

Effect of TMD evolution and partonic flavor on e^+e^- annihilation into hadrons

In collaboration with

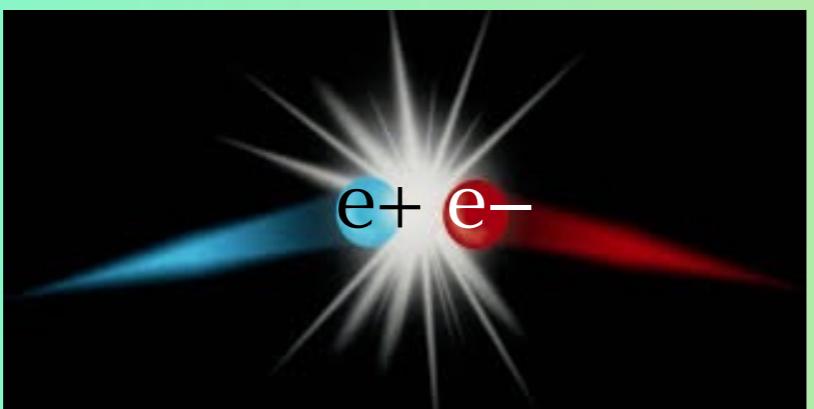
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Marco Radici
INFN - Pavia

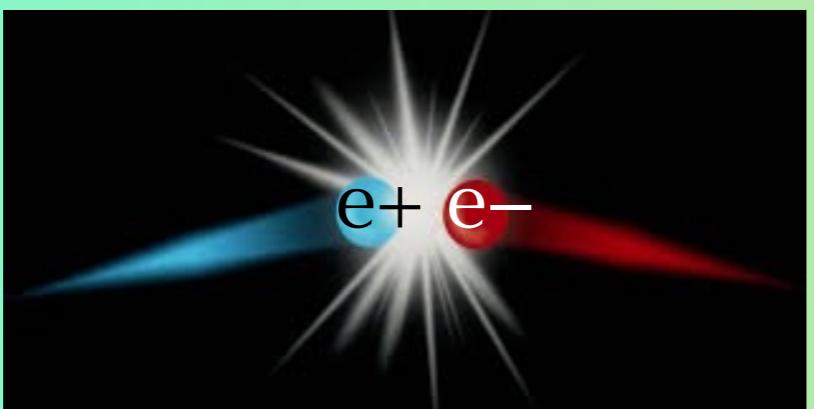




e+e- annihilation
why?

$$1. \text{ SSA} = \frac{(\text{TMD PDF}) \otimes (\text{TMD FF})}{f_1 \otimes D_1}$$

- D_1 affects extraction of all polarized TMD's
- $e+e- \Rightarrow$ clean extraction of D_1



e+e- annihilation
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2. SIDIS — —— $\rightarrow e+e-$



Q^2_{SIDIS}

Q^2_{e+e-}



- best sensitivity to test TMD evolution



Outline

■ different evolution kernels:

1. “new” Collins-Soper-Sterman – nCSS
2. (hybrid) Echevarría-Idilbi-Scimemi – (h)EIS

Collins, *Found. of Pert. QCD*
(C.U.P. 2011)

Echevarría, Idilbi, Scimemi,
P. R. D90 (14) 014003

Outline

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P. R. D90 (14) 014003

■ input TMD FF from Hermes multiplicities

Signori, Bacchetta, Radici, Schnell,
JHEP 1311 (13) 194

Outline

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Collins, *Found. of Pert. QCD*
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- input TMD FF from Hermes multiplicities
- define observables, sensitive also to flavor of TMD FF
- results
- conclusions and outlooks

kernels for TMD evolution

“new” Collins-Soper-Sterman – nCSS

evolved TMD FF

choosing $\zeta_i = \mu_i = \mu_b^2$

Collins, *Found. of Pert. QCD*
(C.U.P. 2011)

$$D_1^q(z, b_T; \zeta, \mu) = e^{S_{\text{pert}}(b^*; \zeta, \mu)} e^{S_{\text{np}}(b_T) \log \frac{\zeta}{\mu_0^2}} \sum_i [C_{q/i} \otimes D_1^i](z, b^*; \mu_b) \widehat{D}_{\text{np}}(z, b_T)$$

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**perturbative
evolution**

$$\begin{aligned} S_{\text{pert}}(b^*; \zeta, \mu) &= -D(b^*; \mu_b) \log \frac{\zeta}{\mu_b^2} \quad \text{evolution with scale } \zeta \\ &\quad - \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left(\Gamma_{\text{cusp}} \log \frac{\zeta}{\mu'^2} + \gamma^V \right) \quad \text{and with } \mu \end{aligned}$$

OPE of pert. part
of collinear D_1
at low b_T

“new” Collins-Soper-Sterman – nCSS

evolved TMD FF

$$D_1^q(z, b_T; \zeta, \mu) = e^{S_{\text{pert}}(b^*; \zeta, \mu)}$$

perturbative evolution

choosing $\zeta_i = \mu_i = \mu_b^2$

$$e^{S_{\text{np}}(b_T) \log \frac{\zeta}{\mu_0^2}}$$

nonperturbative evolution

$$\sum_i [C_{q/i} \otimes D_1^i](z, b^*; \mu_b)$$

Collins, *Found. of Pert. QCD*
(C.U.P. 2011)

$$\widehat{D}_{\text{np}}(z, b_T)$$

OPE of pert. part
of collinear D_1
at low b_T

nonpert. part
of TMD FF
at intrinsic
large b_T

$$S_{\text{pert}}(b^*; \zeta, \mu) = -D(b^*; \mu_b) \log \frac{\zeta}{\mu_b^2} \quad \text{evolution with scale } \zeta$$

$$- \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left(\Gamma_{\text{cusp}} \log \frac{\zeta}{\mu'^2} + \gamma^V \right) \quad \text{and with } \mu$$

“new” Collins-Soper-Sterman – nCSS

evolved TMD FF

$$D_1^q(z, b_T; \zeta, \mu) = e^{S_{\text{pert}}(b^*; \zeta, \mu)}$$

perturbative evolution

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$$e^{S_{\text{np}}(b_T) \log \frac{\zeta}{\mu_0^2}}$$

nonperturbative evolution

$$\sum_i [C_{q/i} \otimes D_1^i](z, b^*, \mu_b)$$

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$$\widehat{D}_{\text{np}}(z, b_T)$$

OPE of pert. part
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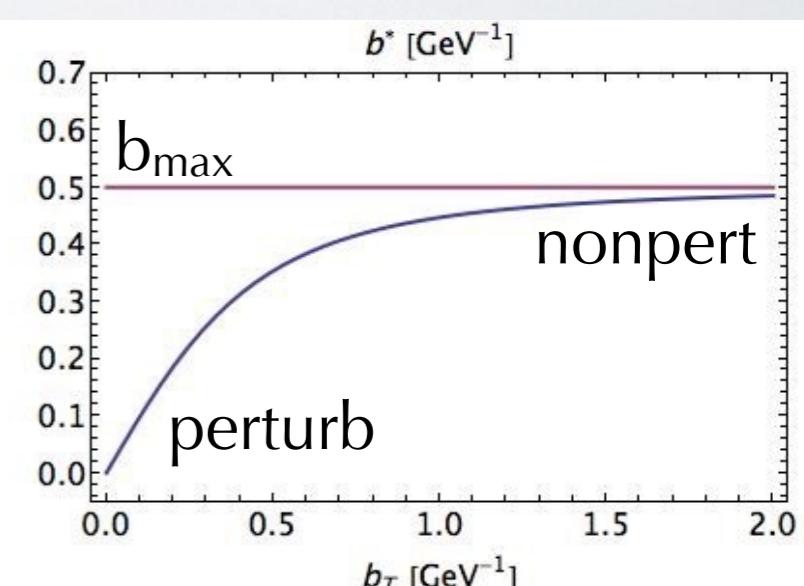
$$- \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left(\Gamma_{\text{cusp}} \log \frac{\zeta}{\mu'^2} + \gamma^V \right)$$

evolution with scale ζ
and with μ

matching
perturb. $\rightarrow \mu_b \leftarrow$ nonpert.

$$\mu_b = \frac{2e^{-\gamma_E}}{b^*} \quad b^* = \frac{b_T}{\sqrt{1 + \frac{b_T^2}{b_{\max}^2}}}$$

b^* prescription



nCSS at NLL

choosing $\zeta = \mu = Q^2$ and $\zeta_i = \mu_i = \mu_b^2$

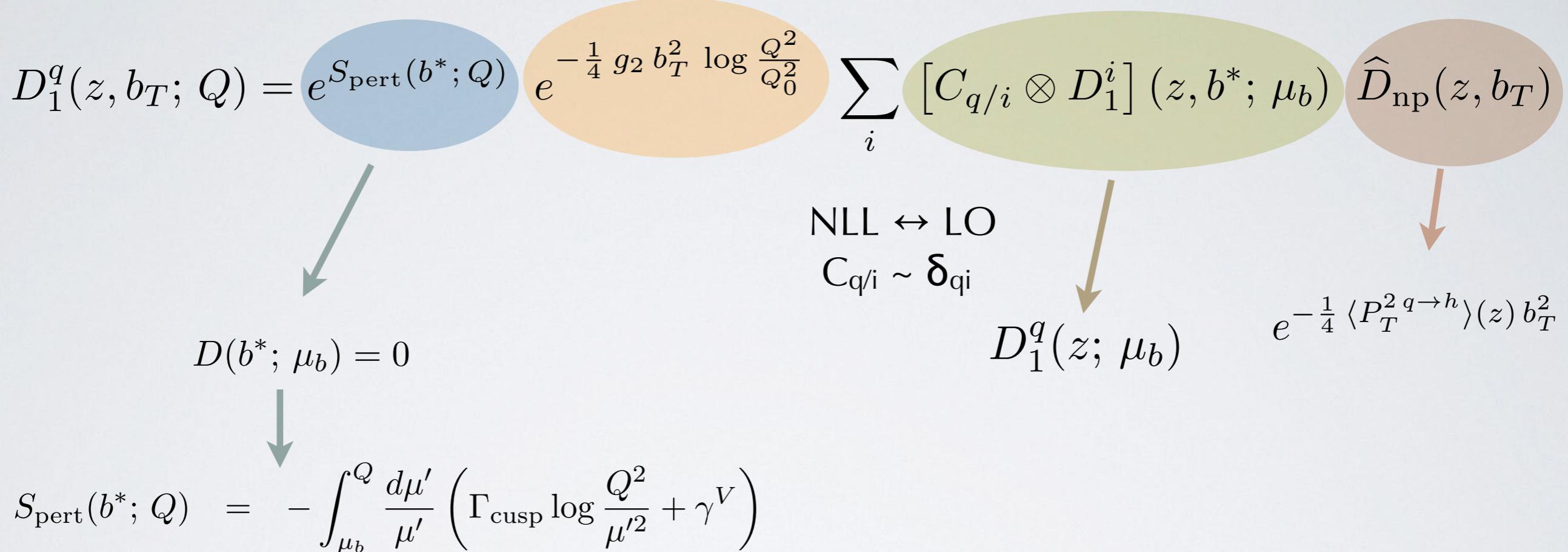
$$D_1^q(z, b_T; Q) = e^{S_{\text{pert}}(b^*; Q)} e^{-\frac{1}{4} g_2 b_T^2 \log \frac{Q^2}{Q_0^2}} \sum_i [C_{q/i} \otimes D_1^i](z, b^*; \mu_b) \hat{D}_{\text{np}}(z, b_T)$$

$$e^{-\frac{1}{4} \langle P_T^{2 \, q \rightarrow h} \rangle(z) b_T^2}$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b^*} \quad b^* = \frac{b_T}{\sqrt{1 + \frac{b_T^2}{b_{\max}^2}}}$$

nCSS at NLL

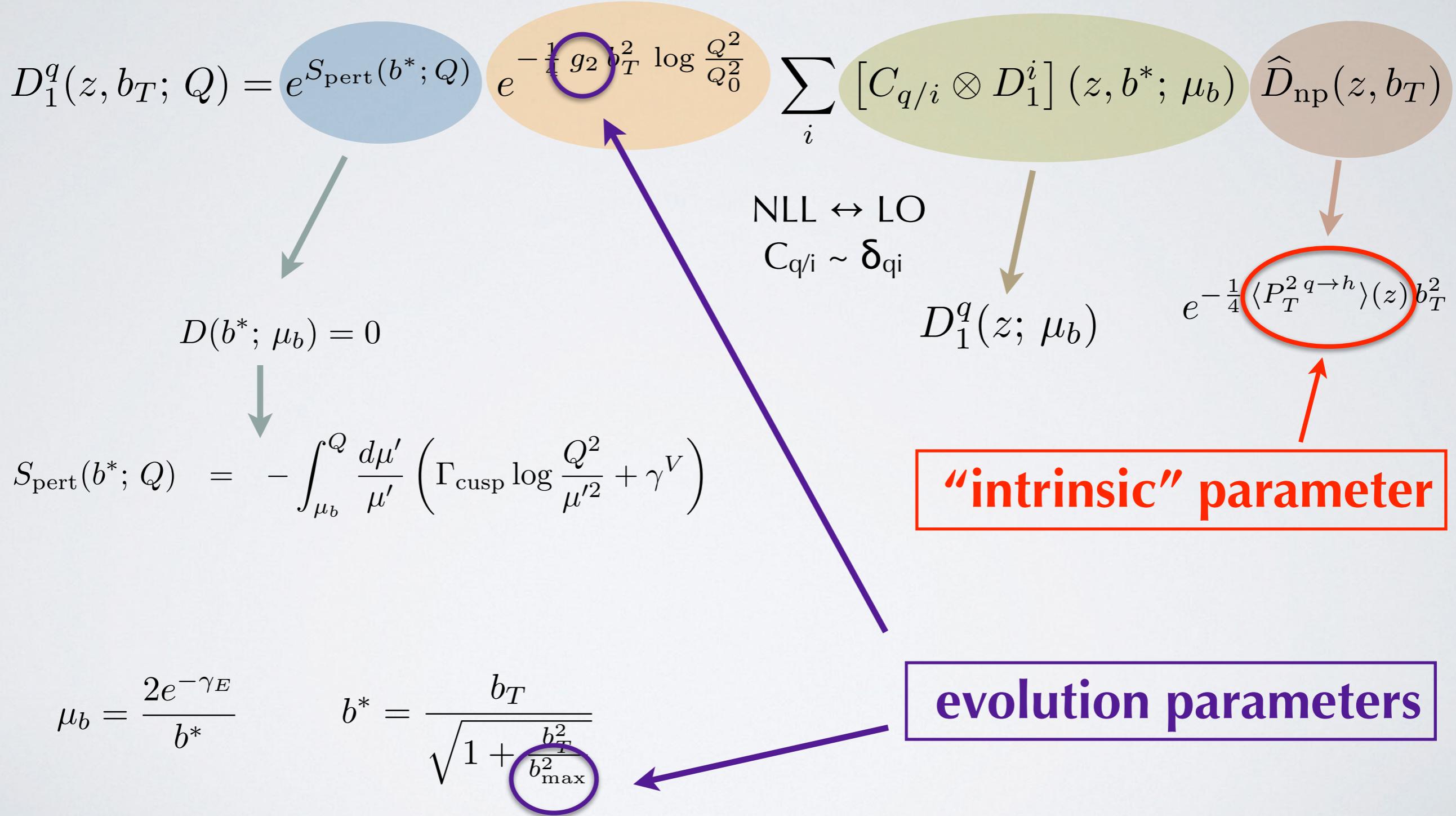
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nCSS at NLL

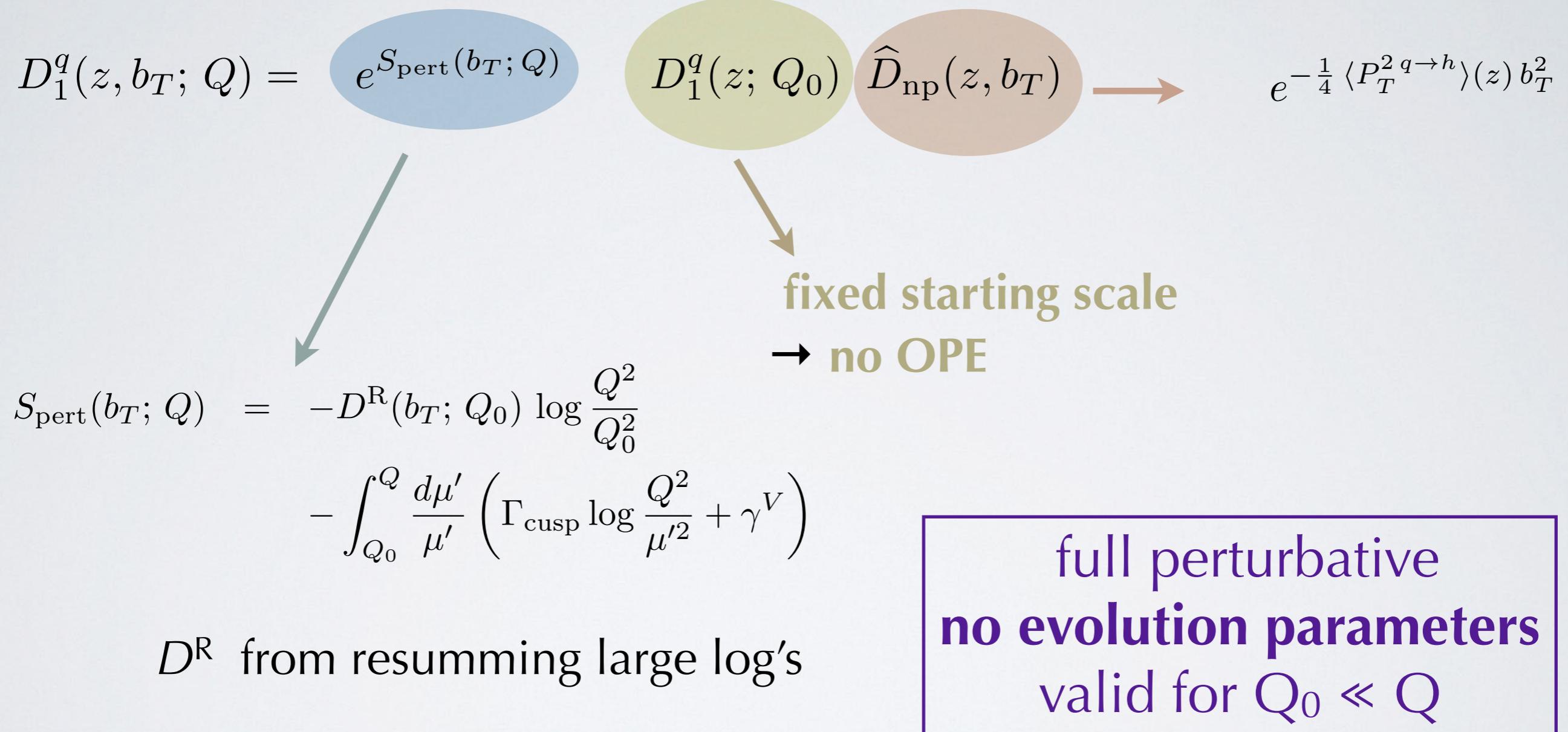
choosing $\zeta = \mu = Q^2$ and $\zeta_i = \mu_i = \mu_b^2$



