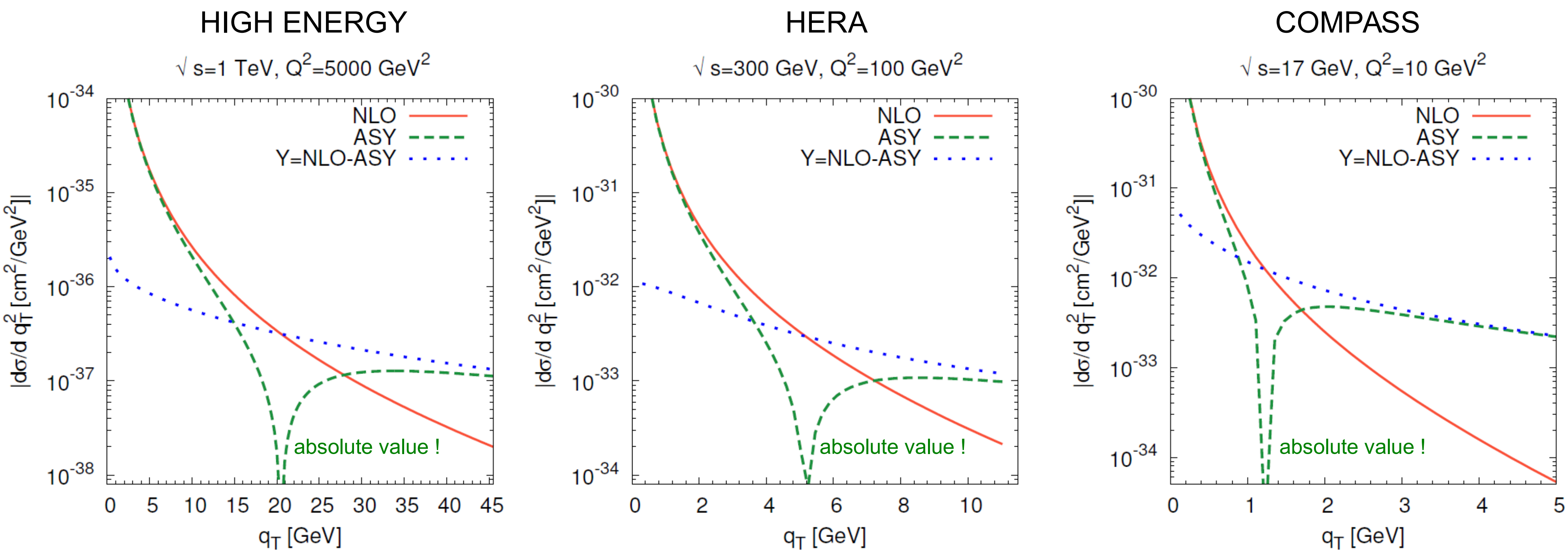
Warning: here NLO means first order in α_s of the collinear QCD cross section

$$\boxed{Y = NLO - ASY}$$

$$\frac{d^5\sigma^{\mathrm{asymp}}}{dQ^2\, dx_{bj}\, dz_f\, dq_T^2\, d\phi} = \frac{\alpha_{em}^2 \alpha_s}{8\pi x_{bj}^2 S_{ep}^2 Q^2} A_1 \frac{2Q^2}{q_T^2} \sum_{q,\bar{q}} e_q^2 \Big[2 f_q(x_{bj}, \mu) D_q(z_f, \mu) \Big(C_F \ln\Big(\frac{Q^2}{q_T^2}\Big) - \frac{3}{2} C_F \Big)$$
$$+ \big\{ f_q(x_{bj}, \mu) \otimes P_{qq}^{\mathrm{in},(0)} + f_g(x_{bj}, \mu) \otimes P_{qg}^{\mathrm{in},(0)} \big\} D_q(z_f, \mu)$$
$$+ f_q(x_{bj}, \mu) \big\{ P_{qq}^{\mathrm{out},(0)} \otimes D_q(z_f, \mu) + P_{gq}^{\mathrm{out},(0)} \otimes D_g(z_f, \mu) \big\} \Big],$$

$$\boxed{ASY = Q^2/q_T^2\ [A\ \ \mathrm{Ln}(Q^2/q_T^2) + B]}$$

The Y factor and the asymptotic part



As ASY becomes negative (i.e. unphysical) at large q_T,
Y = NLO - ASY can become much larger than NLO

WARNING: the Y-term does not tend to zero at small q_T, where ASY and NLO seem to be very close , as Y still contains terms ~ $\log(1/q_T)$, which become large at small q_T.

Matching with the Y-factor

$$\frac{d\sigma}{dx\,dz\,dQ^2\,d^2q_T} = \sigma_0^{SIDIS}\left\{\int \frac{d^2\boldsymbol{b}_T\, e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 W_j^{SIDIS}(x,z,b_T,Q) + Y^{SIDIS}\right\}$$

$$\boxed{Y = \text{NLO} - \text{ASY}}$$

At small q_T , if W >> Y then
W + Y → W

At q_T ~ Q, if W → ASY then
W + Y → ASY + NLO – ASY = **NLO**

This prescription provides a smooth matching only when W → ASY over a sufficiently large q_T region

W	W + Y	NLO

W >> Y

W ~ ASY

Does a kinematical range in which W ~ ASY exist ?

- Before we can answer this question we should worry about the non-perturbative contributions to the Sudakov factor

Non perturbative contributions

$$\frac{d\sigma^{total}}{dx\,dy\,dz\,dq_T^2} = \pi\sigma_0^{DIS}\int_0^\infty \frac{db_T b_T}{(2\pi)} J_0(q_T b_T) W^{SIDIS}(x,z,b_*,Q)\exp\left[S_{NP}(x,z,b_T,Q)\right] + Y(x,z,q_T,Q)\,,$$

- W, the perturbative part of the Sudakov factor, is a function of b*

$$b_* = \frac{b_T}{\sqrt{1+b_T^2/b_{max}^2}} \qquad \mu_b = C_1/b_*$$

- S_{NP}, the non-perturbative part of the Sudakov factor, accounts for the **non-perturbative** behavior at large b_T (i.e. small q_T)