Echevarría-Idilbi-Scimemi – EIS at NLL

choosing $\zeta = \mu = Q^2 \gg \zeta_i = \mu_i = Q_0^2$

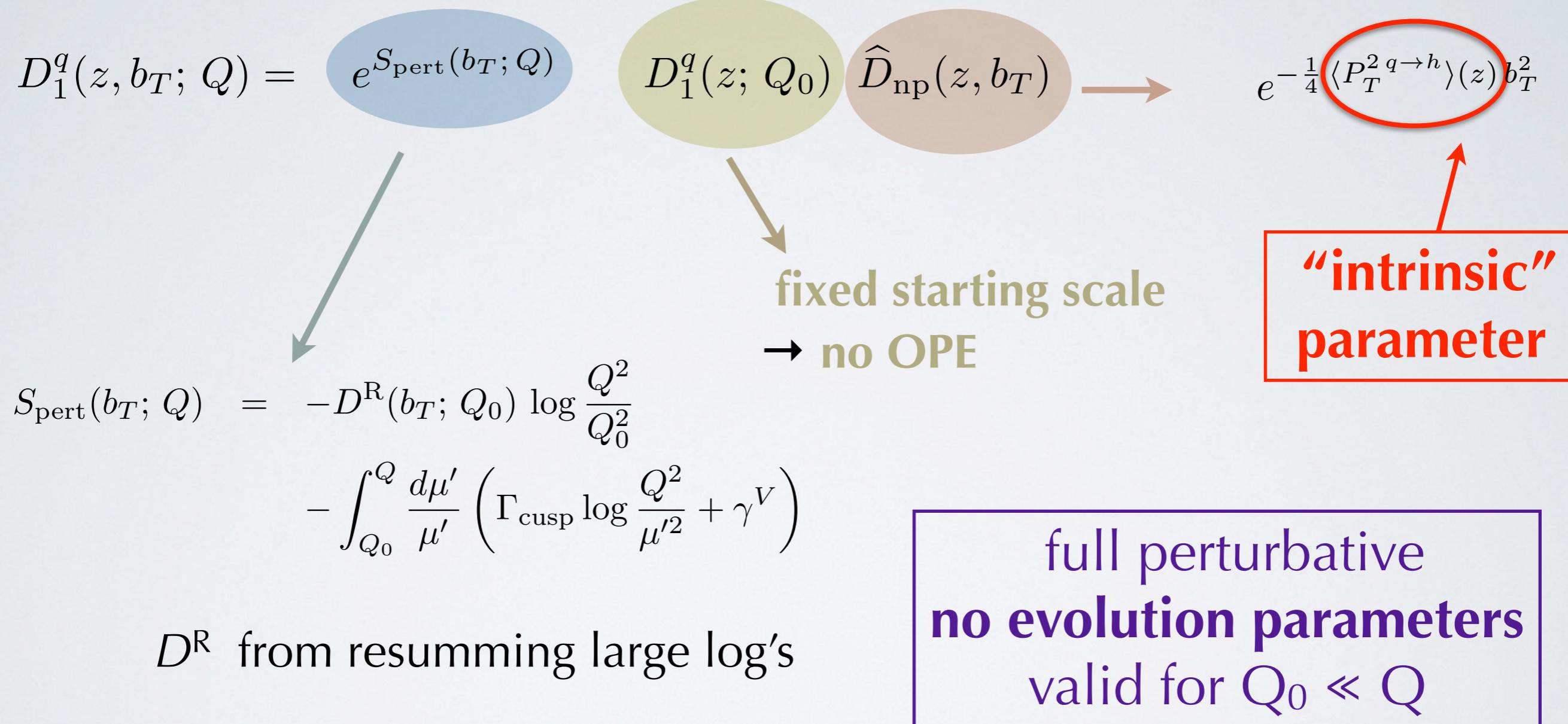
Echevarría, Idilbi, Scimemi,
P. R. D90 (14) 014003



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choosing $\zeta = \mu = Q^2 \gg \zeta_i = \mu_i = Q_0^2$

Echevarría, Idilbi, Scimemi,
P. R. D90 (14) 014003



hybrid EIS – **hEIS** at NLL

$$D_1^q(z, b_T; Q) = e^{\tilde{S}_{\text{pert}}(b_T; Q)} e^{-\frac{1}{4} g_2 b_T^2 \log \frac{Q^2}{Q_0^2}} D_1^q(z; Q_0) \hat{D}_{\text{np}}(z, b_T) \rightarrow e^{-\frac{1}{4} \langle P_T^2 q \rightarrow h \rangle(z) b_T^2}$$

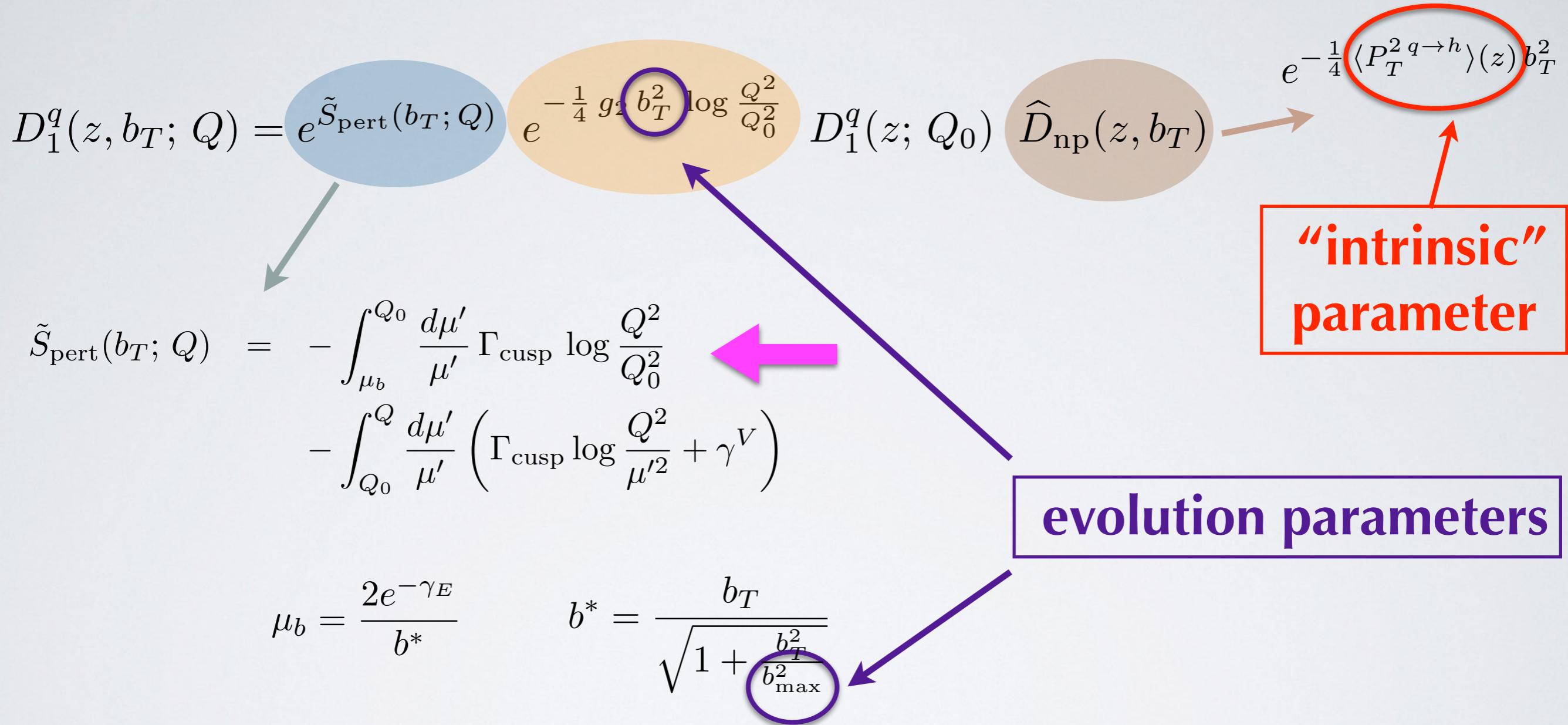
“intrinsic” parameter

$$\begin{aligned} \tilde{S}_{\text{pert}}(b_T; Q) &= - \int_{\mu_b}^{Q_0} \frac{d\mu'}{\mu'} \Gamma_{\text{cusp}} \log \frac{Q^2}{Q_0^2} \\ &\quad - \int_{Q_0}^Q \frac{d\mu'}{\mu'} \left(\Gamma_{\text{cusp}} \log \frac{Q^2}{\mu'^2} + \gamma^V \right) \end{aligned}$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b^*} \quad b^* = \frac{b_T}{\sqrt{1 + \frac{b_T^2}{b_{\max}^2}}}$$

$$\mathbf{hEIS} = \text{EIS with } D^R \leftrightarrow D(\mu_b) (=0 \text{ at NLL}) + \int_{\mu_b}^{Q_0} \frac{d\mu'}{\mu'} \Gamma_{\text{cusp}} + S_{\text{np}}(b_T)$$

hybrid EIS – **hEIS** at NLL



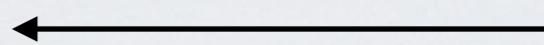
hEIS = EIS with $D^R \leftrightarrow D(\mu_b)$ ($=0$ at NLL) + $\int_{\mu_b}^{Q_0} \frac{d\mu'}{\mu'} \Gamma_{\text{cusp}} + S_{\text{np}}(b_T)$

starting scale for evolution:
input TMD FF

input TMD FF D_1 at starting scale

(h)EIS : ok

evolved



starting

$$D_1^q(z, b_T; Q) = E(b_T; Q, Q_0, \mu_b) D_1^q(z, b_T; Q_0)$$

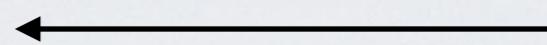


$$D_1^q(z; Q_0) e^{-\frac{1}{4} \langle P_T^{2q} \rangle^h(z) b_T^2}$$

input TMD FF D_1 at starting scale

(h)EIS : ok

evolved



starting

$$D_1^q(z, b_T; Q) = E(b_T; Q, Q_0, \mu_b) D_1^q(z, b_T; Q_0)$$

$$D_1^q(z; Q_0) e^{-\frac{1}{4} \langle P_T^2 q \rightarrow h \rangle(z) b_T^2}$$

nCSS : ?



$$D_1^q(z, b_T; Q) = E(b^*; Q, Q_0, \mu_b) D_1^q(z; \mu_b) e^{-\frac{1}{4} \langle P_T^2 q \rightarrow h \rangle(z) b_T^2}$$

input TMD FF D_1 at starting scale

(h)EIS : ok

evolved



starting

$$D_1^q(z, b_T; Q) = E(b_T; Q, Q_0, \mu_b) D_1^q(z, b_T; Q_0)$$

nCSS : ?

$$D_1^q(z, b_T; Q) = E(b^*; Q, Q_0, \mu_b)$$

$$D_1^q(z; Q_0)$$

$$e^{-\frac{1}{4} \langle P_T^2 q \rightarrow h \rangle(z)} b_T^2$$



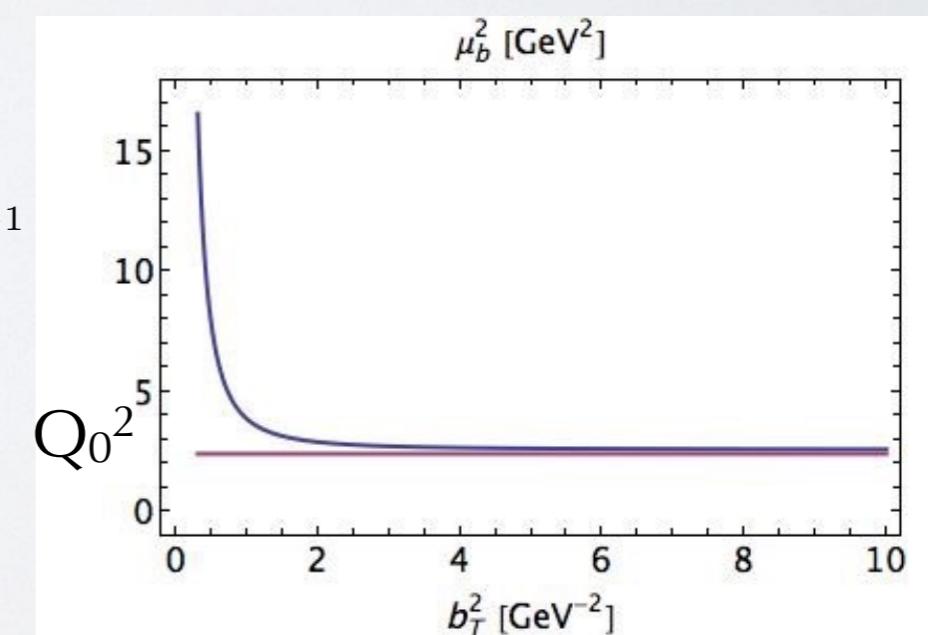
$$D_1^q(z; \mu_b) e^{-\frac{1}{4} \langle P_T^2 q \rightarrow h \rangle(z)} b_T^2$$

if $b_T \gg b_{\max}$ (where SIDIS data are)

$$b^* \approx b_{\max} \quad \mu_b \approx \frac{2 e^{-\gamma_E}}{b_{\max}} \approx Q_0$$

$$b_{\max} = 0.7 \text{ GeV}^{-1}$$

$$Q_0^2 = 2.4 \text{ GeV}^2$$



“intrinsic” parameters

DSS

De Florian, Sassot, Stratmann
P.R. D75 (07) 114010

$$D_1^q(z; Q_0) \quad e^{-\frac{1}{4} \langle P_T^{2q \rightarrow h} \rangle(z) b_T^2}$$

flavor- & z-dependent Gaussian width

$$\langle P_T^{2q \rightarrow h} \rangle(z) = \widehat{\langle P_T^{2q \rightarrow h} \rangle} \frac{(z^\beta + \delta)(1-z)^\gamma}{(\hat{z}^\beta + \delta)(1-\hat{z})^\gamma}$$

$$\widehat{\langle P_T^{2q \rightarrow h} \rangle} \equiv \langle P_T^{2q \rightarrow h} \rangle(\hat{z}) \quad \hat{z} = 0.5$$

$$\beta, \gamma, \delta$$

$$\widehat{\langle P_T^{2 \text{ fav}} \rangle} \quad \text{favored } \pi \text{ (u} \rightarrow \pi^+, \text{ d} \rightarrow \pi^-, \dots)$$

$$\widehat{\langle P_T^{2u \rightarrow K} \rangle} \quad \text{favored u} \rightarrow K \text{ (u} \rightarrow K^+, \bar{u} \rightarrow K^-)$$

$$\widehat{\langle P_T^{2s \rightarrow K} \rangle} \quad \text{favored s} \rightarrow K \text{ (s} \rightarrow K^-, s \rightarrow K^+)$$

$$\widehat{\langle P_T^{2 \text{ unf}} \rangle} \quad \text{unfavored (all the rest)}$$

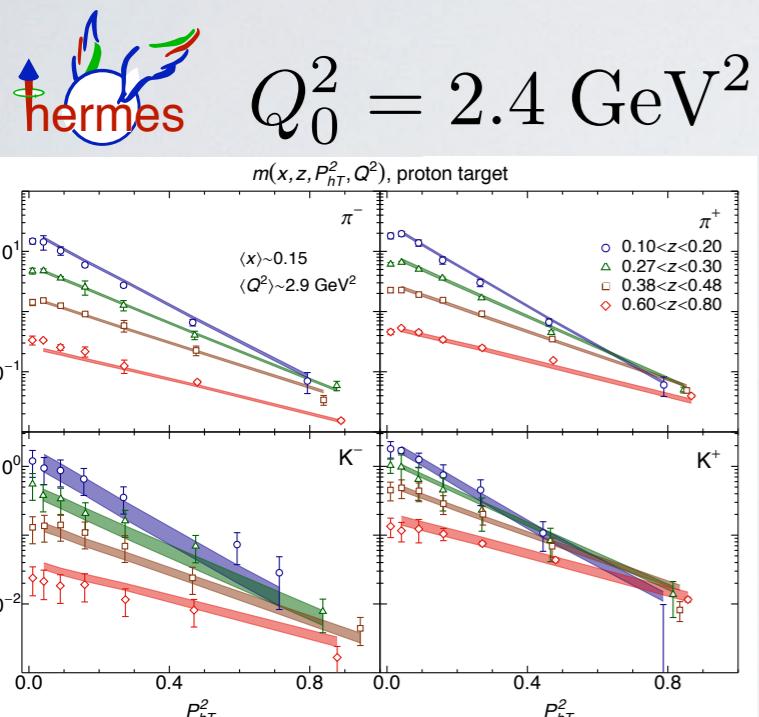
7 parameters

for more details, see

Signori, Bacchetta, Radici, Schnell,
JHEP 1311 (13) 194

fitting Hermes multiplicities

fit 200 random replicas of data
**we will use all 200 values
 for each parameter**



Airapetian *et al.*,
 P.R. D87 (13) 074029

for more details, see
 Signori, Bacchetta, Radici, Schnell,
 JHEP 1311 (13) 194

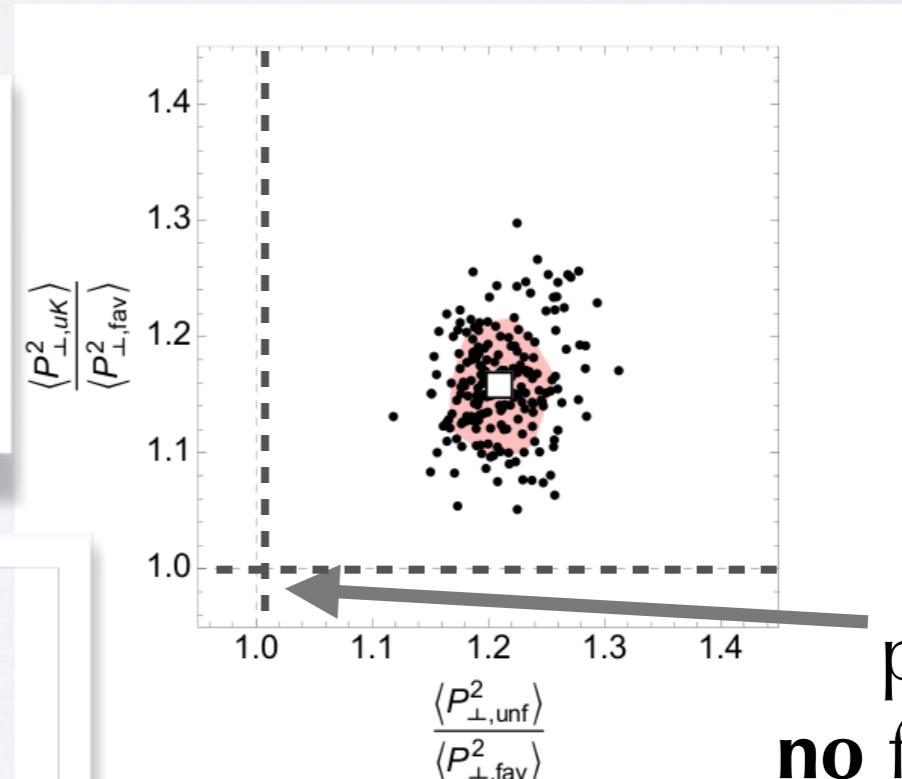
$\langle \hat{P}_{\perp, \text{fav}}^2 \rangle$ [GeV ²]	$\langle \hat{P}_{\perp, \text{unf}}^2 \rangle$ [GeV ²]	$\langle \hat{P}_{\perp, sK}^2 \rangle$ [GeV ²] (random)	$\langle \hat{P}_{\perp, uK}^2 \rangle$ [GeV ²]	β	δ	γ
0.15 ± 0.04	0.19 ± 0.04	0.19 ± 0.04	0.18 ± 0.05	1.43 ± 0.43	1.29 ± 0.95	0.17 ± 0.09

global $\chi^2/\text{d.o.f.} = 1.63 \pm 0.12$
 no flavor dep. 1.72 ± 0.11 flavor-indep. fit
 not excluded

$q \rightarrow K$ favored
wider than
 $q \rightarrow \pi$ favored

unfavored
wider than
 $q \rightarrow \pi$ favored

BUT

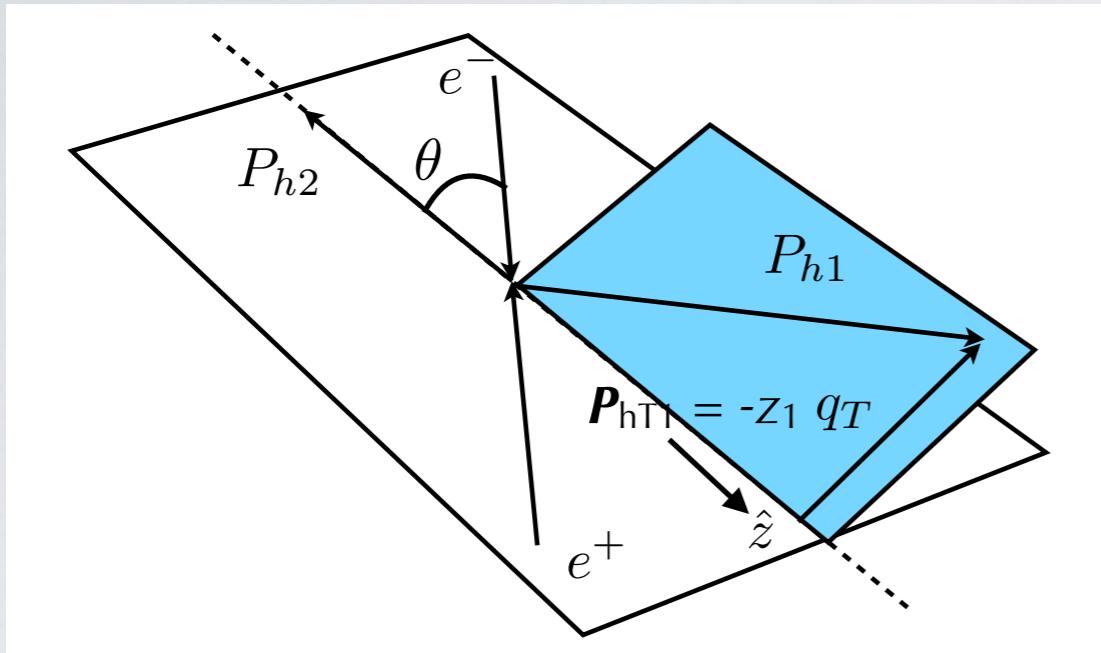


point of
no flavor dep.



observables

e+ e- multiplicity



$e^+ e^- \rightarrow h_1(P_{h1}) \ h_2(P_{h2}) \ X$

$$Q^2 = 100 \text{ GeV}^2 \quad q_T^2 \ll Q^2$$

$$A(y) = \frac{1}{2} - y + y^2$$

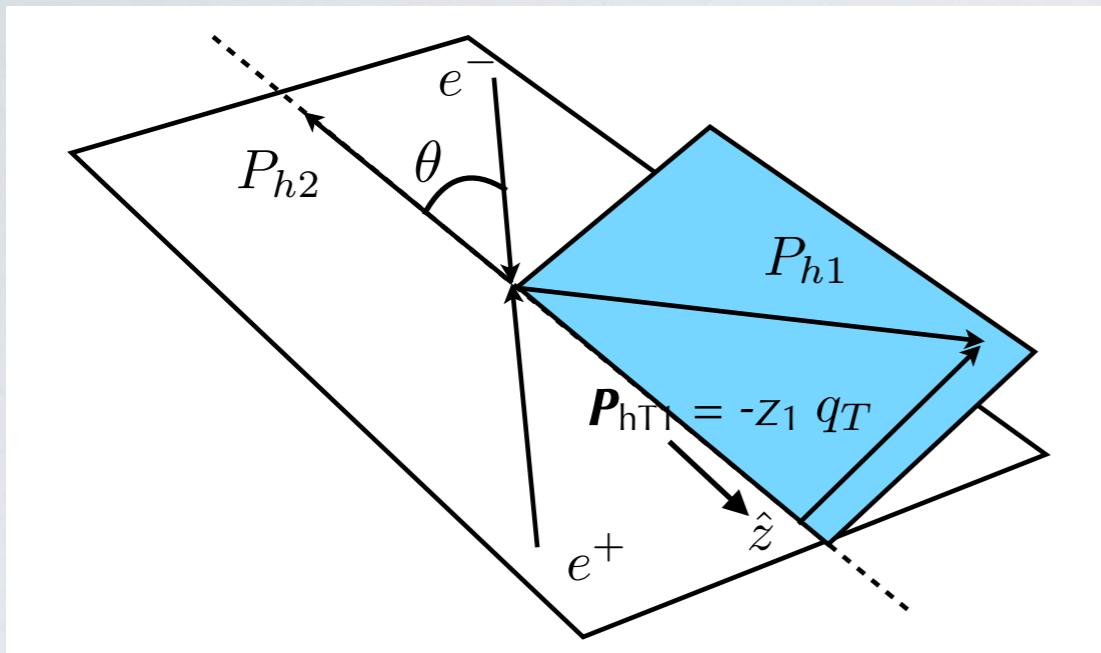
$$y = \frac{1 + \cos \theta}{2}$$

cross section

$$\begin{aligned} \frac{d\sigma}{dz_1 dz_2 dq_T^2 dy} &= \frac{6\pi\alpha^2}{Q^2} A(y) \mathcal{H}(Q, \mu) \\ &\times \sum_q e_q^2 \int_0^\infty db_T b_T J_0(q_T b_T) \left[z_1^2 D_1^{q \rightarrow h_1}(z_1, b_T; \mu) z_2^2 D_1^{\bar{q} \rightarrow h_2}(z_2, b_T; \mu) + q \leftrightarrow \bar{q} \right] + Y(q_T^2, Q^2) \end{aligned}$$

Boer, Jakob, Mulders,
N.P. **B504** (97)

e+ e- multiplicity



cross section

$$\frac{d\sigma}{dz_1 dz_2 dq_T^2 dy} = \frac{6\pi\alpha^2}{Q^2} A(y) [\mathcal{H}(Q, \mu) \sim 1] \times \sum_q e_q^2 \int_0^\infty db_T b_T J_0(q_T b_T) \left[z_1^2 D_1^{q \rightarrow h_1}(z_1, b_T; \mu) z_2^2 D_1^{\bar{q} \rightarrow h_2}(z_2, b_T; \mu) + q \leftrightarrow \bar{q} \right] + Y(q_T^2, Q^2)$$

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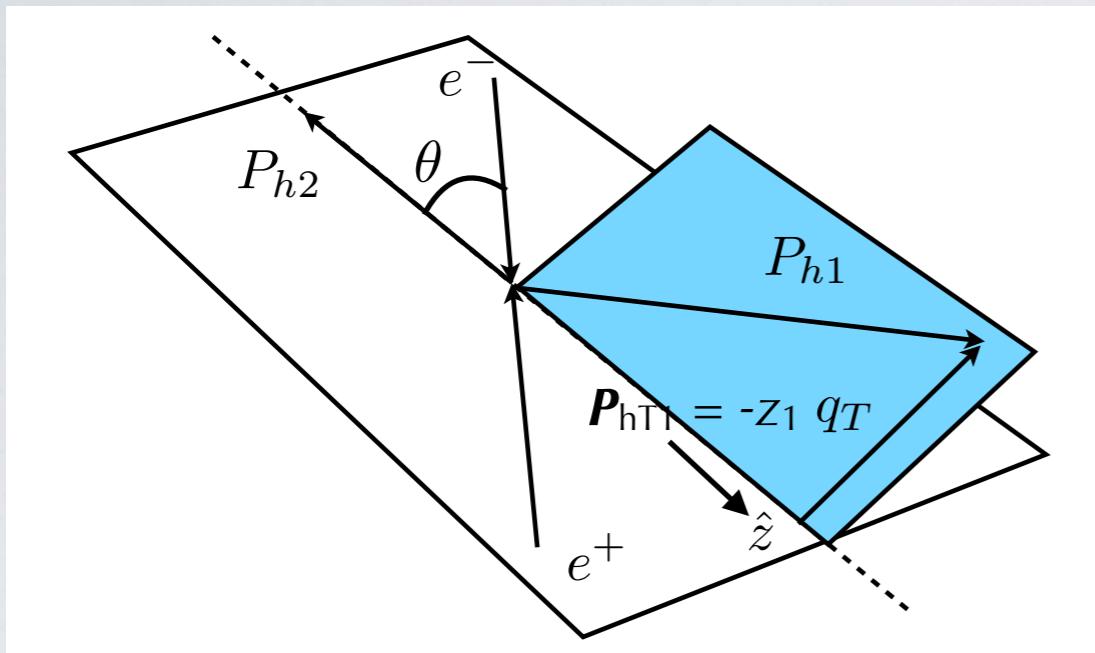
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LO \leftrightarrow NLL

Boer, Jakob, Mulders,
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e+e-
multiplicity

$$M(h_1, h_2) = \frac{d\sigma^{h_1 h_2}}{dz_1 dz_2 dq_T^2 dy} / \frac{d\sigma^{h_1}}{dz_1 dy}$$

TMD
(analogue of SIDIS def.)

$$e^+ e^- \rightarrow h_1(P_{h1}) \ h_2(P_{h2}) \ X$$

$$Q^2 = 100 \text{ GeV}^2 \quad q_T^2 \ll Q^2$$

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$$y = \frac{1 + \cos \theta}{2}$$

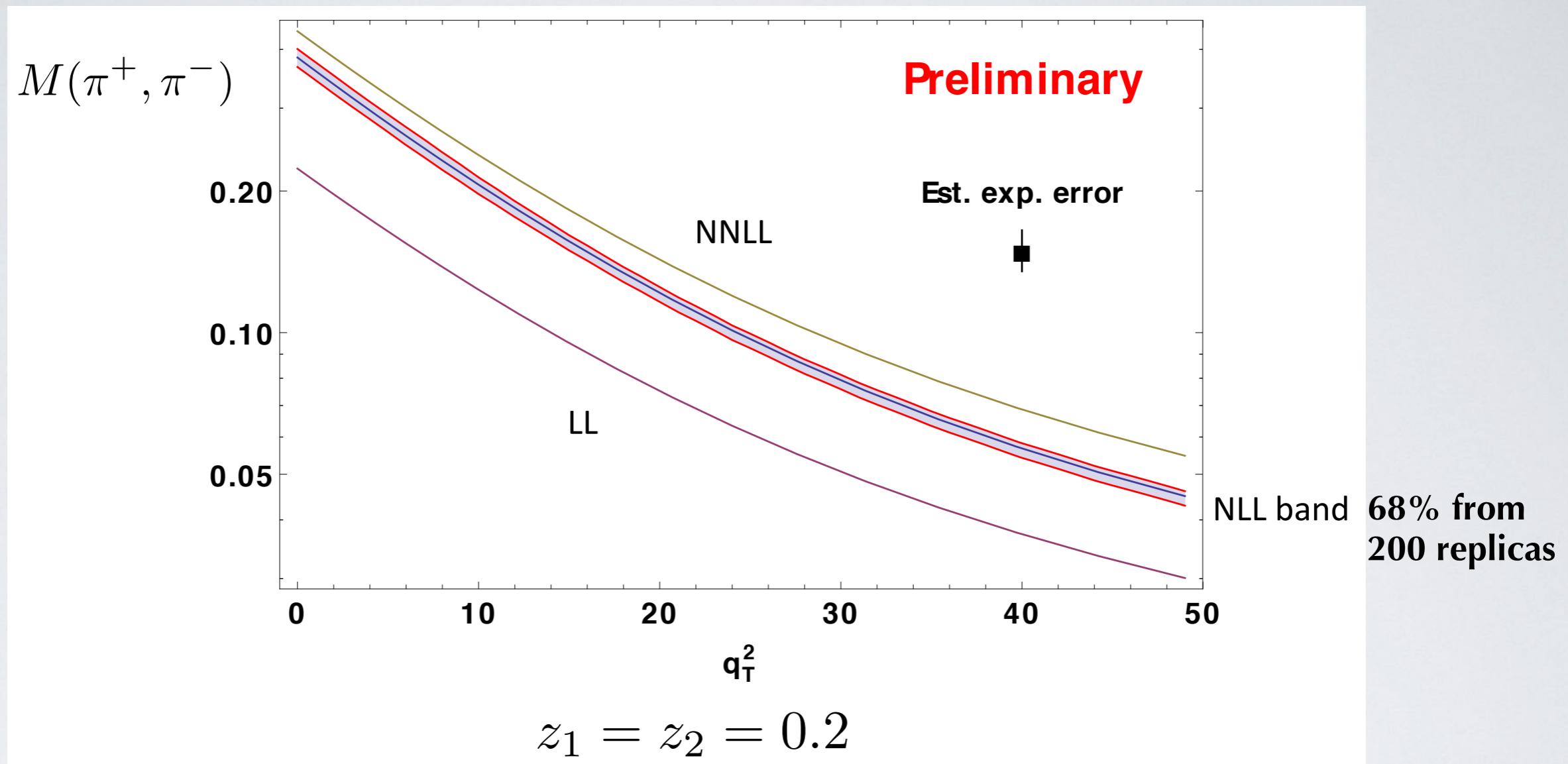
LO \leftrightarrow NLL

Boer, Jakob, Mulders,
N.P. B504 (97)

results

(if not specified, always for $y=0.2$)