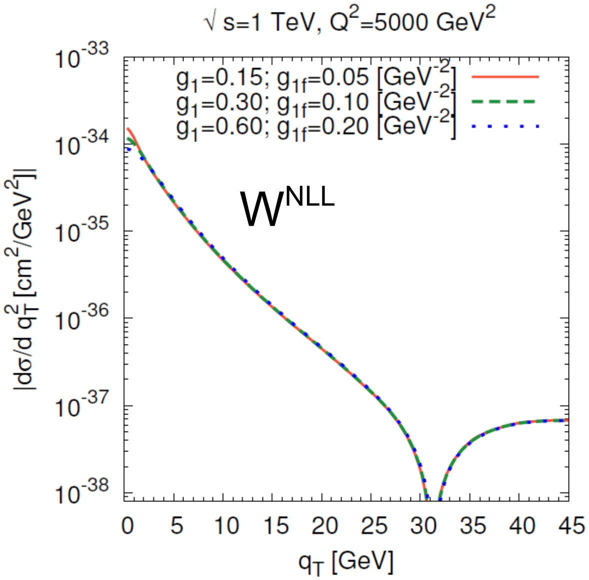
- Just for illustration, let's consider a simple (Gaussian) model for S_{NP}

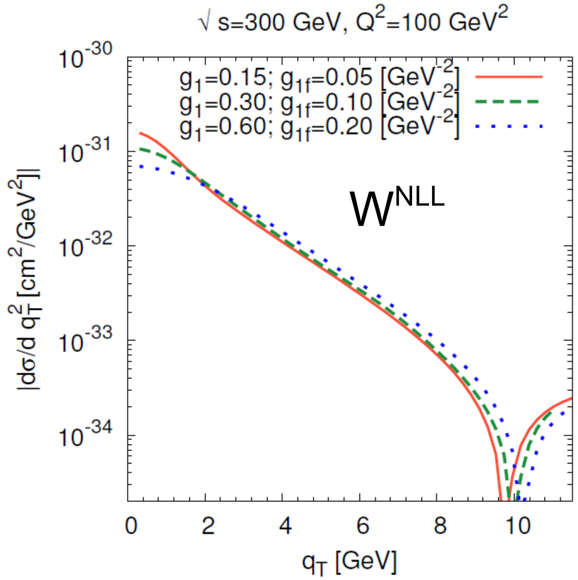
$$S_{NP} = \left(-\frac{g_1}{2} - \frac{g_{1f}}{2z^2} - g_2 \ln\left(\frac{Q}{Q_0}\right)\right) b_T^2$$

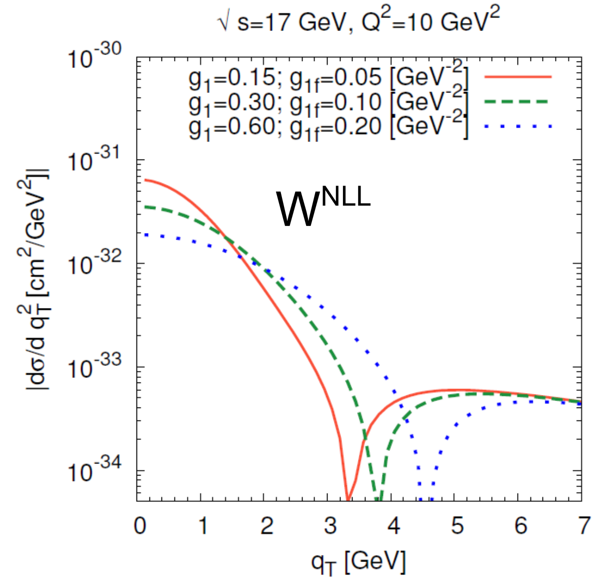
Non perturbative contributions

$$W^{NLL} = \pi\sigma_0^{DIS}\int_0^\infty \frac{db_T b_T}{(2\pi)} J_0(q_T b_T) W^{SIDIS}(x, z, b_*, Q)\exp\left[S_{NP}(x, z, b_T, Q)\right]$$

- S_{NP} takes into account the **non-perturbative** behavior at large b_T ($\rightarrow$ small q_T)





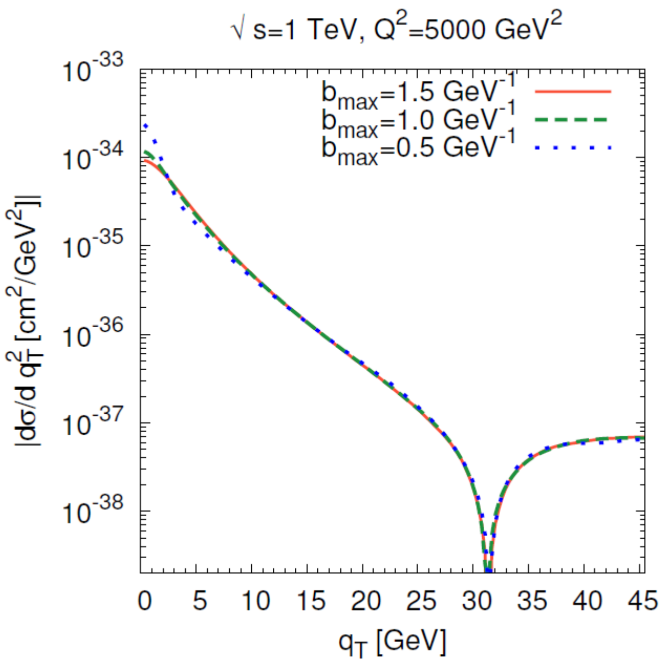


- The dependence on the parameters of S_{NP} is limited to the small q_T region
- The three curves change sign at the same q_T

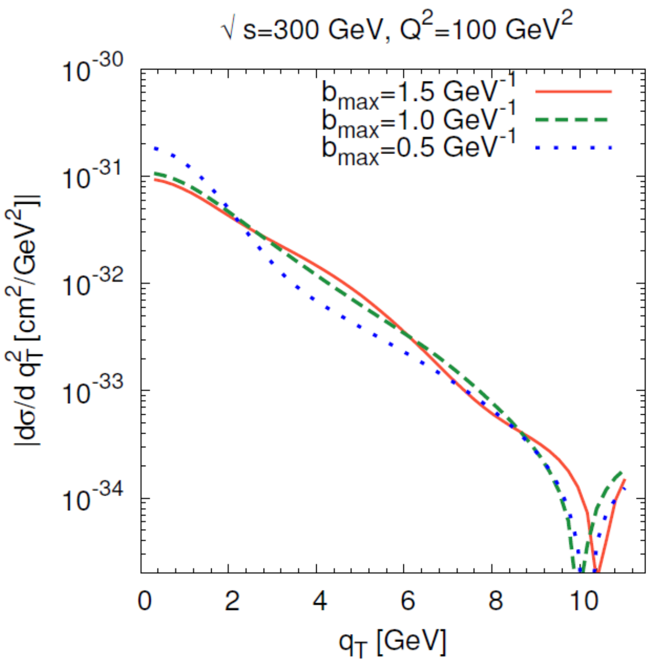
- The dependence on the parameters of S_{NP} stretches to the wholel q_T region
- The three curves change sign at slightly different values of q_T

- S_{NP} induces a **VERY STRONG** dependence on the parameters of the non-perturbative model
- The three curves change sign at **very different** values of q_T

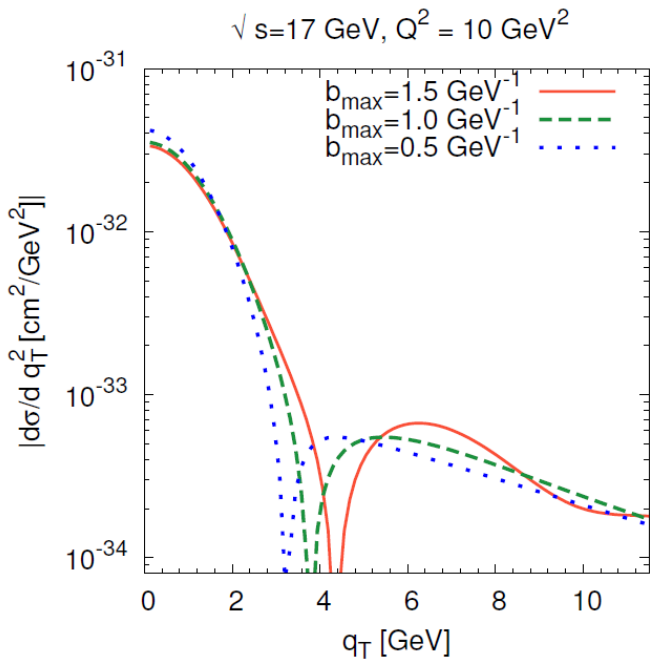
Dipendence on the b_{max} parameter



HERA



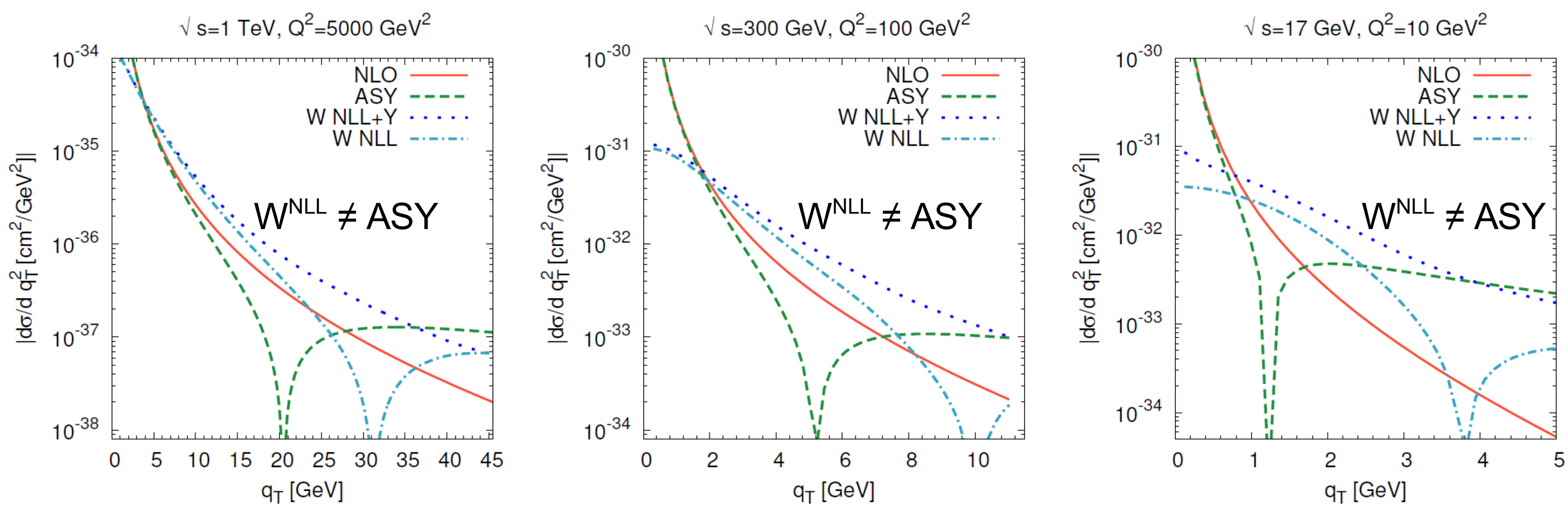
COMPASS



- ✔ The dependence on b_{max} is limited to the small q_T region
- ✔ The three curves change sign at the same q_T

- ✔ **VERY STRONG** dependence on b_{max}
- ✔ The three curves change sign at **very different** values of q_T