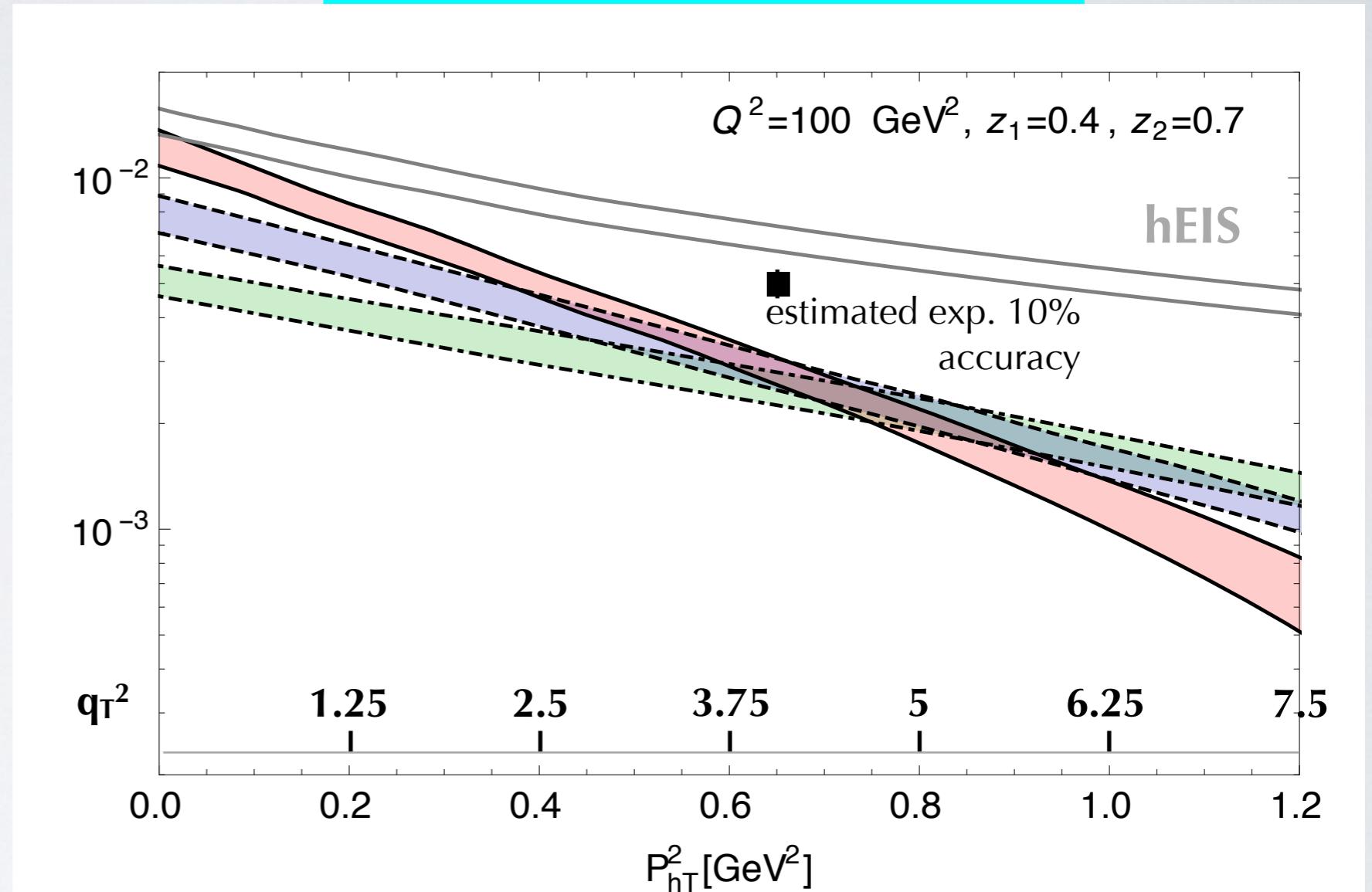
theoretical accuracy



at least,
NLL required

sensitivity to evolution parameters

$\mathcal{M}(\pi^+\pi^-)$ nCSS

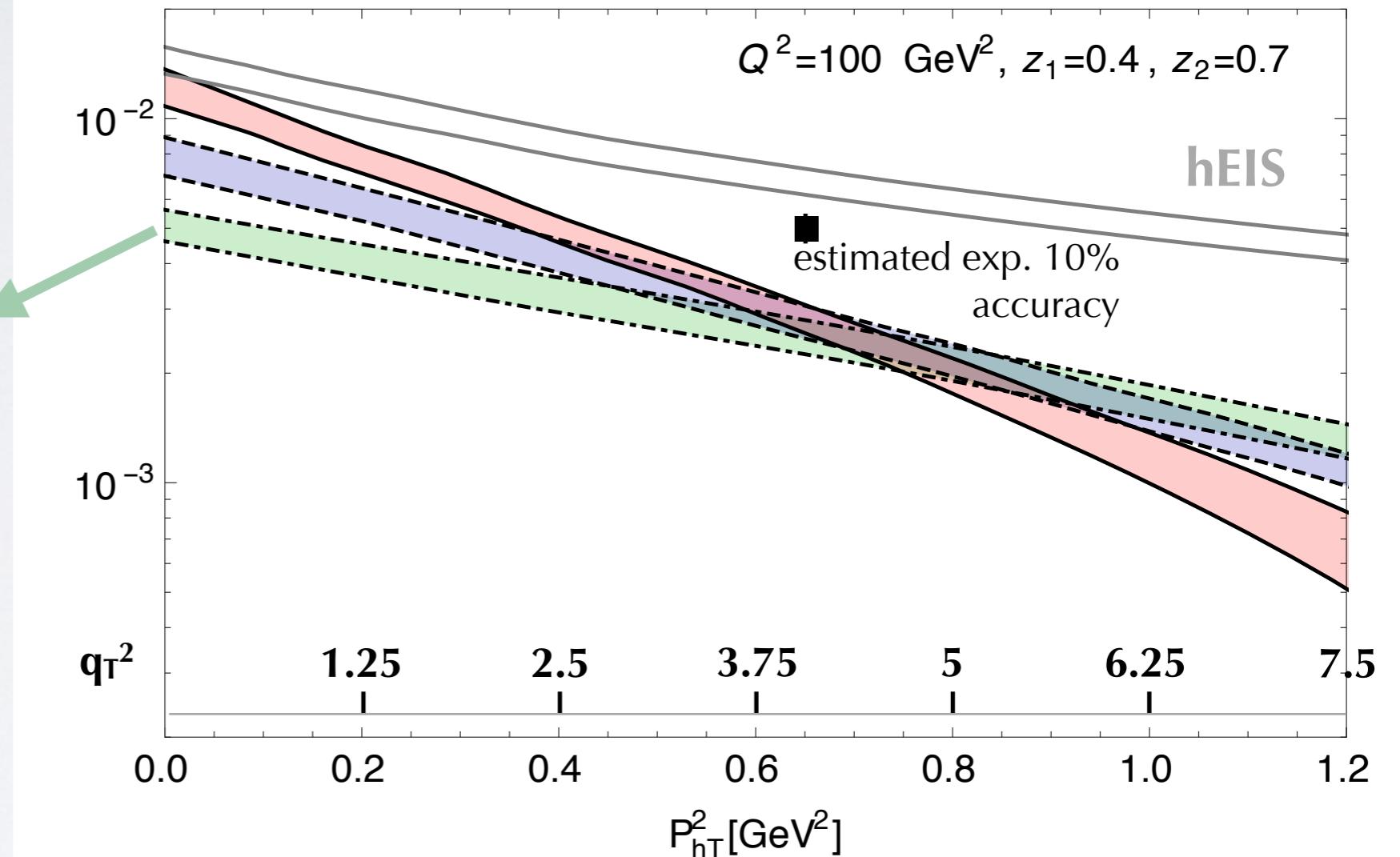


sensitivity to evolution parameters

$\mathcal{M}(\pi^+\pi^-)$ nCSS

$b_{\max} = 0.5 \quad g_2 = 0.64$

Landry, Brock, Nadolsky, Yuan,
P.R. D67 (03) 073016



sensitivity to evolution parameters

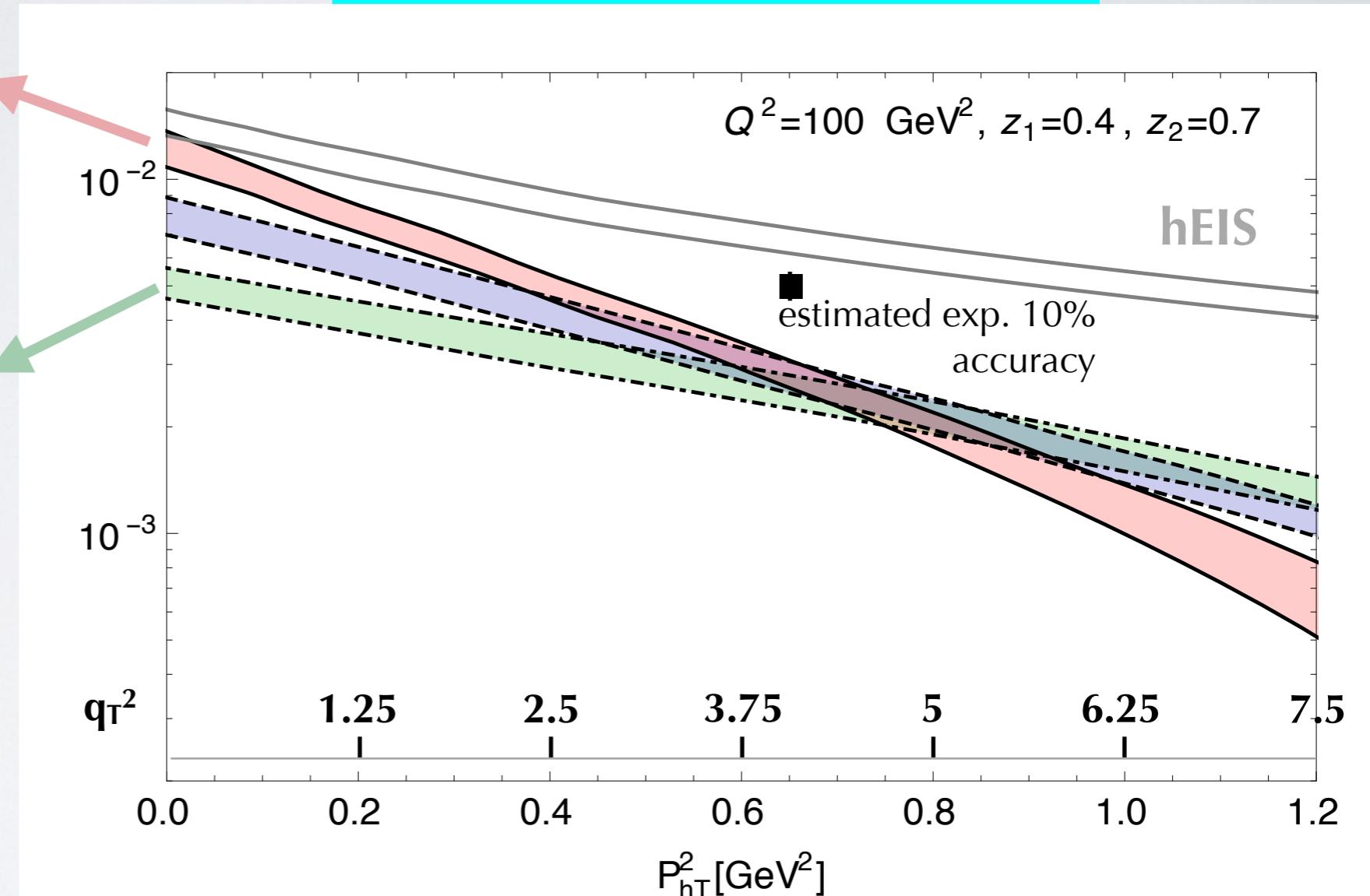
Konychev, Nadolsky,
P.L. B633 (06)

$$b_{\max} = 1.5 \quad g_2 = 0.18$$

$$b_{\max} = 0.5 \quad g_2 = 0.64$$

Landry, Brock, Nadolsky, Yuan,
P.R. D67 (03) 073016

$\mathcal{M} (\pi^+ \pi^-)$ nCSS



sensitivity to evolution parameters

Konychev, Nadolsky,
P.L. B633 (06)

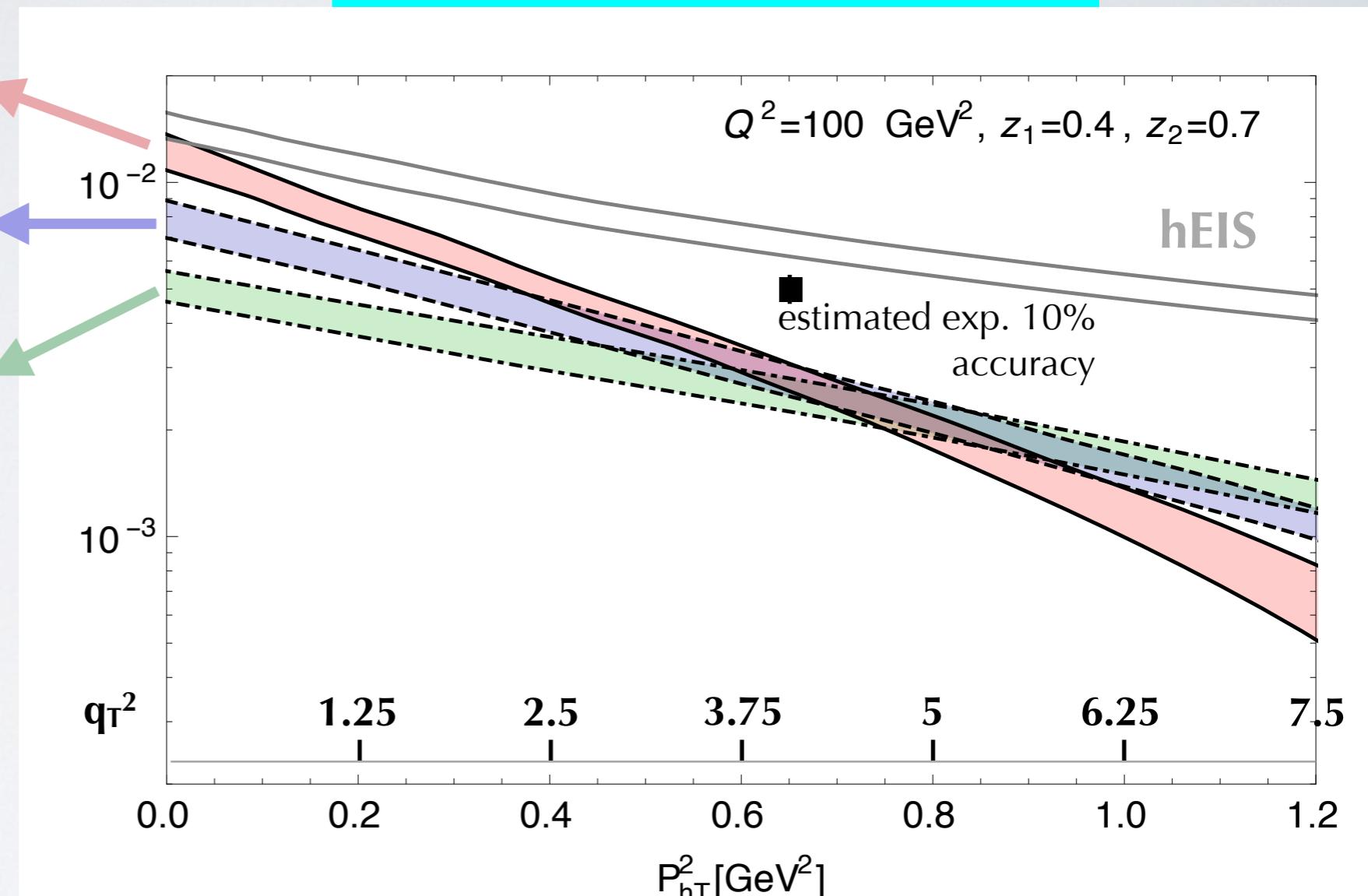
$\mathcal{M} (\pi^+ \pi^-)$ nCSS

$$b_{\max} = 1.5 \quad g_2 = 0.18$$

$$b_{\max} = 1 \quad g_2 = 0.41$$

$$b_{\max} = 0.5 \quad g_2 = 0.64$$

Landry, Brock, Nadolsky, Yuan,
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sensitivity to evolution parameters