Interplay between perturbative and non-perturbative contributions

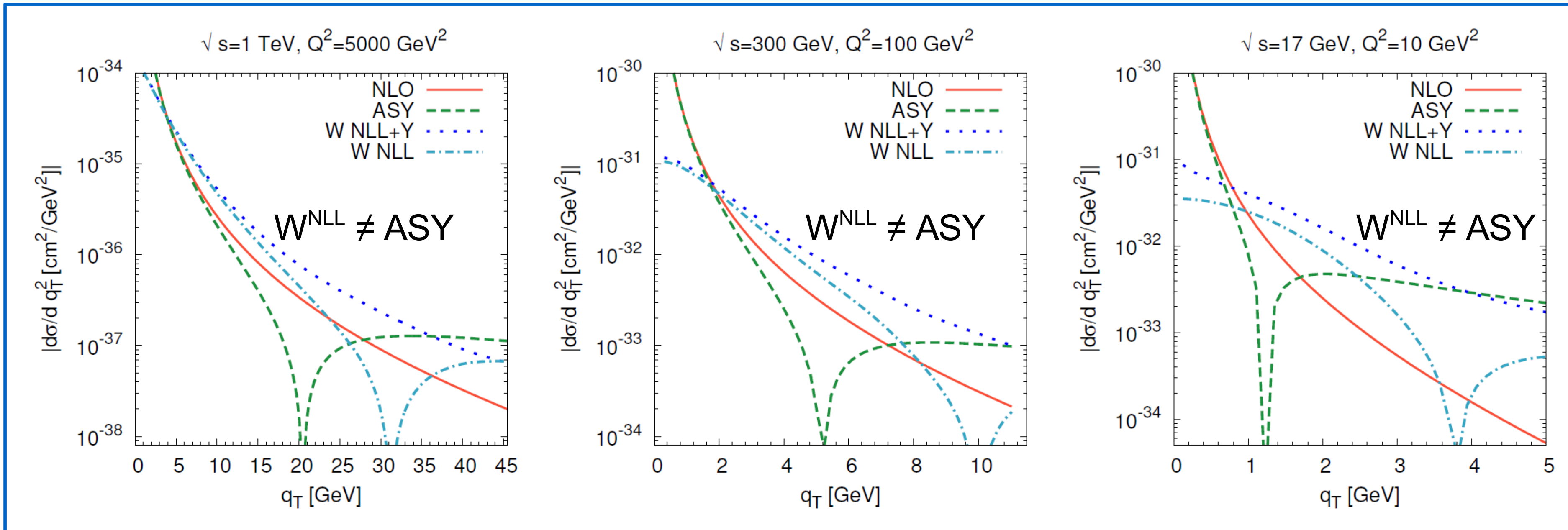


At $q_T \sim Q$, if $W^{NLL} \to$ ASY then $W^{NLL} + Y \to$ NLO

At small q_T , if $Y << W^{NLL}$ then ASY then $W^{NLL}+Y \to W^{NLL}$

- ✔ Notice that ASY and W become negative at different values of q_T
- ✔ Y can become large compared to W^{NLL} and $Y \not\ll W^{NLL}$ at small q_T
- ✔ The q_T values at which ASY and W become negative depend strongly on the considered kinematics

Interplay between perturbative and non-perturbative contributions



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At small q_T, if $Y << W^{NLL}$ then ASY then $W^{NLL}+Y \rightarrow W^{NLL}$

IS ANY MATCHING POSSIBLE ???

Fixed order cross section

The fact that $W^{NLL} \neq ASY$ is (partly) due to non-perturbative contributions
Therefore, instead of setting $d\sigma = W^{NLL} + Y$, let's try a different matching prescription which takes into account the non-perturbative content of the Sudakov factor

$$\boxed{d\sigma = W^{NLL} - W^{FXO} + NLO}$$

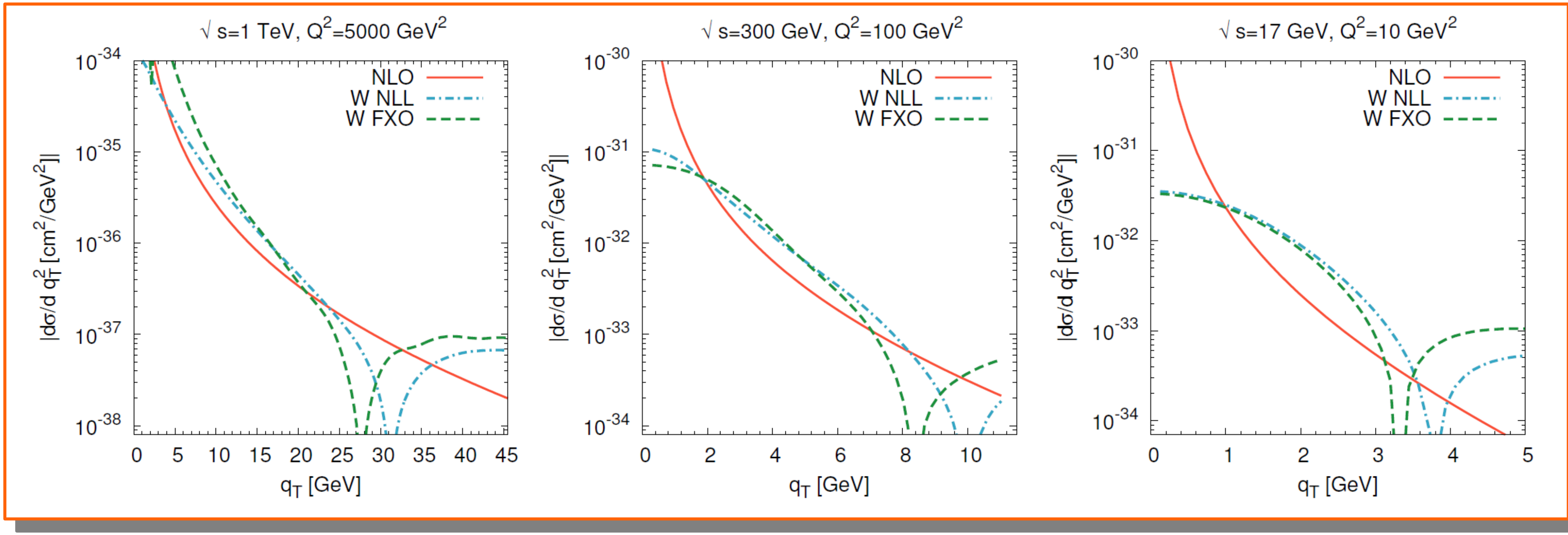
- ✔ W^{FXO} is the NLL resummed cross section approximated at first order in α_s, with a first order expansion of the Sudakov exponential, $\exp[\, S_{pert}(b_*)\,] \rightarrow 1 + S_{pert}(b_*)$

- In principle, in the absence of non-perturbative content and in the limit $b_T \rightarrow 0$ (and $q_T \rightarrow \infty$) then one can show that $W^{FXO} \rightarrow ASY$, so that when this happens this matching prescription reduces to the Y-term procedure

- ✔ In general W^{FXO} contains the same non-perturbative content as that we give to W^{NLL}

> Therefore, with this prescription we might be able to find kinematical regions in which $W^{FXO} \sim W^{NLL}$

Matching with the inclusion of non-perturbative contributions



At 1 TeV and at HERA there are regions in which W^{FXO} and W^{NLL} are crossing (although not at $q_T \sim Q$!)

W^{NLL} does not tend to W^{FXO} asymptotically

No continuous and smooth matching can be performed.

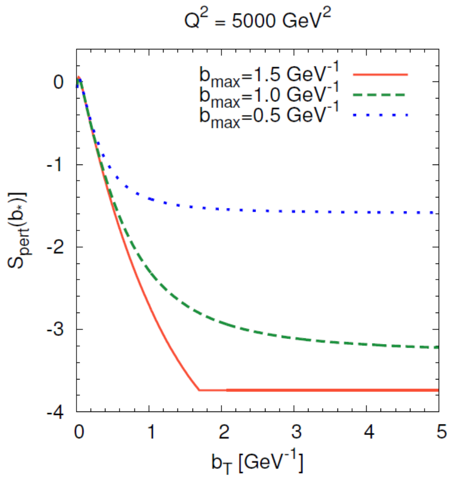
At Compass the non-perturbative regime dominates the whole cross section

W^{NLL} and W^{FXO} never cross

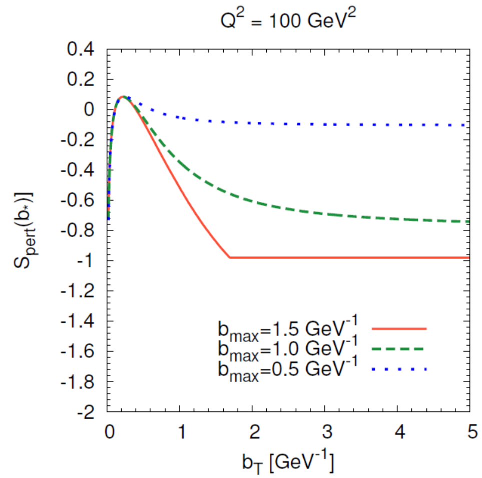
NO MATCHING whatsoever

Why does the matching fail ?

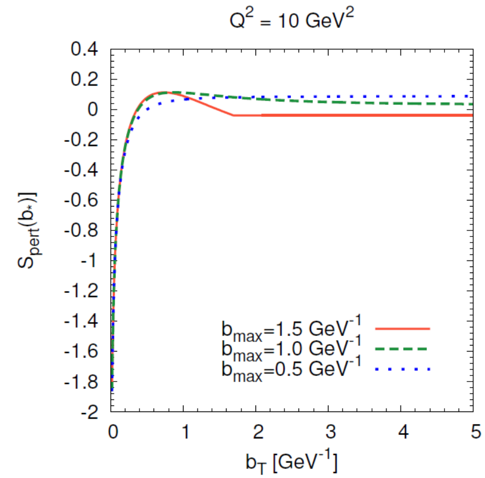
b_T behaviour of the perturbative Sudakov factor



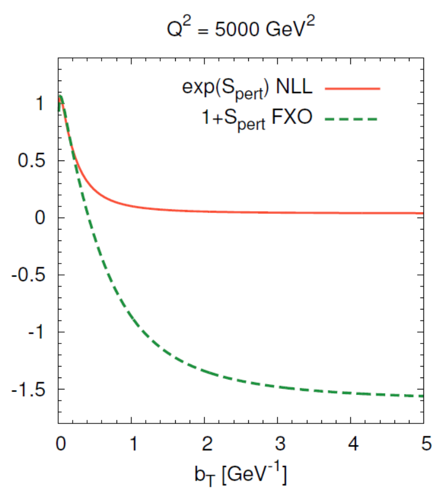
The Sudakov factor is small only over a very limited range of small b_T



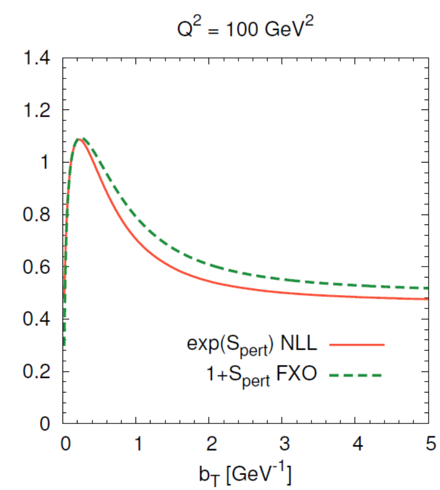
The Sudakov factor is small only over a very limited range of intermediate b_T



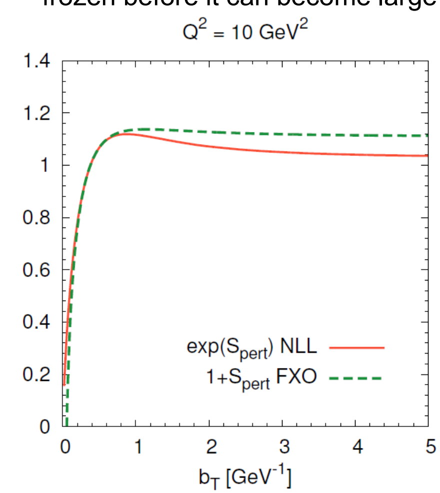
S is very large at small b_T, small at intermediate b_T values and it gets frozen before it can become large again



At 1 TeV the perturbative fixed order expansion of the Sudakov exponential breaks down