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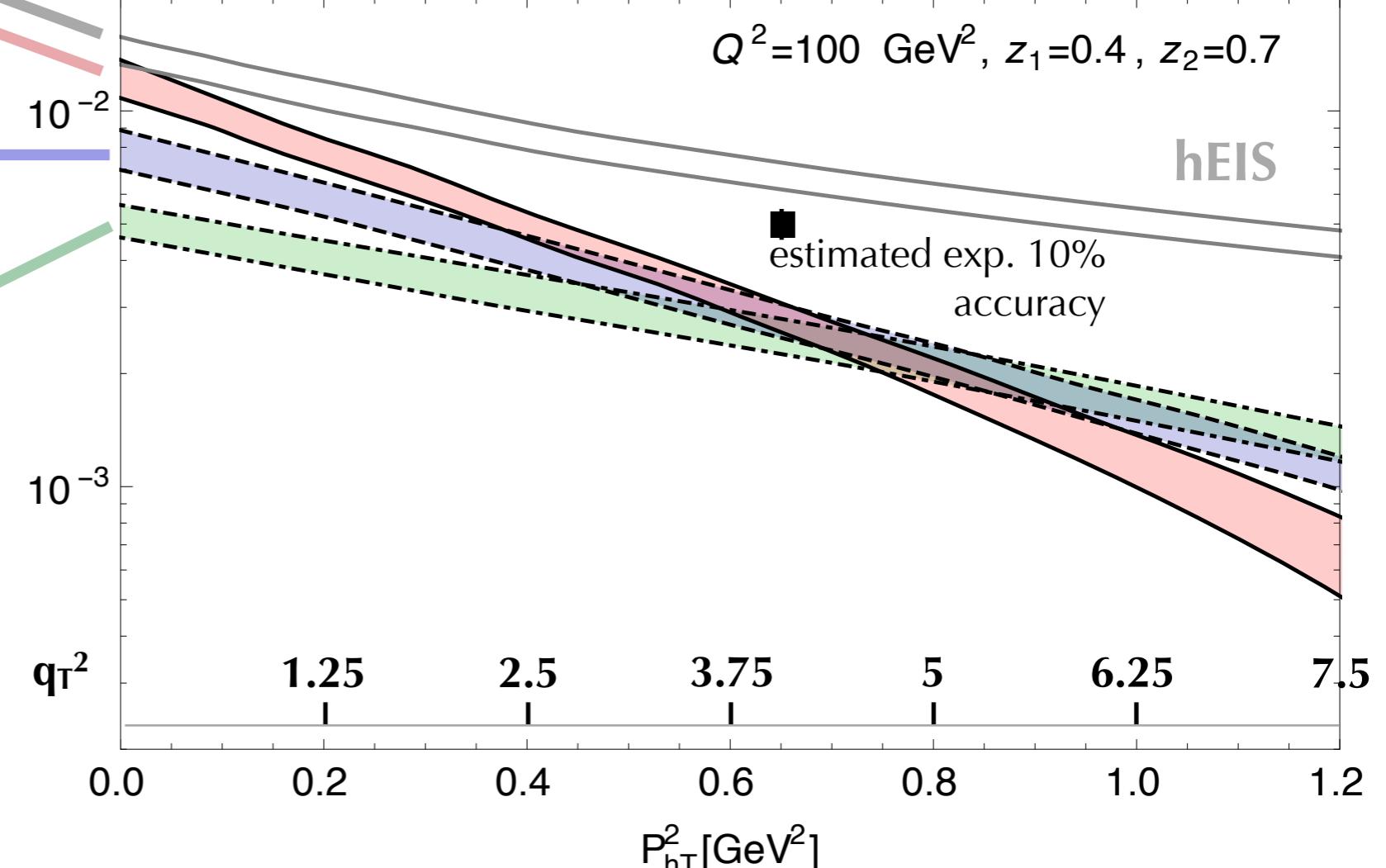
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sensitivity to evolution parameters

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$$b_{\max} = 1.5 \quad g_2 = 0.18$$

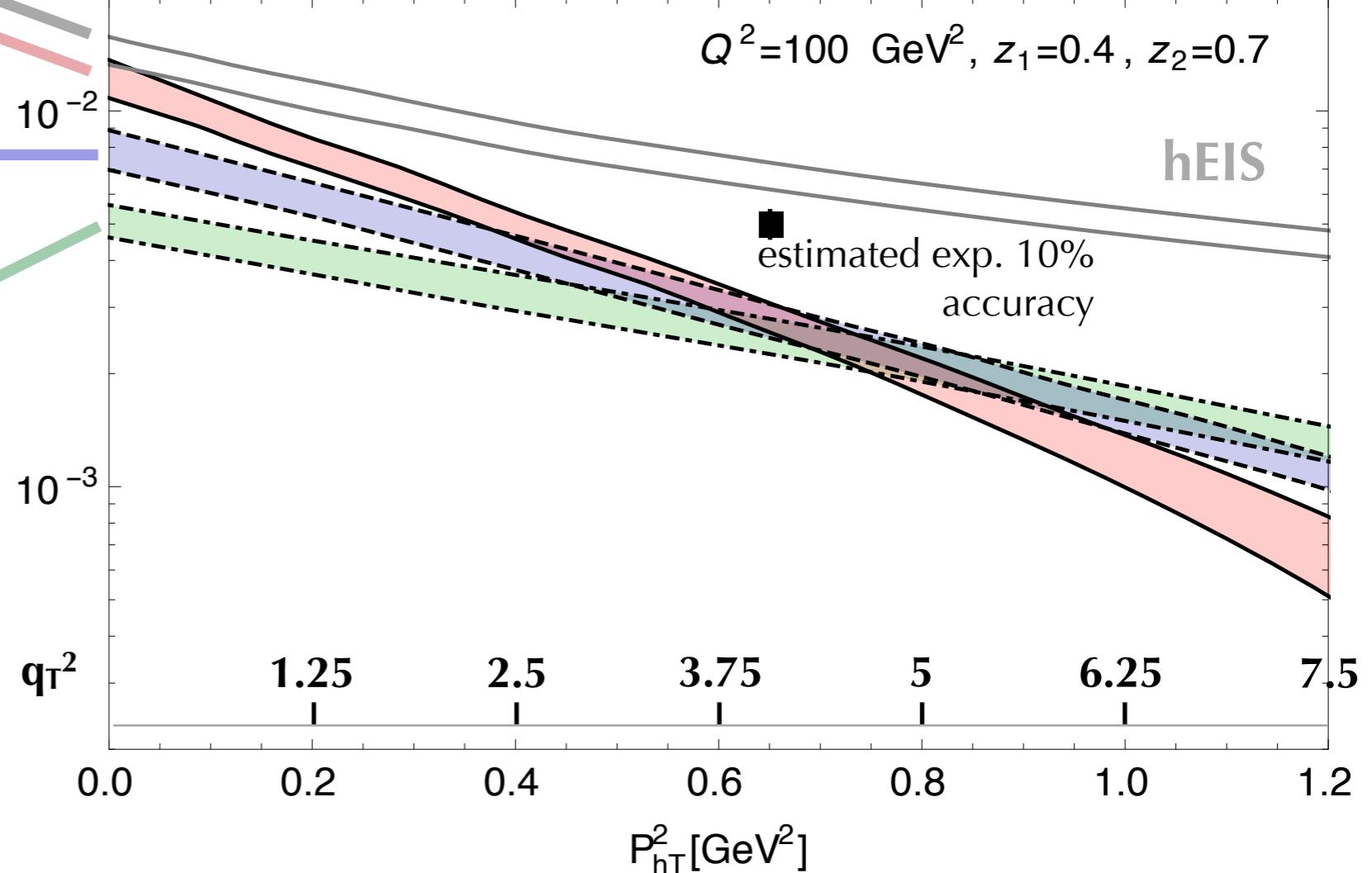
$$b_{\max} = 1 \quad g_2 = 0.41$$

$$b_{\max} = 0.5 \quad g_2 = 0.64$$

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data can constrain
evolution
parameters

$\mathcal{M} (\pi^+ \pi^-)$ nCSS



sensitivity to evolution parameters

Konychev, Nadolsky,
P.L. B633 (06)

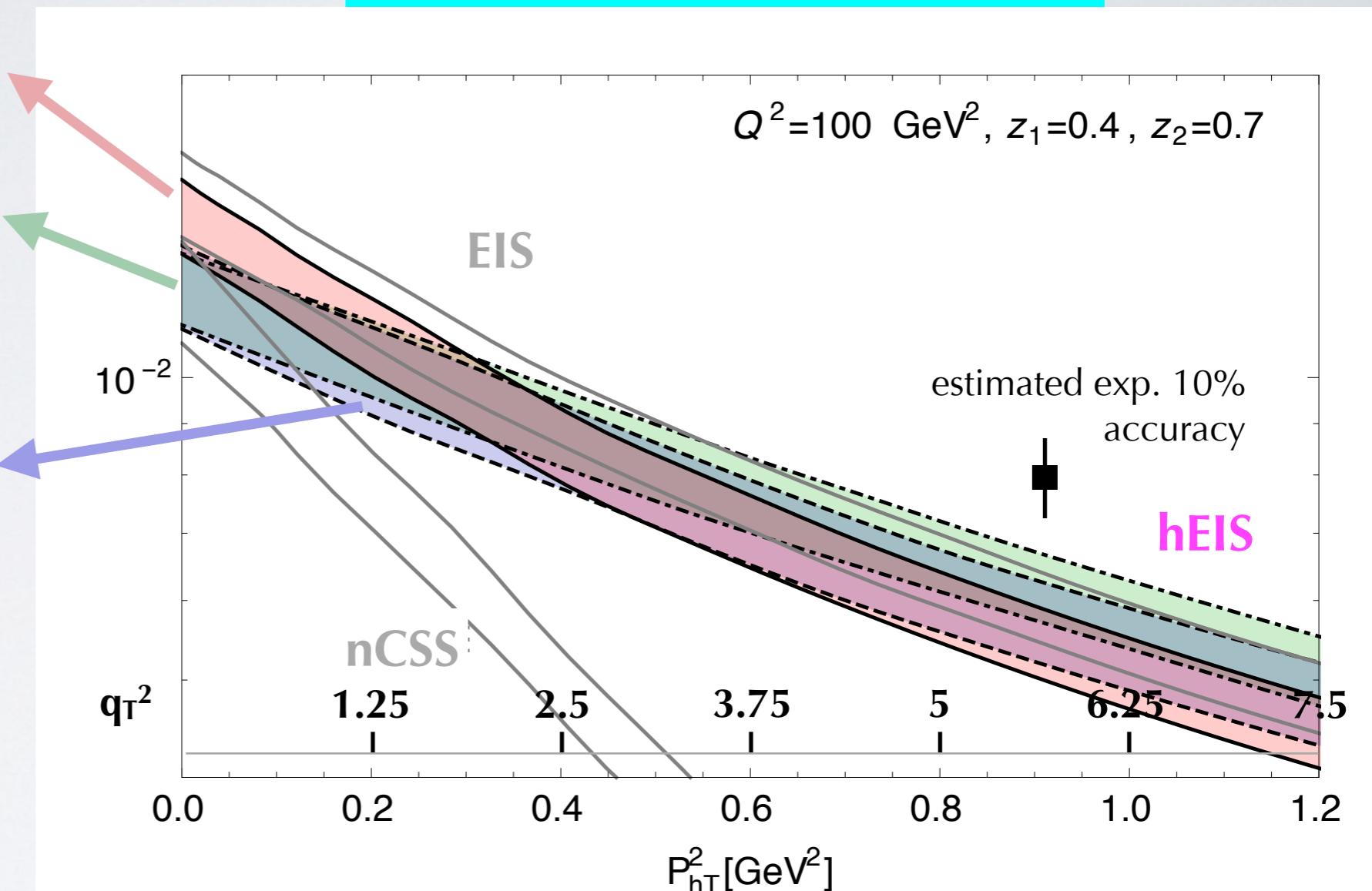
$\mathcal{M}(\pi^+\pi^-)$ (h)EIS

$$b_{\max} = 1.5 \quad g_2 = 0.18$$

$$b_{\max} = 0.5 \quad g_2 = 0.64$$

Landry, Brock, Nadolsky, Yuan,
P.R. D67 (03) 073016

$$b_{\max} = 1 \quad g_2 = 0.41$$



sensitivity to evolution parameters

Konychev, Nadolsky,
P.L. B633 (06)

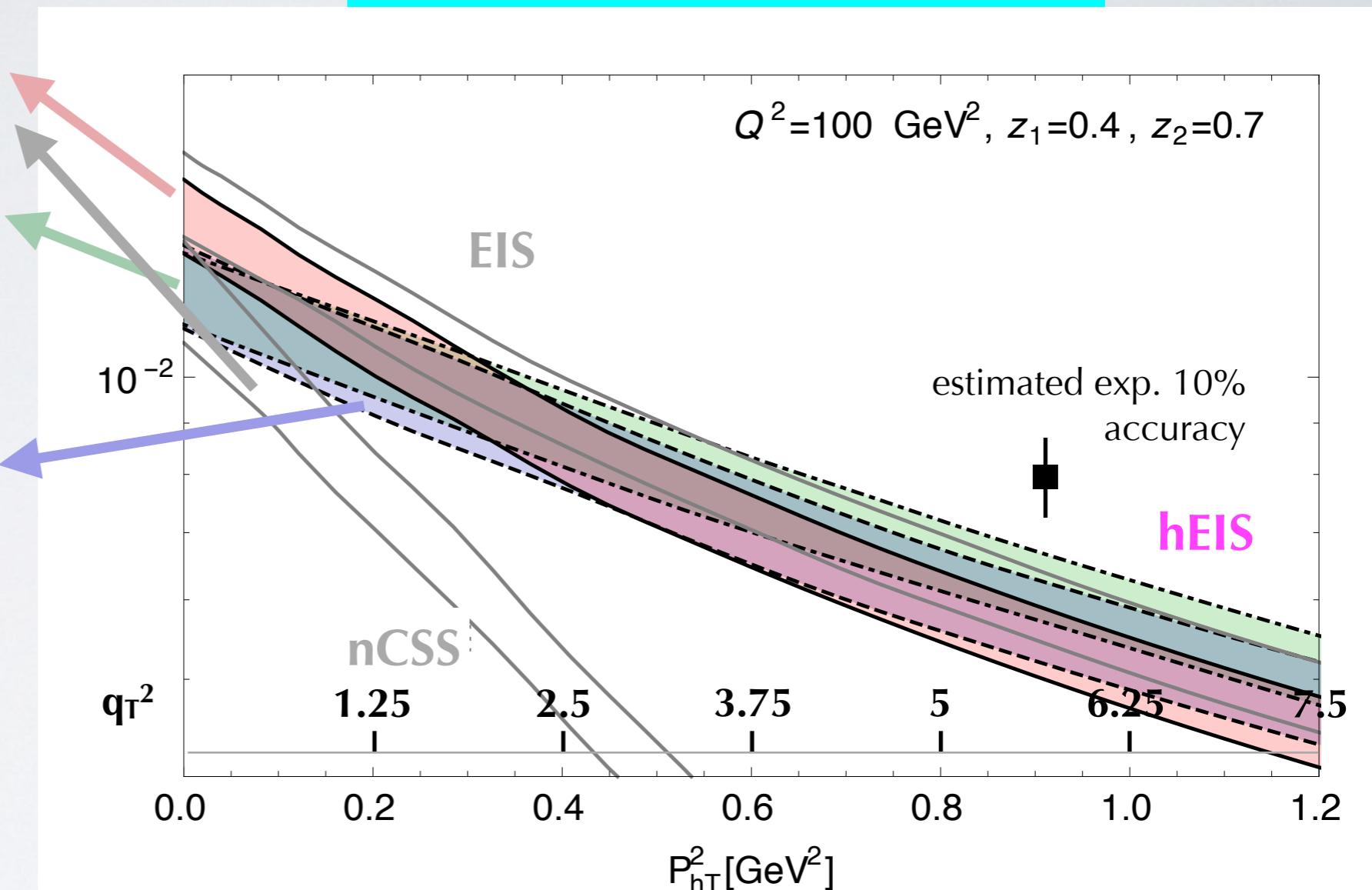
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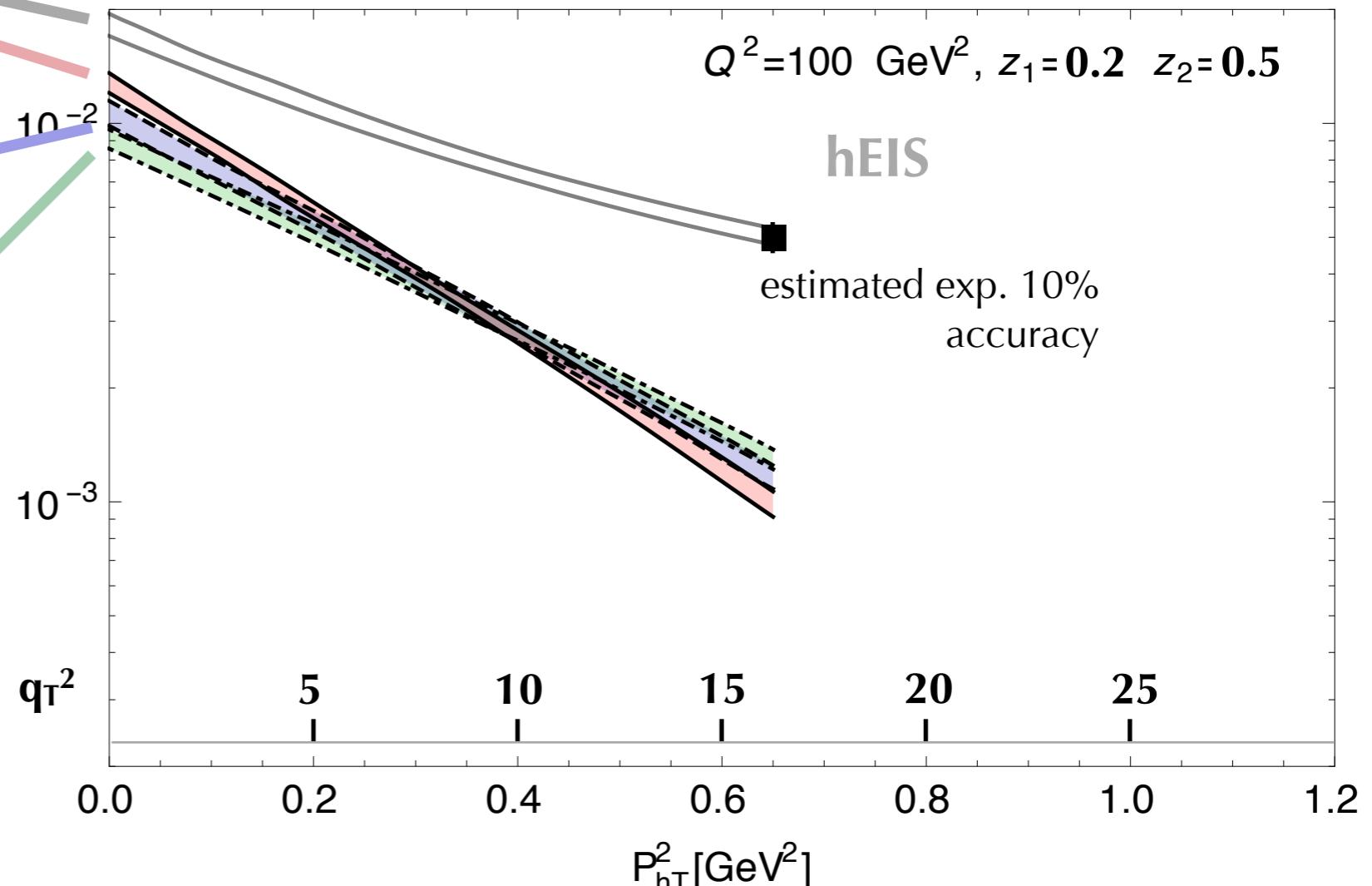
less clear
at lower z
but
distinguish
kernel evolutors