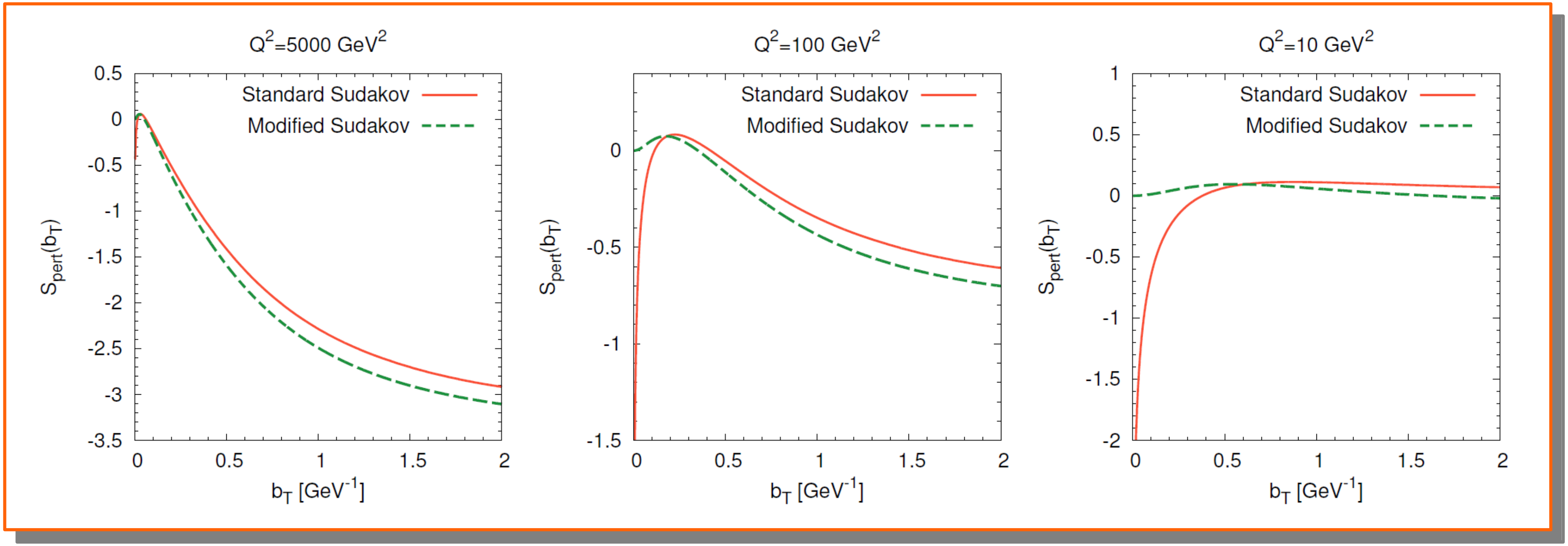


$$\exp[S_{pert}^{NLL}] \rightarrow \exp[S_{pert}^{FXO}] \rightarrow 1 + S_{pert}^{FXO}$$



At COMPASS both the Sudakov and its perturbative expansion are unphysically enhanced (> 1)

b_T behaviour of the *modified* perturbative Sudakov factor



The unphysical behaviour of the perturbative Sudakov factor at $b_T \to 0$ can be corrected by replacing

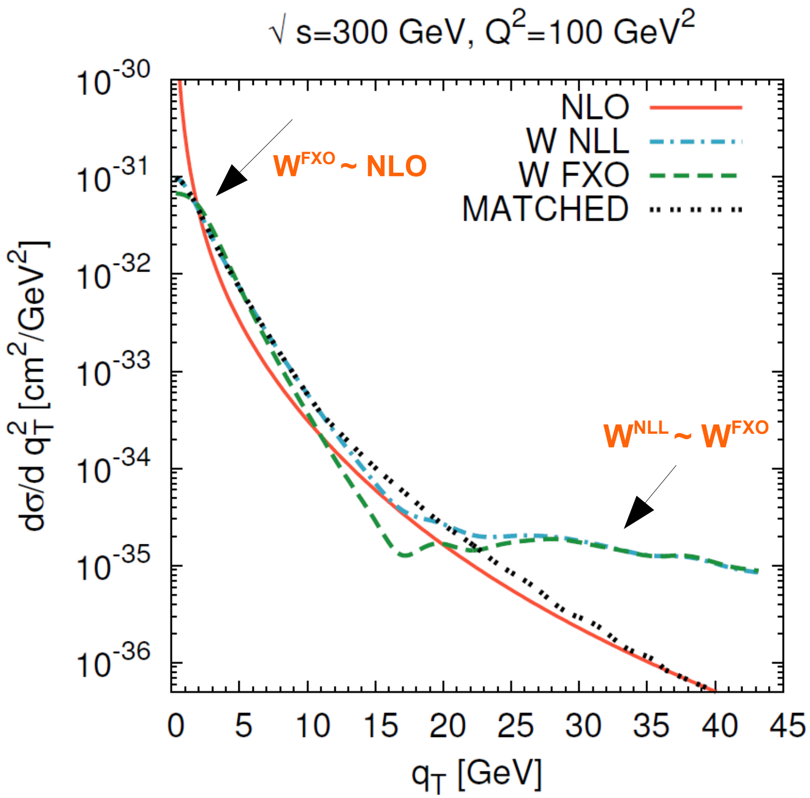
$$\log(Q^2/\mu_b^2) \to \log(1+Q^2/\mu_b^2)$$

Can we finally obtain a successful matching ?

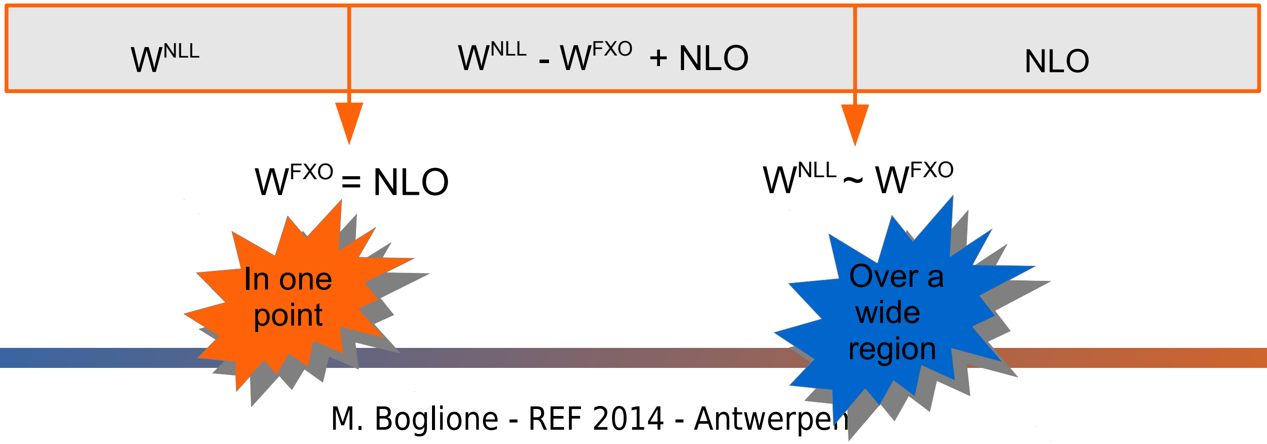
Can we finally obtain a successful matching ?

In general the answer is NO

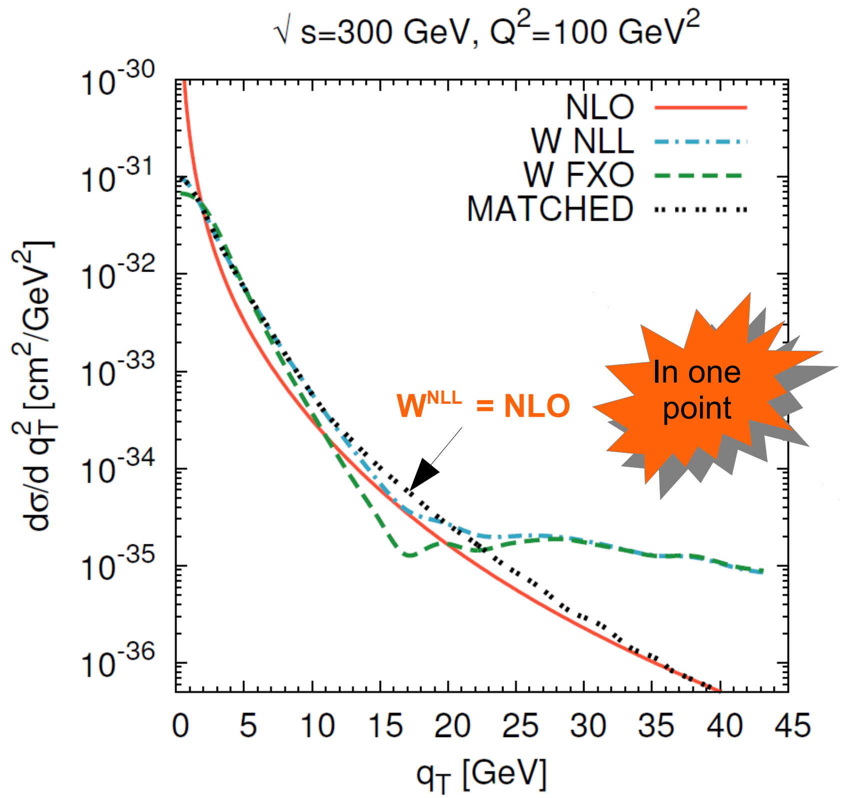
HERA
A case when this matching works … *roughly*



Here W^{NLL} and W^{FXO} are roughly the same over a range wide enough to allow for a safe matching

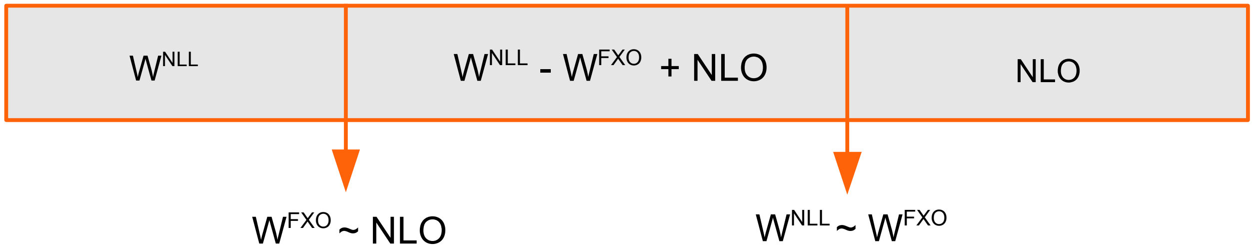


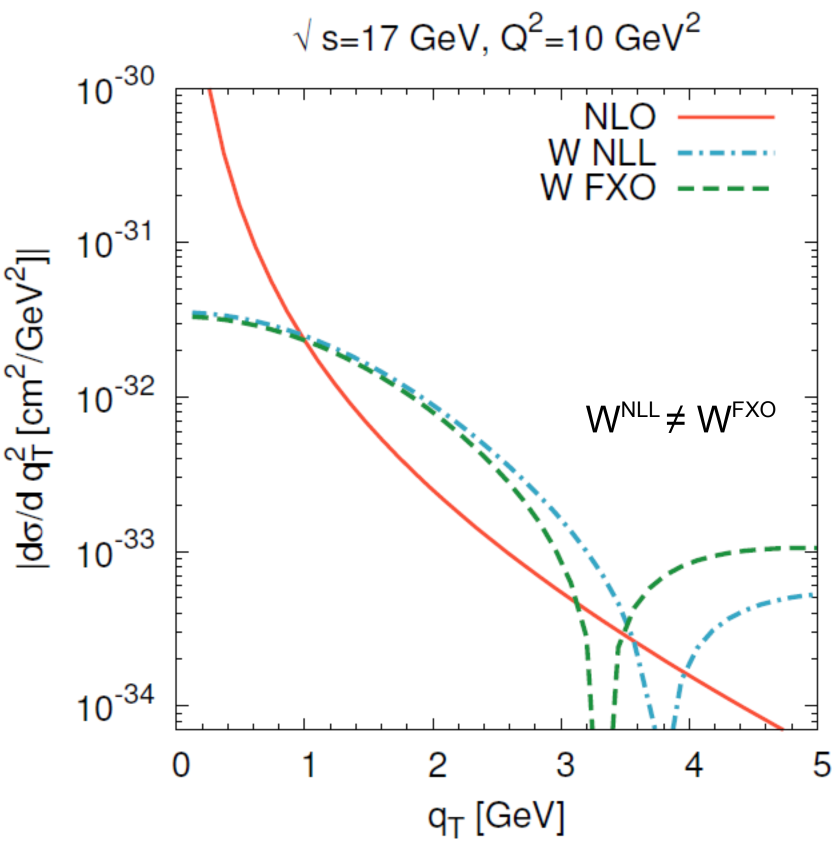
HERA
A case when this matching works … roughly



Notice that one could join W^{NLL} (blue line) directly to NLO (red line) at q_T~ 15 - 16 GeV

COMPASS … a case when the matching does not work





At q_T ~ Q the curves are far from each other, they have different curvatures and they never cross

✗ No way to realize a smooth matching, without jumps and cusps.