$\mathcal{M} (\pi^+ \pi^-)$ nCSS

$Q^2 = 100 \text{ GeV}^2, z_1 = 0.2, z_2 = 0.5$

hEIS

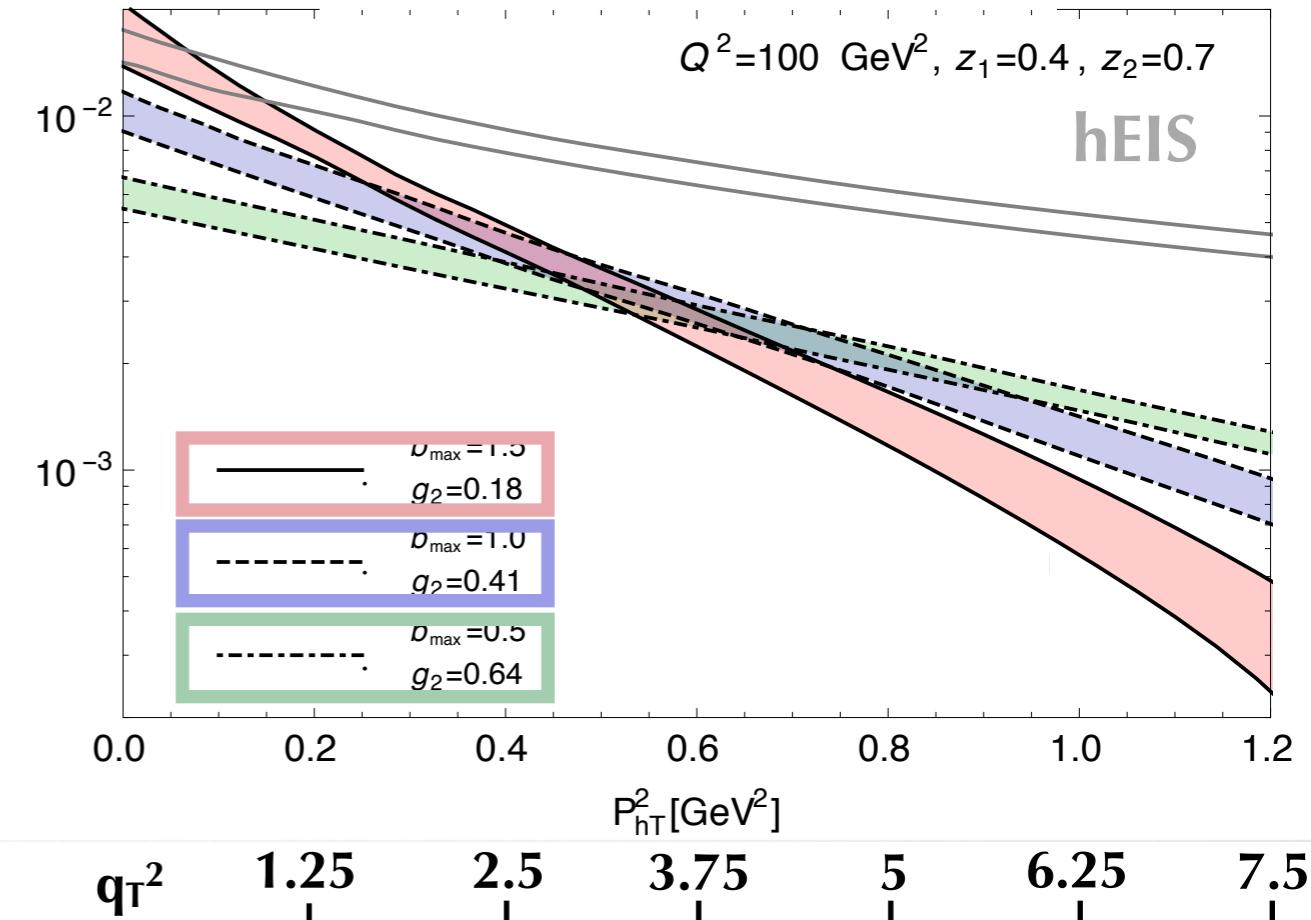
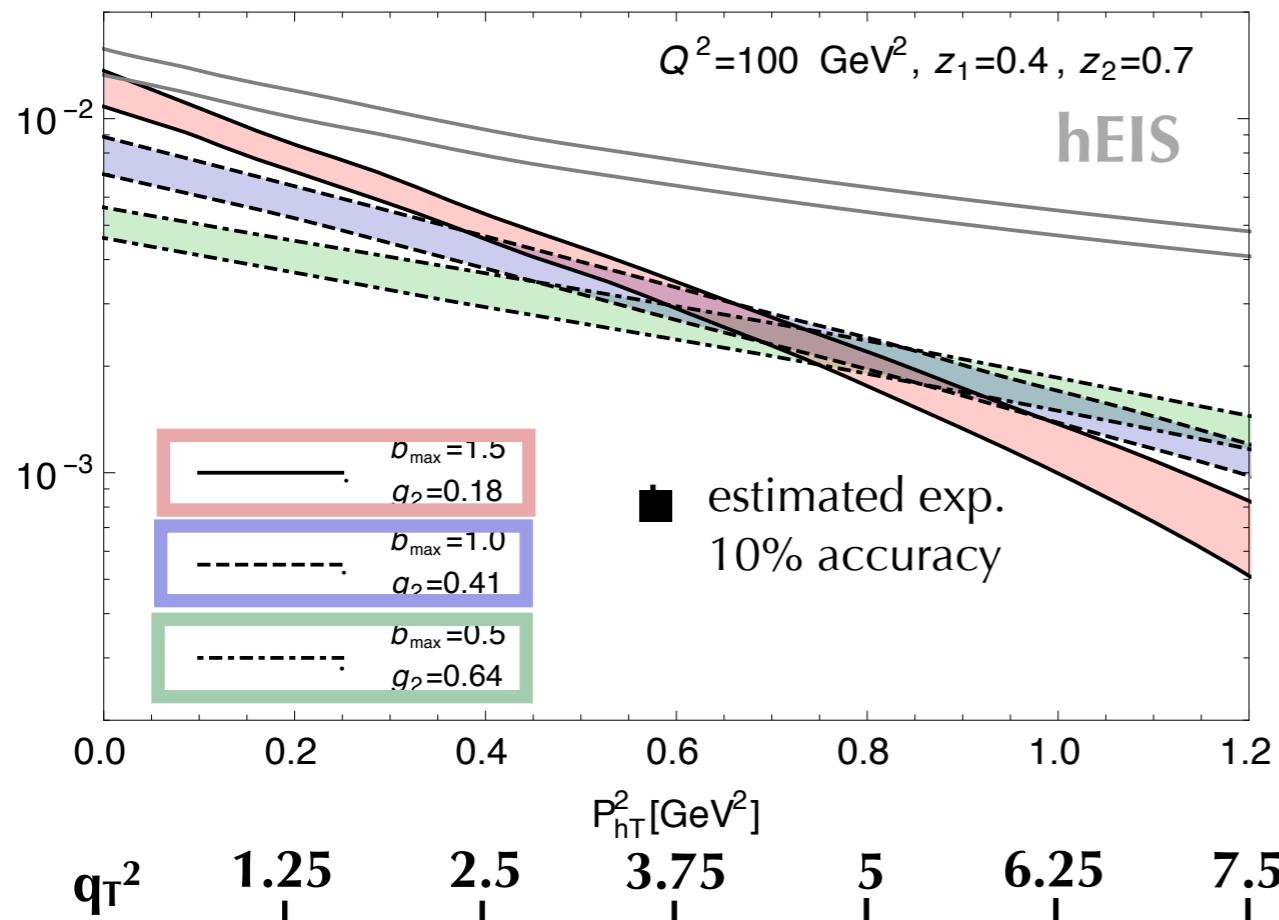
estimated exp. 10%
accuracy



$$\mathbf{P}_{hT} = -z_1 \mathbf{q}_T$$

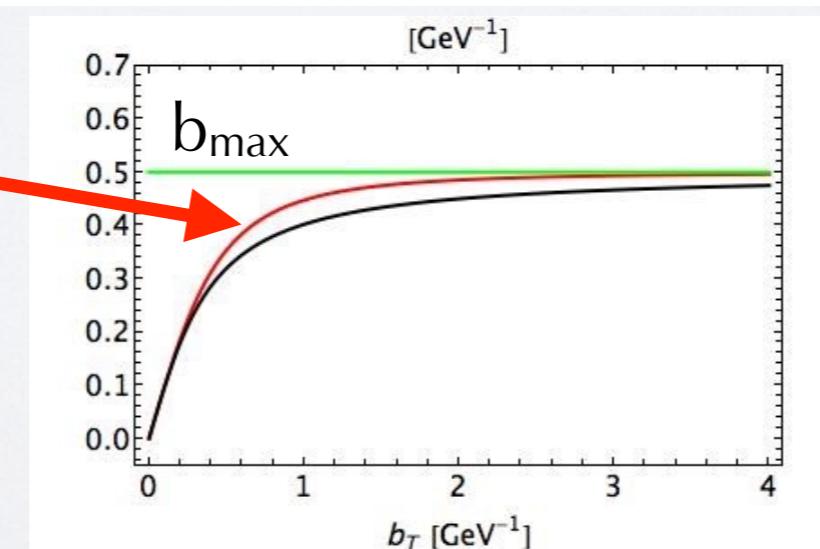
sensitivity to prescriptions for μ_b

$M(\pi^+\pi^-)$ nCSS



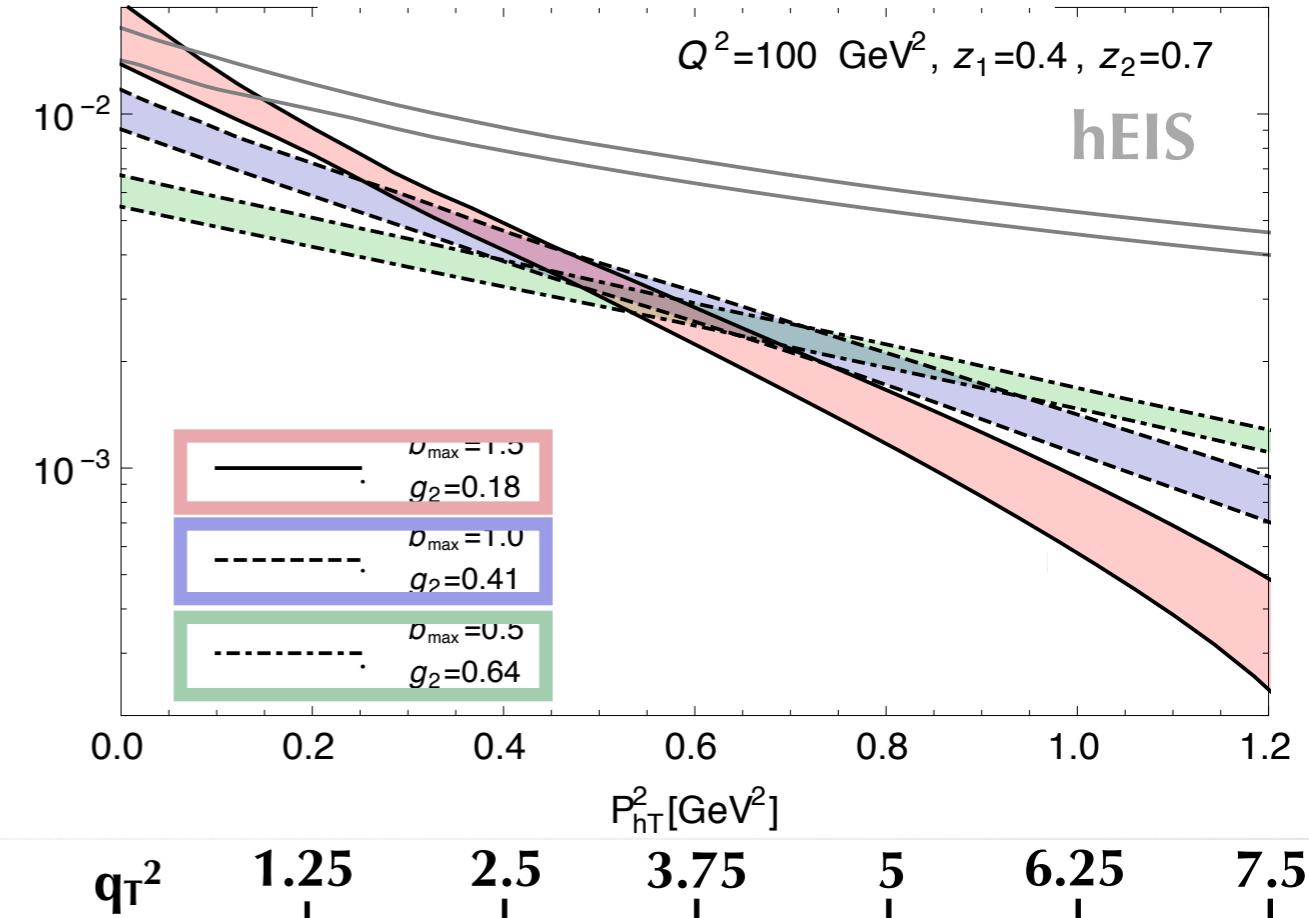
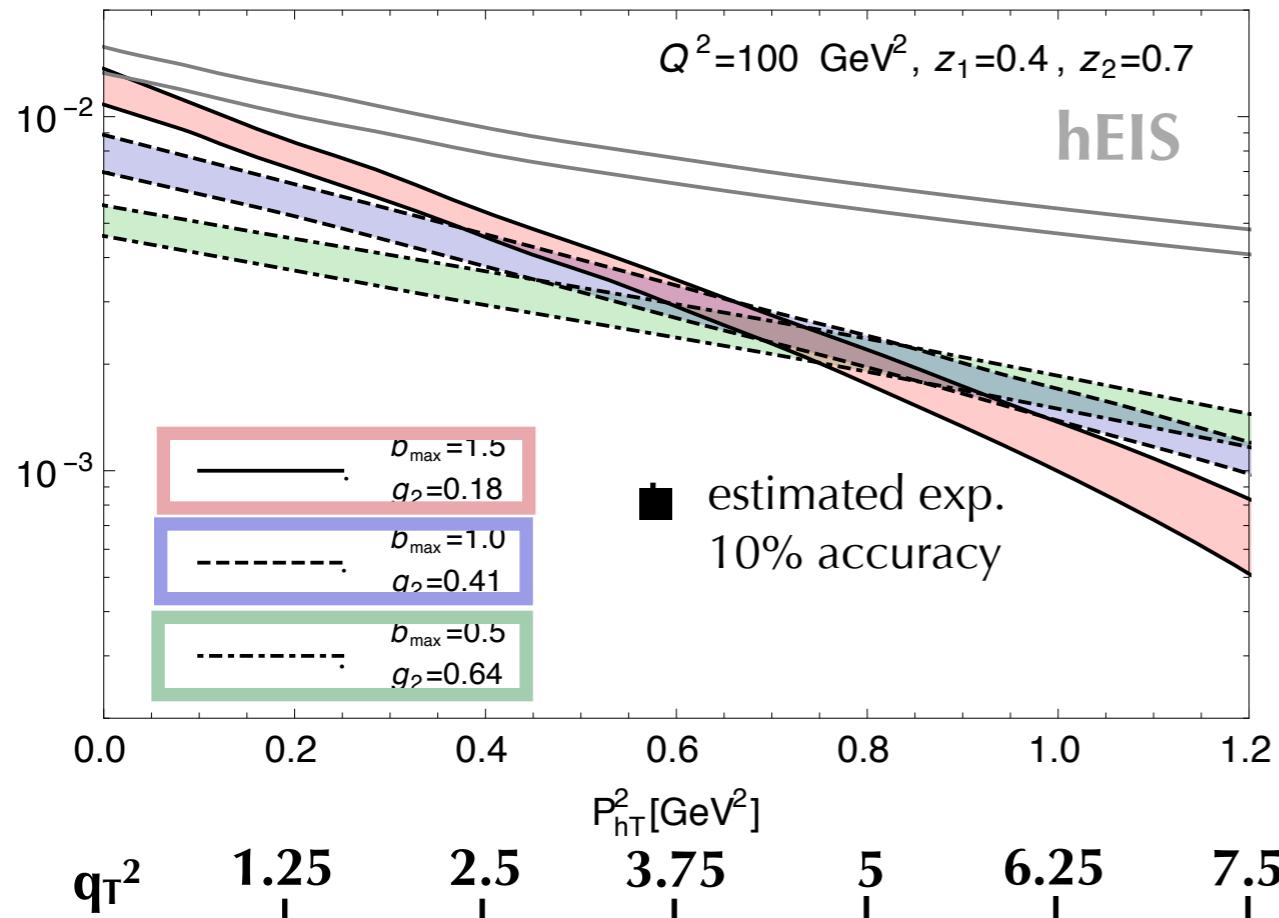
b^* prescription

$$b^* = \frac{b_T}{\sqrt{1 + \frac{b_T^2}{b_{\max}^2}}}$$



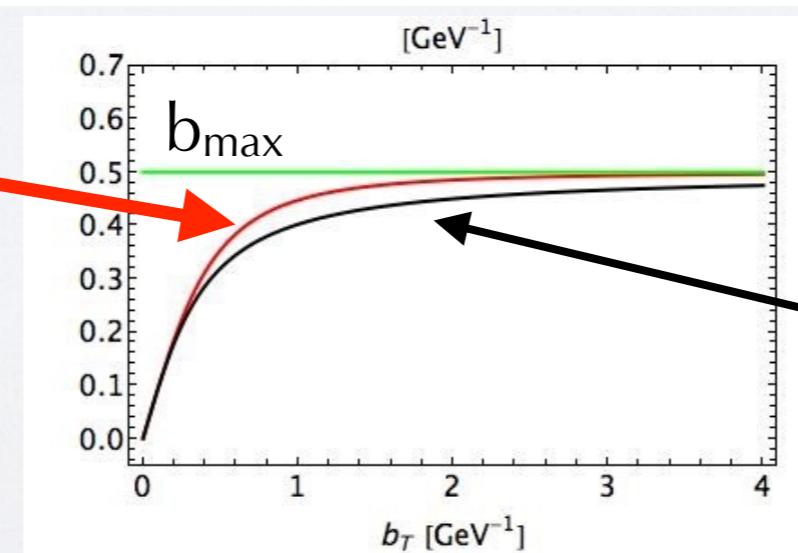
sensitivity to prescriptions for μ_b

$M(\pi^+\pi^-)$ nCSS



b^* prescription

$$b^* = \frac{b_T}{\sqrt{1 + \frac{b_T^2}{b_{\max}^2}}}$$



arctan prescription

$$b^{\text{atan}} = \frac{2 b_{\max}}{\pi} \arctan \left[\frac{b_T \pi}{2 b_{\max}} \right]$$