Results

- *We studied the matching between the region where fixed order perturbative QCD can successfully be applied and the regions where soft gluon resummation is necessary.*

- *We found that the commonly used matching prescription through the Y-factor fails in the kinematical configurations considered.*

- *The non-perturbative component of the resummed cross section plays a crucial role even at high energies.*

- *The perturbative expansion of the resummed cross section in the matching region is not as reliable as it is usually believed and its treatment requires special attention.*

Conclusions

- Resummation in the impact parameter b_T space is a very powerful tool. However, it's successful implementation is affected by a number of practical difficulties (the kinematics of the process, the parameters used to model the non-perturbative content of the SIDIS cross section, etc ...).

- Performing phenomenological studies in the b_T space is rather difficult, as we loose the direct connection of our inputs to the exact outcome in the conjugate q_T space.

  It becomes hard to define the boundaries of the three regions of interest:

  $$q_T \sim \lambda_{QCD} << Q\,,\quad \lambda_{QCD} << q_T << Q\,,\quad q_T \sim Q\,,\quad q_T > Q\,.$$

- Matching prescriptions have to be applied to achieve a reliable description of the SIDIS process over the full q_T range, going smoothly from one region to the following.

Back up

Non perturbative contributions

$$\frac{d\sigma}{dx\,dz\,dQ^2\,d^2q_T} = \sigma_0^{SIDIS}\left\{\int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 W_j^{SIDIS}(x,z,b_T,Q) + Y^{SIDIS}\right\}$$

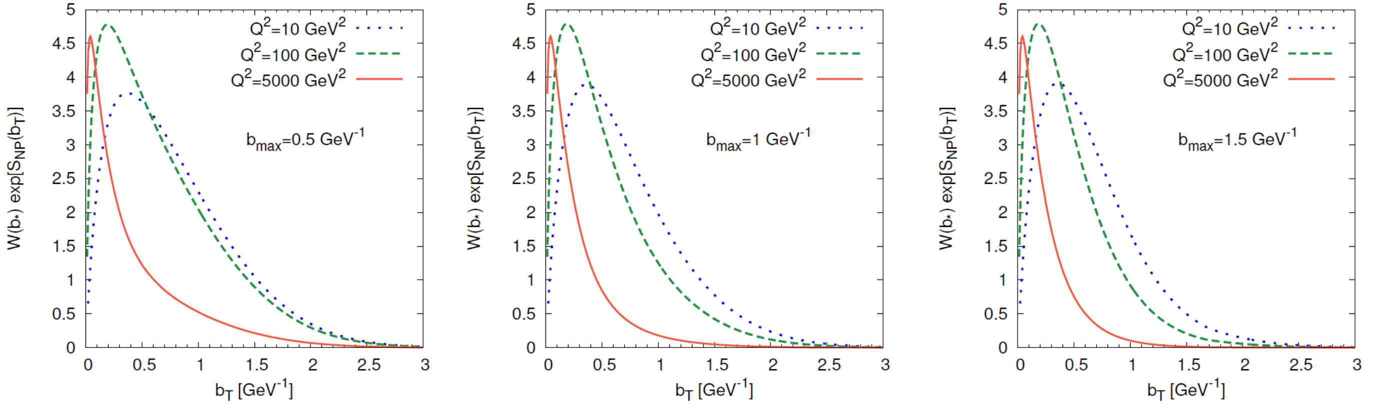
- The CSS formalism relies on a Fourier integral which runs from 0 to ∞
  No prediction can be made without an ansatz prescription for the non–perturbative region, where b_T is large (and q_T is small ♫).
- The Sudakov factor diverges at large b_T

$$S_j(b_T,Q) = \int_{C_1^2/b_T^2}^{Q^2}\frac{d\kappa^2}{\kappa^2}\left[A_j(\alpha_s(\kappa))\ln\left(\frac{Q^2}{\kappa^2}\right) + B_j(\alpha_s(\kappa))\right]$$

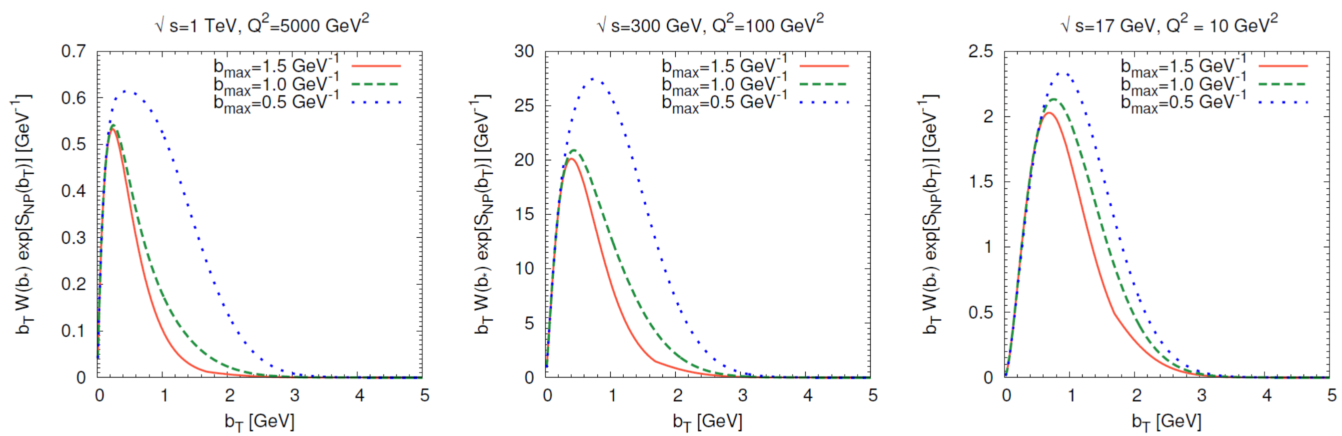
- In the CSS scheme a prescription is used to avoid entering the non-perturbative region, such that

$$\boxed{b_* = \frac{b_T}{\sqrt{1+b_T^2/b_{max}^2}} \qquad \mu_b = C_1/b_*}$$

b_T behaviour of the integrand



These b_T distributions become increasingly peaked and narrow as Q^2 grows, reflecting the dominance of smaller and smaller b_T contributions at growing energies and Q^2



At each fixed kinematical configuration, the peak moves towards larger values as b_{max} decreases, reflecting the fact that the distributions obtained from large b_{max} die faster in b_T , as non-perturbative contributions set in at larger values of b_T