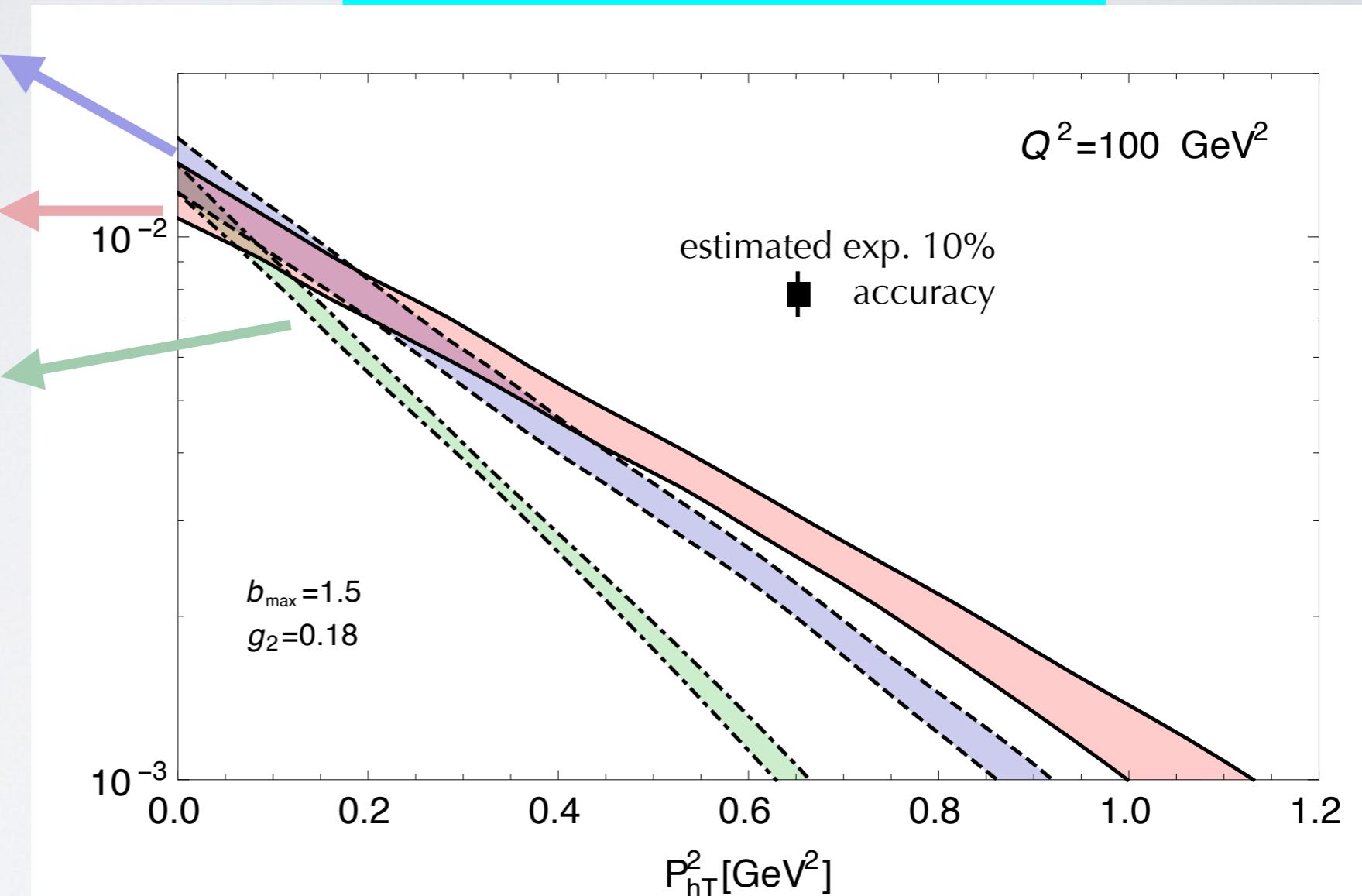
sensitivity to “intrinsic” parameters

$M(\pi^+\pi^-)$ nCSS

$Z_1 = 0.3 \quad Z_2 = 0.6$

$Z_1 = 0.4 \quad Z_2 = 0.7$

$Z_1 = 0.2 \quad Z_2 = 0.5$



sensitivity to “intrinsic” parameters

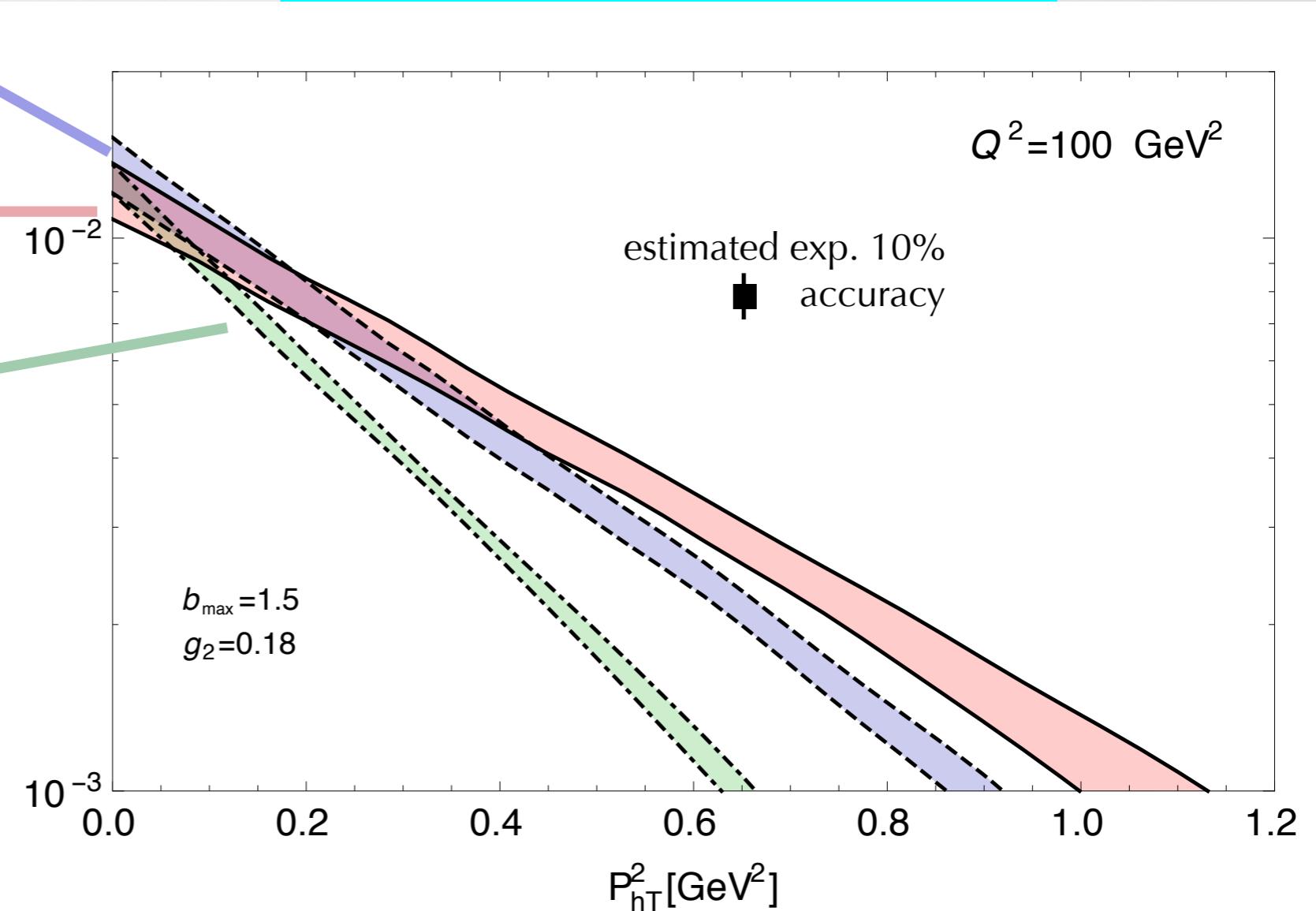
$M(\pi^+\pi^-)$ nCSS

$z_1 = 0.3 \quad z_2 = 0.6$

$z_1 = 0.4 \quad z_2 = 0.7$

$z_1 = 0.2 \quad z_2 = 0.5$

data can constrain
z dependence of
 $\langle P_T^2 q \rightarrow h \rangle(z)$
at large z



sensitivity to flavor dependence

$$\mathsf{M}(\pi^+\pi^-)$$

fav
+
unf

$$\begin{aligned} & \left[\frac{4}{9} D_1^{u \rightarrow \pi^+} D_1^{\bar{u} \rightarrow \pi^-} + \frac{1}{9} D_1^{\bar{d} \rightarrow \pi^+} D_1^{d \rightarrow \pi^-} \right] \exp \left\{ -\frac{1}{4} \left[\langle P_T^{2 \text{ fav}} \rangle(z_1) + \langle P_T^{2 \text{ fav}} \rangle(z_2) \right] b_T^2 \right\} \\ & \left[\frac{4}{9} D_1^{\bar{u} \rightarrow \pi^+} D_1^{u \rightarrow \pi^-} \right. \\ & \left. + \frac{1}{9} \left[D_1^{d \rightarrow \pi^+} D_1^{\bar{d} \rightarrow \pi^-} + D_1^{s \rightarrow \pi^+} D_1^{\bar{s} \rightarrow \pi^-} + D_1^{\bar{s} \rightarrow \pi^+} D_1^{s \rightarrow \pi^-} \right] \right] \exp \left\{ -\frac{1}{4} \left[\langle P_T^{2 \text{ unf}} \rangle(z_1) + \langle P_T^{2 \text{ unf}} \rangle(z_2) \right] b_T^2 \right\} \end{aligned}$$

sensitivity to flavor dependence

$$\mathcal{M}(\pi^+\pi^-)$$

fav
+
unf

$$\begin{aligned} & \left[\frac{4}{9} D_1^{u \rightarrow \pi^+} D_1^{\bar{u} \rightarrow \pi^-} + \frac{1}{9} D_1^{\bar{d} \rightarrow \pi^+} D_1^{d \rightarrow \pi^-} \right] \exp \left\{ -\frac{1}{4} [\langle P_T^{2 \text{ fav}} \rangle(z_1) + \langle P_T^{2 \text{ fav}} \rangle(z_2)] b_T^2 \right\} \\ & + \frac{4}{9} D_1^{\bar{u} \rightarrow \pi^+} D_1^{u \rightarrow \pi^-} \\ & + \frac{1}{9} \left[D_1^{d \rightarrow \pi^+} D_1^{\bar{d} \rightarrow \pi^-} + D_1^{s \rightarrow \pi^+} D_1^{\bar{s} \rightarrow \pi^-} + D_1^{\bar{s} \rightarrow \pi^+} D_1^{s \rightarrow \pi^-} \right] \exp \left\{ -\frac{1}{4} [\langle P_T^{2 \text{ unf}} \rangle(z_1) + \langle P_T^{2 \text{ unf}} \rangle(z_2)] b_T^2 \right\} \end{aligned}$$

$$\mathcal{M}(K^+K^-)$$

fav
+
unf

$$\begin{aligned} & \frac{4}{9} D_1^{u \rightarrow K^+} D_1^{\bar{u} \rightarrow K^-} \exp \left\{ -\frac{1}{4} [\langle P_T^{2 \text{ u} \rightarrow K} \rangle(z_1) + \langle P_T^{2 \text{ u} \rightarrow K} \rangle(z_2)] b_T^2 \right\} \\ & + \frac{1}{9} D_1^{\bar{s} \rightarrow K^+} D_1^{s \rightarrow K^-} \exp \left\{ -\frac{1}{4} [\langle P_T^{2 \text{ s} \rightarrow K} \rangle(z_1) + \langle P_T^{2 \text{ s} \rightarrow K} \rangle(z_2)] b_T^2 \right\} \\ & \left[\frac{4}{9} D_1^{\bar{u} \rightarrow K^+} D_1^{u \rightarrow K^-} + \frac{1}{9} D_1^{s \rightarrow K^+} D_1^{\bar{s} \rightarrow K^-} \right. \\ & \left. + \frac{1}{9} \left[D_1^{d \rightarrow K^+} D_1^{\bar{d} \rightarrow K^-} + D_1^{\bar{d} \rightarrow K^+} D_1^{d \rightarrow K^-} \right] \right] \exp \left\{ -\frac{1}{4} [\langle P_T^{2 \text{ unf}} \rangle(z_1) + \langle P_T^{2 \text{ unf}} \rangle(z_2)] b_T^2 \right\} \end{aligned}$$

hEIS no flavor dependence

$$\frac{M(\pi^+\pi^-; P_T^2)}{M(\pi^+\pi^-; P_T^2 = 0)} \Bigg/ \frac{M(K^+K^-; P_T^2)}{M(K^+K^-; P_T^2 = 0)}$$

$$\mathcal{M}(\pi^+\pi^-) / \mathcal{M}(K^+K^-)$$

hEIS

no flavor dep. of
“intrinsic” parameters

$$\exp \left\{ -\frac{1}{4} \langle P_T^2 \rangle(z) b_T^2 \right\} \Bigg/ \exp \left\{ -\frac{1}{4} \langle P_T^2 \rangle(z) b_T^2 \right\}$$



no P_T^2 dep.

hEIS no flavor dependence

$$\frac{M(\pi^+\pi^-; P_T^2)}{M(\pi^+\pi^-; P_T^2 = 0)} \Bigg/ \frac{M(K^+K^-; P_T^2)}{M(K^+K^-; P_T^2 = 0)}$$

no flavor dep. of
“intrinsic” parameters

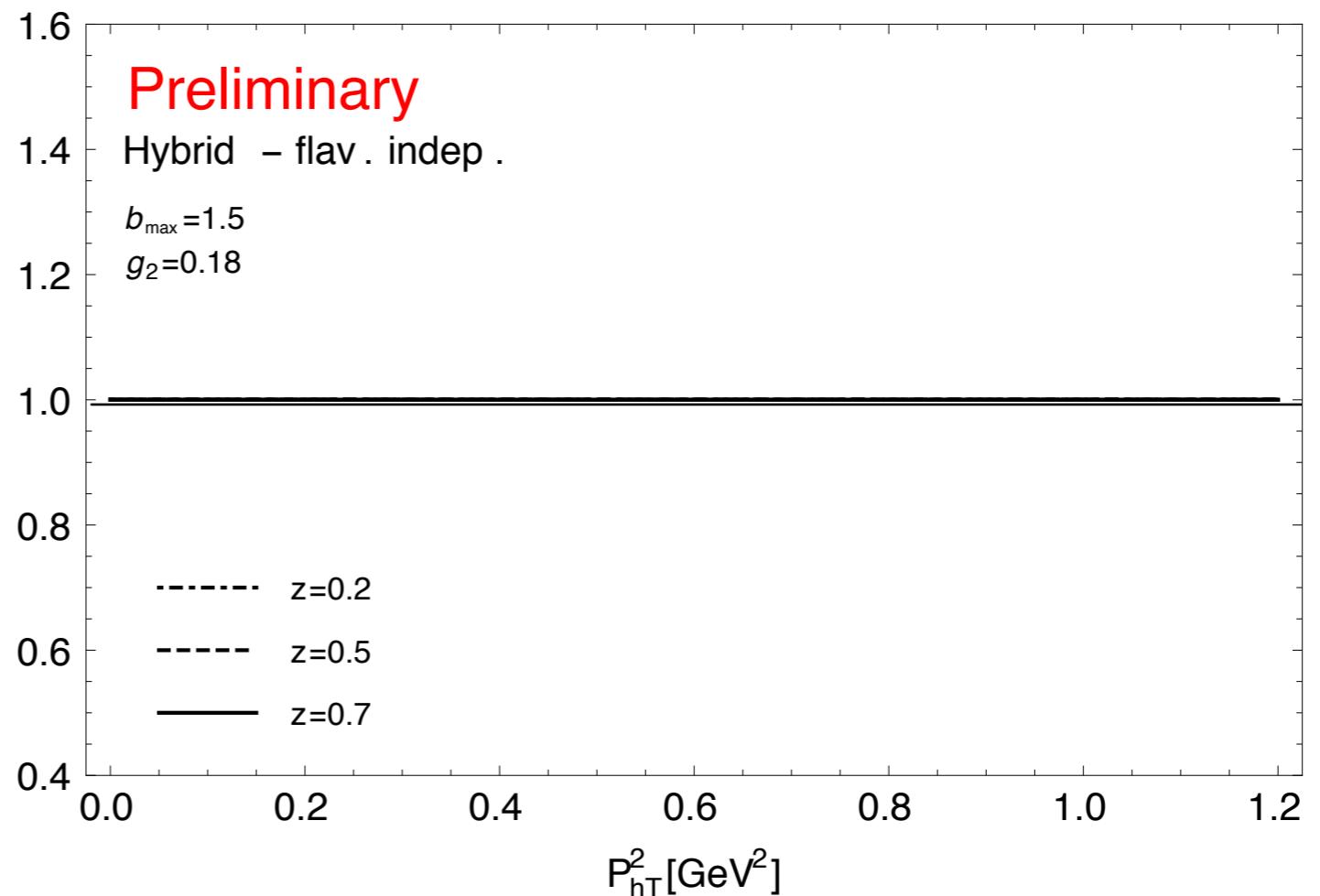
$$\exp \left\{ -\frac{1}{4} \langle P_T^2 \rangle(z) b_T^2 \right\} \Bigg/ \exp \left\{ -\frac{1}{4} \langle P_T^2 \rangle(z) b_T^2 \right\}$$



no P_T^2 dep.

$$\mathcal{M}(\pi^+\pi^-) / \mathcal{M}(K^+K^-)$$

hEIS



nCSS no flavor dependence

$$\frac{M(\pi^+\pi^-; P_T^2)}{M(\pi^+\pi^-; P_T^2 = 0)} \Bigg/ \frac{M(K^+K^-; P_T^2)}{M(K^+K^-; P_T^2 = 0)}$$

$$\mathcal{M}(\pi^+\pi^-) / \mathcal{M}(K^+K^-)$$

no flavor dep. of
“intrinsic” parameters

$$\frac{D_1^{q \rightarrow \pi^+}(z_1; \mu_b) D_1^{\bar{q} \rightarrow \pi^-}(z_2; \mu_b)}{D_1^{q \rightarrow K^+}(z_1; \mu_b) D_1^{\bar{q} \rightarrow K^-}(z_2; \mu_b)}$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b^*} \leftrightarrow b_T$$



$P_T^2(z)$ dep.

nCSS

no flavor dependence

$$\frac{M(\pi^+\pi^-; P_T^2)}{M(\pi^+\pi^-; P_T^2 = 0)} \Bigg/ \frac{M(K^+K^-; P_T^2)}{M(K^+K^-; P_T^2 = 0)}$$

no flavor dep. of
“intrinsic” parameters

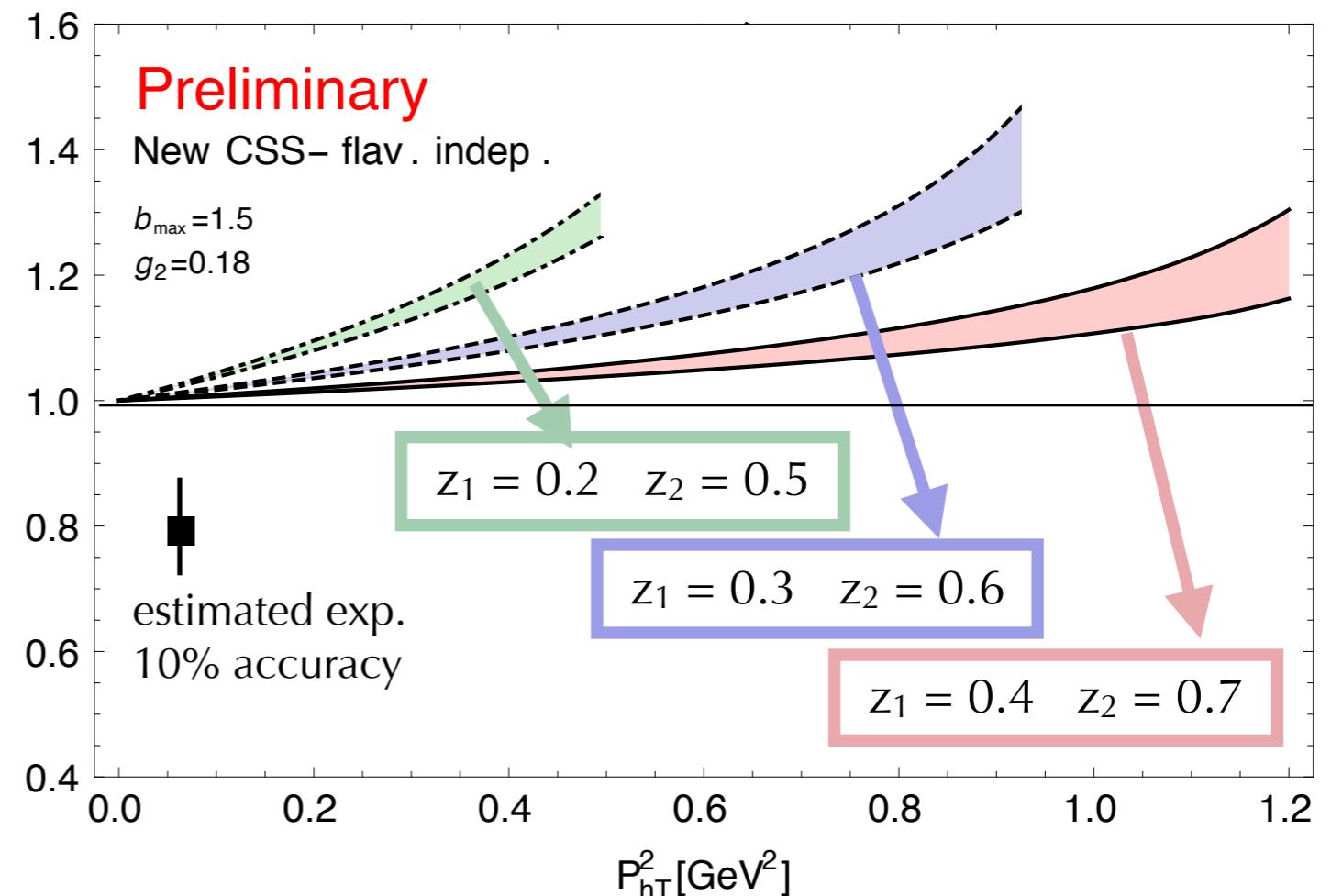
$$\frac{D_1^{q \rightarrow \pi^+}(z_1; \mu_b) D_1^{\bar{q} \rightarrow \pi^-}(z_2; \mu_b)}{D_1^{q \rightarrow K^+}(z_1; \mu_b) D_1^{\bar{q} \rightarrow K^-}(z_2; \mu_b)}$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b^*} \leftrightarrow b_T$$



$P_T^2(z)$ dep.

$$\mathcal{M}(\pi^+\pi^-) / \mathcal{M}(K^+K^-)$$

nCSS

hEIS with flavor dependence

$$\frac{M(\pi^+\pi^-; P_T^2)}{M(\pi^+\pi^-; P_T^2 = 0)} \Bigg/ \frac{M(K^+K^-; P_T^2)}{M(K^+K^-; P_T^2 = 0)}$$

with flavor dep. of
“intrinsic” parameters

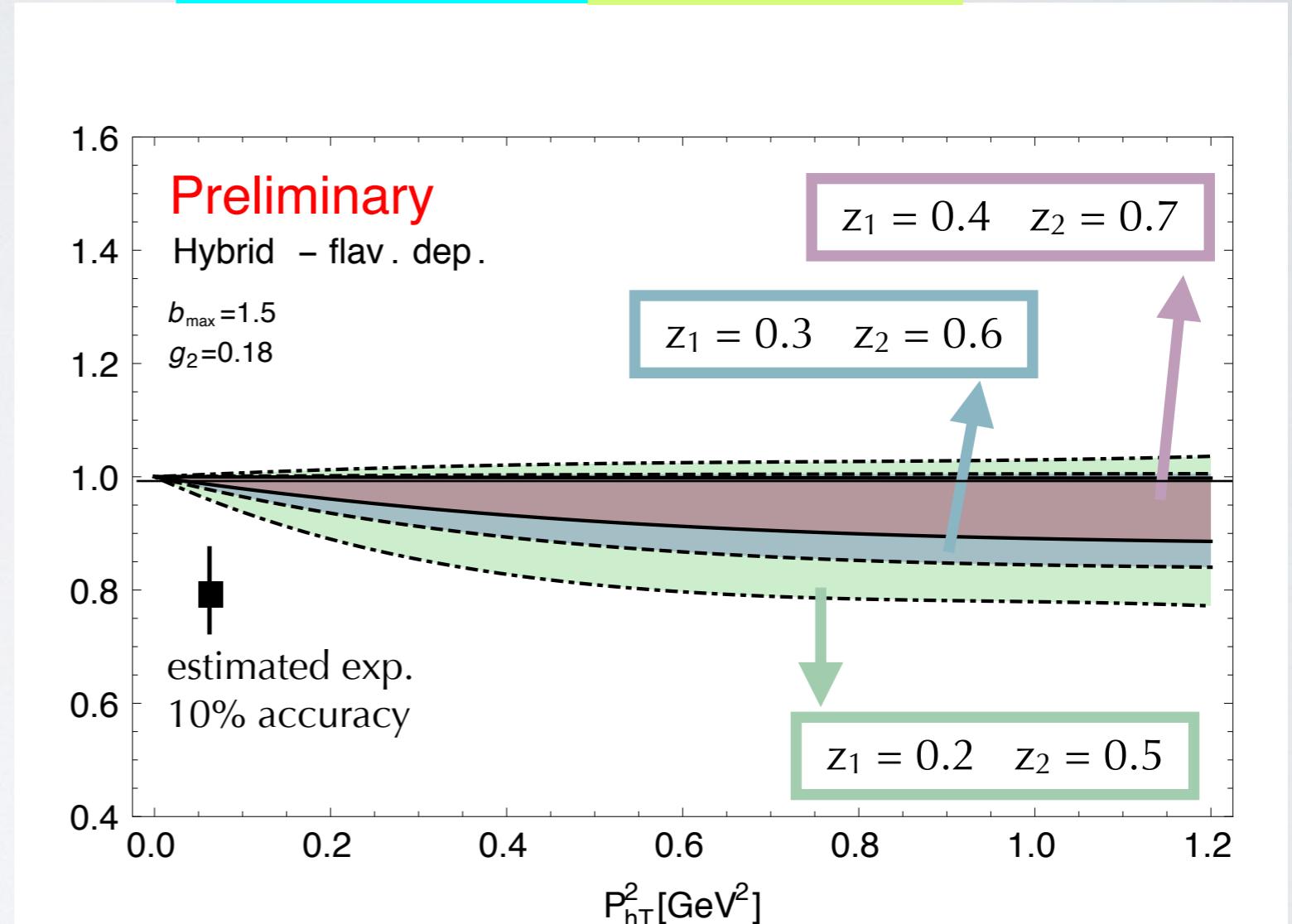
$$\frac{\exp \left\{ -\frac{1}{4} \langle P_T^{2q \rightarrow \pi} \rangle(z) b_T^2 \right\}}{\exp \left\{ -\frac{1}{4} \langle P_T^{2q \rightarrow K} \rangle(z) b_T^2 \right\}}$$



$P_T^2(z)$ dep.

$$\mathcal{M}(\pi^+\pi^-) / \mathcal{M}(K^+K^-)$$

hEIS



nCSS with flavor dependence

$$\frac{M(\pi^+\pi^-; P_T^2)}{M(\pi^+\pi^-; P_T^2 = 0)} \Bigg/ \frac{M(K^+K^-; P_T^2)}{M(K^+K^-; P_T^2 = 0)}$$

with flavor dep. of
“intrinsic” parameters

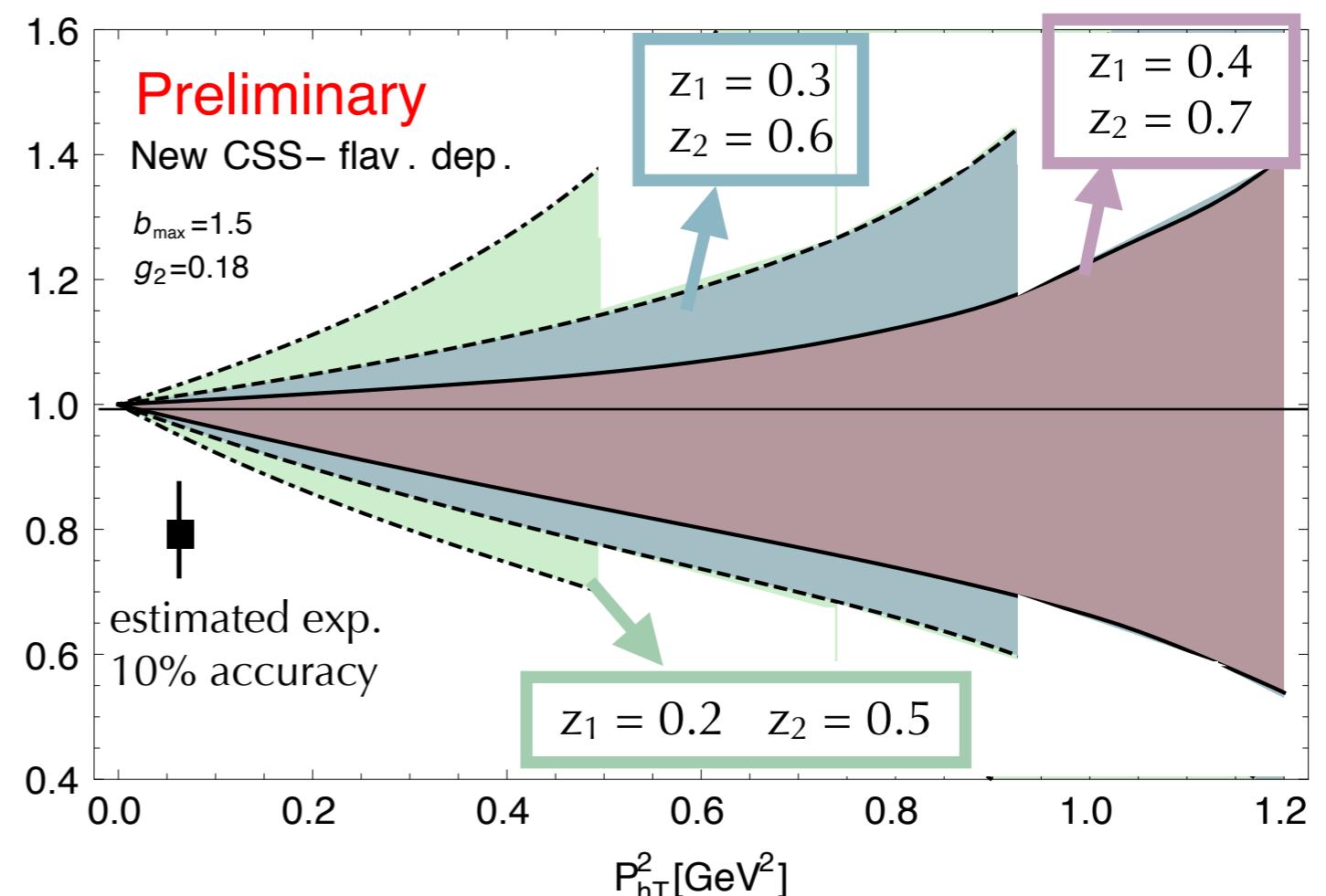
$$\frac{D_1^{q \rightarrow \pi}(z; \mu_b) \exp \left\{ -\frac{1}{4} \langle P_T^{2q \rightarrow \pi} \rangle(z) b_T^2 \right\}}{D_1^{q \rightarrow K}(z; \mu_b) \exp \left\{ -\frac{1}{4} \langle P_T^{2q \rightarrow K} \rangle(z) b_T^2 \right\}}$$



$P_T^2(z)$ dep.

$$\mathcal{M}(\pi^+\pi^-) / \mathcal{M}(K^+K^-)$$

nCSS



Conclusions

1. $e^+ e^-$ annihilations at Belle scale (100 GeV^2) can be very useful to **test evolution** and **pin down evolution parameters**
2. $e^+ e^-$ data needed to determine nonperturbative **intrinsic parameters** of TMD fragmentation functions
3. $e^+ e^-$ data useful to **constrain flavor dependence** of TMD fragmentation functions
4. knowledge of TMD FF D_1 helps in **constraining both (polarized) TMD PDF's and TMD FF's**